Fluid Dynamics And Turbo Machines.
Professor Dr Shamit Bakshi.
Department Of Mechanical Engineering.
Indian Institute Of Technology Madras.
Part C.
Module-1.
Lecture-1.
Differential Analysis.

Good morning and welcome back to this course to the 3rd week of the course on fluid dynamics and Turbo machines. In the last 2 weeks we have looked at introduction to fluid flow and the integral approach for analysing fluid flows. In this week we will take up the another approach for analysing fluid flows, that is the differential approach. So, the differential approach actually builds on the integral approach, that is why we have studied it before coming into the differential approach. We will use the concepts introduced in the last week during getting the differential equations using the differential analysis of fluid flows. So, let us go to the slides now.

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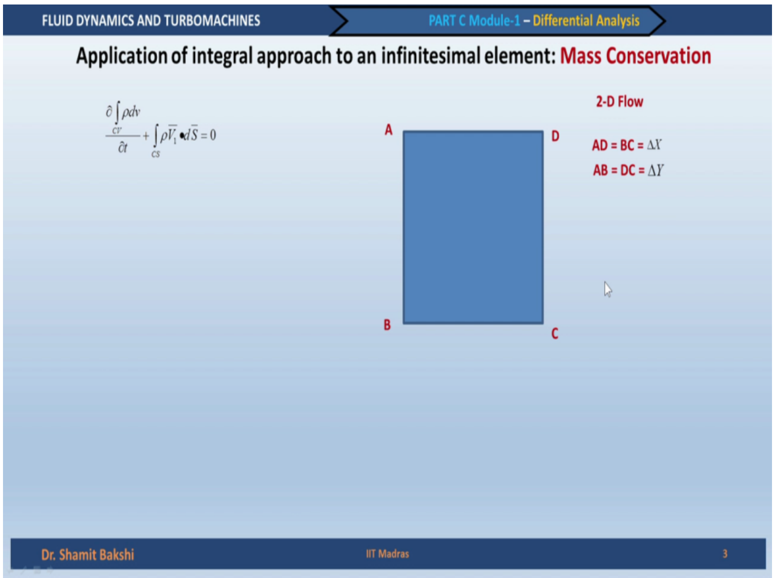

So this is the first lecture of the 3rd week. So, what we see here, we see how the, we look at how we can use the integral approach to get the conservation equations which were defined before. Now if you see we have looked at mass momentum angular momentum conservation during the last chapter. We will start with the mass conservation. Just to remind ourselves, the difference between integral and differential approach is, in the integral approach we write the conservation equation for a finite size control volume like demonstrated in the tutorial problems in the last chapter. And in the differential approach we look at an infinitesimal control volume, very small control volume. The objective is to get differential equations for the entire flow field, not the overall quantities as such which is obtained from the integral approach like the force, thrust, etc. or torque.

Here we want a, complete the full information on the velocity field. To get that, we have to solve appropriate differential equations for the entire field. The basic conservation equations are same, conservation principles are same, so that is mass momentum and angular momentum conservation, only to the applied to a very small control volume. So, let us start with the mass conservation equation. If you look at the mass conservation equation given by the integral approach, it has the first term which talks about the rate of change of mass in a particular control volume. And the 2^{nd} term is the rate of mass exiting the control volume. The total mass exiting the control volume because there is flow through the control surfaces. In a control volume there can be mass exchange, so there is a flow through the, there is velocity at the control surfaces.

To simplify our derivations in this chapter we always consider 2-D flow, a two-dimensional flow. We can extend it easily, if we understand concepts, we can extend this easily to a three-dimensional flow. So, will begin with a small infinitesimal control volume which is given as this letter ABCD, the vertical direction of the Y axis and the horizontal direction is the X axis. Now, the elemental size in the X direction that is AD or BC, they are given as Delta X and the elemental size in the Y direction, that is AB or DC, they are given as Delta Y. So, this is basically the size of the control volume. As we are dealing with a 2-D flow, so the size of the control volume perpendicular to this slide is actually 1.

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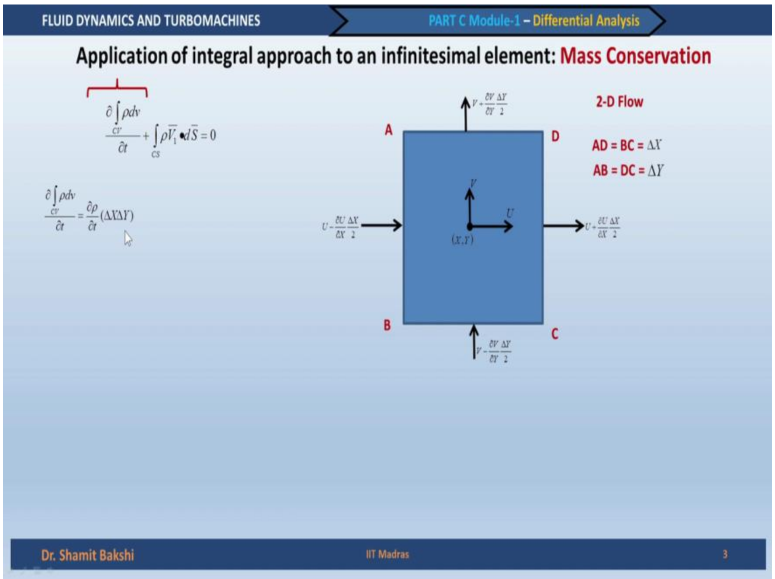

So, if you want to obtain the volume of this control volume, it is just Delta X multiplied by Delta Y multiplied by 1, that is Delta X into Delta Y. So, let us keep that in mind and proceed with the application of this equation to this control volume. So, let us consider the location of the control volume with respect to the XY axis, so let us say X, Y is the location of this control volume and if we look at the velocity coming into the control volume through the control surface AB, it is U minus Dell U by Dell X into Delta X by 2. How do I to we write this expression? So to answer that question first let us see, like we have defined this point X, Y, we have defined velocities at this point, that is the centre of this control volume. The X directional velocity is U and the Y directional velocity is V, if we know the velocity, as velocities are continuous functions, so as we know the as we know the velocity at the point X, Y, we can find the velocity at the control surface from the same velocity and the velocity gradient information by using Taylor’s Taylor’s series expansion.

