Acoustics & Noise Control
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Module – 30
Lecture - 35
Monopole and Dipole

Friends; hello friends, we will still our class for the NPTEL course on acoustics and noise control in this last module where, we are looking through this spherical waves.

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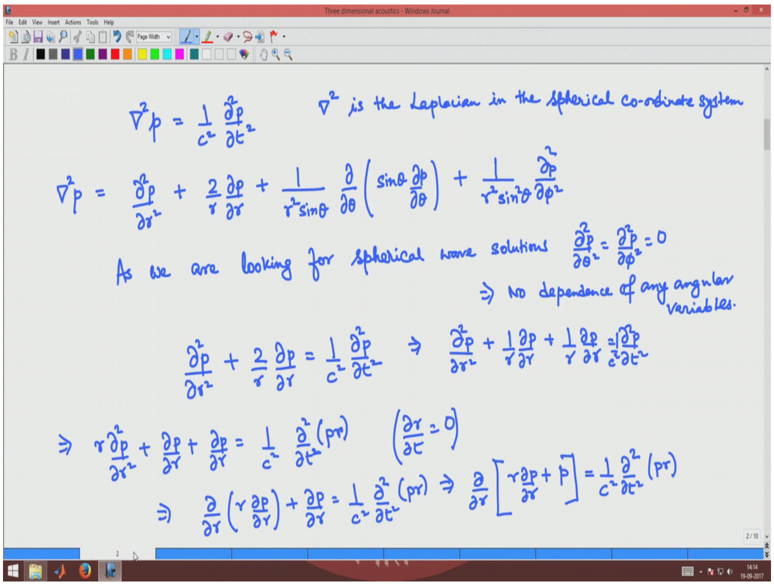

In the last class, we derived the notions of spherical waves. We derived the equations governing the propagation of spherical waves.

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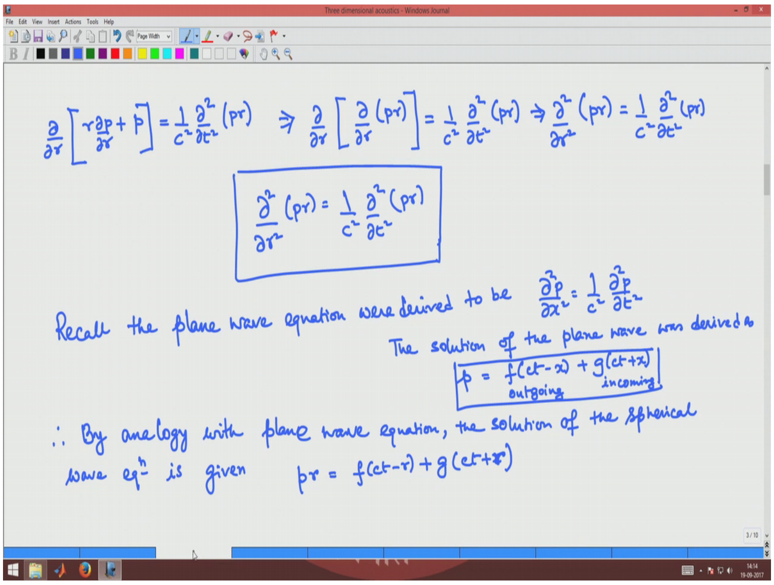

This was the equation for the spherical waves. We saw that there are outgoing and incoming waves possible.

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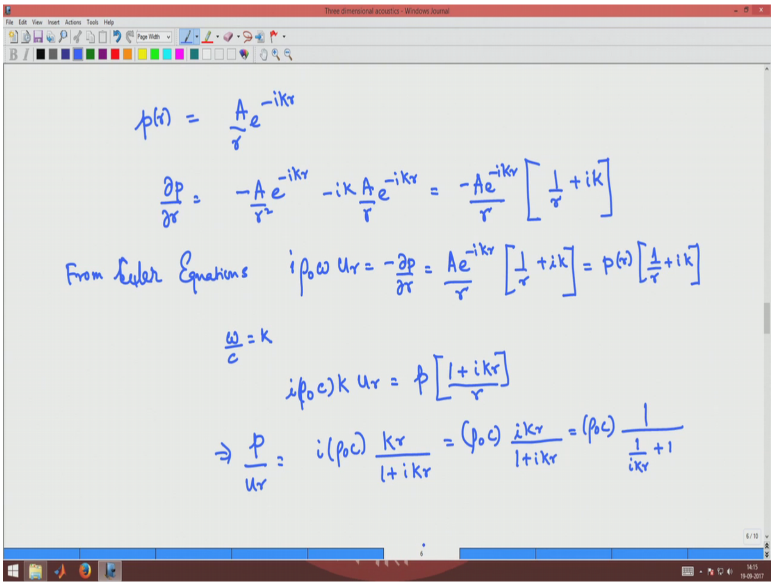

