X-Ray Crystallography

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Lecture – 04
Stereographic Projection

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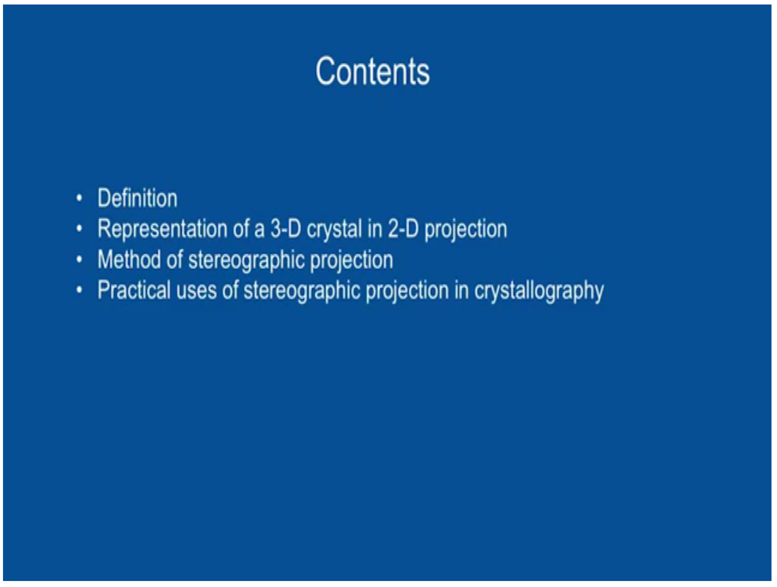

In this lecture I shall deal with the principles and techniques of stereographic projection and we will also talk about some important applications of stereographic projection. Now what is meant by stereographic projection? You see when we deal with three dimensional crystals; very often we have to deal with angles between crystallographic planes, angle between crystallographic direction, angle between a crystallographic plane and a crystallographic direction etcetera etcetera.

Now how this can be done in a very easy method. You see human beings find it very difficult to comprehend a three dimensional object that is the reason why the information of a three dimensional object is resolved into two dimensional form for example, when you look a globe you know that geographic atlases are nothing but some kind of two dimensional projection of a three dimensional globe, but the principle behind that kind of projection is that this projection is area true projection; that means, a country which is really big on the surface of the globe, will occupy a bigger space than a country which has a smaller area on the globe in the geographical atlases.

So, geographical atlases are so to say area true projection.

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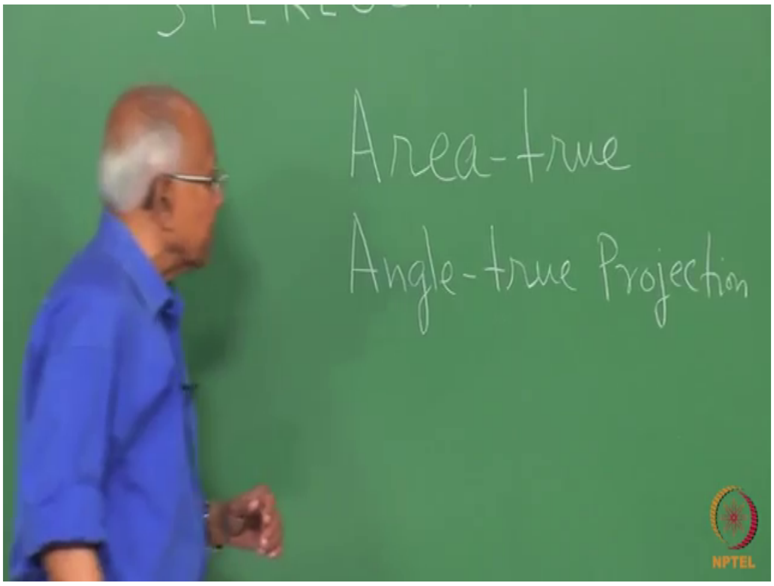

So, geographical atlases are area true projection, but stereographic projection means it is an angle true projection. So, a stereographic projection means an angle true projection now let us see how we can bring all the data regarding a three dimensional crystal into some two dimensional form, and this is what is known as the stereographic projection.

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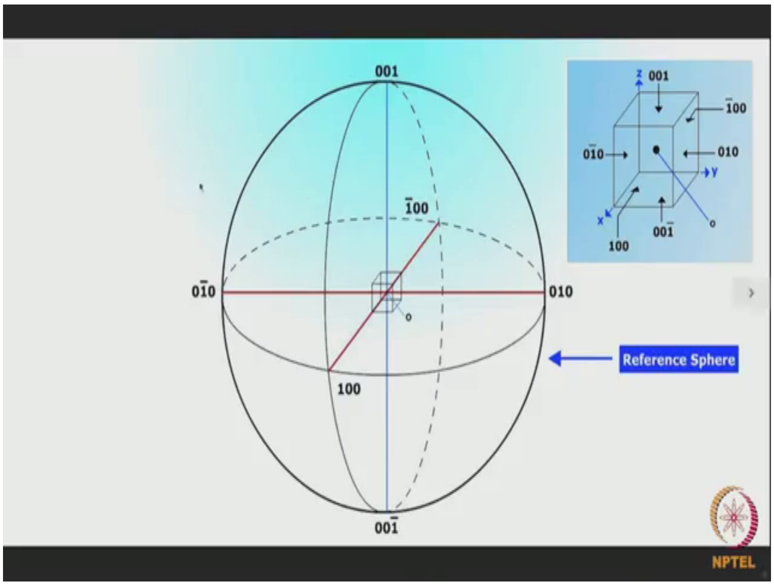

Now, let us assume that we have a unit cell of a crystal and here we consider a simple cubic unit cell, the 6 planes of this unit cell are shown here, the front plane is 100 the back plane is bar 100, the right hand side plane is 010, the left hand side plane is 0 bar 10, the top plane is 001, the bottom plane is 00 bar 1. Now if we consider the unit cell to be very very small, so small that it can be consider as nothing but a single point and then what we do keeping that point unit cell as the center, we construct a big sphere around it and that sphere is known as the reference sphere. So, we put the point unit cell at the centre and construct a big sphere around that we call it the reference sphere.

The next step is to draw perpendiculars to the six faces of the unit cell. So, the perpendicular from the front plane 100 will cut the reference sphere at the 0.1 0 0 here. So, this point is known as this point on the sphere is known as the pole of the 100 plane. Now similarly the perpendicular to the back plane bar 100 cuts the reference sphere over here, and this is the pole of the bar 100 plane. Similarly the perpendicular to the 010 plane cuts the reference sphere here. So, this is the pole of the 010 plane this is the pole of 0 bar 10 plane similarly this is the pole of the 001 and this is the pole 00 bar 1 plane. So, what we have done we have reduced the 6 planes of the unit cell into 6 points lying on the surface of the reference sphere.

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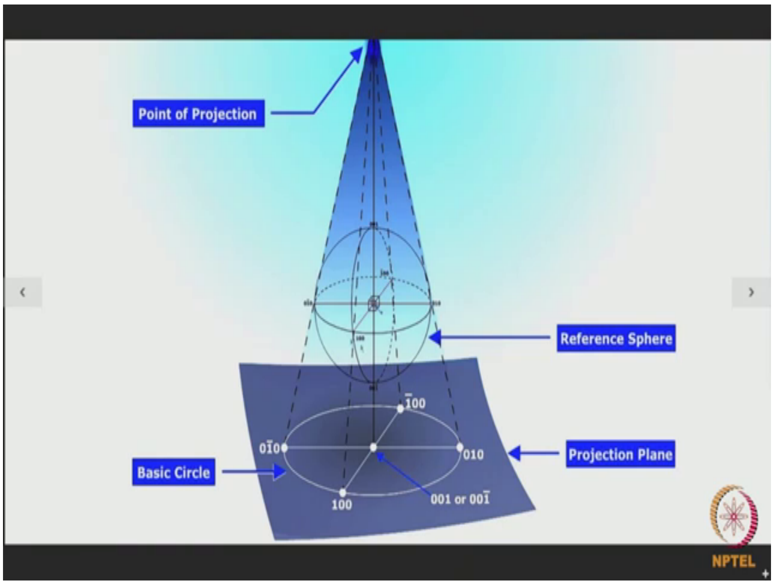

Then what we do? We put a source of light right at the top we call it the point of projection and allow the rays of light to pass through all the different poles lying on the surface of the reference sphere, and allow these lights to fall on a piece of paper which is lying perpendicular to the main direction of the light. So, the situation is we have got let me draw it over here it will be easier. So, we have got a reference sphere.

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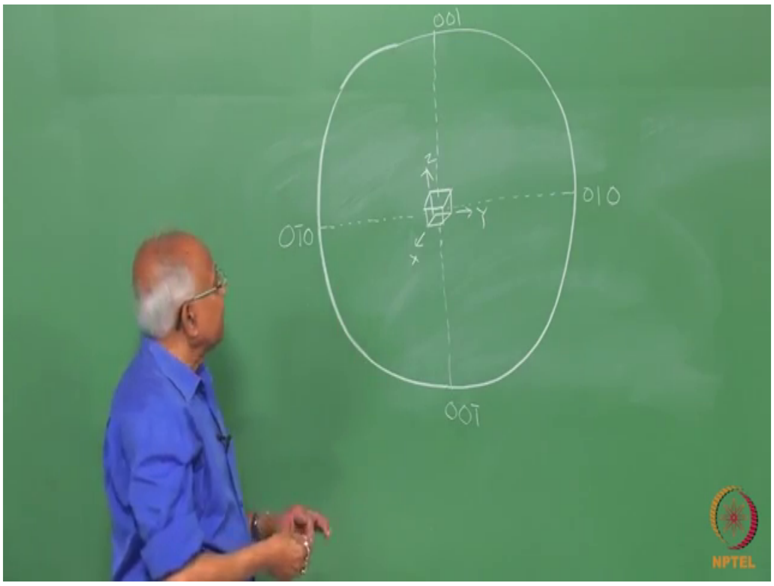

So, the situation is we have got the point unit cell at the center of a big reference sphere. So, this reference sphere is such that half of the reference sphere you see on the top of the board, the remaining half of the sphere will be at the bottom of the board and at the back side of the board I am sorry.

