Theory of Mechanism

Lecture 10

Function generation using relative poles

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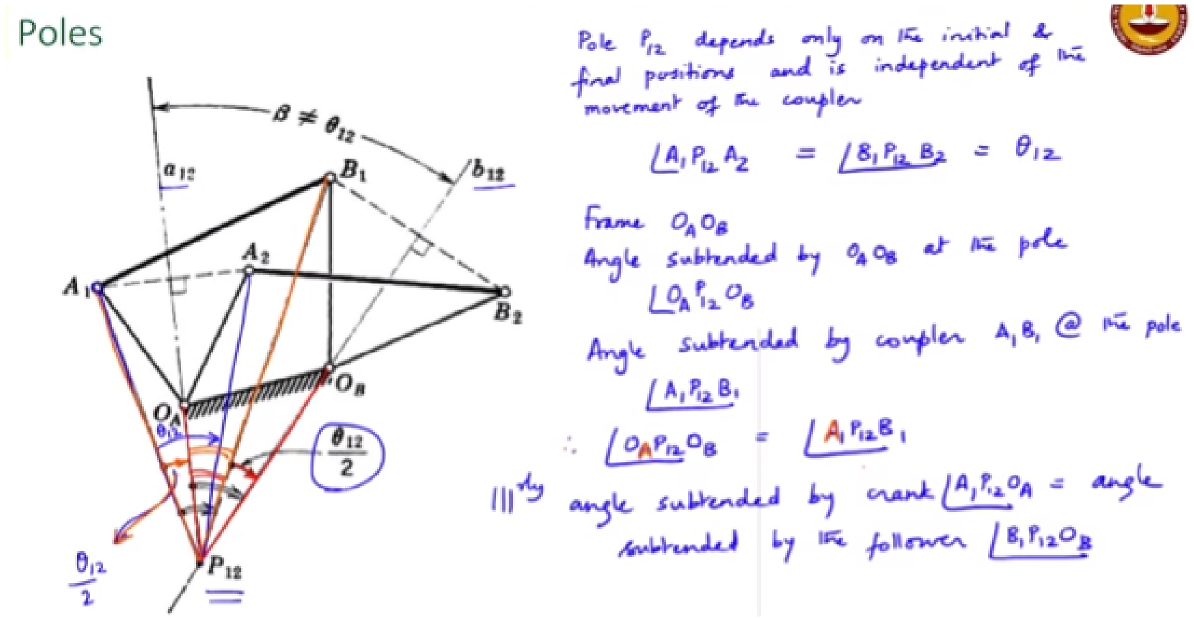

Today we look at the concept of poles and relative poles and see how we can use that for the synthesis of function generation mechanisms, okay. so we saw earlier so if you look at two positions of the coupler, a1, b1 and a2, b2, right? Say if I have a 4 bar, then if I look at, so this is the perpendicular bisector of a1, a2, b1, 2, is the perpendicular bisector of b1, b2 and we saw that the two perpendicular bisectors intersect at a point, which we call the pole, okay. and what the pole essentially tells us? is that I can move pole is the point on the coupler, that does not change, when you move it from when you move the coupler from position 1 to position 2, or in other words a1, b1 will move to a2, b2 about a pure rotation, with the pure rotation about p1, 2, okay. So this pole p1, 2 depends only on the initial and final positions, it's independent of what happens in between, ok so, the pole is pole p1,2, depends only on the initial and final positions and is independent of the actual movement of the coupler, of the coupler. So now, if the pole, means that okay I can move from A1, B1 to a2, B2, by a pure rotation, then so I have when a1, B1, moves to a2, B2, the rigid body is rotating about an angle, theta 1, 2 ,okay? So if I take any line on this rigid body and move it to the new position and this line okay, so essentially P1 to a1 is a line on the rigid body b1, a1, b1, 2 right if I look at that. so then this has moved through an angle it has rotated about the pole, by an angle theta 1 to 2, theta 1, 2, okay. So then if I look at this angle which is the angle to the perpendicular bisector, right so that angle is half of the total angle that the rigid body rotates by, ok. So a1, b1 also I can say has rotated by the same angle, any point on any line on that rigid body, would have rotated by that same angle, when it goes from position 1 to position 2.