So, how to do that, if we take this to this side, it is that a, because the size of the control volume in this direction is Delta X and the control surface is located at a distance Delta X by 2 from the Centre of the control volume. So we can write the expression for velocity is U minus Dell U by Dell X into Delta X by 2. So, basically we do a Taylor’s series expansion and get this expression for velocity. Of course the higher-order terms can be neglected because this control volume size is very small. So, Delta X square, Delta X cube, etc. whatever appears in the expansion can be neglected. Now this Dell U by Dell X is of course velocity gradient at the same point X, Y. Now this is a velocity coming into the control surface, the velocity going out can be obtained similarly, that is U plus Dell U by Dell X into Delta X by 2.

So, this is at minus Delta X by 2 distance from the centre and this is at plus Delta X by 2 from the Centre of this control volume, so we can easily write these 2 expressions. Writing the expression for any quantity defined at the centre of a control volume at the control surfaces like this will be utilised throughout the derivations in this particular chapter. Now as we have done in the case of U velocity we can do the same in the case of V velocity. So one thing you can notice here that we have made a little change in the symbols because we have now defined the Y component of velocity by symbol V, that is why we have defined the velocity vector in this equation as V1 bar. So, V1 bar and 2 components, the X component, it is for this particular case it is a two-dimensional velocity field and so the X component is U and the Y component is V.

And we also should keep in mind that this small v appearing over here represents the volume. So, small v represents the volume which we have also indicated before. Now this is the scenario of the mass of the velocities at the control surfaces of this control volume. This is what we used to do even in the case of a finite size control volume, like a plate moving by a jet which is coming onto it. So, after doing this, now let us see term but how we can determine the value of this parameter. So, the first term is this, the unsteady term which is given as Dell Dell T of integral over the CV rho dV. So this can be written as, so rho in this particular control volume is not constant but we can consider the rho at the centre of this control volume because this is a very small volume, we can consider the Centre density at the centre which is rho X, Y. In this derivation we are also considering density as a variable quantity, not a constant quantity.

So we can write this, we can replace density with rho at the point X, Y and assume that the density is not varying at least for this expression it is not varying within the control volume. So, if you assume that, then you can take density out, rho out of this integral sign and you can integrate over volume. So, if you integrate over volume what you get is the total volume. Integrate dV over the entire volume, you get the total volume which is Dell X into Dell Y. So, this comes out to be the first one comes out to be Dell rho by Dell T into Dell X into Dell Y. So, this is quite easy to understand. Let us see the next part.

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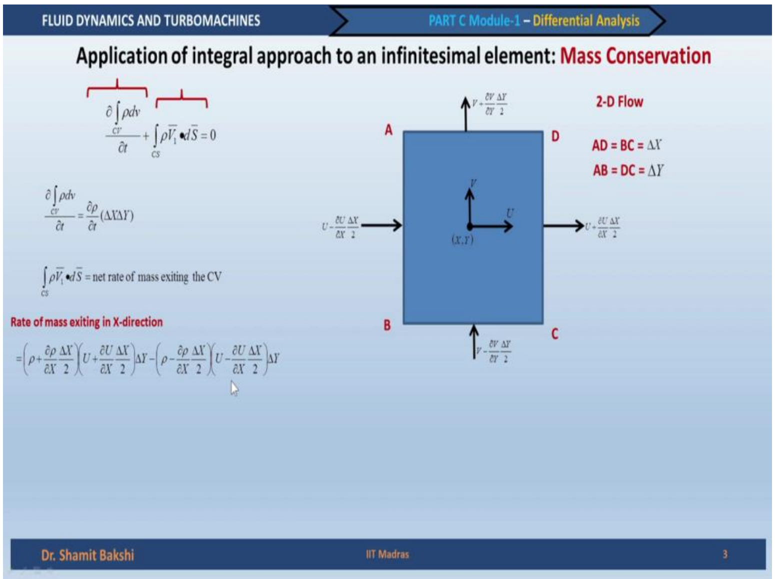

The next part is this part, this part is actually net rate of mass exiting the control volume, that means the rate at which mass is going out of the control volume, for this case, through this surface, that is surface AD and surface DC minus the rate at which the mass is coming into the control volume through surface AB and surface BC. So, we do that systematically, we first find out rate of mass exiting in X direction. So, this is the X direction, to find out the rate of mass exiting the X direction we can write it like this. So, now you can observe here that the density is a variable quantity. To find the 2^{nd} term, to find the 2^{nd} term in this equation, so rho as the similar to velocity, rho is a continuous function and is defined as rho plus by expanding it into a Taylor's series we can define rho, similarly as velocity, all the functions are continuous functions, so they can be, the first derivatives are also continuous.

So assuming that we can actually write it as rho plus Dell rho by Dell X into Delta X by 2. So, this is the density at the exiting surface at the X direction, that is the surface DC multiplied by velocity U, that is what we have written here, U plus Dell U by Dell X into Delta X by 2. So, this is rho, density at the exiting surface, velocity at the exiting surface multiplied by the surface area. The surface area is this, just this length multiplied by 1. We have not written 1 here because it is two-dimensional analysis, so we can just replace this by Delta Y. So, this is basically the mass which is exiting minus mass of fluid which is coming into the control volume in X direction. So, this can be again similar to this written like this form.

