Theory of Mechanisms

Lecture 16

Dyad Form Synthesis: Motion Generation

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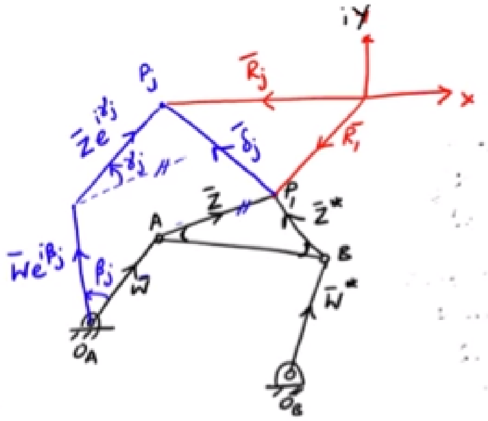

$$\bar{W}e^{i\beta_j} + \bar{Z}e^{i\gamma_j} - R_j + R_1 - \bar{Z} - \bar{W} = 0$$

$$\bar{W}(e^{i\beta_j}-1) + \bar{Z}(e^{i\gamma_j}-1) = \bar{R}_j - \bar{R}_1 = \bar{\delta}_j$$

$$\boxed{\bar{W}(e^{i\beta_j}-1) + \bar{Z}(e^{i\gamma_j}-1) = \bar{\delta}_j}$$

Standard or dyad form equation

$$W(e^{i\beta_j}-1) + Z(e^{i\gamma_j}-1) = \delta_j$$

Okay, so last class we were looking at the dyad or the standard form for these, this is called the standard form for the synthesis equation using complex numbers, and essentially we say that a four-bar can be represented as two dyads, and we looked at just one side of the dyad right, and I'll probably skip using the bar. You know from now on it's understood that if I write this okay, this is understood that these are vectors or complex numbers. The reason I want to avoid using the bar is, in complex numbers, the bar indicates the conjugated, so I don't want confusion with that. The textbook, they can make it bold, but you know, but in the context of what we are doing you should know that this is a vector equation, so it's an equation, its two scalar equations and this is called the dyad or the standard form. So how do we apply this to a four bar? That's what we will look at today and then the different types of synthesis problems that we have seen.

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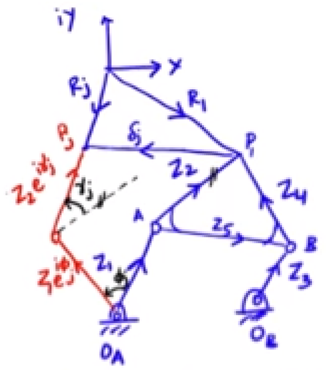

4-bar linkage seen as two dyads

$\underbrace{Z_1, Z_2}_{\text{left}}$ and $\underbrace{Z_3, Z_4}_{\text{right}}$

Path point P moves along a path from position P_1 to P_j defined by R_1 & R_j, resp.

Prescribed two positions : R_1, R_j, ϕ_j, γ_j

$$Z_1 e^{i\phi_j} + Z_2 e^{i\gamma_j} - R_j + R_1 - Z_2 - Z_1 = 0$$

$$Z_1 (e^{i\phi_j} - 1) + Z_2 (e^{i\gamma_j} - 1) = R_j - R_1 = \delta_j$$

$$j \geq 2$$

So let's say a four bar is represented by two dyads. I say this is Z1. This is Z2. This is C3. And C4 is a point P, and this coupler okay? I can, once I know Z1, Z2, Z3, Z4, then this Z5 can be expressed in terms of the other two. It's nothing but Z2 minus Z4. If this is Z5 right, so the four bar linkage is seen as two dyads, Z1, Z2, and Z3, Z4. So let's say the point P from the first position, it moves to some other point. Let me draw the coordinate system. iy, so I have, this is R1 and then at some other Jth position point P, so let's say this moves by an angle Phi, draw that. That means like that, and so this, so this is being the first position, this will be in the Jth position and if this, this is them, so these two are parallel okay? So let's say this has rotated by gamma J, this has rotated by Phi J, so this one here, the red one, this vector is Z1, e power i, Phi J, this vector is Z2, e power i, gamma J and this is PJ, then this is RJ, go A,B, OA,OB So z1, z2 form the left dyad. This is the right side of the four bar Z3 and Z4. So the point P on the part on the coupler path, the path point P moves along some path from position P1 to Pj. The positions are defined by R1 and Rj respectively. So that means we have prescribed two positions here. So if you have the case where two positions are prescribed, which means, I know R1, Rj, Phi j, and gamma j, well, I don't necessarily know all of them. Let's say, okay, these are the quantities that I require okay? So based on this, so now I can write like we did when we derived the standard form. I'll just write down the loop closure equation for this loop here. So I have X1, e power i, Phi J, plus Z2, e power i gamma j minus Rj, going the other way, plus R1, minus Z2 minus Z1, is equal to 0. So all these are vectors okay, or I have Z1 e power i Phi J, minus 1, plus Z2 e power i gamma J minus 1, equal to RJ minus R1, which is nothing but Delta J. That is Delta j okay, so this is how we got the standard form J is greater than or equal to two. Now let's look at the case of motion generation okay?