So this is Theta 1, 2 by 2. So here you have so if I look at these two triangles, I have a1, P12 to a2, that angle is the same as the angle that B1, P12, B2 makes and this is Theta 12. And therefore this angle or this angle B1, P12 to the perpendicular bisector the angle which it makes with the perpendicular bisector, is Theta 12 by 2, okay. so now if I look at the frame, okay, frame, Oa,OB angle subtended, by it at the pole, is Oa this angle, Oa, p12, OB, that's the angle I want to look at, okay. So if I do that, I have this is what I am looking at, angle subtended by the frame at the pole. here I have this angle is Theta 12 by 2 okay. and then I have this part of the angle okay, I have this angle that I want to determine or so it's this angle plus theta 12 by 2 is the angle subtended by the frame at the pole. Similarly if I look at, angle subtended by the coupler, a1, B1 at the pole that angle is a1, P12, B1 okay, which is a1, P1 to b1. So again here I have this angle is Theta 12 by 2 and then I also have this angle, which is the same as the previous angle the red and the orange, those two angles that are the same. So I'm adding theta 12 by 2 to both. so this means Oa, p12, OB, equals a1,B12, B1, ok? You have to make sure that you follow the same order, if this is away this should be a1, you're measuring from this line, to that, same order okay. Oa, p12, OB this should not be B1, B12 a1 okay then the sine will change, follow the same order. So this these two angles are the same. So essentially the angle subtended by the frame at the pole is equal to the angle subtended by the coupler at the pole and this is a fact that is quite useful for dimensional synthesis. That we will use to similarly if you look at the angle subtended by the crank, similarly angle subtended by crank, which is a1, P12, Oa, equal to angle subtended by the follower angle B1, P12, OB, ok. So be careful in measuring the angle you have to if his is A this should be A, ok not the other way round. So you will see that in the next one when we, when they sometimes, so it would either be equal or will differ by 180, as you will see in the next one. So this is the property we will use, then we do the synthesis.

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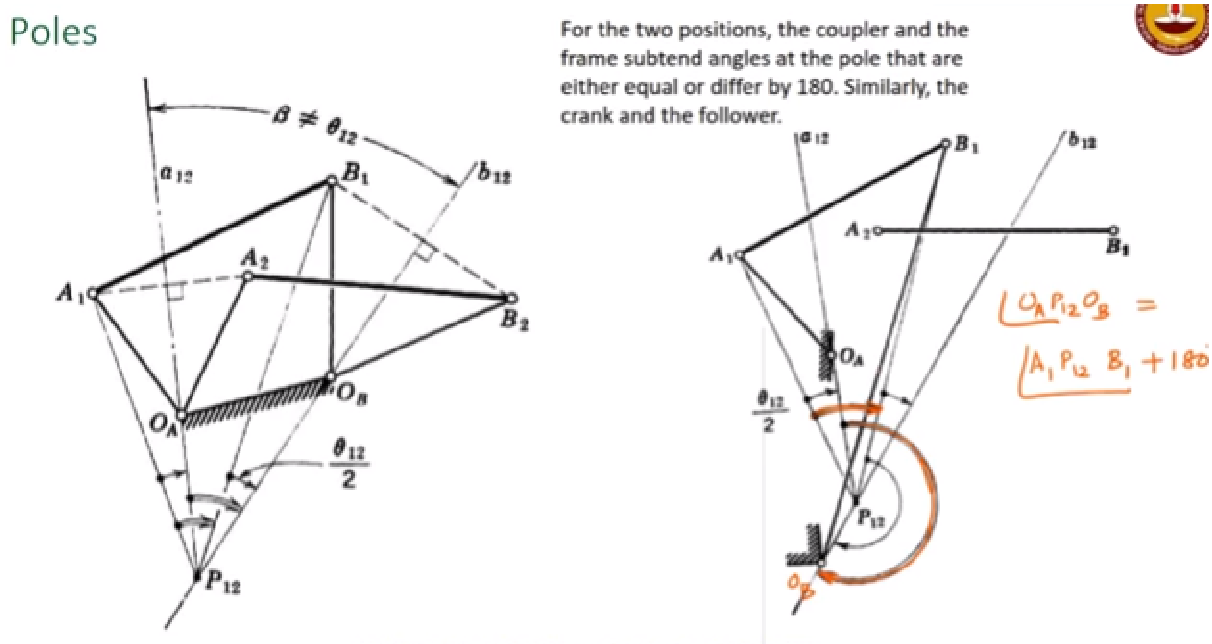