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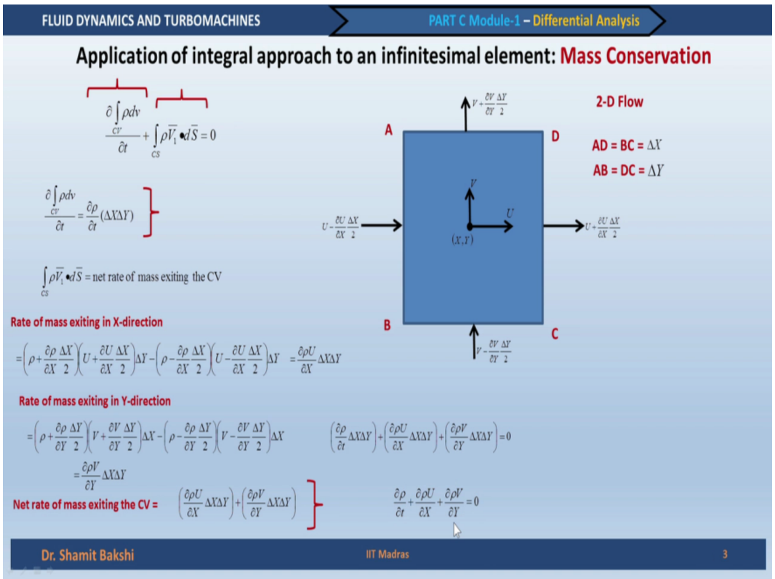

If we, if we multiply these 2 quantities and simplify this expression, then what we will get is Dell rho U by Dell X into Delta X into Delta Y. So, this is very simple, we can directly simplify this and get this expression. So, this is the rate of mass exiting in the X direction, similarly we can get the rate of mass exiting in Y direction, so we can write whatever mass is going out in the Y direction from the surface AD, the velocity there is V plus Delta V by Dell Y into Delta Y by 2 and the area is Delta X now, because this is the area perpendicular to the V velocity. So the area perpendicular to the V velocity in this control volume is basically Delta X multiplied by 1, so Delta X. This is 1, this expression is the mass, rate of mass entering the control volume in Y direction, so it comes in through here, so like here we get a similar expression Dell rho V by Dell Y Delta X into Delta Y. So, we have everything now, whatever is required to get the differential equation for mass conservation.

So we, this is the first part, so net rate of mass exiting, so before going into that, these 2 can be clubbed together to get the net rate of mass exiting the control volume which can be written as a sum of this and this, so Dell rho U by Dell X into the volume plus Dell rho V by Dell Y into the volume. Now we can club this part and this part, these 2 parts, this part forms the unsteady part of the equation, this part forms the the part which includes the convective component. Convective means the one which exchanges through the control surfaces, the velocity or the mass exchange by velocity through the control surfaces. Now we club these 2 parts together, if the club these 2 parts together what we get is like this, so this is our final expression.

Of course Delta X by Delta, Delta X multiplied by Delta Y is not zero, this is a infinitesimal but nonzero sized control volume, so it can be taken out, that is not equal to 0, so what is zero is Dell rho by Dell T plus Dell rho U by Dell X plus Dell rho V by Dell Y is equal to 0. So, this is basically our continued the equation which is or the mass conservation equation which is applicable for a two-dimensional compressible flow and unsteady flow. So, 2-D unsteady, unsteady because the unsteady term is returned here, two-dimensional because only 2 dimensions are concerned considered and compressible because density has been considered as a variable quantity in this expression.

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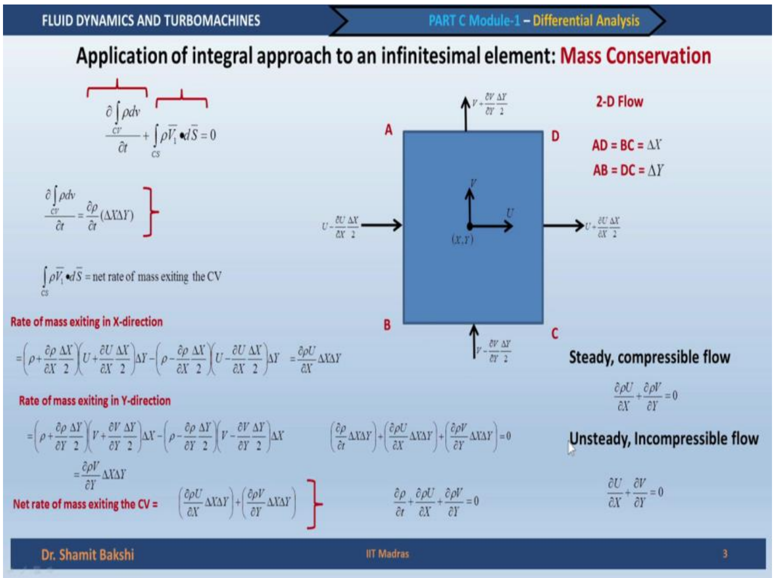

We can reduce it to a steady and compressible flow situation. So if you consider the flow as steady, of course the first term drops out and you get this expression. So, this basically, this is the continuity equation or mass conservation equation for a compressible, steady compressible flow. Dell rho U by Dell X plus Dell rho V by Dell Y is equal to 0. Now we can also find out for an unsteady incompressible flow. We keep the flow unsteady but we make it incompressible, but if you notice the only unsteady term appearing in the continuity equation, that is, that means in this equation is concerned with density. And by making the assumption that it is incompressible, this density is anyway constant, so this drops out. So, this drops out even for a unsteady flow and what we are left out with is this equation. As density is constant it can be taken out, so we can get this equation Dell U by Dell X plus Dell V by Dell Y is equal to 0.