So, now what happens that the pole of 100 plane will be over here, and exactly at the opposite point backwards the pole of bar 100 will be. The pole of 010 will be here pole of 0 bar 10 here, pole of 001 plane will be there, pole of 00 bar 1 will be there. Now what we do we take a piece of paper and put it perpendicular to this axis and put a source of light over there, and allow the source of light rays to pass through all the poles and fall on this piece of paper. Now if we do that the whole of the reference sphere it will be projected as a circle.

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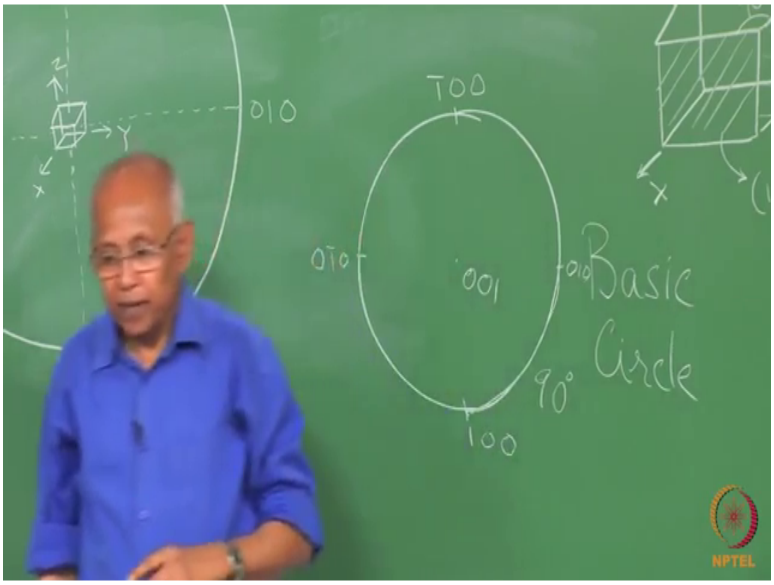

The whole of the reference sphere will be projected as a circle, and we call it the basic circle. So, we put a source of light on top and allow the light rays to pass through all the poles of the various planes and fall on this piece of paper which is taken perpendicular to this line. So, the entire sphere reference sphere will be projected as what is known as a basic circle.

Now, in the basic circle we know that the 100 pole which is over here this light ray will pass through that 100 and cut it over here. So, if we draw a basic circle this is where the 100 pole will be projected by the light rays, similarly on the back side the bar 100 pole will be falling. So, here the bar 100 will lie on the basic circle. So, on the basic circle where the 010 pole will lie it will lie on the right hand side here. So, this point is the 010 pole on the basic circle, where this one lie it will be at the left hand side. So, here there will be the 0 bar 10 pole, and the 001 and 00 bar 1 they will be right at the centre.

So, you see that in this kind of a projection now we have replaced the 6 atomic planes by 6 points on the basic circle, and these are nothing but the projections of the poles of the 6 planes. Now this diagram is known as a stereographic projection. So, this is what is known as a stereographic projection. So, you see that this is the diagram which is now which is known as the stereographic projection, as I said before this is the basic circle and the projected 100 point will be here, the projected bar 100 point will be here, the projected 010 will be here, the projected 010 will be here, the projected 001 or 00 bar 1 will be here.

Now, we call it stereographic projection what is the reason? Because this projection is actually an angle true projection; how come? You see if you look at the angle between say 100 pole and 010 pole this is 90 degree, and if you look at the actual unit cell the 100 plane makes a 90 degree angle with the 010 plane.

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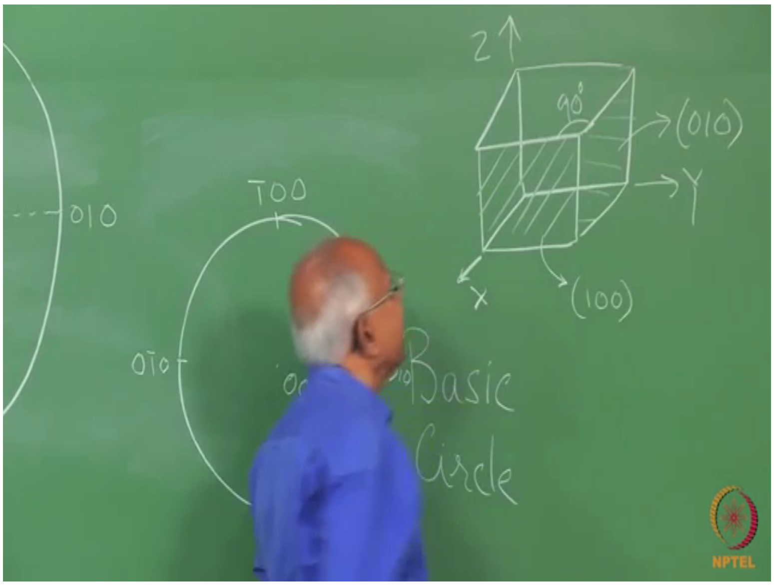

So, let us draw it once again. So, if we draw a unit cell on the board now this front plane will be the 100 plane of the unit cell, this right hand side plane will be the 010 plane of the unit cell. So, how much is the angle between these two planes that actually it is 90 degree. So, the angle between these planes is 90 degree.

And when we look at the projected poles of 100 and 010 they are also making an angle 90 degree between them. So, you say that this projection that we have drawn is angle true, angle between 100 pole and 010 pole is 90 degrees and the actual angle between the planes 100 and 010, 90 degree. Now you take any two pair suppose and find out the angle and you see that you know they will tally with the angular relationship these planes have between themselves. So, we can now say that our projection which we have drawn here is nothing but a stereographic projection because it is an angle true projection.

Now, in this stereographic projection, what kind of poles we have plotted?

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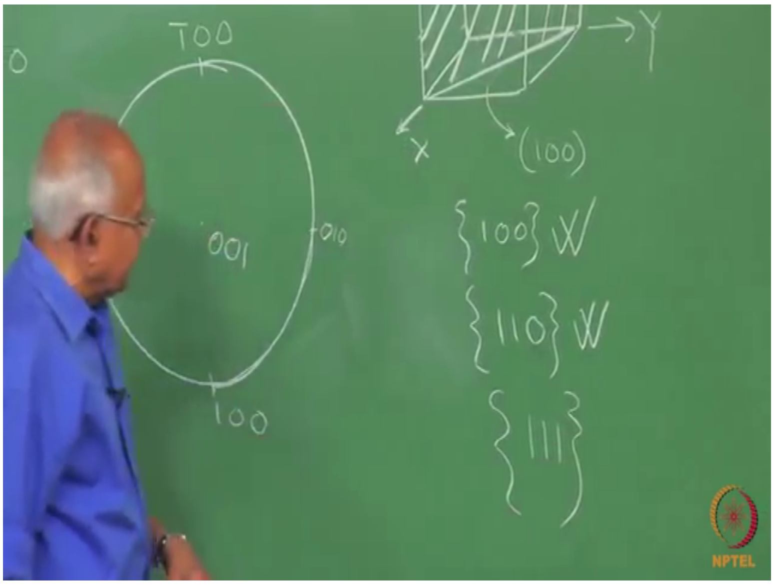

We have plotted the poles of 100 type planes only; in this stereographic projection we have plotted the poles of 100 types of planes only. Now if we want to plot the poles of other type of planes say for example, if we want to plot the 110 type of poles in the same stereographic projection then how to proceed.

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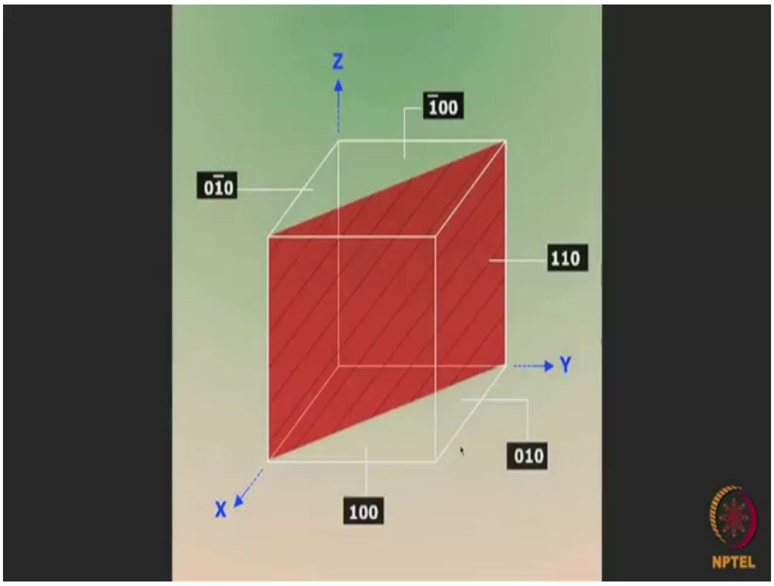

Now, look at the 110 plane in the unit cell. So, in this red colored plane this is the 110 plane this is the 110 plane in the unit cell.