so if you see here, if you measure the angle subtended by the frame at the pole, how would you measure it? If you take it as Oa, p12, this is OB, then OA, p12, OB, would be this angle here, okay and you compare with A1, P12, B1, A1, P12, B1, so this is A1 to P12 B1, which is this angle, okay. So you can see that these two angles differ by 180. Okay, so this equals this plus 180. okay so when the poles on the when the pivot is on the other side of the pole you'll find that they differ by 180 when they are on the same side they are equal, okay in this case. so now because this angle is equal to this angle, theta-12 by 2. So this is what we will use for the dimensional synthesis, and we can also define a relative pole, which would be between the two moving links, which are the crank and the follower. So between the coupler and the frame, that point is the pole, to relate the motion of the crank and the follower, we call that point the, relative pole. we define points similar to that called the relative pole. Ok.

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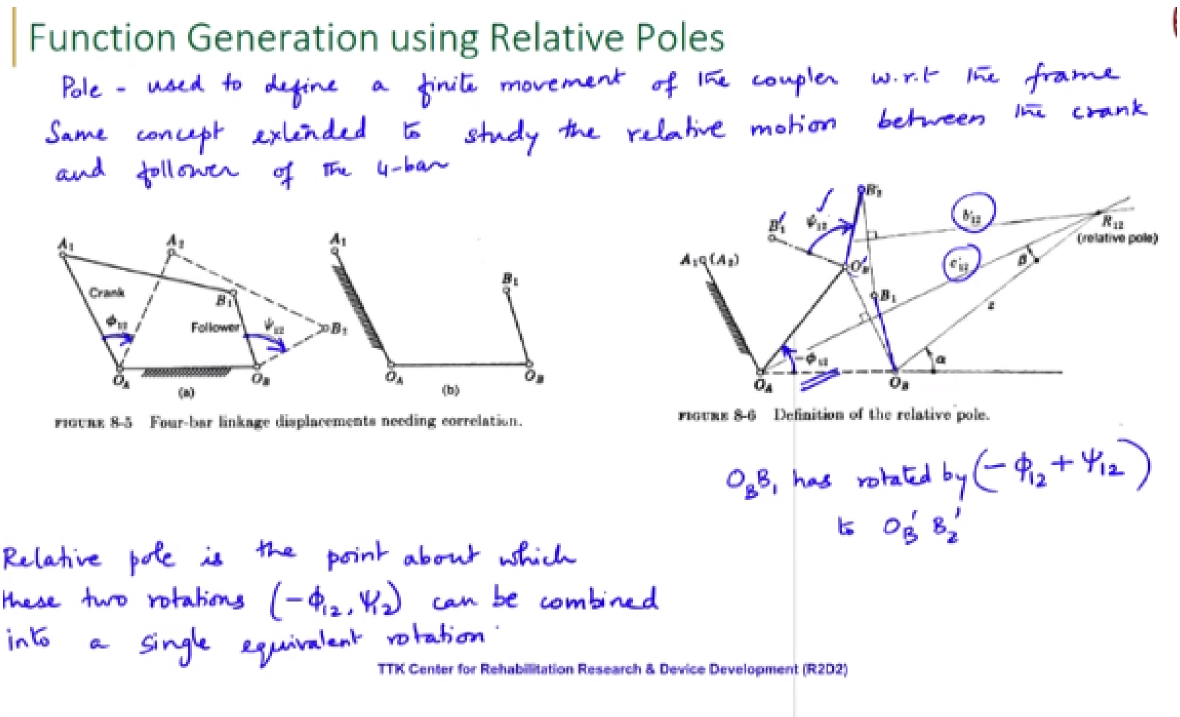