This is, this equation is same for a steady and a unsteady flow for an incompressible flow situation because the only unsteadiness in the continuity equation pertains to the density. So this is the equation which is valid for a two-dimensional or unsteady incompressible flow. If we want a three-dimensional flow, the simple difference will be, we will have another derivative that is let say W is the velocity in the Z direction, so we will have Dell U by Dell X plus Dell V by Dell Y plus Dell W by Dell Z equal to 0. So we have got the mass conservation equation from by using the integral analysis to a infinitesimal control volume.

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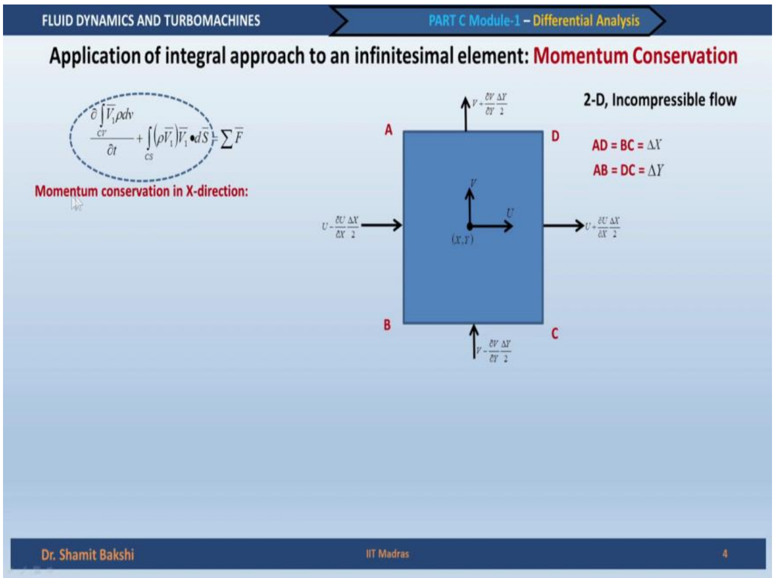

Let us now go to the momentum conservation equation. We take the same approach even for the momentum conservation equation. What we do is we start from the integral approach. In the integral approach, the velocity, the momentum conservation equation is written in this form, so the left-hand side comes from the Reynolds transport theorem and the right-hand side is basically the sum of all the forces acting on the control volume. This is a vector equation, so again we consider this as a 2-D flow and if we consider it as a 2-D flow, this has 2 components. So, unlike the continuity equation, the right-hand side of the momentum equation, momentum conservation equation is non-zero. So, we will deal the left-hand side and right-hand side separately while arriving at the differential equation for momentum conservation equation.

So first in this slide we will take up the left-hand side which is the unsteady part of the momentum and the net momentum exiting the control volume. So, we take up this part, again we take 2-D and we take incompressible flow. In the last case for mass conservation we have considered compressible flow but in this case we considered incompressible flow only because in the case of compressible flow the density variation will make our expression very big. So we want to keep that expression simpler, that is why we have assumed incompressible flow but by considering density as a continuously varying function through the flow field we can extend the same approach to the compressible flow as well.

Okay so now let us look at this part, the control volume is the same, again we have these 2 edges of the control volume is defined as Delta X and Delta Y, this is the point and in this point we know the X, Y, this control volume is located at a point X, Y U velocity, V velocity are X and Y components of velocities, so these were all defined in the last slide. So we can quickly go over this. Now let us see how to write the item conservation equation in X direction.

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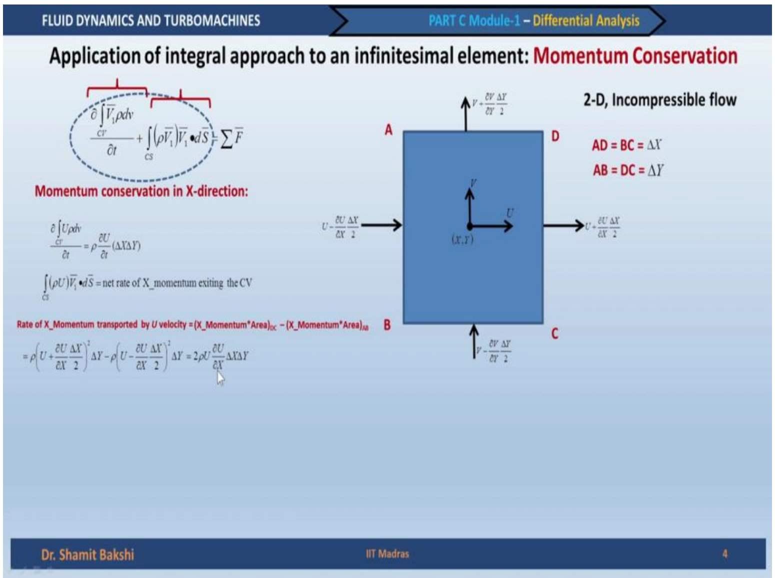

Again we will only make the derivation for momentum conservation in X direction and in Y direction the same principle can be extended, same procedure can be extended. So we will look at the momentum conservation equation in X direction. In the momentum conservation equation in X direction we have the first term, that is the unsteady term, let us take up that term first. In that term, again density is constant, so density can go outside this derivative, so we have taken out density and within the control volume velocity for this unsteady term, we can consider it as constant and then we can again take this out, we can write this as Dell U by Dell T, integral of dV will be Delta X into Delta Y. So, basically this is the expression which we get for the first term or unsteady term or rate of change of momentum within the control volume.

