Lecture 13- Theory of Mechanism

Analytical Linkage Synthesis-I:

Vector Loop Closure

Freudenstein's method

So, we looked at the representation of vectors, using complex numbers and that is something we will use extensively, in this course both for synthesis and analysis. Some of you are already familiar with the analysis part using complex numbers; here we will use it for synthesis.

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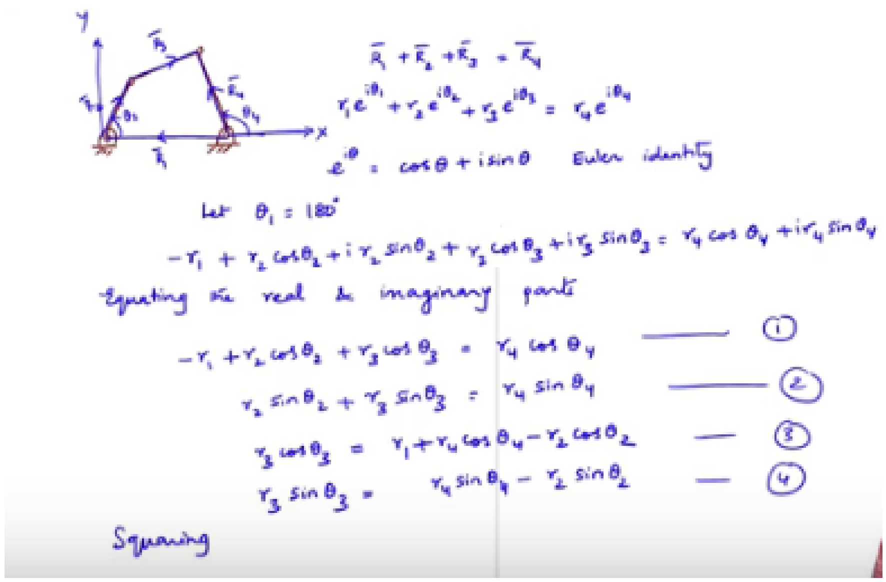

So essentially in a four-bar linkage, you would represent the links as vectors. So for instance, this yes, yes, yes so I found some but, I'm still not completely clear on that. One thing, he asked about Maxima in between ,that is not possible, so that is not possible because, see are assuming, a linear relationship between the output and the function, so if the function has a maxima in between, it means you're going to an extreme position and then return. Okay? So that ,we don't do synthesis for those kinds of conditions using this method, the a bishop polynomial, essentially tries to, so if you have the error function, if you draw the error, structural error, then what it does is. Okay? So let's just take, say three precision points, so you have error here, here, here and here. Okay?

What the polynomial tries to do is to basically, make the errors in between the precision points, equal to the errors at the ends. Okay? So some of these errors between the precision points becomes equal, so that is how you minimize and then if you if their choice of precision points, is such that, we call this a first step because, if this is big. Okay? You would have to try to bring these two precision points closer, so that overall at the expense of some, so you can think of this, as like a wire that you know of constant, so that if you lower it somewhere, it's going to increase somewhere else. Okay? So that's the best thing that you can do with this, this is what you are trying to do with this Chevy chef spacing. Okay? So the Maxima that you were talking about, that we cannot, you have to choose the range of the function such that, there are no math maxima or minima in between. Because you're taking your output to be proportional to the function, so which means, you're you know going to an extreme then coming back and that's not how we, how we can synthesize for the function generator. Okay? I would have to talk to somebody in, apparently the Chevy chef polynomial is used a lot in this kind of error approximation sort of applications, but I'd have to talk to somebody in the math department, to really find out where it applies, where it doesn't I looked up some things and some of it was not my cup of tea. So I just thought. Okay? But as far as application goes, this is what it's trying to do essentially, so if you have three precision points, it's trying to, the errors between the precision points that, so this in this case it's very easy, to see this Plus this, this

Plus this, should be equal to this Plus this. That's what you're trying to do essentially. Okay? So the negative error, so overall it gets minimized, that's what, the roots of the Chevy chef polynomial give you these precision points, the spacing such that this error is minimized in this fashion. But the range you choose, such that it's not, you know it goes back and forth, then again you have a problem, because your see you can always assume, a nonlinear relationship between your output variable and e then you can do other things, for the simplified case where we've assumed a real, linear relationship between the output variable and the angle, you can't have that kind of a, it has to be like monotonically increasing or decreasing function. Okay? So coming back to the vector loop equation, so you can express a 4-bar, in terms of vectors, so this would be say theta 2 and say theta 4.Okay?