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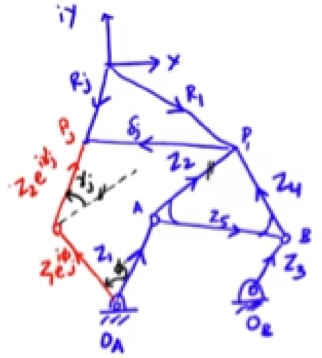

Motion generation : $R_1, \dots, R_j$ prescribed — or $\delta_2 \dots \delta_j$ prescribed
$\gamma_2, \dots, \gamma_j$ prescribed

# of positions	# of scalar eqns	Unknowns	# of scalar unknowns	# free choices	# solutions for unknowns
2	2	Z_1, Z_2, ϕ_2	5	3	∞^3
3	4	" $+ \phi_3$	6	2	∞^2
4	6	" $+ \phi_4$	7	1	∞
5	8	" $+ \phi_5$	8	0	finite

$$Z_1(e^{i\phi_j}-1) + Z_2(e^{i\gamma_j}-1) = \delta_j \qquad j \geq 2$$

for right-side dyad,

$$Z_3(e^{i\psi_j}-1) + Z_4(e^{i\gamma_j}-1) = \delta_j$$

2-posn

$$Z_1(e^{i\phi_2}-1) + Z_2(e^{i\gamma_2}-1) = \delta_2$$

3-posn

$$Z_1(e^{i\phi_3}-1) + Z_2(e^{i\gamma_3}-1) = \delta_3$$

So I have the equation, the standard form Z1, e power i, Phi J minus 1, plus Z2, e power i, gamma J minus 1, equal to Delta J okay? This is my standard form equation for the left dyad. Now let me look at the case for motion generation, let me see whether I should do this okay, so for motion generation, what would be the quantities that would have to be prescribed? You need the location of a point on the coupler and you need the orientation of the coupler. Those are the two things for motion generation, so I need R1 to Rj depending on the number of positions they should be given okay? Similarly gamma 1, the orientations, corresponding orientations of the coupler are also given. So if I am given, so if I'm given these two, then the synthesis I have to perform is form a motion generation problem, okay? That's what I am trying to find. I need to find the vectors which will give me this, which will solve this motion generation problem. So anyway, when when we write it okay, gammas know it, gamma is J is greater than or equal to 2. It's always with respect to, so I should say yeah, gamma 2 - you will need R1 because you will, you use Delta to determine from the first position, but J greater than or equal to 2 means, gamma is always from the first position okay? So let's say, let's make a table where I have number of positions, then number of scalar equations for the dyad. It's actually, this will be gamma 2 to gamma j, yeah, number of scalar equations and then the unknowns, the scalar unknowns, or first we look at the vector unknowns, then the number of scalar unknowns. Number of free choices scalars and then number of solutions for the unknowns, okay? So you can also look at this as R1 to Rj, prescribed or delta 2 to delta j prescribed okay? the displacements of that and gamma 2 corresponding orientations. So let's say it's a two position problem, that we are trying to solve. We know this and we know that, so how many scalar equations? So if I write the standard form, okay my j is two position problem. I have position 1 and position 2, so the equations that I get will be,for the two position problem, I have e power i, Phi 2 minus 1, plus Z2, e power i gamma 2 minus, 1 equal to Delta 2, are there any other equations? It's a two position problem. I get this, a single vector equation which corresponds to, two scalar equations, what are my unknowns? I have Z1, Z2, okay? In this equation gamma 2 is given, Delta 2 is given, because that's my