And most importantly we derived what is known as the impedance for this spherical wave formula, which is what you see in the slides right now.

So, today we will go ahead and use these relations to discuss certain spherical wave sources. The most important spherical wave source as I made a passing remark the other day is that of a pulsating sphere.

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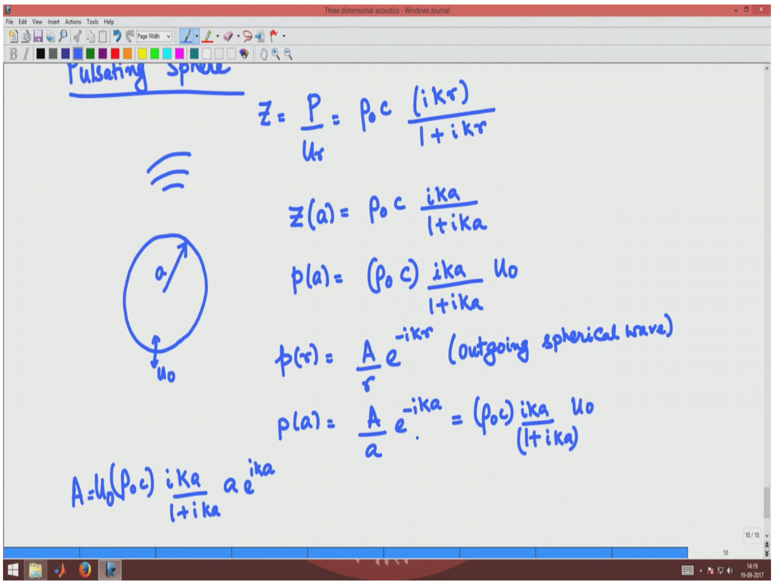

So, the problem here consist of finding the acoustic radiation due to a sphere of some radius let us say a. So, here we have a sphere which is of radius a and this sphere is pulsating in and out. So, this is a sphere of radius r, sorry radius a and this sphere is bulging in and out with the certain velocity let us say u 0 and the question is, what is the acoustic pressure field that is radiated because of this bulging sphere? Towards this end we will simply apply the impedance relation that we have derived in the previous class. You will recall the pressure by radial velocity formula, that we had derived in the previous class states as rho 0 c into i k r divided by 1 plus i k r, is the spherical wave impedance; which is basically the ratio of the pressure to the radial velocity. And then if we go, if we apply this formula now to this problem at hand we will need to evaluate this quantity I call this as Z. So, we wish to evaluate this quantity at r equals to a. So, at r equals to a we should have Z is equals to rho 0 c into i k a divided by 1 plus i k a.

So therefore, the acoustic pressure that is expected on the surface. So, acoustic pressure on the surface is supposed to be rho 0 c into i k a divided by 1 plus i k a into u 0. So, this is the acoustic pressure on the surface of this sphere. Next we can also look at the form of the acoustic pressure purely from physical principles. Purely from physical principles we understood even in the other day when we introduced this problem to you we were convinced that this situation is going to lead to outgoing spherical waves. Remember there is no boundary that is there in this problem at present, which means the form of the solution that we are going to expect here is A by r into e to the power minus i k r. So, this spherical wave, the outgoing spherical wave solution is given by this condition.

However in this form, as we saw the other day A is an undetermined constant and this constant has to be determined from the appropriate boundary condition. And now if we look at P at r equals to a, we get to see that we have this condition here and that in turn will be given as rho 0 c i k a divided by 1 plus i k a into u 0. So, this implies this undetermined constant can now be determined and the value of this will be rho 0 c i k a divided by 1 plus i k a into a into e to the power i k a.