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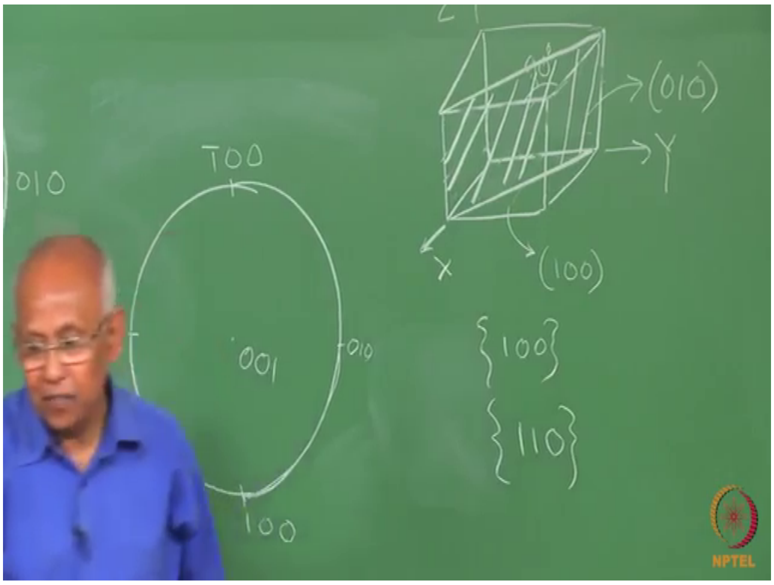

Now naturally if we look at perpendicular to that plane.

So, in this diagram where is ours. So, coming back to this diagram on the board we see that this is the 110 plane. So, naturally if we draw the perpendicular to the 110 plane, where it is going to be you see the, perpendicular to 100 is coming like this perpendicular to 010 is going like that and since this plane bisects these you know the angle between this and this the perpendicular to that will lie midway between the perpendiculars to 100 and 010.

If that is the case we will have the perpendicular to this plane, you know coming light you know making the same angle with the pole of with the perpendicular to the 100 plane and the perpendicular to 110 planes.

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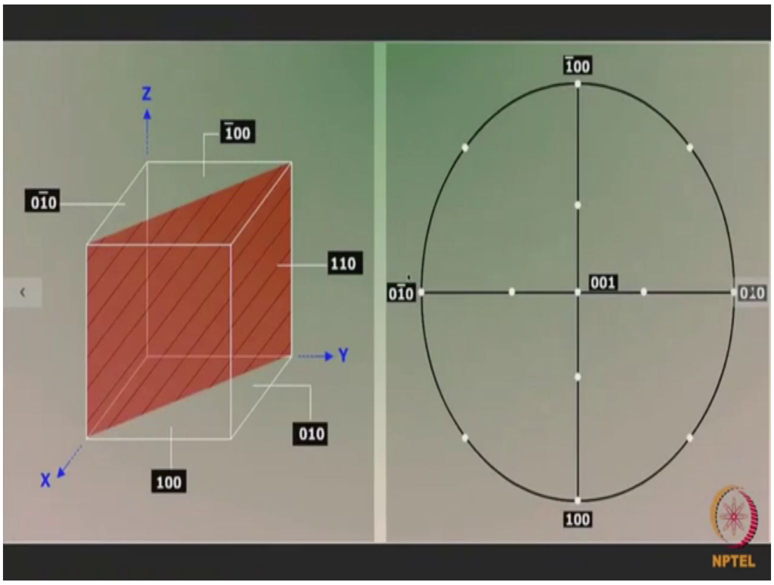

If that be the case where do expect to get a pole get the pole of 110. So, if this is the pole of 100 plane and that is the pole of 010 plane, then the pole of 110 the projected pole of 110 must be lying midway between the two because the perpendicular to the 110 plane lies exactly midway between the perpendicular to the 100 and 010. So, the position of the projected pole of 110 will be here. In a similar way where are the poles of all the other 110 type poles going to be now we have found out a simple reason that it will be lying in between the poles of any 2: 100 type of poles. So, if that be the case the other locations of the 110 type poles will be this one, will be this one here, here, here, here, here, and here.

So, you see that 110 type poles will be lying between exactly midway between the poles of 2 100 type poles. So, if that be the case this is the 110 pole this is another location another 110 type pole this is another location of another 100 type of pole like this, this, this, and this. Now how to find out the indices of all those poles for example, the indices of this pole here be you know midway between the poles 100 and 010 is 110. So, how we get to that now, look at this.

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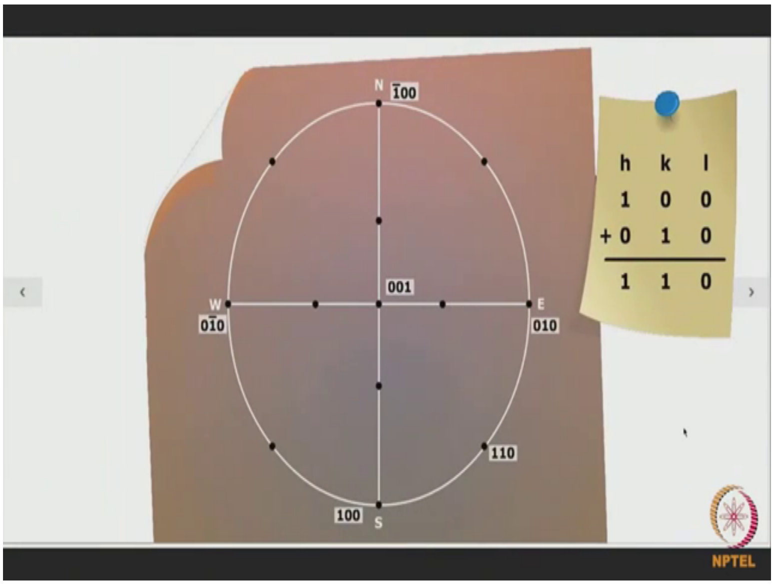

So, the pole between the 100 and 010 is the 110 type, and how we find out the indices? Now we add up the indices of this pole as well as that pole; so 100 h k l, plus 010. So, you add up it is 110. So, you see that in a very simple manner it is possible to find out the indices of this pole. Similarly if we want to find out the indices of this pole how we are going to do that now it is lying midway between a bar 100 pole and a 010 pole. So, if we add up the indices it will be bar 1 plus 0 that is bar 1 1 plus 0 1, then 0 plus 0 0. So, it will be bar 110.

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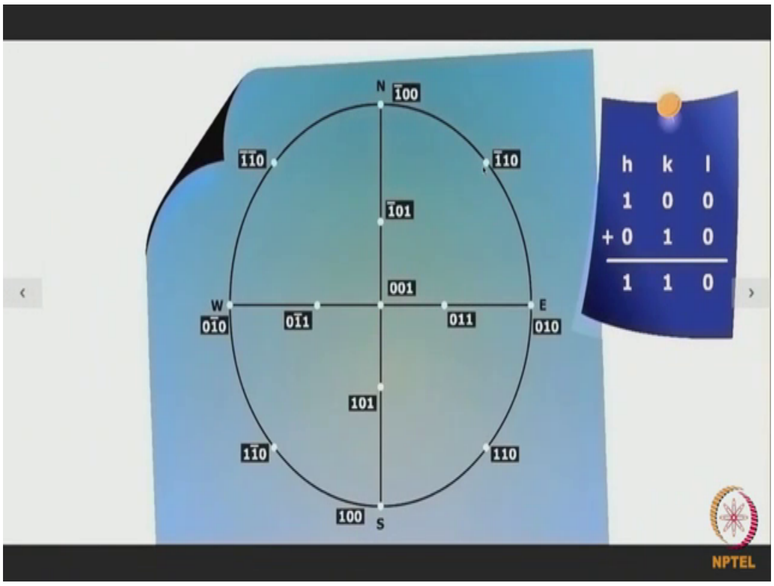

So, in this manner it is possible to find out the indices of all the other 110 type poles. So, this one will be bar 110 as we have seen already, this one will be 1 bar 10, this one will be bar 101, this one is 0 bar 11 this one will be 101 and this one will be 011, and finally this one will be 1 bar 10 1.

So, what we have done? Now in the same stereographic projection we have now plotted not only the poles of the 101 type of planes, but also the poles of the 110 type planes.

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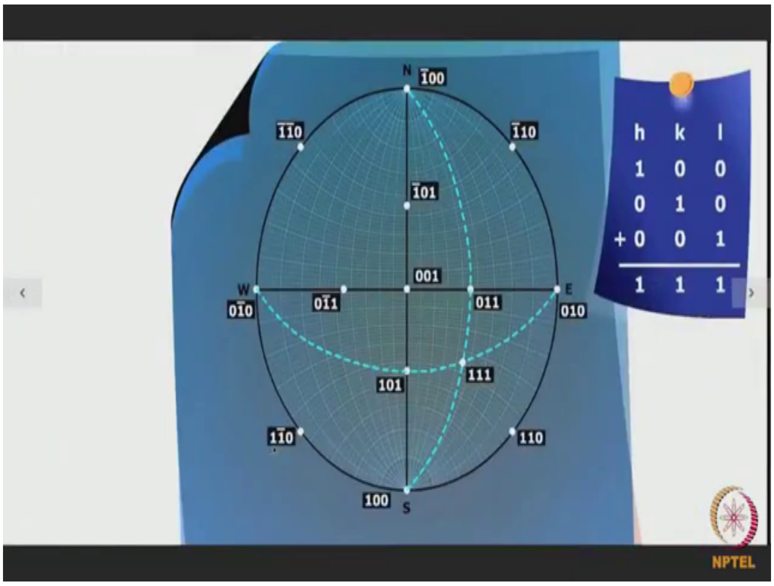

Now, let us try to figure out how in the same pole figure in the same stereographic projection, we can plot the poles of the 111 type of planes. Now how we can plot the poles of the 111 type of planes in the same stereographic projection; now in order to do that we will have to take the help of the zonal relationship which we derived in the previous course in the previous course we have seen that.