So let's look at the definition of the relative pole. So here you have a function generation, a linkage where the crank rotation, of PHI 12, results in a follower rotation of SHI 12. Okay this is your these are the linkage displacements that need correlation. So you need to design a function generation linkage, for to accomplish just goal. You have a question in the previous one? They are different, the pole relates the coupler with the fixed link, the relative pole relates the crank and the follower. So the four bar you have the four links, right? Coupler with respect to the fixed link is what we call the pole relative pole is, relates the crank with the follower. So let me just, pole is, use to define a finite movement of the coupler, with respect to the frame. Same concept, extended to study the relative motion between the, crank and the follower of the 4-bar. So let's first look at again to understand this, we look at inversion. Okay so this is the four bar linkage you want, which correlates a crank displacement of Phi 12, with a follower displacement of SHI12, now instead I make the crank the fixed link. Okay so I sit on the crank, OA, A1, becomes my fixed link and what did we do? we looked at so if you look here now, by inversion once I sit here, this OA, OB, B1, okay, first rotates by minus phi 12, okay I rotate by the angle minus Phi 12, to bring the frame closed in the second position and then, so this is OB dash and initially B1 if I move this as a rigid body, it goes to B1 dash, but B1 Dash should actually OB, B1 dash should actually have moved the SHI 12, in the second position. So this becomes my B2 dash, this is what we did last class, with the inversion. okay so B1 moves to B2 Dash, right and if you look at this thing, now this is my fixed link, I can look at the displacement of the follower as, moving from OB, b1 to OB dash, B2 dash, inversion again, I'm just so this now becomes the coupler, essentially for this link, I've moved from OB, b1 to OB dash, B1 dash. So if I find the perpendicular bisector, so OB has moved to OB dash, again we want to find the point, about which this motion has happened as a pure rotation, right so OB, OB dash, perpendicular bisector, which is this one, then B1, B2 dash perpendicular bisector, this one where they intersect, this point I call it the relative pole, because it's basically the pole for this linkage, when the crank is the fixed link. the only reason we call it the relative pole is in my original linkage, it's not the fixed link. Okay it's the same, same concept OB, B1 moves tube so these two rotations can be combined to a single rotation about this relative pole, the two rotations which are the two rotations? Minus Phi 12, this one and SHI

12, this one. So if I look at OB, B1, first it's moved by minus Phi 12 and then by SHI 12, okay? Think about it initially as, welded like that, I move that by minus Phi 12 and then I remove that weld and then move that alone, OB, B1, move it relative to OA, OB dash, by SHI 12. So if I look at the net rotation of this body OB, B1, it is in the plain, in it is, minus Phi 12, plus SHI 12. So that is my theta 12 for OB, B1. OB, B1 has rotated by this angle, yes rotated by this to OB dash, B2 dash, initial position final position about this point the total rotation is minus Phi 12, plus SHI 12. So that is the relative pole is the point about which this rotation is happening this net rotation, okay so relative pole, is the point about which, these two rotations minus Phi 12, SHI 12, can be combined into a single equivalent rotation. In the planar case it's very easy, you're just adding them algebraically, the two rotations for this rigid body, okay. So now let's see how we can use this for Synthesis. We will develop some relations between these various angles and then use that to for synthesis, okay.