So, I will possibly write this little about clearly, by shifting this a little further yeah. So, this is the amplitude, the undetermined amplitude which now gets determined through I forgot a u 0. So, there is a, I simply substitute in this expression, I simply get the value of A by multiplying this small a on the other side and e to the power i k a. So, thereby I get this formula for the undetermined constant A, which is what I will cut and paste in the next page.

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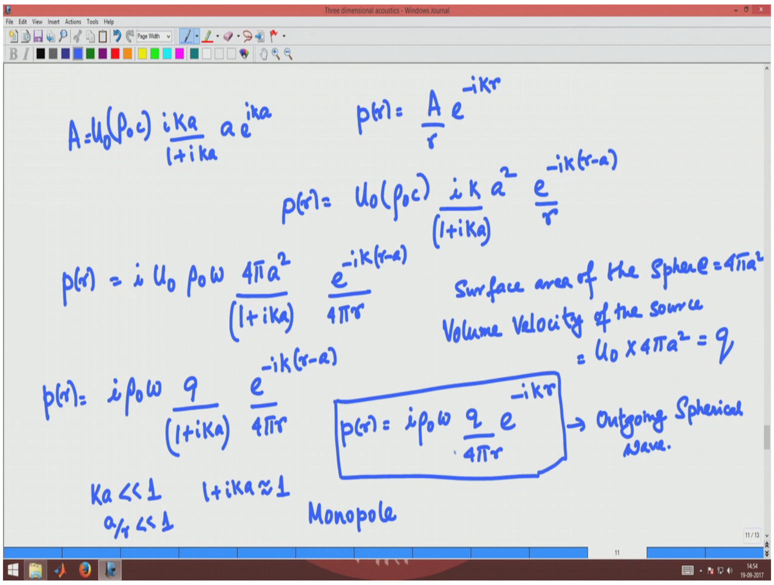

So, therefore, the pressure will be given as follows. So, therefore, the pressure relation P as a function of r, we have seen is A by r e to the power minus i k r and that in turn now that A has been determined, can be written in the following fashion. P by r is u 0 rho 0 c into i k a square divided by 1 plus i k a into e to the power minus i k r minus a to account for e to the power plus i k a in the expression for capital A. We are introducing this quantity e to the power minus i k minus a. So, that the effectively is e to the power i k and that should be divided with r. So, that is what we have at this stage, capital A has been only substituted; the only simplification that we have done at this stage is we have substituted the derived formula for capital A which is u 0 rho 0 c into i k a divided by 1 plus i k a into a, but that a and a taken together is a square and there is a division by r e to the power i k r minus a. So, this is the expression for pressure at any radius r. We will carry this a little simplification, little bit more by noting that this is u 0 k c, we can write it as omega and we will bring the i in front.

So, what we have as a constant popping out is i times u 0 rho 0 into omega and then there is an a square and there is also a 1 plus i k a and e to the power minus i k r minus a remains as in the previous step and divided by r. So, what we will is we will multiply and divide by this number 4 pi and as you know 4 pi a square is the surface area of the sphere; that is 4 pi a square. So therefore, we define a term which is called the volume velocity of the source and the volume velocity of the source is defined to be this radial velocity of the sphere multiplied by the surface area of this sphere and this will denote it as small q.

So, therefore, pressure at any radius can now be written in an even more compact form as i rho 0 omega into q which stands for u 0 times 4 pi a square divided by 1 plus i k a into e to the power minus i k r minus a divided by 4 pi r. So, this is what we have for the acoustic pressure, that is radiated due to a pulsating sphere of radius a. Now if we take a very crucial limit in case of this pulsating sphere, we get to a very fundamental source in acoustics which is called the monopole. So, by setting K a to be much less than 1. Remember K is the acoustic wave number and a is the radius of this sphere that we are talking about. So, this in other words means that the radius of this sphere is very small in compassion to the acoustic wavelength.