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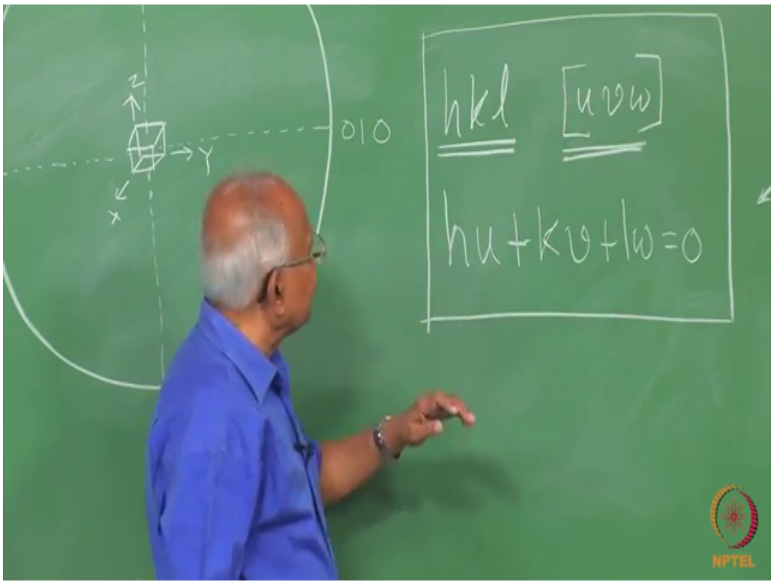

If a plane h k l belongs to a zone for which the zone axis is u v w, in that case this relation h u plus k v plus l w must be equal to 0. So, we know the zonal relationship that a plane h k l will belongs to a zone for which u v w is the zonal axis only and only when h u plus k v plus l w becomes equal to 0.

Now say for example, if we draw a longitude line passing through the bar 100 plane the 011 plane, and the 100 plane then all the planes which will lie on this longitudinal line they will form a zone and what will be the zone axis? The zone axis will be 90 degree from each point of this longitude line. So, you see that if you move 90 degrees for example, 10, 20, 30, 40, 50, 60, 70, and 80.

So, from here to here if we move 90 degrees then we can say that the planes which will lie the (Refer Time: 24:38) will lie on this longitude line, they must form a zone because each zone has a distance of 90 degree from this point; that means, all those planes will form a zone with this as the zone axis. So, if we draw a longitude line passing through bar 100, 011 and 100 then you know each longitude line there are plenty of planes which will lie on this longitude line and if we get away from this along the equator to the point 0 bar 11, then we find that this point is making 90 degree angle with each and every point lying on this longitude line.

So, you see that this pole here is at 90 degrees with all the poles lying on this longitude line; that means, the planes which lie this poles will lie on this longitude line they will form a zone and this will be the zone axis. Now if that be so we can say that we can draw a second longitude line in this manner passing through 010, 101 and 0 bar 10. Now if we go 90 degree along that then this pole will be making 90 degree with all the poles lying on that longitude line; that means, all the planes whose poles lie on this longitude line they will form a zone, and this will be the zone axis.

So, the intersection between this longitude line and this longitude line is this point. So, this pole will be such that it belongs to both this longitude line and both and this longitude line; that means, this pole must be the one which belongs to the zone for which this is the zone axis, and also belongs to the zone for which this is the zone axis; that means, 0 bar 11 and also bar 101.

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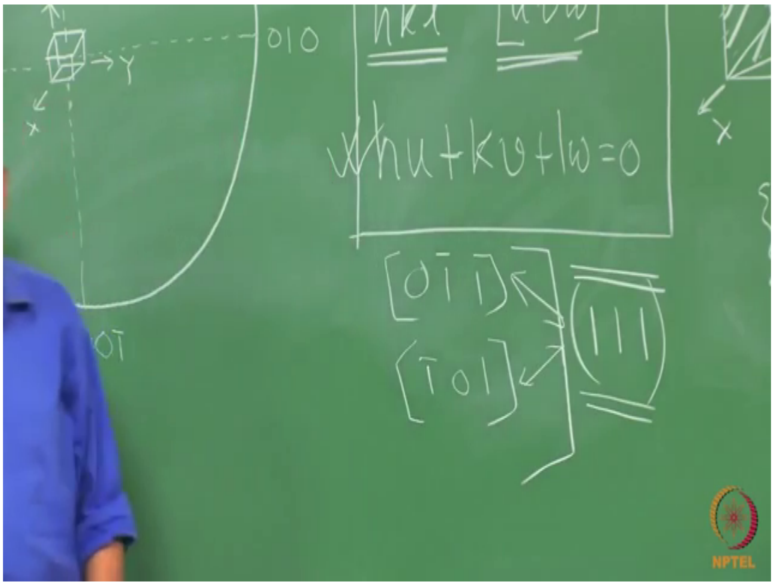

That means, this is a point this is a pole of a plane and the plane is such that it will belong both to this zone as well as to this zone; that means, it must satisfy the h u plus k v plus l w equal to 0 relationship with this axis as well as this axis.

Now, what is the plane which makes this kind of a relationship with both the axis, now an inspection will always tell us that it is the 111 plane, because if h k l are 111 and this is the u v w, you can see that 1 multiplied by 0 is 0 1 multiplied by bar 1 is minus 1, 1 multiplied by 1 is plus. So, it does have a zonal relationship with this axis similarly this will also have a zonal relationship with this axis because 1 into bar 1 minus 1, 1 into 0 is 0 1 into 1 is plus 1. So, h u plus k v plus l w is equal to 0. So, it is the 111 plane which really is the one that belongs to both this zone as well as this zone. So, that is the reason why the intersection point is the 111 pole.

Now, we can find a thumb rule how to get the indices of the pole? So, it is in this quadrant and you see that if we add up the indices of this, this and this we get the indices of that. So, that is exactly what we do h k l 100 is this one, then 010 this one, 001 is this one. So, add them up you get 111 in a similar way it is possible to find out the other locations of the 111 type of poles.

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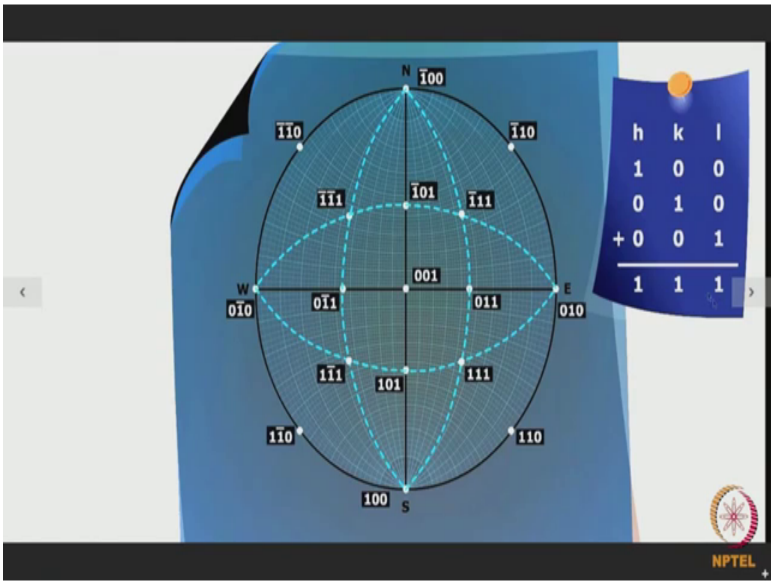

And this is what we have done it here for example, a second 111 type of pole will be lying here, at third 1 1 one type of pole will be lying here, a fourth 111 type of pole will be lying here and in a manner similar to what are described previously you add up the indices. For example, indices of this pole this pole, and this pole and arrive at the indices of this, add up the indices of this pole this pole and this pole arrive at the indices of this, add up the indices of this pole this pole and this pole and arrive at the indices of this pole.

So, in this way we can find a stereographic projection where 100, 110, 111 all the poles have been drawn and you can continue doing like that put the poles of 112, 123 type of planes, etcetera, etcetera.