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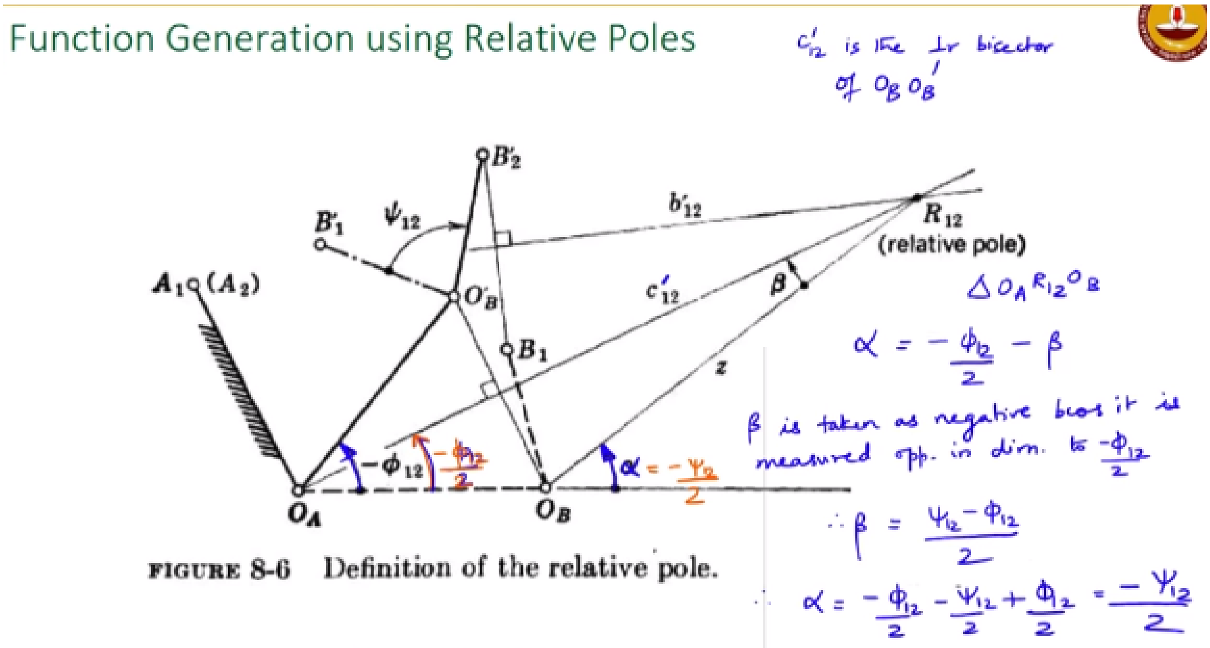

FIGURE 8-6 Definition of the relative pole.

So let's, let me call this angle alpha here, the angle which, so I have the relative pole, OB to the relative pole, the line from OB to the relative pole, let's says it makes an angle alpha. Okay this I know is minus Phi 12, this angle here is minus Phi 12 and OB, B dash, this line here C dash 12, is the perpendicular bisector of which line OB, OB dash, okay. so this angle here, is going to be half of minus Phi 12, this angle is that now beta, okay if I look at the rotation of OB, B1 to OB dash, B2 dash, so I'm a, this is there are half the rotation, from the first position to the second position. So if I said, if you have theta 12, which is the rotation of OB, B1 to OB dash, B2. Okay because R12, OB would have rotated to R12, OB dash, by a certain angle and half of that angle is beta, in this direction, I'm moving from position 1 to position 2 Yes, So now if I look at this triangle, in triangle OA, R12, OB, alpha is an external angle, okay. So I can write alpha equal to minus Phi 12 by 2 minus beta, because I have taken beta in this direction which is from position 1 to position 2 ok because I take beta as the angle of rotation from position 1 to position 2, its opposite its measured opposite to the direction of minus Phi 12 so it is minus beta, ok. So beta is negative, is taken as negative because it is measured opposite in direction to PHI 12 sorry, minus PHI 12 right. okay so that means alpha is this now I already showed you that the rotation, beta is essentially half the angle, that any line on this has rotated by when this is the coupler OB,B1 rotates to OB dash, B2 dash, half which is we saw in the previous slide that angle is SHI 12, minus Phi 12, that was the angle that OB, B1 rotates to OB dash, B2 dash, therefore beta is, half of this angle okay, because the net rotation of OB, B1 to get to OB dash, B2 dash, is SHI 12, minus Phi 12, minus Phi 12 plus SHI 12, as a showed. So this is beta is the angle with that perpendicular bisector, so it is half that angle, okay. so now if I substitute I get alpha equal to minus Phi 12 by 2, minus SHI 12 by 2, plus Phi 12 by 2, minus SHI 12 by 2. So if Phi 12 is clockwise, minus Phi 12 will be counter clockwise, if SHI 12, is counter clockwise, minus SHI 12 would be clockwise. So you would take the direction based on what the original required rotations are, okay. So if you look at the relative Pole now, what is the relative Pole? if I have the frame OA, OB, I can directly find the relative pole, knowing my design criteria, Phi 12 and SHI 12, because alpha I have found is nothing but minus SHI 12 by 2, so I can draw a line from OB, at minus SHI 12 by 2, I can draw a line from OA, at an angle of minus Phi 12 by 2 and where those two lines intersect, gives me the location of the relative Pole. Okay let's do a synthesis then it will be clear.