And then write the vector loop equation as R 1 plus, R 2 plus, R 3 in this case, I've chosen the direction, such that R 1 plus, R 2 plus, R 3 equal to R 4. Okay? And this holds true, for all the positions of the linkage, so in fact, this is how we do the position analysis, we start from this, to do the position analysis for a 4-bar and so, we then write this in terms of complex numbers, as I theta 4 and then. I can now look at, if I expand this, I use the Euler identity e^ (I theta) RI theta equal to, cos (- theta) plus IC sine theta, so the real part will be the COS theta terms, the imaginary parts will be the ones with the operator I. So I can split this equation now, this is essentially a vector equation, so it is two scalar equations. Okay? This is a vector equation, I can split it into the two scalar equations as, so let's say here in this particular case, let I am taking theta 1 to be equal to 180 degrees, because that's my angle from the access, x-axis that I have chosen. Okay? So I've chosen this is. Okay? So theta 1 is 180, so I can write this as, minus R 1, plus R 2, cos theta 2, plus ice R 2, sine theta 2, plus R 3 cos theta 3, cos theta 4 + 5 I r sine theta, so this gives rise to the scalar, two scalar equations, which I get by equating the, real and imaginary parts. I get minus R 1, plus R 2 cos theta 2, plus R 3 cos theta 3, equal to R 4 cos theta 4. So this is one equation and then, by equating the imaginary parts, I get I R 2 sine theta 2, so I can remove I all over.

Because I am just taking the terms with the I operator, R 3 sine theta 3, equal R 4 sine theta 4. Okay? Now for synthesis, so say I'm looking at a function generation, I want to relate theta 2 and theta 4, that's my function generation mechanism. So I want to eliminate theta 3 from these two equations, if you look at the form of the equation, the easiest way to do it, is to move all the terms containing theta 3 alone to one side. Theta 2, so now if I Square and add, these two equations, I will essentially eliminate theta 3, because cos square theta 3 plus, sine square theta 3 will be one, squaring and adding 3 & 4.

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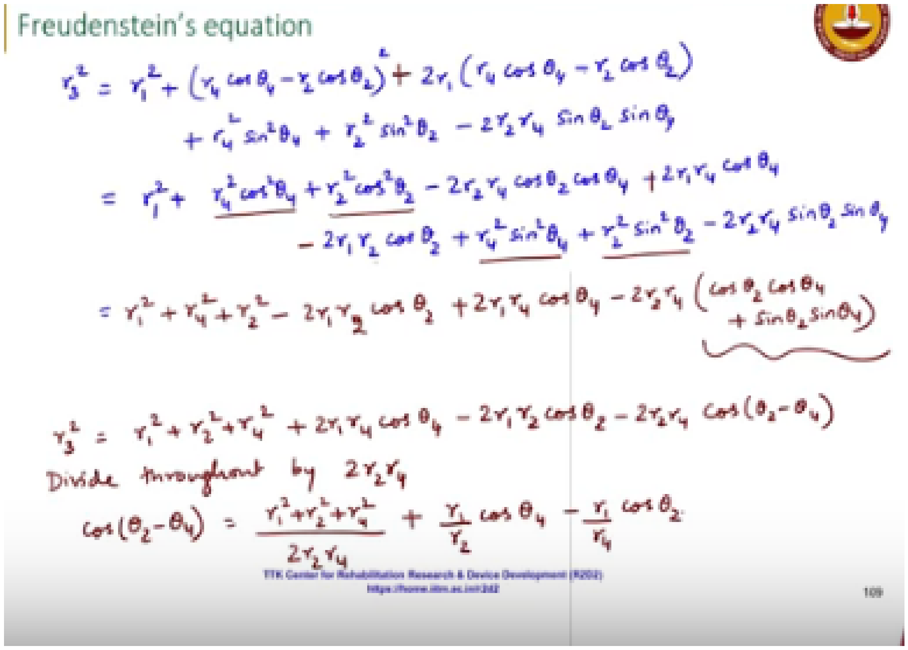