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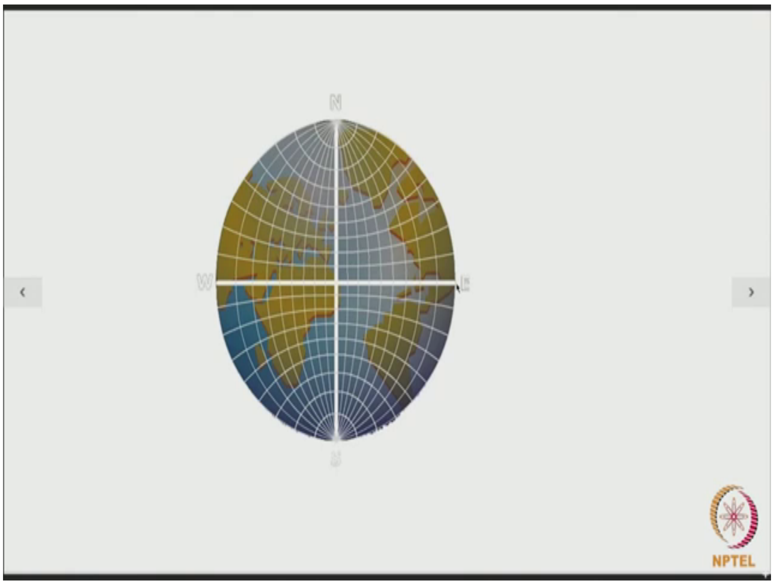

Now, you see on the globe you will find there are two types of lines drawn for example, you know we know that in the middle is the East West axis which is called the equator, and there is an axis connecting the North and the South poles. And you can see that there are two types of lines drawn on the surface of the globe, and this diagram is a projection of that. So, there are two kinds of lines which are drawn one connecting the North and the South Pole over the globe. So, there are lines connecting the North and the South Pole over the globe and all these lines are of the same size, and these are also the biggest circles that can be drawn on the surface of the globe. So, all the lines are known as the longitude lines. So, all the longitude lines are of the same length and they are the biggest they are big circle so to say.

And then a second set of lines are drawn parallel to the equator. So, if you have the equator you draw another set of parallel lines parallel to the equator at regular intervals along the North, along the South, and those are known as the latitude lines though. So, the big circles you know drawn from the North and South over the surface of the globe, these are known as the longitude lines and the smaller circles drawn parallel to the equator they are known as the latitude lines. We have to remember that this is a two dimensional projection showing all the latitude and the longitude lines. Now the equator itself is also a big circle. So, we distinguish between the two types of lines the longitude lines and latitude lines.

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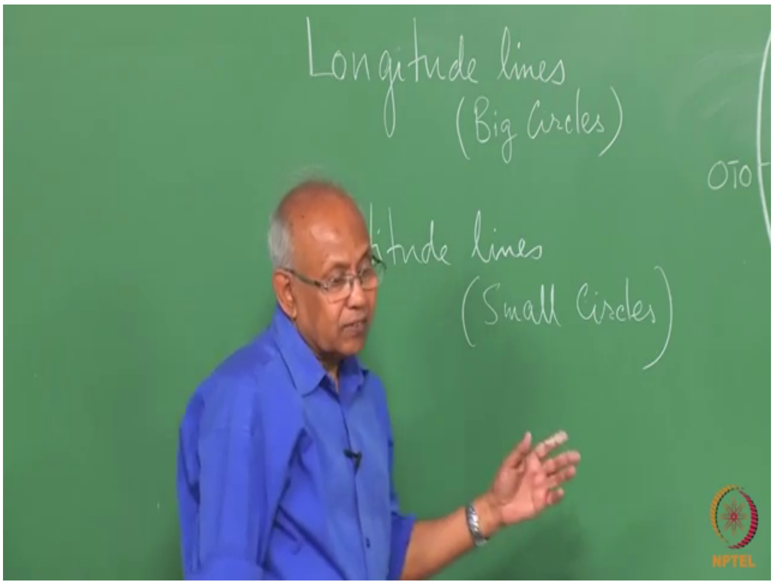

So, we distinguish between the two types of lines the longitude lines and the latitude lines. So, the longitude lines are drawn by connecting the North and South Pole over the surface of the globe. So, these are all which are known as big circles these are the big circles.

Now, the equator also is a big circle because if the size of the equator if you look is same as that of the size of any of those longitude lines. Now what about latitudes lines these are called the small circles and actually the size of the circles decreases as we move away from the equator to the North or from the equator to the South. Now these lines latitudes and longitude lines these are used to measure angle between poles, angle between a plane and another plane angle between a plane and direction, etcetera, etcetera.

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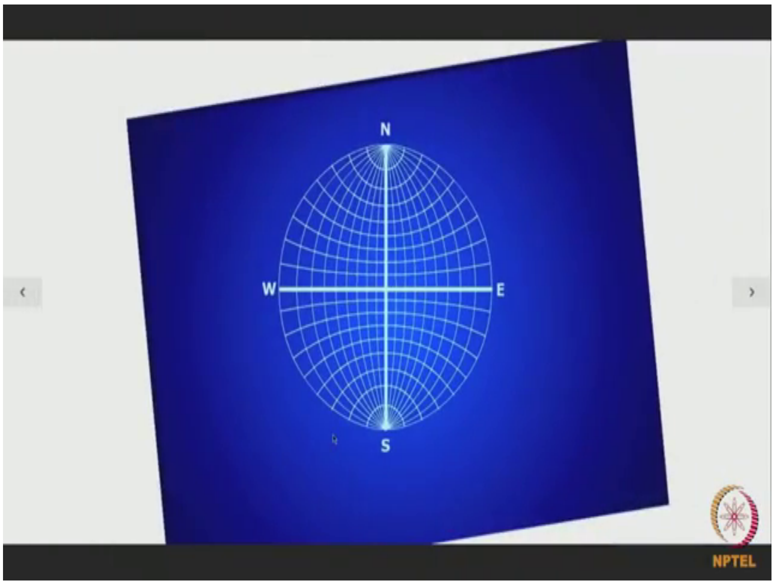

So, in order to do that we use what is known as a Wulff net.

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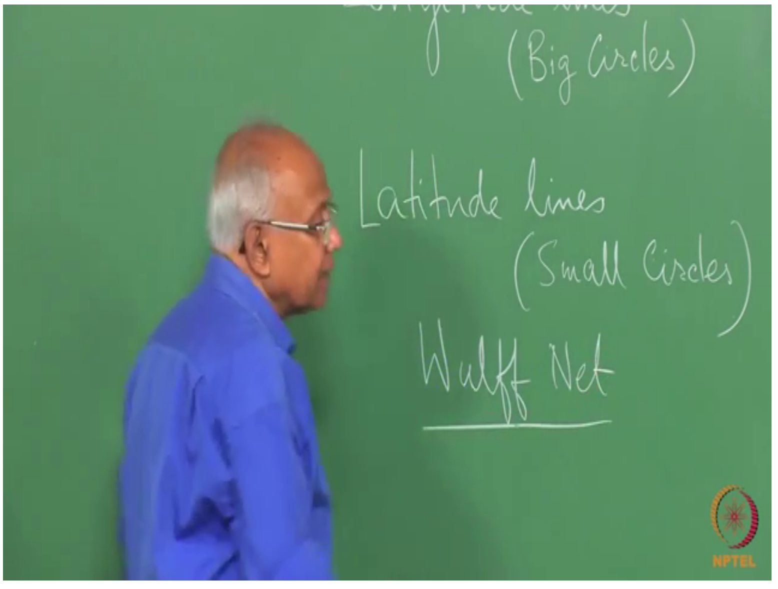

So, we use what is known as a Wulff net. So, what is the Wulff net? The Wulff net is nothing but a projection of the globe on which all the longitude and latitude lines are drawn. So, these are the longitude lines connecting the North and South over the surface of the globe, and these are the latitude lines which are drawn parallel to the equator. Now we can draw the lines at different intervals. So, in bigger Wulff net we can have the longitude and latitude lines drawn at an interval of 2 degrees, in someone's they can be drawn at intervals of 5 degrees or 10 degree according to what we need.

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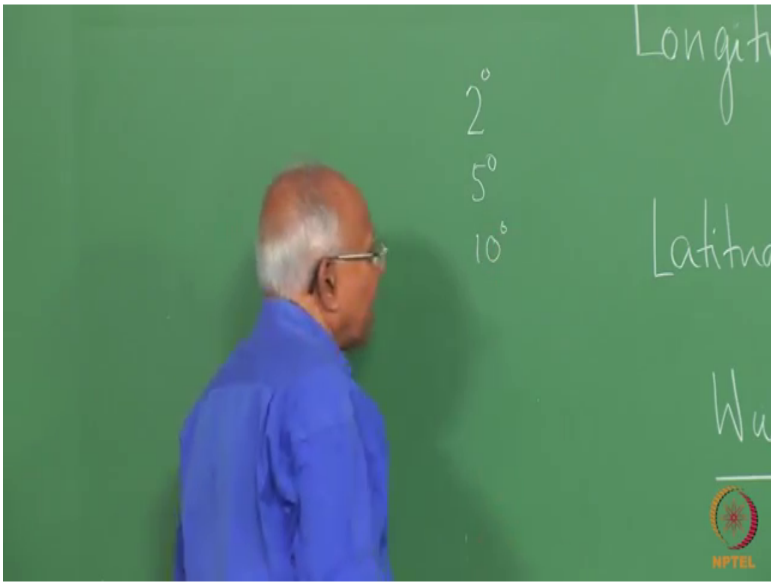

Now, once we have such a Wulff net as we show see red line that it is possible to measure the angle between poles or the angle between planes quite easily.