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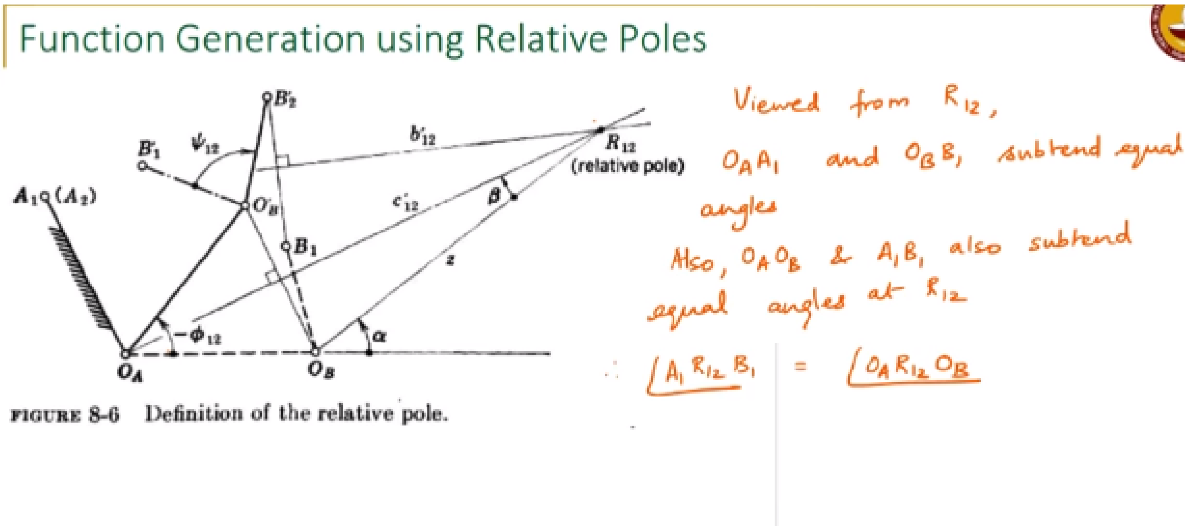

One more thing if you look here, because this is just the same as the initial case with the pole where you looked at the coupler and the fixed link, right. So now what do you know this the angle that OB, B1, makes at the pole, ok or sorry, the angle that OA, OB, makes at the pole again is going to be the same as the angle that a1, b1 makes at the pole, at the relative Pole. We saw earlier because that becomes the crank and the follower for mechanism. We saw earlier with the poles that the crank and the follower also make, so if you see here at the pole, this angle made by the crank, is the same as the angle made by the follower, okay. So, so what we will use is the fact that OA, OB makes a certain angle, at the relative pole and that is going to be the same angle made by the coupler a1, b1, at that relative pole. Okay so viewed from R12, OA, A1 and OB B1, subtend equal angles, also it's the same with the frame and the coupler, OA, OB and A1, B1 also subtend equal angles at R12. Therefore we will use angle A1, R12, B1 equal to angle OA, R12, OB. So now if I know OA, OB and I know Phi 12 and SHI 12, I can find the relative pole, okay I can assume some A1 for the crank length same as what we did for inversion okay I need one and I need to find B1 and how can I find B1? If I know this angle once I find the relative pole I know the angle subtended by OA, OB, at that point. B1 should be such that A1, B1, subtends that same angle at that relative point okay. So that's how we will use it for the Synthesis.