design condition. I don't know Z1, I don't know Z2, I don't know Phi 2 okay? So of these Z1 and Z2 are vectors or complex numbers. So I need, so if I look at the number of scalar unknowns I have 5. I have two equations, five scalar unknowns, so how many free choices do I have? Three, the number of free choices is three, which means number of solutions that I have is infinity okay? Take three positions. Now what happens? I have this equation, I have in addition to that I also have this plus, I mean I'm using plus rather loosely here, but I have that equation. I will also have this equation for the third position here. What are the specified quantities? Gamma 2, gamma 3, and Delta 2, Delta 3, that's my three position problem. 3 position motion generation problem okay? So here now I will have number of scalar equations becomes four, unknowns, Z1, Z2. So I have all these. Basically these three are unknown, so I'll just put a little plus in addition to that. What else is unknown? Phi 3, Phi 3 is the only other unknown because again gamma 2, gamma 3, Delta 2, Delta 3, are given, so the number of scalar unknowns is now 6. My number of free choices come down to 2 and I have infinity square solutions that are possible for position scalar equations. One more the equations go up by two, I have six. The unknowns, all these data plus five, four okay? So this is seven number of free choices goes down to one and I have an infinity of solutions. Five positions, this goes up to eight, this goes up to eight again okay? So I am no longer left with any free choices and we say there are a finite number of solutions okay? Not necessarily a unique solution, you would have a finite number of solutions, you don't have an infinite. I will see later when we do that, this is only for one dyad we are talking about, only one side of the four bar right? For the other side of the four bar you again have for the right side dyad, Z3 e power i, let's say the angle is, Shi, like we've been using plus, Z Phi e power i, What would be the power Z4 e power i? What would be the angle of Z4? you're trying to form a four bar? It'll still be gamma, that is what links the two Dyads are linked together because Z2 and Z4 are going to be part of the same rigid body and they're going to undergo the same rotation. That's how you're building the 4 bar, with this dyad form, so e power i, gamma J minus 1 and it's still the same point P equals Delta J okay? So here the unknown, so I can write, make a similar table like this okay, and in those cases instead of file it's going to be safe, otherwise the form stays the same, that's why it's called the standard form okay? So so this gives me what for, if I want to create a 4 bar with these two dyads it's infinity cube for one dyad. This one also, it has the same form, so, another infinity for a two-position problem right? So I get infinity cube infinity cube. I can, so it's infinity to the power 6. Now go back to our graphical synthesis that we did okay, and see if it is, if you got the same result there. What did we do with the graphical synthesis?

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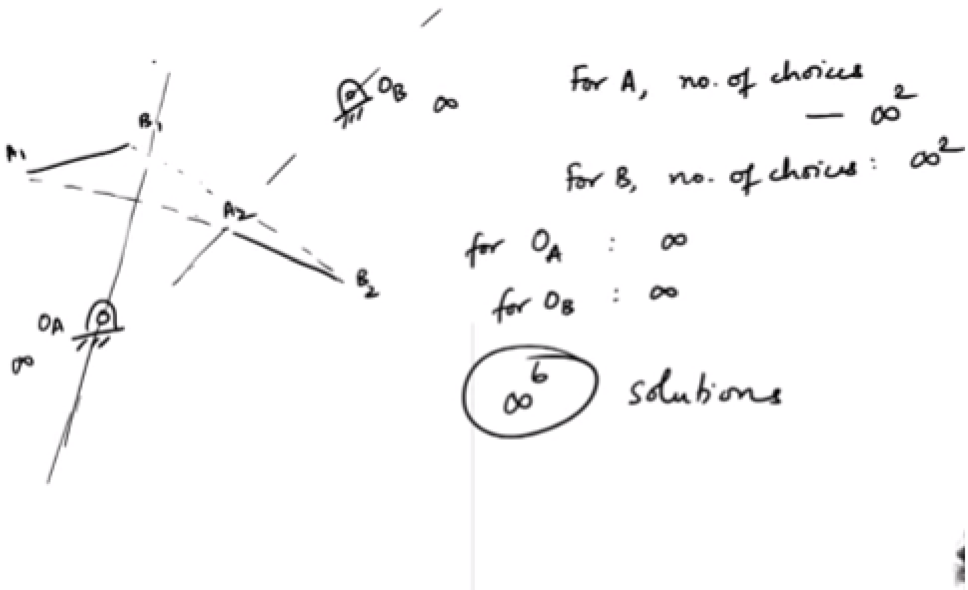