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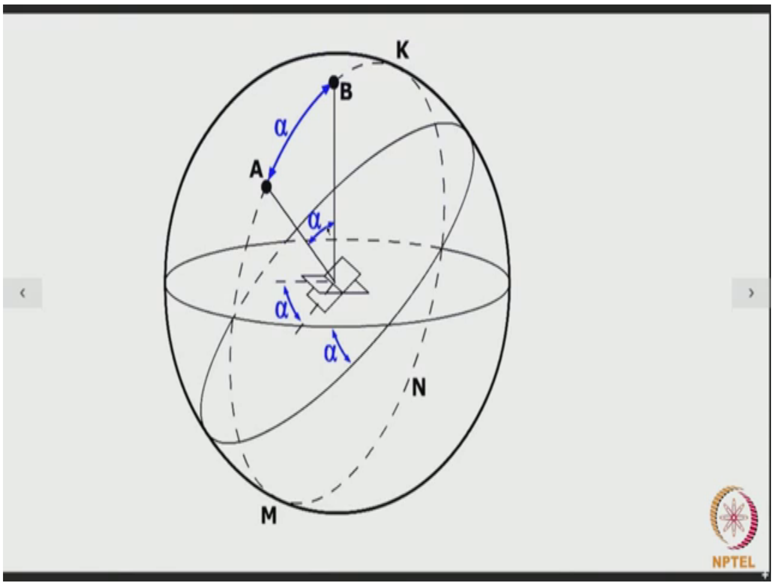

Say for example, if we have got two planes as show over here if we have got two planes and this is the perpendicular to one plane, and this is the perpendicular to the other plane then say this angle is alpha, so it is the actual is the angle between the actual angle between the planes one and two. So, if we extend these perpendiculars they will touch what is known as a longitude line, and as you can see in the longitude line we can easily measure it out. So, we can always measure the angle between two planes say here the two planes are one is this one, this is one of the plane you know it is the extended plane the other plane is this one the extended plane.

So, these two planes make an angle alpha say. So, this is alpha now if we draw perpendiculars to these planes then it will cut a longitude line as shown here. So, this angle is a same as this angle; that means, the angle between two planes can be easily read out from the angle between the two corresponding poles of the place. So, these are A and B are nothing but the two poles of the two planes. So, it is quite possible to measure the actual angle between two planes from their respective poles.

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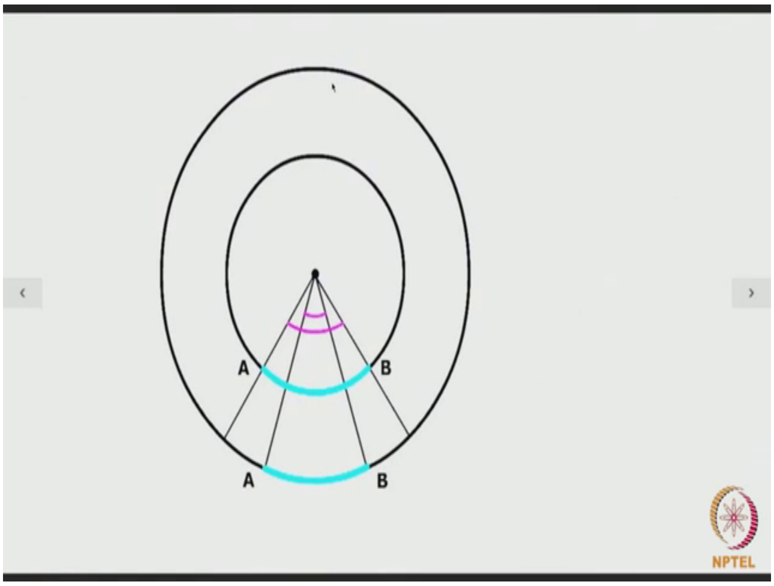

Now, the question may be asked that can we measure angle between two poles using a latitude line. As I have already shown we can do it by taking into consideration a longitude lines say this is a longitude line which is the which is the big circle. So, from here if you can plot two poles of the two planes, we can measure out the angle and that will be the angle between planes.

Now is it possible to use a latitude line to do the same job, now you see that it is quite a different proposition say for example, if we have a distance between two poles A and B this much is a distance, now if we are on this small circle and if on the other hand we are on this big circle then what you find? The same distance between the two poles will give you a much bigger you know will give you know for example, here the angle between these two poles will sustain a much smaller angle when we consider measurement along this line, but at the same time we if we consider this smaller you know line then here this, this distance the same distance will sustain a much larger angle.

You see the latitude lines are not all of the same size, you know they have varying sizes. So, you see that the same distance between two poles can be interpreted as making an much bigger angle when we are on a larger sized latitude lines, then much you know I am sorry the same distance between the two poles will give you a much smaller angle when the measurement is made on a big size latitude line than when the measurements are made on a smaller size latitude lines, that is the reason why we never use a latitude line for measuring the angle between poles.

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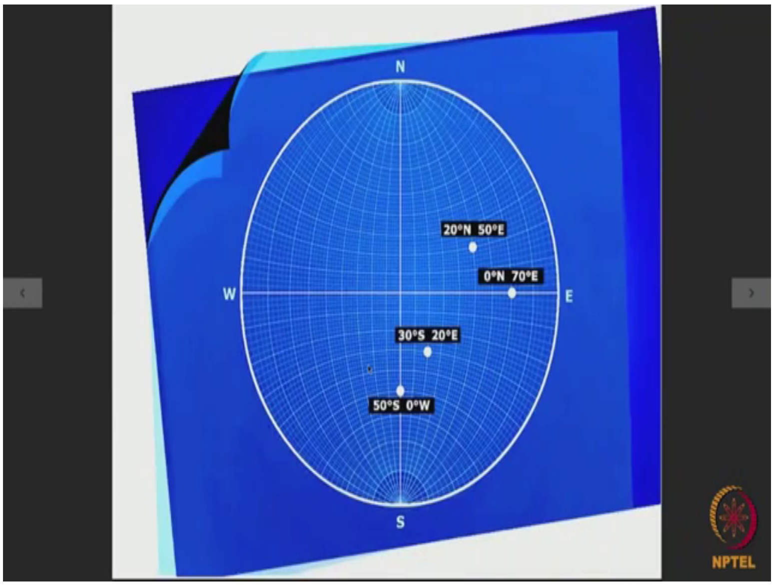

Now, once we have got the Wulff net, we can trace out the entire wulff net on a tracing paper and put it on a top on the net as we have done over here. Now any pole can simply be plotted in this manner say for example, if we want to plot a pole which is 20 degree North 50 degree is East what does it mean? That the pole is such that it lies on the 20 degree North latitude and 50 degree East latitude; you see 50 East longitudes sorry. So, you say- pole which is 20 degree North, 50 degree East, what it means? That means, you know as you go from equivator to the North you will be encountering all the latitude lines. So, when I say 20 degree North means I am on the 20 degree North latitude.

So, if this is 10, 20, 30, 40, 50, 60, 70, 80, 90. So, will be the 20 degree North latitude will be somewhere here, but only one coordinate is good enough we have to give the other coordinate on the longitudinal line. So, if it is a 50 degree East latitude longitude then what it means that we have to go you know this is the long this is the 10 degree East longitude, this is the 20 degree East longitude, 30 degree East longitude, etcetera, etcetera and this is the and this is the 90 degree East longitude.

So, this point 20 degree North 50 degree East we can easily locate by moving 20 degree along the latitude line on the 20 degrees North latitude line, and then move to the 50 degrees East point we comes over here. Similarly if we have a pole as 0 degrees North 70 degrees East, then how we plot it? We go you know this is 0 degrees North or 0 degrees South; that means, the right at the centre and then you are we are here. So, 0 degree North means it is 0 degree North means right at the centre of the Wulff net, then seventeen degree East. So, you go along the equator by 70 degrees. So, you are here.

So, this is a 0 degree North 70 degree East pole. In a similar way we can plot the pole 30 degree South 20 degree East. So, you can go to 30 degree South latitude and then move to 20 degree East longitude and you come over here similar way if we want to plot the pole 50 degree South 0 degree West we simply come from the centre to 50 degree South and do not go anywhere either to the right or the left and we get the pole 50 degree South 0 degree West.

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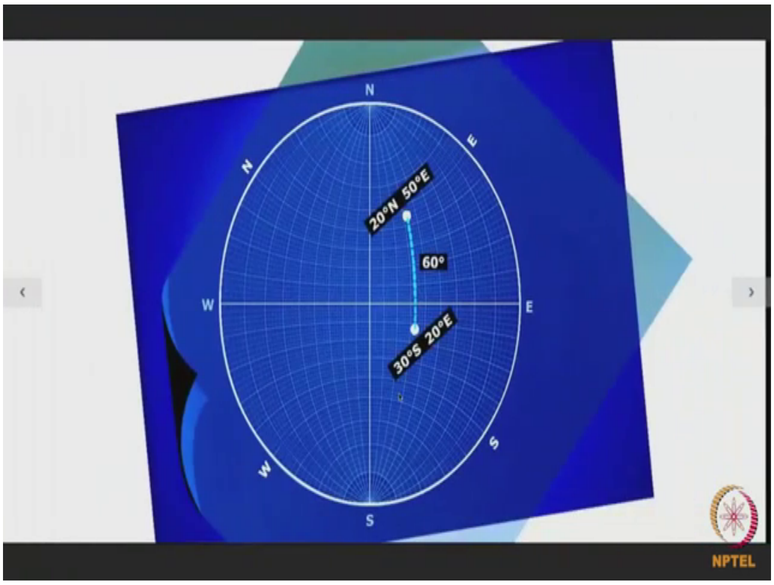

Now, suppose we want to find out the angle between two poles 20 degree North, 50 degree East and 30 degree South 20 East as we have shown here, you see we want to find out the angle between this pole and this pole. So, how we can measure the angle between two poles we know that if we can bring the two poles of the same longitude line then we can measure out the angle straight away. So, that is exactly what will do. So, what will do for the purpose? You see all this poles are plotted on a tracing paper and this is put on top of the fixed Wulff net.

So, now we what we do once you have got these two poles we will rotate the tracing paper with respect to the Wulff net. Once we do that it is quite possible to bring the two poles lying on the same longitude line and then we can simply measure out the angle from the graduated Wulff net which is 60 degree. So, you see that it is quite easy to figure out the angle between two poles by taking both the poles on the same longitude line and then simply measuring out according to the Wulff net. So, angle between the poles 20 degree North 50 degree East and 30 degree South 20 degree East is 60 degrees as we see here.