So I get R 3 square, equal to R 1 square, plus R 4 cos theta 4, minus R 2 cos theta 2 square. Okay? Please stop, correct me if I make a mistake, cos theta 2 plus R 4 square, sine square theta 4 plus, R 2 square sine square theta 2, minus 2 R 2, R 4 sine theta 2 ,sine theta 4. So if I expand this further, I get R 1 square, plus R 4 square, cos square theta 4, plus R 2 square, cos square theta 2, minus 2 R 2, R 4, cos theta 2, cos theta 4, minus 2 R 1, R 4 cos theta 4, plus 2 R 1, R 2 cos T 2. Okay? Plus this, our four square, sine square theta 4, plus R 2 square, sine square theta 2, minus 2 R 2, R 4, sine theta 2, sine theta 4, so now I can start combining some of these terms, so I have our 4 square, cos square theta 4, R 4 square, sine square theta 2. Okay? So this is R 1 square, plus R 4 square, again I have R 2 square, cos square theta 2, R 2 square sine square theta 2, plus R 2 square.

Then I have plus 2 R 1, R 2 cos theta 2, minus 2 R 1, R 4 cos theta 4, minus 2 R 2 R 4 into cos theta 2, cos theta 4, plus sine theta 2, sine theta. Okay? This one now, that should be plus, correct. Okay? Is that right now? Before I do that, this term, here simplifies to cos of theta 2, minus theta 4. Okay? So I get R 3 square, equal to R 1 square, plus R 2 square, plus R 4 square, - r1r 4 cos theta 4, minus 2 R 1, R 2 cos theta 2, minus 2 R, 2 R 4 cos of theta 2 minus theta. Okay? Now I divide throughout by 2 R, 2 R 4. Okay? And in this, I take the course of theta2 minus, theta4, to the other side. Okay? And divide throughout by this, so I get, cos of theta 2, minus theta 4 ,equal to R 1 square, plus R 2 square, plus R 4 square, by 2 R 2, R 4. Okay? Plus, if I divide this by r 2, I get R 1, by r 2 cos theta 4, minus r1 by r4 cos theta 2, that's it. I get it in this form, minus R 3 square, yes so I can write this in the form, so if you look at this, these are all linked length, this is ratio of link lengths and it relates theta 2 and theta 4. So, or my Phi n site, as we have been, using in the function generation problems, so I could this equation, is called, so trodden Stein wrote this in the form K naught.

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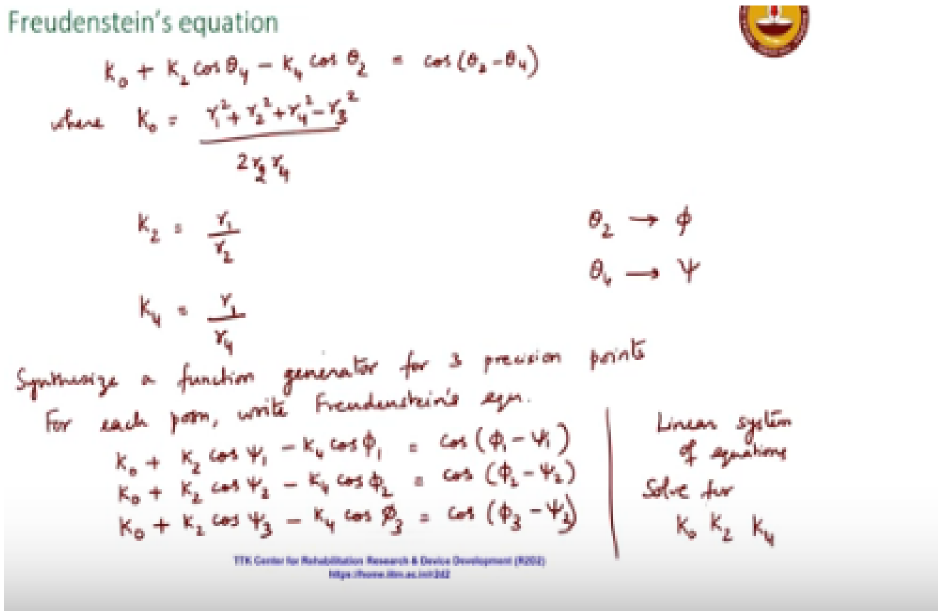