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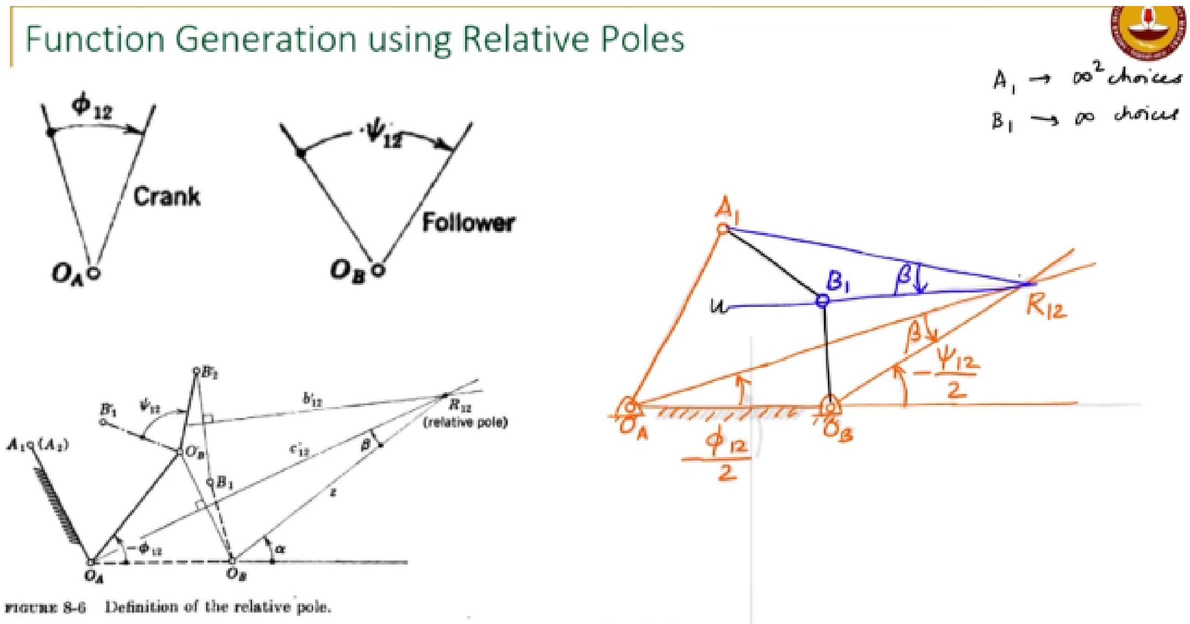

So I am given OA, OB okay and I know this alpha is, so I want to find the relative pole, I want to coordinate these two motions, motion of Phi 12 on the crank, should result in a motion of SHI 12, of the follower okay. So to do that I have my OA, OB picked out, so I draw a line from OA, at an angle what will this angle be if this is clockwise so this this is the angle minus Phi 12 by 2, okay. Then from this point OB, I draw a line at an angle to the frame of minus SHI 12, by 2, where those two points intersect is my relative pole R12, okay. So now, I pick some point, so some crank length, A1, so I pick A1 is a choice I make, I am not used it so far, I only worked with the frame, right I found this angle so this is the angle Subtended, let's let us call this angle beta the angle subtended by the frame which is OA, OB, at the relative pole, that's my beta, if I measure from, OA, R12, say I know A, that is why I'm measuring it in this direction now, because now I have to from A, I connect A to from A, I connect to R12, okay I know that the angle that A1, B1 will subtend at the relative pole, should be the same beta, same direction when I measure from A ok from A to B, should so this line, I construct at an angle beta, if I pick any point on this line my A1, B1 will subtend the same angle at the relative pole. So I have an infinity of choices for B1, I can pick B1, any place on this line to complete my linkage. So that's how I design my follower length and my coupler length. Now this linkage if I move the crank by Phi 1 the follower will move by SHI 1, Okay. Any questions? A1 was also a free choice or it may have been specified because of other considerations, but essentially I have a A1 have infinity square choices it could be anywhere on the plane, then B1, I have how many choices? I have an infinity of choices because it has to be somewhere on this line U, which is at an angle beta to the line joining A1, R2. Any questions? so this is the way you would use the relative pole to design a function generation mechanism, you can do the same thing this is a two position function generation, that's the solution.