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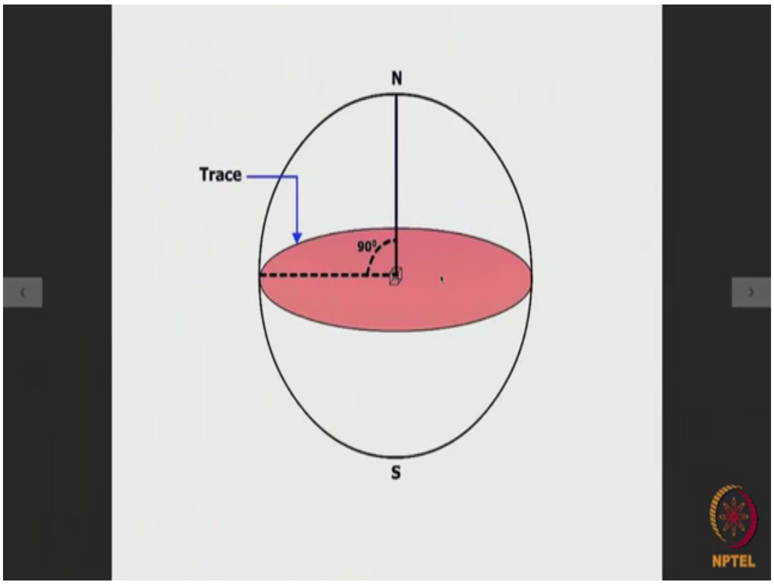

Now, very often we come across a torque called a trace of a plane. So, what is the trace of a plane? So, if you have a plane in the unit cell and allow the plane to extend in space it will cut the reference sphere you know along a circle like that. So, this is the trace of a plane, and if we look at the trace this is the trace of the top plane here that is the 001 plane. So, this is the trace of the 001 plane; that means, if we take the 001 plane and allow it extends this is where it will made the reference sphere and so it is the trace of the plane.

Now if we take the perpendicular to the 001 plane it will go and meet the point North over here; that means this will be the pole of the plane. So, if this is trace of the plane this will be the pole of the plane. So, if we have you know the trace of a plane and the pole of the plane they can make an angle 90 degree between them. So, this is a very important relationship.

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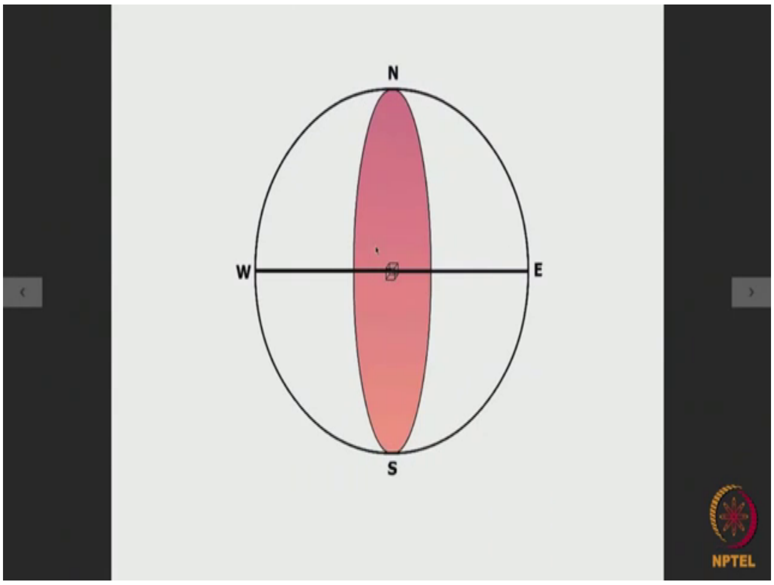

Now, if we have another plane say for example, the 010 the right hand side plane. So, this will be the trace and you know the pole can be either this or this. So, again the relationship is the trace of a plane and the pole of a plane make 90 degree angle with each other.

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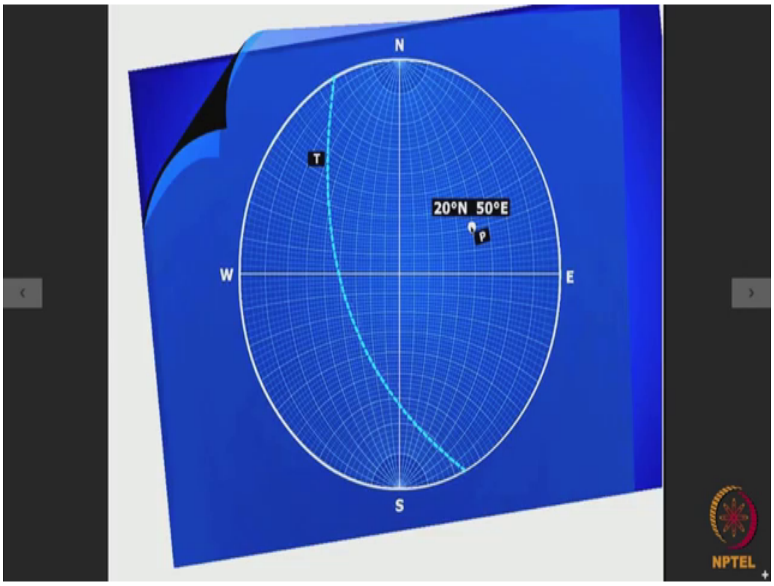

Now, let us suppose that this is the trace of a plane as we find you know as we plot on the tracing paper, and then put it on the top of the Wulff net. Now say we have to find out the plane. So, what we do you know what is the our next step. So, we know that the pole will be at 90 degree to the trace. So, first of all what to do we rotate the tracing paper on which the trace has been drawn with respect to the underline (Refer Time: 46:30) so that the trace lies on a longitude line. So, this is what is show in the next figure.

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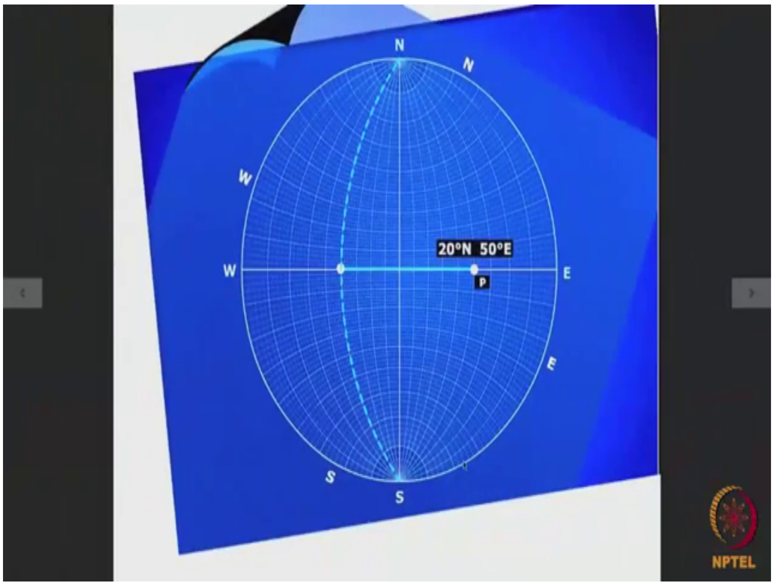

So, there you see that we rotate the tracing paper with the trace you know so that it lies on a longitude lines, and then simply go 90 degree along the equator; so 10, 20, 30 40, 50, 60, 70, 80, 90. So, you see that this is the pole of the trace, so the trace of a plane which is this, you know the pole of the plane is over here. So, now, we have to get back to the original position of the trace. So, that is what we do.

So, we go back to the original position by changing the by rotating the tracing paper with respect the Wulff net, and the pole moves over here we read out the location it will be 1, 2, 3, 4, 5 50 degrees East and 20 degrees North. So, if you have a trace of a plane you can easily find out the pole of the plane in this manner. So, what are the steps involved? You have the trace first you want to if you want to find out the pole, the next step will be to rotate the tracing paper on which the trace has been drawn in such a manner with respect to the Wulff net so that it coincides with a longitude line. So, this is the situation then you go 90 degree away from the trace and you find out the pole, and then move back again to take the trace to the original position the poles travels over here and you read out the coordinates which is 20 degrees North 50 degrees East.

Again another problem may arise that we have been given the pole of a plane we have to figure out what is the trace of the plane. So, how we are going to do that? Again what we do the tracing paper on which the pole is plotted is rotated in such a manner that it comes on the equator line. So, the next figure shows that. So, you have brought it to the equator and then move 90 degree away from this 2, 3, 4, 5, 6, 7, 8, 9.