The 2nd term is the net rate of X momentum exiting the control volume. So the first was the rate of change of X momentum within the control volume, the 2nd term is that rate of X momentum exiting the control volume. Now this we have to do it very carefully. Let us see what is the, we do it in a stepwise manner so that we understand all the considerations here. So first the thing is, first thing is rate of change of X momentum transported by U velocity. So, this is very important to consider that X momentum, that is the momentum of the fluid in the X direction is transported by U velocity and it is also transported by the V velocity by the Y component of velocity. So, we deal with these 2 parts separately. First part is how, what is the rate of X momentum transported or what is the rate of X momentum exiting the control volume which is transported by U velocity.

So this U velocity brings in the momentum of the fluid through this control surface and takes out the momentum in X direction momentum through this control volume, through this control surface. So we write it in that manner X momentum multiplied by area in the control surface DC which is going out subtracted to X momentum multiplied by area coming in through the control surface AB, so this surface. Now we just need expression for these 2, so for expression for these 2, we can write X momentum is basically rho velocity at the control surface multiplied by velocity at the control surface. So this is the exiting surface that means in the surface DC, so in the surface DC rho multiplied by U plus Dell U by Dell X into Delta X by 2 whole square. So basically that is the X momentum at the exiting surface multiplied by area, Delta Y is the area.

Now if we would have taken it as a compressible flow, we have to, we have to replace this density with a variable quantity. That means rho plus Dell rho by Dell X into Delta X by 2 for this control surface. But we want to keep the expression simple and basically demonstrate the procedure. So this is basically the X momentum out of this control volume, X momentum out of this control volume with this transported by U velocity and then this is what is coming in. So now if we simplify this equation, we can write it as 2 rho U Dell U by Dell X into Delta X multiplied by Delta Y. So this can be simply simplified to this form. Basically if you take rho into Delta Y out, then what you will be left out with is, you can see this looks like A plus B whole square minus A minus B whole square which should be 4 into AB.

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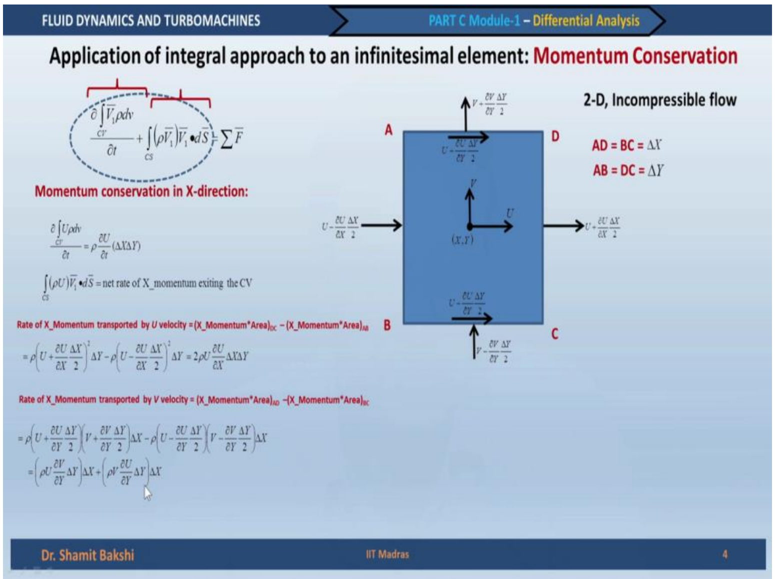

So, 4 multiplied by U multiplied by Dell U by Dell X into Delta X by 2. So that comes out to be 2 into rho 2 into U into Dell U by Dell X into Delta X and rho into Y was already there, so we get this expression. So we get the first component of the contribution of the first component that means the X momentum passport it by U velocity. Now the 2nd component talks about the rate of X momentum transported by V velocity. So this is a very important thing to understand in this in this analysis that although you might not have a variation in U velocity but a variation in V velocity means I I mean the spatial variation in U velocity is not there but a variation of V velocity can bring about a rate of change of momentum of rate of change of X momentum in the control volume. And why is that, that is because we have some X momentum in these control surfaces.

So in the control surface BC and AD, there is an X momentum so which is given as this. So, U velocity here is, if you know the U velocity here, like we did for these surfaces, we can do it in these surfaces also by using Taylor's series expansion and we can write the velocity as U minus Dell U by Dell Y into Delta Y by 2. So, instead of Dell U by Dell X, it is Dell U by Dell Y here because we are talking about the variation in Y direction. And U is continuous U is continuous and its derivative with respect to X and Y both are continuous. Now we can write this and we can write a similar expression on the surface that AD. Now if you see, even if you forget this part, let us say these 2 are same, the momentum coming through the, the X momentum coming in through AB is same as momentum going out through CD, that means these 2 velocities are same.

Even under that condition the V component of velocity can transport momentum. Even a constant momentum can be transported. Even if you consider this velocity this U velocity is same as this velocity but the variation of V velocity can result in a rate of change of X momentum in the control volume. That is very important to understand. And that is the quantity which we are trying to derive here that rate of X momentum transported by V velocity. So, if you do that, again we can write in this form, this is the X momentum multiplied by area which is exiting means which is going out through the surface AD subtracted to X momentum multiplied by area which is coming in through the surface BC. Now to write an expression for this X momentum multiplied by area, here we have to remember that this is the X momentum transported by V velocity.