k 2 cos theta 4 ,minus k 4 cos theta 2, equal to COS of theta 2 minus theta 4, where this constant, K naught is R 1 square plus R 2 square, plus R 4 square, minus R 3 square, by 2 R 1, 2 R 2, R 4 ,K 2 is r1 by r2. Okay? Let me just write this as, yaw that's fine and K 4 s r1 by r4. Okay? So these are all the constants K, are functions of the link lengths and so now if I have the relationship, between the input angle input and output angle specified. Okay? Say let's say, I have a, I am given theta to say corresponds to Phi, from my previous and I have Psi, so I am given. I want to synthesize a function generator, with for 3 precision points. Okay? So, I can now set up this equation, for these three precision points and solve for, so I will get three equations, in the constants, is what you will determine from those equations. Okay? So once you determine the constants, then you're basically assuming some value for the say r1. Which is what we usually do, for the fixed length I take certain unit, say a unit length, then I find r2, r3 and r4 from these three equations. Once I do that, I have my linkage. Okay? So, if you look at the process, if you are given say, if you want to say, do a function generator, you'll be given x and y. Okay? So to design a linkage for that, you first determine the precision points, then find the corresponding phase and size, then analytically you can solve for the link length, so you can do the graphical method, or analytically you can use these equations, you can program these equations and then find the link lengths from this equation, so this is called, this is using Freudenstein's equation, for function generation. Okay?

Let me see, if I have one , so for each position we would write, K naught plus, K 2 cos Phi 1, minus K 4, cos Psi 2 ,equal to cos of Phi 1 minus, sine K naught plus K 2 cos Psi 2 minus K 4 cos, Phi 2 equal to cos of Phi 2, minus Psi 2, k naught plus, K 2 cos site 3, minus K 4, cos Phi 3, equal to cause of, so since, Phi 1, Phi 2 ,Phi 3, Psi 1, Psi 2, Psi 3 are known, this is just a system of linear equations, in K 1, K 2 and K 3. Okay? So it's not, you know all the costs terms. Okay? So you solve for, so this becomes a linear system of equations, solve for K naught, K2, k4. Assume one of the link-lengths, remember it's a function generator, so I can scale it up or down, sir zoom one link-lengths AR, one then solve for R 2, R 3, R4, to get your this, then you can use the same equation, same set of equations, to do your displacement analysis. Okay? Using so, in a different form so, if I want to look at this, so here I have eliminated theta 3, I have a relation between, theta 2 and theta 4. Now when I do analysis, then I know all my case. Okay?

In fact we would put it in a different form to do the analysis, but essentially, in that case once you know the link-lengths, you would use, you would manipulate the same equation in a different form, to do the displacement analysis. So you give theta 2, in the range that you want, you look at theta 4, so you can even compute the errors, so in the range that you are looking at, you can compute the error in the structural error, in the mechanism for the entire range that you are looking at. Okay? You would essentially, because you've eliminated theta 3 from the equation, theta 2 will be your input angle, so you can rewrite this equation, do you want me to do that, or all comfortable with, you can rewrite the same equation, Freudenstein's equation, in terms of just theta 4, to solve for theta 4, for a specific angle theta 2, and you will actually get, two values for theta for the two values, correspond to the two configurations of the linkage. Okay? So you pick the configuration that suits you. And see if it meets the requirements, see look at the error, over the range of the function that you are looking at, so you would go back and use the same equation for analysis, so you can set it up as a loop. Okay?