So, therefore, in this case therefore, what will happen is that this quantity P r will be reduced to i rho 0 omega into q, 1 plus i k a now can be approximated as 1 because i k a is much lesser than 1. So, therefore, there is only a 1 in the denominator which we may choose to even ignore and the 4 pi r in the denominator though remains and similarly within the exponential, we can neglect the effect of K a in comparison to K r. So therefore, another quantity that is neglected. So, a by r is also small. So, this time again we are saying that a is small in comparison to r. So, we are looking at distances which is far away from this radius of this sphere; obviously, you cannot look at distances which are very close to this sphere using this formula, but if you choose to look at distances which are far away from the surface of this sphere or from the source where it is generated and you also choose the radius of this sphere to be much lesser than the acoustic wave length at the particular frequency, then you get this nice compact formula and this situation is called a monopole.

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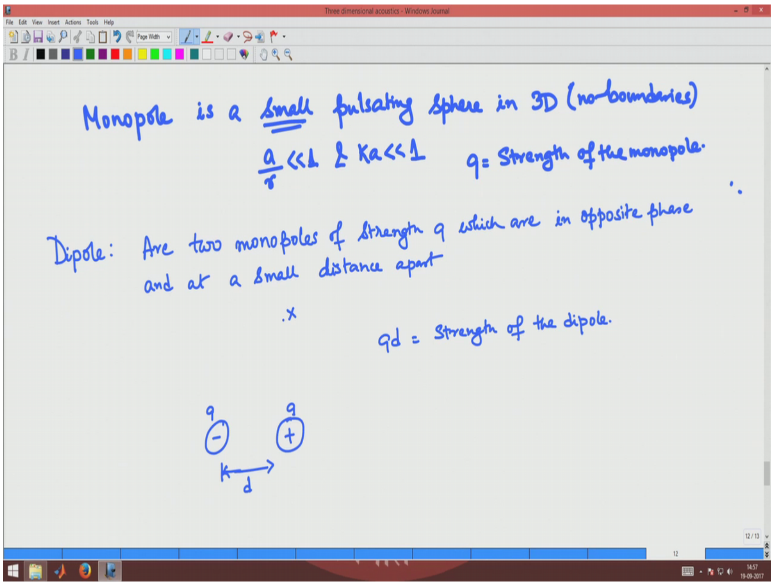

So, monopole is defined monopole is a small pulsating sphere in 3D. That is there is no boundaries at there in this analysis, an completely unbounded three dimensional domain. If you take this pulsating sphere and by smallness we essentially mean that the radius of this sphere is small in comparison to the distances that we are looking at and also the radius of this sphere is small in comparison to the acoustic wavelength. So, if this two smallness criteria are met then we are going to have this pulsating sphere which is extremely small and that special kind of source is known as monopole source. And as expected if you have a pulsating sphere big or small it is going to lead to a spherical wave condition and we have already seen that the expression for the pressure due to a pulsating sphere, which is very small has been obtained in this form which is directly seen in comparison to our spherical wave equation; this is an outgoing spherical wave.

So, this is the first source that we are dealing with in this course; in spherical wave acoustics. So, this is an outgoing spherical wave. We may choose to construct a monopole, this is the expression for monopole and please note here in this expression q is called the strength of the monopole. So, the reason why this monopole source is very significant in acoustics is because as we will show that together with the dipole monopole and dipole can be used to represent any vibro acoustic source and aero acoustic source will possibly include one more type of source which is called the quadrupole.

So, essentially what we are trying to point is that these three fundamental sources comprises all the different forms of sources that could possibly be there in acoustics and therefore, it make sense to study these sources. In fact, dipoles and quadrupoles are also super position of monopoles. So, in our next topic we are going to study dipoles and as the name suggests dipoles are two monopoles of same strength of strength q, which are in opposite phase and at a small distance apart.

So, what we have in the case of dipole is two monopoles; one is the negative monopole and the other is the positive monopole. So, what I mean by that is when one of these monopoles is pulsating outwards the other is pulsating inwards. Similarly when one of the monopoles is pulsating inwards the other is pulsating outwards. So, this comprises a dipole situation and this distance of separation which we can call as d, both of these monopoles are given by a strength q and there is a separation d between these two dipoles and again d is very small in comparison to the acoustic wavelength, in comparison to the distances that we are interested. And q d as we will see is got to be the strength of this dipole and we will see that the acoustic pressure that is available at any point, any field point is directly proportional to q d. So, q d will, as it will turn out will be the strength of the dipole.

So, what we will do now is that we will estimate the acoustic pressure due to a dipole at an arbitrary field point. Towards this end here is the schematic figure.