So the expression for the momentum is rho multiplied by U velocity multiplied by the V component of velocity because it is transported by the V component of velocity. Similarly we can write multiplied by Delta X because of the area and then we can write the one which is coming into the control volume. Rho multiplied by U component of velocity multiplied by the V component of velocity into area. So this is basically the X momentum transported by, this is the X momentum but it is transported by V velocity. This is the X momentum and transported by U velocity itself, so that is the expression which we got in the first case, the 2nd case is this. If we simplify this expression, what we get is like this. If we **we** have to just simplify this entire expression by multiplying these 2 things and then if we 2 expressions, algebraic expressions and if we do that, we will get a final expression in this form.

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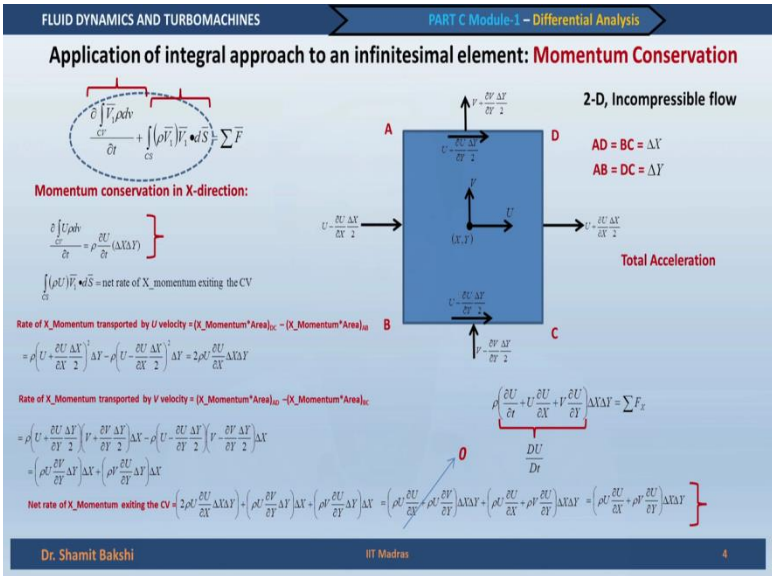

So now by combining these 2 by adding these 2 parts we get the net rate of X momentum exiting the control volume. What is that, it is basically this part and here also if you see, Delta X into Delta Y, that is the volume appears in the equation. So we get something like this. So, let us see this expression carefully. This part directly comes from the X moment rate of X momentum transported by U velocity, this is, this has directly come from the rate of X momentum transported by V velocity. So, it has come from there.

Delta X into Delta Y is a common factor in all these expressions. So, now we can define this further, what we can do is we can write it in this form, to explain how to write it in this form, we have actually broken down this first component, that is rho U, twice rho U Dell U by Dell X into Delta X Delta Y into 2 parts. First part has been shown here, that is rho U Dell U by Dell X and the 2nd part is shown here, that is rho U, another rho U Dell U by Dell X clubbed with the this component that means rho U Dell V by Dell Y plus this component that is rho V Dell U by Dell Y.

Dell X by Dell Y appear in as a multiplier of this expression 2 expressions in both the factors. Now, if we observe carefully we will see there is the reason for writing it in this form. If you take rho U out of this expression, what you get is Dell U by Dell X plus Dell V by Dell Y and that is essentially our continued equation. And you can directly remove this from here because we have taken this flow as a 2-D and incompressible flow and for a 2-D incompressible flow, Dell U by Dell X plus Dell V by Dell Y is zero.

So, in this way while deriving these equations we have to make use of the continuity equation and simplify the expressions so that we can finally get an expression which is meaningful. So we will get the net rate of X momentum exiting the control volume simply as rho U Dell U by Dell X plus rho V Dell U by Dell Y Of course multiplied with the elemental volume. So this is the final expression.

Now in this integral approach if we go back, the first term was this, it was obtained as rho Dell U by Dell T into the volume and the 2nd term, that means the net rate of momentum, X momentum exiting the control volume has come out to be this. So we club these 2 parts, let us club these 2 parts, so this is the first part and this is the 2nd part, if we club these 2 parts we get this equation. This equation is basically the X momentum equation for a 2-D incompressible flow. But of course the right-hand side has not been simplified.

So we will do that but before going into that lets understand the meaning of this equation or the physical meaning of this equation. So if you look at this equation what you get is this is density and this is volume, if you multiply density with volume you get mass. So what this equation actually tells you is the total force of the sum of all the forces acting on the control volume is mass multiplied by a particular quantity and that quantity has to be acceleration as we all know. But this acceleration so basically the, why doing all this what we have got is an expression for acceleration.

So this part is actually the acceleration of the flow written in terms of the Eulerian velocity. So we can write this because this is essentially, we know acceleration is rate of change of velocity with time. So in that sense we can write it as a derivative like this. So this derivative is different from the partial and it is called total derivative. So this is capital D dt of U. So this is basically the total acceleration of the fluid.

What is the total acceleration of the fluid, we see it has 2 components, the first component is an unsteady component and the 2nd component has no time involved in it which also explain that even in your Eulerian field or in your Eulerian description of velocity which is the velocity is defined at a particular point and not for a particle, that particular velocity which is only defined at a particular point even if it is not a function of time you can have acceleration. How can you have acceleration, you can have acceleration because of this 2nd component.