Look at the error, or you know, you may want to look at say, the link ratios, so you, you get these in terms of link length ratios right. R1 by r2, if you get very small values for K, then you might end up with a linkage, where two of the links are very small and two are very large, typically that's in any linkage where the link lengths are so off, you know in terms of their dimensions, you will not get good trans, you will not get good transmission angles, you not get good force transmission in the linkage. So you, it will be, so you may want to then go back and the way you would do that, is to change, so suppose you have three precision, points if you go back to our function generation, so you know ,she asked me this ,so here is my fiber.

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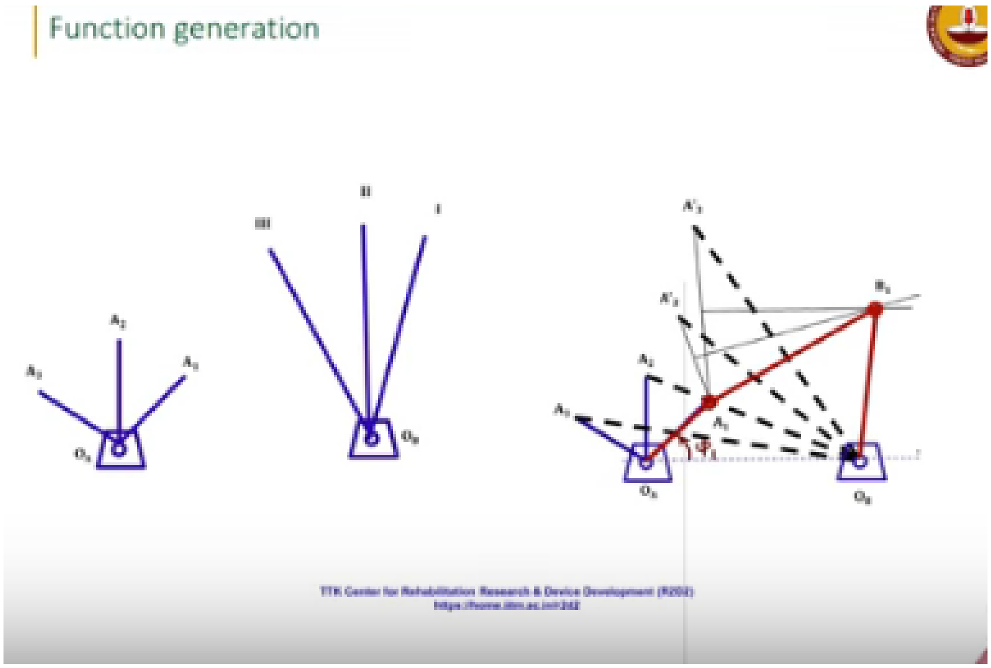

Okay? And that gave me a particular solution, for three precision points. Right, this will be, Phi 2, 1, Phi 3, 1. Now if I choose a different Phi 1, I'm going to get a different linkage, so it does matter, where you choose so Phi 1 is what you can manipulate, in order to get a better solution. Okay? So you can, so you can still get the same range, that's why when we say, you know, we when we talk of three precision points, we are talking about displacements, two displacements, two angular displacements. It's because, this may not give you what you want, so you may have to use a Shay Bishop spacing again, to modify your starting angle Phi 1. Okay? Change Phi1, in order to get, but these two will remain the same, so Phi 2, 1, 5, 3, 1 Phi 2 and Phi 3 will change. So that Phi 2, 1 and Phi 3, 1 remain the same. Because you're talking about angular displacements in this function generation, so for instance, if I choose both, Phi 1 equal to 0 and C 1 equal to 0. Okay? In my design, then what happens? I'm having the initial configuration as, so I'm going to get a Grashof Newton linkage, so because that will essentially be at the change point, Phi, so that's not desirable so typically you want to choose, at least Phi 1 or siphons one of them nonzero, so that you get a more desirable linkage ,the desirability you can find out, analytically it's easier, so here I would have to redraw, redo the whole thing, if I choose a Phi 1, that's not suitable in the case of the analytical method. Once i setup program the equations, then it's very simple for me to run it for other things. I can give ,you know, I can say that, if the solution, if the ratio between the links, is more than 2 or 4 or something like that the ratio of the links, if I get a larger ratio than that, which means the values of K 2 and K 4, then I don't want that solution, so I, I trait again, I can do that, much more quickly, once I set it up, in the analytical ,I can also look at ,I can also program ,the calculation of the transmission angles, because, again that depends on the link lengths and the input and output angle. Okay? So this is the way I would, so analytical synthesis can be very much more powerful, then graphical synthesis for the same thing, when I want to look at, these other quantity sizes, but I want to look at say the quality of the linkage it's much easier for me to cycle through multiple solutions, to meet my requirements, as opposed to the graphical method. Graphical method if I get lucky, get a good solution, the first or second time, then I have saved a lot of time, because the initial investment for the analytical method is much more. Okay?