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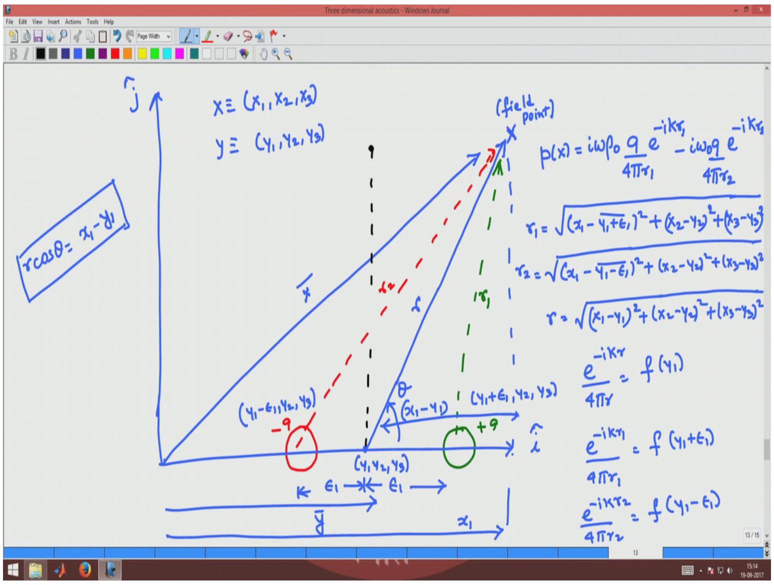

So, what I have done is I have taken two monopoles plus q and minus q as I had drawn and these two monopoles are at a distance 2 epsilon 1 apart. I have also align my code net axis i in the direction of this two in the line joining these two monopoles and the perpendicular access is j, k axis is perpendicular to the plain of the paper.

So, the center of these two monopoles is denoted by the position vector y and epsilon 1 units on either side of this vector I, we have the positive and the negative monopoles as has been drawn in this case. So, the pressure at and this is our field point x. So, this is the field point where we wish to evaluate the acoustic pressure. So, the acoustic pressure at this point x or I will call this x is given by the following. So, you have to understand that this acoustic pressure is got to be a super position of the acoustic pressures due to each of these two monopoles and we know how to exactly calculate the acoustic pressure for each monopole the formula is already sitting here.

So, if we apply this we just need to keep into mind that the positive monopole is at a distance r 1. So, r 1 is the distance vector from the positive monopole to the field point and r 2 is distance vector from the negative monopole to the same field point. So, therefore, P x will be i omega rho 0, that is the what the formula says multiplied by q in divided by 4 pi r 1 e to the power minus i k r 1. So, that is what is contributed by the positive monopole and similarly the negative monopole will contribute an identical, have an identical contribution except for the fact that negative sign will appear associated with the opposite phase of this monopole. So, that will be given by this quantity 4 pi r 2 in the in the denominator and e to the power minus i k r 2. Please note r 1 is given by if the coordinates of x can be written as x 1, x 2 and x 3; the coordinates of y could also be written as y 1, y 2 and y 3.

So therefore, the distance r 1 could be written as. So, before doing r 1 we need to find the coordinates of this positive monopole. So, the coordinates of the positive monopole is y 1 plus epsilon 1, y 2 and y 3 and the coordinates of the negative monopole is y 1 minus epsilon 1, y 2 and y 3 minus epsilon 1, y 2 and y 3. So, therefore, r 1 is given by the following fashion, r 1 is given as square root of x 1 minus y 1 plus epsilon 1 whole square plus x 2 minus y 2 whole square plus x 3 minus y 3 whole square. And similarly r 2 is given by the following which is x 1 minus y 1 minus epsilon 1 whole square plus x 2 minus y 2 whole square plus x 3 minus y 3 whole square.

Now, this expression might look intimidating, but we will have an easy way to simplify this expression. Towards that end we will try to solve out for r, which is the distance from the center of the field point from the center of these two monopoles which the center of the monopole is given by y 1, y 2, y 3. So, therefore, what we have as r is x 1 minus y 1 whole square plus x 2 minus y 2 whole square plus x 3 minus y 3 whole square. So, here we have the expressions for r, r 1 and r 2. Please note that I could as well write e to the power minus i k r by 4 pi r as a function of y 1, there is nothing wrong if I do that. I could also write e to the power minus i k r 1 by 4 pi r 1 as a function of y 1 plus epsilon 1 because it is the same function the only difference between the two functions that I have written in these two consecutive lines lies in the fact, that instead of where ever I was using y 1 I am using y 1 plus epsilon 1.