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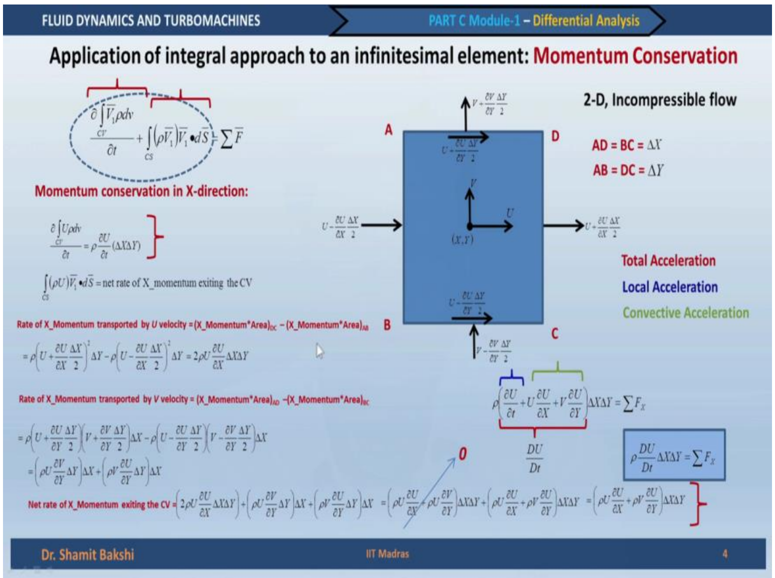

So the first component is the local acceleration which considers the rate of change of velocity within the control volume with time and the 2nd part constitute of the convective component of velocity. In the convective component of velocity what we have is basically the, along with the density if we consider, it is the momentum exiting the control volume subtracted to the momentum which is coming into the control volume.

Now this part of velocity is very important to consider in the case of a description of a flow using a Eulerian field. This was demonstrated in our first chapter when we considered a flow field which is independent of time but when we consider a fluid particle, the velocity of a fluid particle, it was changing with time. So the fluid particle definitely experiences and acceleration but the velocity field in terms of our Eulerian description is independent of time.

If we now consider that velocity field which was independent of time and plug-in the velocity values in this expression, we will see the local acceleration is zero, definitely because it is independent of time but the convective acceleration is nonzero. And this contributes to the or this explains the acceleration of a fluid particle as the fluid particle moves through this control volume. So, this is a very important thing to consider in our description of Eulerian field. So finally we can write our X momentum equation as rho dU by dt multiplied by the volume with the forces is equal to the sum of the forces. Now let us look at the 2nd part which is basically this part. We have already described the left inside, let us look at the right-hand side now. The right-hand side is the sum of the forces. We will do that in the next slide.

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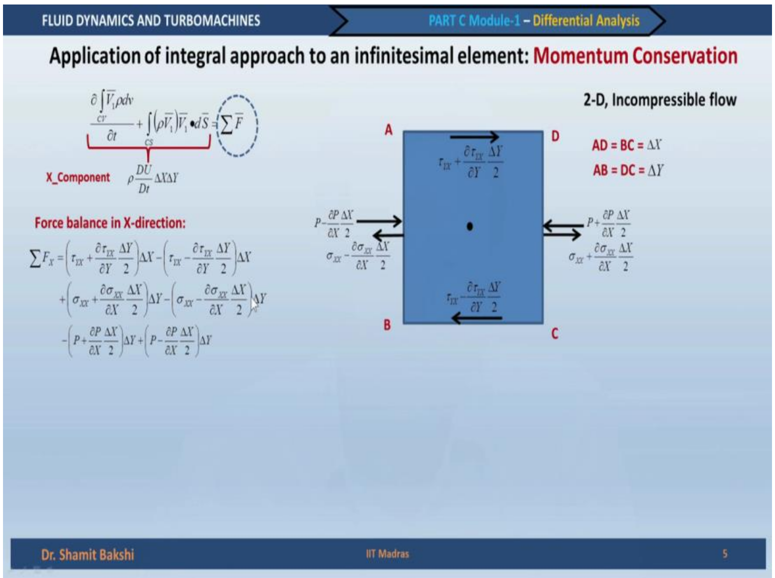

So this is our complete expression, we have got the, for the X component we have got an expression for the left-hand side which is basically mass into the total acceleration. We have to find the forces now. So we do a force balance in the X direction because we are considering the X momentum equation in this derivation. Of course 2-D incompressible flow, now these are things which are known, if we look at, so while doing the force balance we should look at all the forces acting in the X direction which is the horizontal direction here. First considers a shear forces which are acting on this particular fluid element. The shear forces like we did for components of velocity and density, we can write the shear stress Tao YX in this way, so the shear stress on the top surface is different from the shear stress at the bottom surface.

This difference in their shear stress will bring a net force in the X direction which will be given by subtracting these 2 quantities. So this is the first contribution, the difference in the shear stress is the first contribution considered in this presentation to the forces, to the net forces acting on the control volume. The 2nd force which is considered here is the normal stresses, the force is due to the normal stresses, the stress acting on the surface CD and AB, they are not same, again we are different like we had defined the other quantities and the difference of the stresses will multiplied by the area will bring about a force acting in the X direction. So this is the 2nd consideration.

And this is not all because we have a pressure field. So in the pressure field we have a pressure component acting on these 2 surfaces like we did while drawing, while finding the forces in an integral approach. The sum of the forces when we obtain we considered the pressure field in the control surface. So the pressure of course is acting inward into the control volume always and then this at the forces? So shear, the forces coming from the shear stresses, the forces coming from the normal stresses and the pressure forces, the forces coming from the variation or distribution of the pressure in the flow field. So now we club them, sum of forces, the first component is the shear stress component, because multiplied by the area.

What we observe here is that in the case of shear stress the area is Delta X, whereas because it acts on the surface, on this surface, the other forces, that that is the normal, the forces due to the normal stresses acts on the Delta Y, the surface with area Delta Y, both these cases. So we have multiplied this with Delta Y and the 3rd component is the pressure which again acts on the surface Delta Y.

So these are the sum of all the forces acting on the control volume. Although this expression looks complicated but it can be simplified because if you look at it carefully, we see these 2 components cancels out and these 2 components can be clubbed together. And if they are clubbed together for the shear stress, it can written as Delta YX by Dell Y multiplied by telex into Dell Y.