So this is Braden Stein's equation, now the other, disadvantage with graphical synthesis is that, you may not be able to relate derivatives of the, derivatives of the displacement. Okay? You're only looking at displacement, when you do graphical synthesis, it could be an angular displacement, it could be displacement of a point it could be the displacement of a point and the orientation of a link. So we are looking at only quantities related to position, not related to derivatives of position and there are some special cases where you could relate the two, but in most cases it's difficult to relate higher derivatives, with but that's where again the analytical synthesis, especially synthesis using these complex numbers comes to mind. Again let's go back to the loop closure equation, so let's say.

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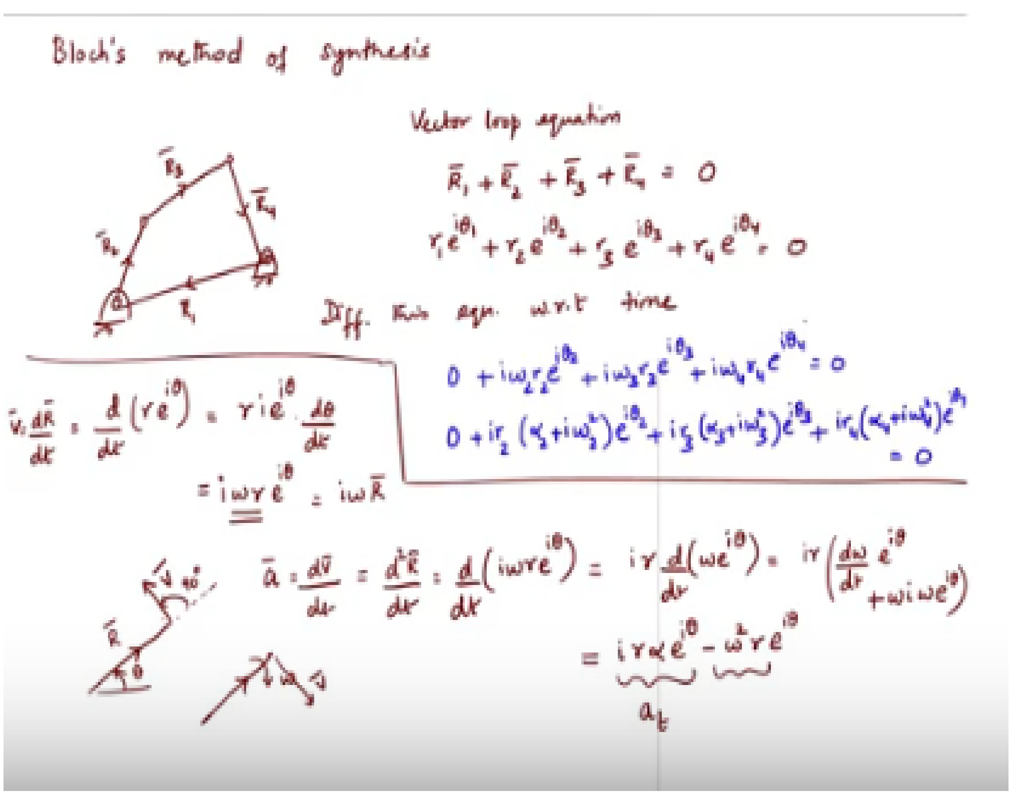