Similarly, e to the power minus i k r 2 by 4 pi r 2, could be written as f with an argument of f of y 1 minus epsilon 1. So, therefore, what we have is the following for the pressure.

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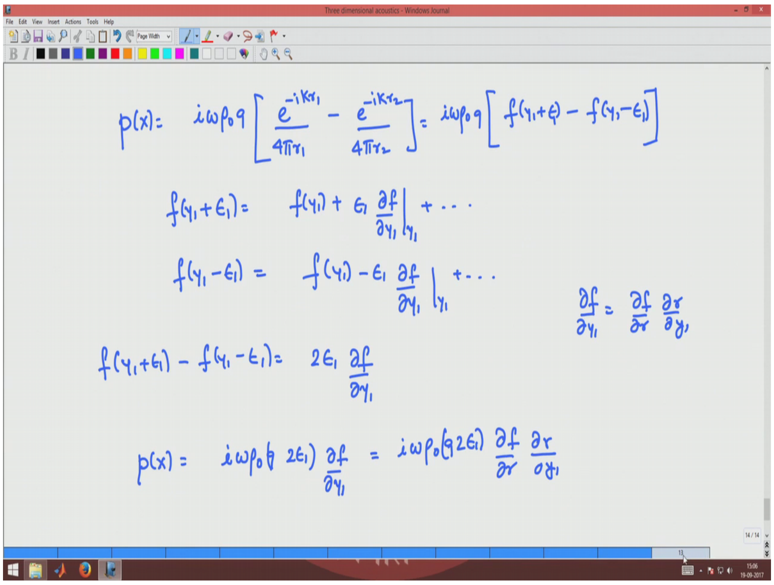

So, pressure at our field point is given by i omega rho 0 q e to the power minus i k r 1 by 4 pi r 1 minus e to the power minus i k r 2 by 4 pi r 2 and as we have decided to call these two expressions as f of y 1 plus epsilon minus f of y 1 minus epsilon 1 will make that substitution here.

Now, we will recall our good old theorem of calculus which is the Taylors Theorem; f of y 1 plus epsilon 1 is basically given by f of y 1 plus Del f Del y 1 into epsilon 1. So, Del f Del y 1 has to be evaluated and that has to be evaluated at the value of y 1, that has to be evaluated at this y 1 value and then there can be other terms, but we choose to ignore those terms. Similarly f of y 1 minus epsilon 1 can be written as f of y 1 plus sorry this time this will be with a minus sign. So, minus epsilon 1 Del of Del y 1 evaluated at y 1.

So, therefore, the subtraction of these two; f of y 1 plus epsilon 1 minus f of y 1 minus epsilon 1 will be 2 epsilon 1 Del f Del y 1 evaluated at y 1 which is implicit . So, therefore, at this stage what we have is pressure at the field point of our interest is actually going to be i omega rho 0 q into 2 epsilon 1 into Del f Del y 1 and remember y 1 is the coordinate, is the first coordinate of the center point of these two monopoles which is basically like the location of the dipole itself. So, y 1 is this location and will continue using that notation and also here we see that it is q times 2 epsilon, which is the distance between the two monopoles which is directly relating to the acoustic pressure and therefore, as I have said q times 2 epsilon will be called as the strength of the dipole configuration here.

So, carrying forward we could write this thing as i omega rho 0 q times 2 epsilon 1 into Del f Del r into Del y 1 Del r Del y 1. So, this is again nothing, but chain rule of calculus. I have chosen to expression Del f Del y 1 as Del f Del r into Del r Del y 1 and what was my f function? The f function exactly had this r expression that is why I am doing it in two parts; f is e to the power minus i k r by 4 pi r in turn r depends upon y 1, but I can do it two steps; I can first take the differentiation with respect to r and then differentiate r with respect to y 1; y one is my fundamental independent variable with which respect to which I am doing the differentiation, but I am just to make life a little more simple for me in terms of this mathematical manipulation. I will do it in two steps; I will first to take the differentiation with respect to r which should be pretty easy and then differentiate r with respect to y 1.