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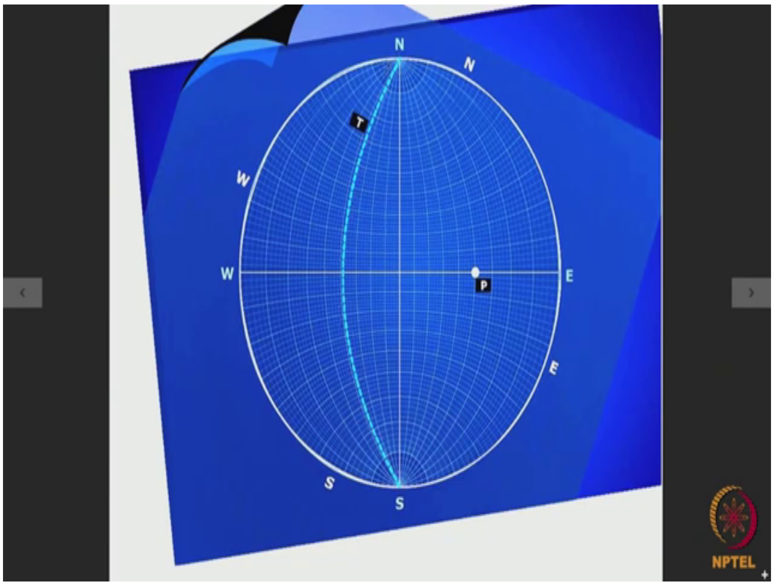

So, this and draw the longitude line through this point and we get the trace of the place. So, we do exactly the opposite what we did earlier in order to plot the trace of a plane from its pole.

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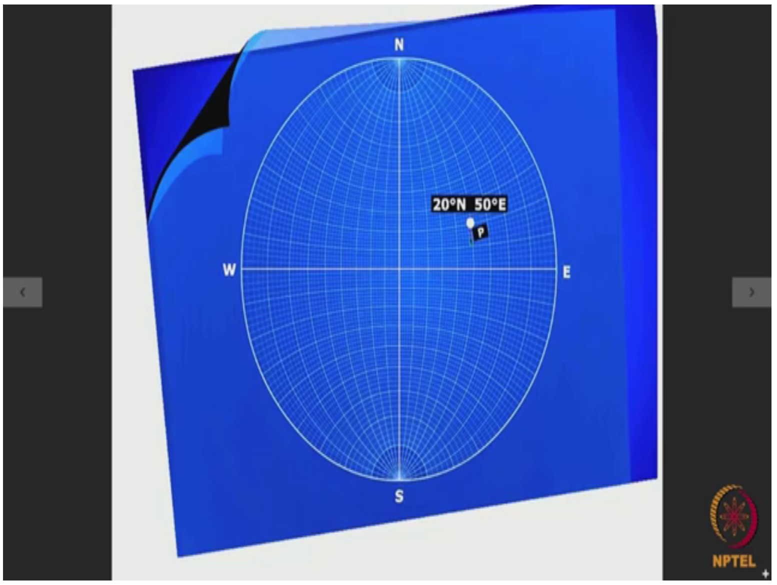

So, what we do we simply rotate the tracing paper in the pole on it such that it comes on the equator on the equator line. So, this is what we have done and then along the equator move 90 degree and draw the longitude line as shown here and then the next step is go back to the original position. So, we go back to the original position, if this is the pole of the plane this is the trace of the plane.

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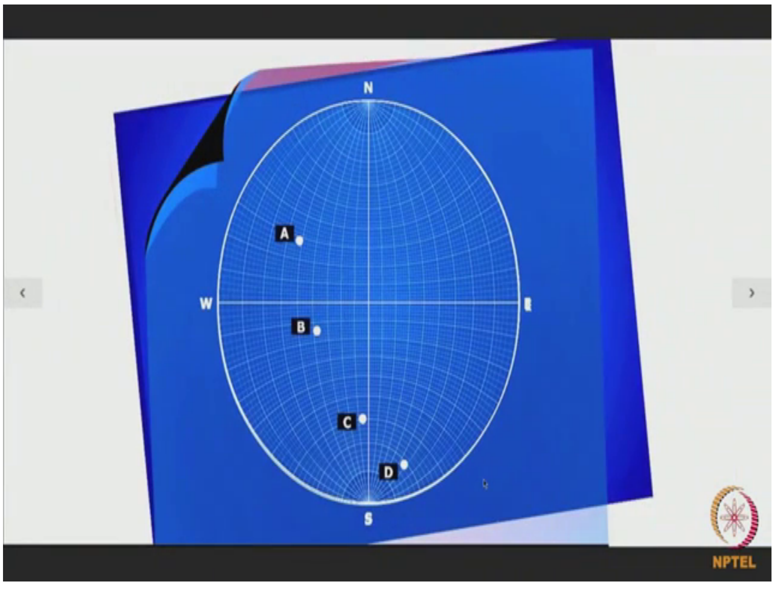

You see several times we find that we come across a number of planes which belong to a particular zone say for example, on a tracing paper we have plotted out four poles, and we can easily check whether they lie on a zone and how to check that? Again we rotate the tracing paper on which the poles have been drawn with respect to the underline Wulff net.

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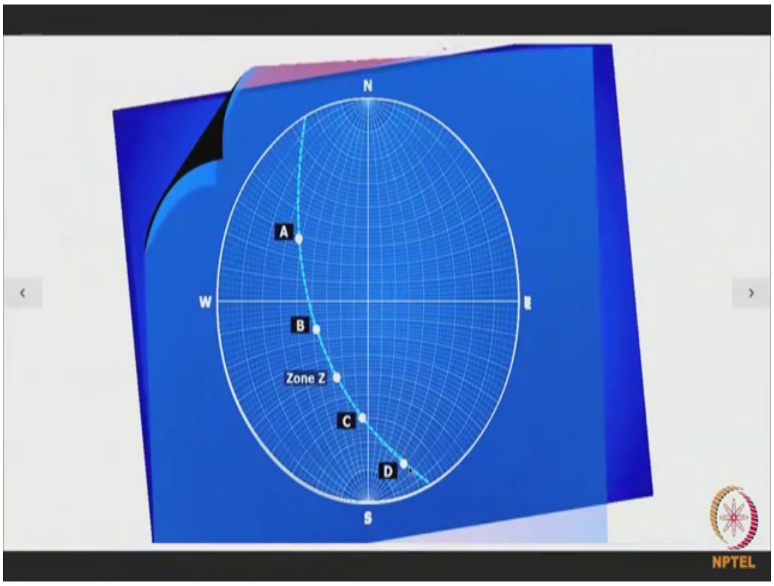

And once we do that you know you find there will be a position you know they are you know when you do that you will find they will be lying on a particular longitude line.

If it is so, then we can easily say that the poles A B C and D they do lie on a zone, if we cannot bring all the poles on a single longitude line then they do not lie on the same zone. Now once we have got the zone of planes how to figure out where the zone axis will be again it is quite simple, we rotate the tracing paper with this longitude line in such a manner that they coincide the end coincide with North and South we do that and then we move 90 degree away from this along the equator and find the zone axis.

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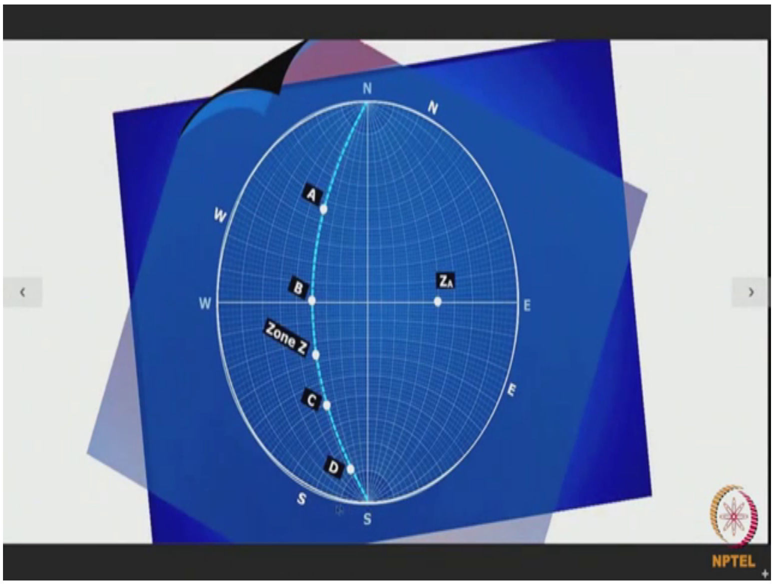

So, in this way if you are given a number of poles you know we can figure out whether the lie on the same zone, if you find that the all the poles can be brought on a single longitude lines then that will show that all those poles are all those planes belong to a particular zone, and one that is found out you simply move along the equator by 90 degrees to find out the zone axis.

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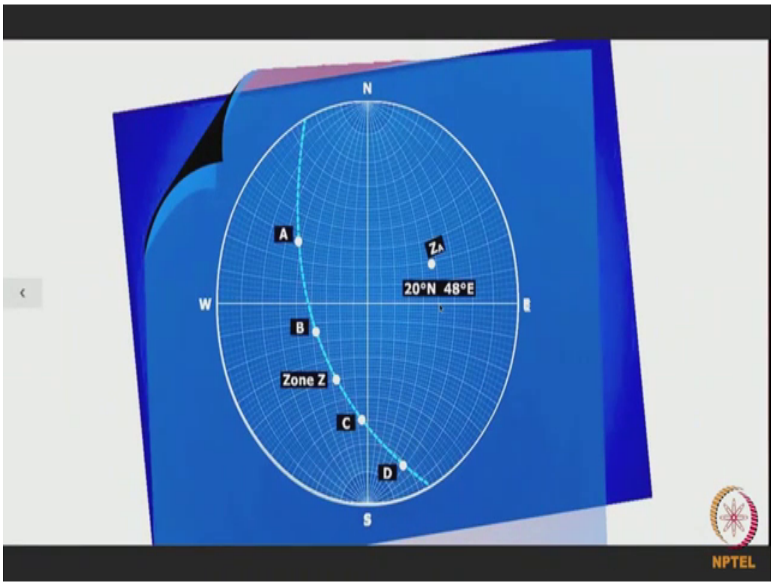

In a similar manner it is also quite possible to figure out the zone from the zone axis.