We write this as and method is called, blocks method of synthesis, Russian Kineme Titian blocks. I'm just choosing the vectors differently each time; just so, you know that, there's no set way off, so

Freudenstein's equation itself, you'll find it in different forms in different books. Depending on what, how they chose their vectors, so it's not something you want to muck up. But once you pick something, stick to that ok, so my vector loop equation here, equal to c and I, have few complex notation ,I can write it as, equal to zero. Now I differentiate this equation, with respect to time, when I do this, my D by DT of R Riper, I theta, will be, our in the case of the 4 bar is a constant. Okay? So I get R, I, E power I theta, into D theta by DT. Which I can write it as Omega, R Omega, I, R Omega, E/ I theta Forbidden it is Omega, harmful and theta so I Omega R. So you see here, one of the reasons the complex number, method is very nice to use with the vectors, is that. Okay?

I have this link R; I am looking at the velocity of this slick. Okay? Because, this is d r by d t, d r by d is essentially, if I look at this equation. I have, the magnitude of that velocity, is omega R. Okay? And the direction of that velocity, because you have I operating on this, re power I theta, what did we see, when you operate on a vector, with the, with I you get a counterclockwise rotation by 90 Degrees. So the velocity vector for this is going to be, this would be the direction of the velocity vector. So from this direction, you rotated by 90 degrees and the magnitude of that velocity vector, is omega times R. Okay? Because this is just I Omega R, so the magnitude of the velocity, so you operate on the vector R, by so you rotate it counterclockwise by 90 degrees. And it's magnitude is scaled by the angular velocity Omega. Okay? So this is assuming ,this is Theta, typically we Assume, theta is positive counter clockwise ,so Omega, Alpha, all those will Be, will follow ,similar this way, so this is an aside, in terms of the analysis, but you will see that, so if I differentiate this equation, what is the derivative of the first term. R 1 e power I theta 1, it's 0 ,because the fixed link does not change ,so I get 0 Plus, this one is I Omega 2, R 2 E power I theta 2, plus I Omega 3, R 3 t power I theta 3, plus I Omega 4, r4e power I theta 4, equal to C. You now let me differentiate the velocity Vector, so DV, by DT, so V is d r, by d t, the acceleration is DV by d t. Okay? Which is basically differentiating this equation Again, d square R by DT square, so if I differentiate this, so d by DT of I Omega RT / height 3 de, is what I want to Differentiate, R is again constant, I can take out I also, I our D by DT of omega t / I theta. Okay? So, if I do it, then I get I R, into B Omega by DT, T power I theta, plus Omega I, I get another Omega t / I Theta, change so this becomes D Omega by DT is my alpha, angular acceleration alpha, so I have I R alpha, t power I Theta, I into I minus 1 minus, Omega square R, T power I theta. So these are my two components of acceleration and you can see here, this component which I call the tangential Component, is again going to be, is of magnitude R alpha and is also perpendicular to the position vector, or the link this are. Okay?

Now if alpha is positive, you will get a Counterclockwise, same thing with Omega here. I forgot to mention if Omega is positive you get a counterclockwise rotation, so if the same link if this was the direction of Omega, then my velocity vector would be like this. I into the negative Omega will give me a clockwise rotation by 90 degrees, similarly here, the tangential component of acceleration will be perpendicular to the position vector and this component Here, you can see the direction of that is opposite to the position vector, it's along the minus E power I theta and that component has a magnitude Omega square R, you already know this ,from a rigid link rotating about a pivot, you would have these two components fixed pivot you would have these two components of Acceleration, the tangential component that's normal to the link and a centripetal component, which is towards the pivot, about which it is rotated, so the third equation in this, would be when I differentiate this again ,so I would Get, zero plus, I, R 2 into alpha 2, plus I Omega 2 square, e power I theta 2 plus Phi R 3, alpha 3, plus I Omega, 3 square, V power I theta 3, plus I R 4, alpha 4, plus I Omega 4 Square, e power I theta 4, equal to. So these are three equations from the same vector loop equation, but they relate the velocities and the accelerations.