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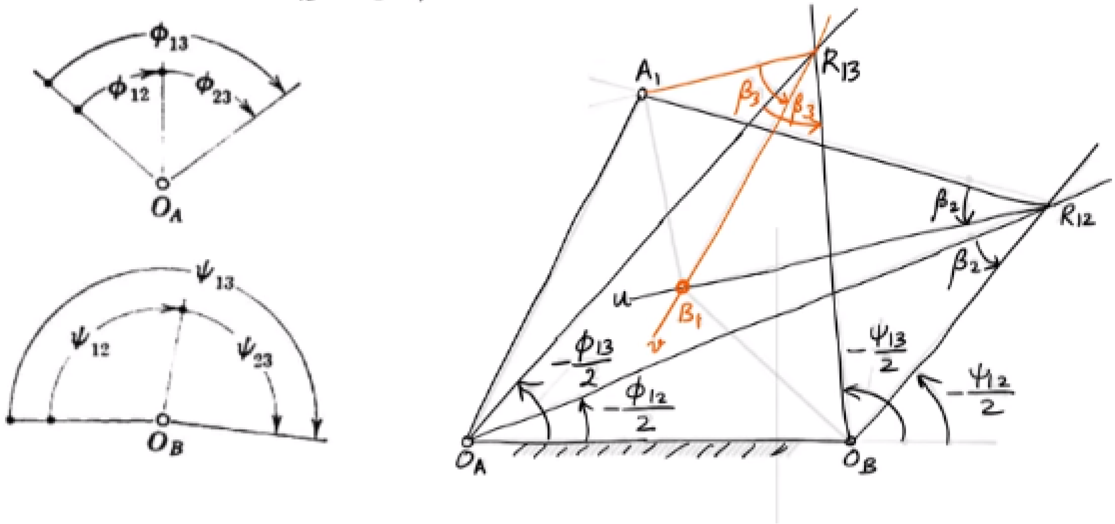
Function Generation using Relative Poles

$$\phi_{12} = \phi_2 - \phi_1$$



I can use it for three position function Generation, I have here the crank has to move Phi 12 and then 2 to 3, Phi 23 so I can get Phi 13 so Phi 12 is essentially Phi 2 minus Phi 1, right that's the definition 1 from 1 to 2, okay similarly Phi 12, Phi 13, I have this So if I pick my OA, OB, what do I do first, and locate which relative pole I can locate R12, so what would this angle be? Minus Phi 12 by 2, then from OB okay so, that gives me relative pole 1,2, okay. So I measured this angle, beta from 1 to 2. so let us call that, then I choose A1 I connect A1 to R12, measure of that same angle beta 2, in this direction same angle and this line, so my B should lie somewhere on this line, but now because I have an additional condition I cannot pick B anywhere on this line, it has to also satisfy the conditions for position 1 and 3, okay. So to do that, I construct the second relative pole, which is at an angle minus Phi 13 by 2, this line intersection of that and what should this angle be? Minus Shi 13 by 2. So this gives me R 13 and if I look at what is the angle made by OA, R 13 with R 13 OB then I get this angle is beta 3. So I join A1, R13, take the same angle beta 3, to construct another line which will intersect the first line at the point what point is this? It's B1, okay. So this is using relative poles instead of inversion using relative poles to construct a function generation mechanism.