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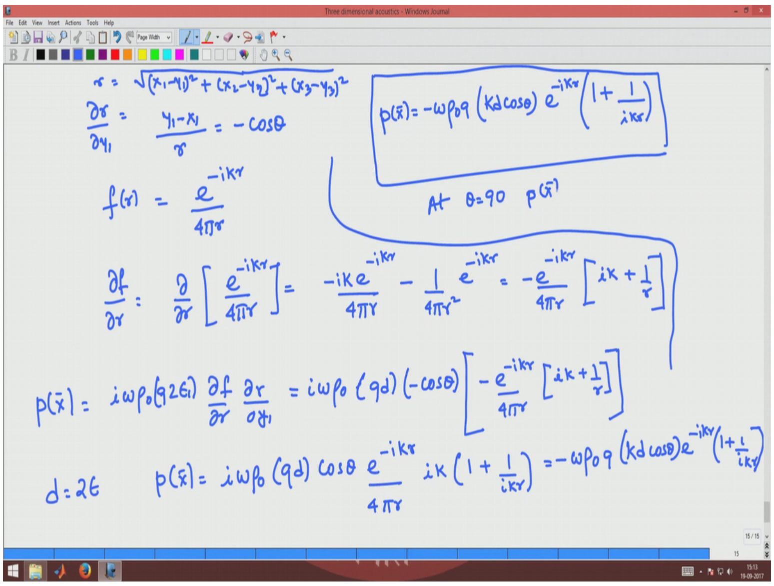

So, this is the formula that I will wish to carry over. So, accordingly I need to calculate Del r Del y 1 where, r you will recall is given by square root x 1 minus y 1 whole square plus x 2 minus y 2 whole square plus x 3 minus y 3 whole square. That was what we had derived a few slides back; this is the expression. So, therefore, Del r Del y 1 is given by we will have exactly the same quantity coming in the denominator, but what we will have in the numerator is y 1 minus x 1 divided by r. And let us see what that is in the drawing. So, this is y 1 and this position if I drop a perpendicular this value is going to be x 1.

So, therefore, this value is going to be x 1 minus y 1 and this is this distance is any way marked as r. So, therefore, you realize that r Cos theta where, theta is this angle; r Cos theta is going to be given by x 1 minus y 1. So, this is a geometrical relation that we have from our drawing and therefore, y 1 minus x 1 by r is going to be given by minus cosine of theta where, theta is the angle that has been drawn in this figure. And similarly, Del f Del r; so f r is e to the power minus i k r by 4 pi r. So, Del f Del r will be given as Del Del r of e to the power minus i k r by 4 pi r and that would have two terms; the first term will have a minus i k popping out e to the power minus i k r by 4 pi r and then we will have minus 1, 4 pi r square e to the power minus i k r. And this if we take this e to the power minus i k r by 4 pi r as common, together with the minus sign we will be left with a minus i k and plus 1 by r. So, Del f Del r is given in this fashion.

So, therefore, what is the expression for P x? P of x we have derived to be this quantity; edit, copy and paste. We have derived to be this quantity. So, this is P of r; P at the field point x and that in turn now can be written as i omega rho 0 q times 2 epsilons. So 2 epsilon I will call as d. So, d is equals to 2 epsilon, d is the distance between the two monopoles. So, here we will have q times d times Del f Del r and Del f Del r was given by that expression and Del r Del y 1 is given by minus Cos theta. So, that is what I am writing now, multiplied by minus e to the power minus i k r by 4 pi r into i k plus 1 by r.

So, then we could write this as P as. So, these two negative signs can get cancelled and we will be left with i omega rho 0 q d Cos theta e to the power minus i k r by 4 pi r and lastly we will take one; the i k common. So, what you will be left out is with 1 plus 1 by i k r and again the i over there are two i’s. So, we can put a minus sign and we can write this in the following fashion minus omega rho 0 q K d Cos theta into e to the power minus i k r into 1 plus 1 by i k r.