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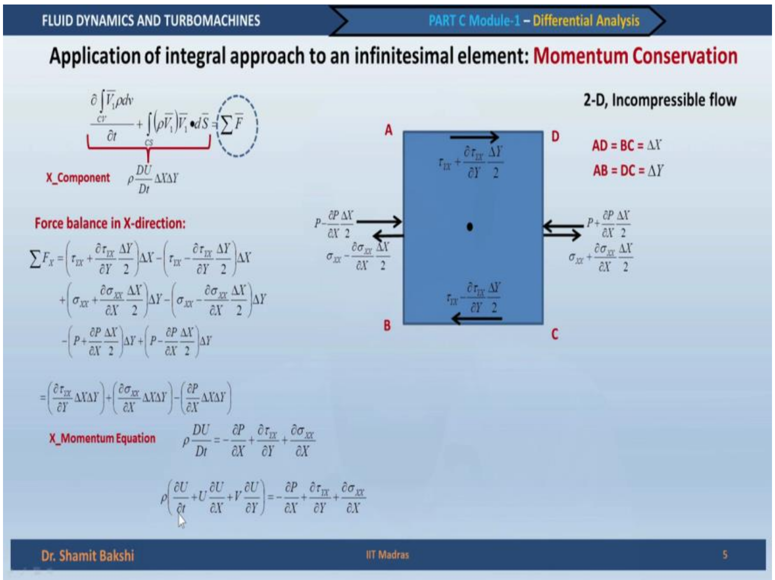

And similarly for the normal stresses under pressure. So if we do that we get this expression. The Delta X by Dell Y into Dell X Dell Y, Delta Sigma XX into Dell X into Dell X Dell Y that is the volume minus Dell P by Dell X into Dell X Dell Y. So this minus sign is because that the pressure is acting into the control volume on the control surface. So, now we have obtained the sum of forces, if we plug-in this value here, what we get, we get the X momentum equation which is the left inside is rho dU by dt and the right-hand side is this. Of course we have cancelled the volume, elemental volume that is Delta X into Delta Y from both sides. We are allowed to do that because Delta X into Dell Delta Y is nonzero quantity. We know that it is a nonzero quantity, if a nonzero quantity appears in the both sides of an equation, we can cancel it, if it is the do we cannot do that.

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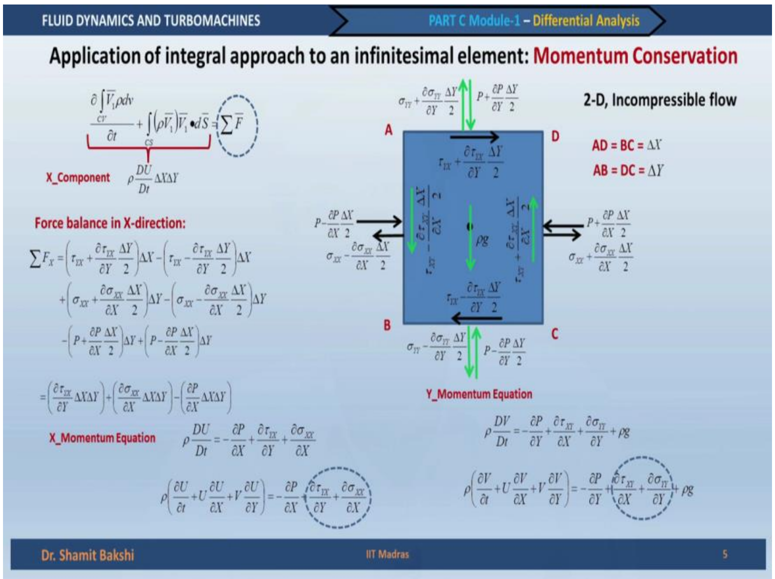

So, it it being a nonzero quantity we cancel it and we get the final expression that is the X momentum equation. So this X momentum equation now is the, almost close to the final form of the X momentum equation, we can write the total derivative in this for in the local acceleration and the convective acceleration and then write it. So in this way we can also get the Y momentum equation. In the Y momentum equation what we get is basically the shear stresses acting on these surfaces, the surface in which are perpendicular to the X axis and the, then we have the normal stresses which are acting on the Y plane, that means Y plane in both sides of the control volume and we have the pressure forces, so it is very similar. So only acting in a perpendicular direction but only with the difference that here we also have rho into g that is the weight of the fluid inside this control volume. So considering that for Y momentum equation what we get is the equation like this.

So if you see it is very similar to this equation only U is replaced with V and the derivatives of pressures, pressure is with respect to Y which is understandable because we have considered the pressure variation in Y direction and similarly for shear stresses and normal stresses, additionally we have the weight of the fluid in the control volume in this expression. So finally this is our Y momentum equation. So now we have almost obtain the X and Y momentum equation to buy this is not sufficient for us to solve or to get the information of the flow field because we still do not know the expression for shear stress and the normal stress.

So this is the subject of our, this will be the subject of our next lecture where we will see how to express the shear stresses and the normal stresses in terms of the velocity component so that we can get a partial differential equation for only in terms of velocities and also pressure is coming in here and we can get the flow field. So this brings us to the end of the first lecture of the 3rd week when we deal with the differential analysis, what we did here is we started with the integral approach and applied to infinitesimal control volume then we looked that the left-hand side of the Reynolds, the left-hand side of the conservation equation from the integral approach, that is the Reynolds transport theorem, then we from there by considering that we get the expression for the total acceleration which constitute of the local and the convective acceleration and then by considering the sum of forces we finally got the X momentum equation and similarly showed how we can get the Y momentum equation. In the next lecture we will take up the how to write the shear stresses in terms of the velocity field. Thank you.