So, this is my final expression for pressure at any point on the dipole which is given by minus omega rho 0 q K d Cos theta into e to the power minus i k r 1 plus 1 by i k r. Please note at theta equals to 90. So, at theta equals to 90, we have pressure equals to 0. That should not come as a surprise because at theta equals to 90, which is precisely this direction. So, this is the 90 degree direction; any point on this line is equidistant from both the positive monopoles as well as the negative monopoles.

So, therefore, the pressure contributions due to the positive monopole and the negative monopole exactly nullify each other and thereby there is no pressure left at this point. So, here what we get to see therefore; is the first instance of what is a directive sound. Remember monopoles where omni directional all the directions had equal sound pressure, equal acoustic velocity, everything was equal in the case of monopole, but in dipoles these are directional fields because there is a Cos, theta dependence as I have pointed out to you.

So, in this derivation what we have done is that, we have made a life little simple by choosing the axis of the dipole to be aligned with one of our coordinate system. In the next class what we will do is we will keep it perfectly general by saying that we will choose the axis to be aligned in an arbitrary direction not necessarily the axis of the I mean the line joining the two monopoles need not align itself with the coordinate axis. These two monopoles can be on any arbitrary line and we will repeat this derivation for dipole, but at this stage you must be able to appreciate that the dipole is the first instance where we are starting to see what is known as directive sound. And as I said and I repeat that remark that, the monopoles and dipoles in totality can be used to general any vibro acoustic source.

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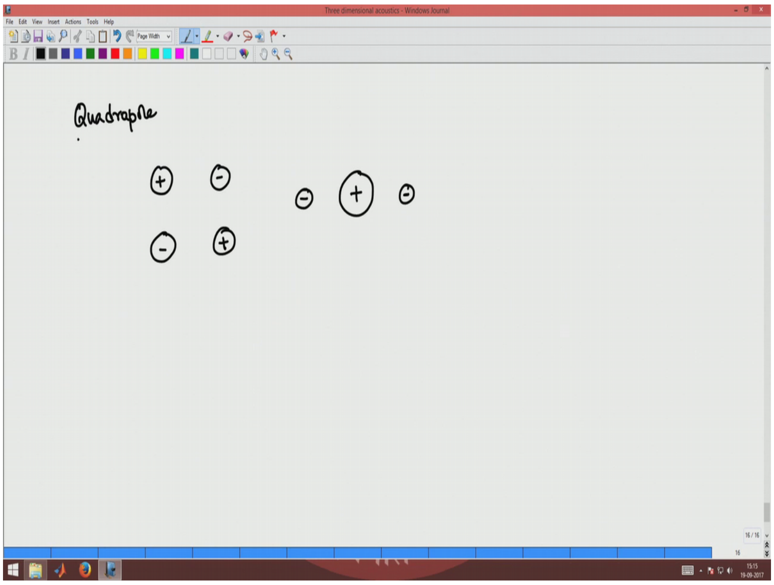

Similarly, in the case of aero acoustic source you will have what are known as quadrapole sources.

So, quadrapole sources in which are common in aero acoustic application will involve monopoles of this kind. You will have a distribution of monopoles of this kind or you could also have a monopole here and two dipoles here. So, this is also a quadrapole. So, there are different kinds of quadrapoles configuration. We would not go into the details again of the derivation of the quadrapole, but the point is in vibro acoustic source which is typically the specialized focus of this course. In vibro acoustics application, you will not you will see and we will show that by the end of this course that everything boils down to a monopole and a dipole contribution and that is a precisely how the numerical method of boundary element method works. And it is only required to find that distribution, once you find that distribution you already know what happens due to one monopole, what happens due to one dipole and then you just need to super pose the two.

So, I would close this class with this remark that, today what we have done is we have derived the monopole and the dipole expression. The dipole expression was derived with the simplistic assumption that the dipole axis which is the line joining the two monopoles is actually aligned with one of the coordinate axis which sort of simplifies the calculation. The next day the calculation will be repeated wherein the dipole axis will be completely along an arbitrary direction, but the important take away from today's lecture should be though it is not shown writing today is that these two type of sources monopoles and dipoles will constitute, will make up all kinds of vibro acoustic sources. Any acoustic radiation problem is essentially a collection of monopole and dipole sources. So, that is left for the later part of this course. So, with that we end today's lecture.

Thank you.