So you had two positions a1, b1, a2, b2, right, and then how, what did I do? I just took the perpendicular bisector of a1, a2 okay, and I picked OA anywhere on that perpendicular bisector, so that gave me how many solutions infinity for b1, b2? Pick the perpendicular bisector, I can pick OB, anywhere on the perpendicular bisector, that gives me how many solutions? Infinity, so total number of solutions for my 4 bar? yeah, so this is infinity square after I have picked A and B to be the moving pivots. It's not necessary, so I have for A number of choices. I can pick any point in the plane number of choices is infinity square, for B number of choices is infinity square. Once I picked my A and B, I still have infinity into infinity, infinity square, So for OA and OB, for OA infinity, for OB infinity. So overall I again have infinity to the power 6 solutions. Got the 2 position, so it does not change okay? The number of solutions that are possible does not change. What may be restrictive is how you can find the solutions because if I'm working, you know if I'm trying to do geometry like for the three position problem we ended up with a unique solution right? Although this tells me here that I would have more,

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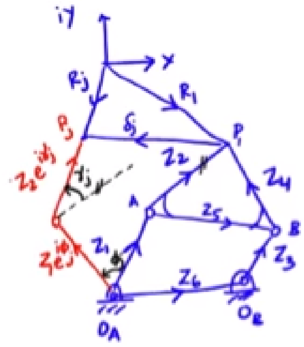

Motion generation : $R_1, \ldots, R_j$ — or $\delta_2 \ldots \delta_j$ prescribed
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$\gamma_2, \ldots, \gamma_j$ prescribed

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$$Z_1(e^{i\phi_j}-1) + Z_2(e^{i\gamma_j}-1) = \delta_j \qquad j \geq 2$$

2-posn
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For right-side dyad,
$$Z_3(e^{i\psi_j}-1) + Z_4(e^{i\gamma_j}-1) = \delta_j$$

Coupler is given by $Z_5 = Z_2 - Z_4$

Fixed link : $Z_6 = Z_1 + Z_5 - Z_3$

I actually have infinity square choices for the 3 position problem, but because of the way that we do the construction and the way we choose, so when I choose a point I immediately lose two, three choices when we do it analytically. You will see I can possibly choose an angle or I can choose a length. Perhaps you know when we look at solving these equations that gives you more flexibility but the number of free choices does not change fundamentally. The problem will have that many number of free choices because you have so many equations, you have so many unknowns but the method that you use may restrict you in some ways to on what free choices you are allowed to be, okay, and you may have to pick free choices in pairs like as a point for instance, so then that becomes a little bit more restrictive. So it's, it's more the method that restricts the free choices then the problem still has that many number of possible solutions, okay? So that's something if you need to keep in mind okay? So once we solve the right side dyad, okay, then to create the 4-bar the coupler is given by selects. We will do it for a couple of problems. So the coupler is given by Z5 equal to Z2 minus Z4, so once I solve Z1, Z2, then Z3, Z4, my coupler is given by Z5 equal to Z2 minus Z4 and fixed link will be Z1, this will be Z1, sorry, I should call it Z6, Z6, Z1 minus, sorry Z1 plus Z5, minus Z3. Just by the loop closure so that completely if I find these two dyads it completely defines my 4 bar. That's, that's the point I'm trying to make, so I can look at the 4 bar as these two separate dyads design, these two separate dyads, put them together to make my 4 bar.

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Synthesis for
2-position motion generation

Given $\vec{\delta}_2, \gamma_2$

$$\vec{Z}_1(e^{i\phi_2}-1) + \vec{Z}_2(e^{i\gamma_2}-1) = \vec{\delta}_2$$

Unknowns: $\vec{Z}_1, \vec{Z}_2, \phi_2$ (5 scalar unknowns)

Say, my free choices are $\vec{Z}_2$ & ϕ_2

$$Z_1 = \frac{\delta_2 - Z_2(e^{i\gamma_2}-1)}{e^{i\phi_2}-1}$$

$$Z_1(e^{i\phi_j}-1) + Z_2(e^{i\gamma_j}-1) = \delta_j$$

3-position motion generation

Given $\delta_2, \delta_3, \gamma_2, \gamma_3$

$$Z_1(e^{i\phi_2}-1) + Z_2(e^{i\gamma_2}-1) = \delta_2$$

$$Z_1(e^{i\phi_3}-1) + Z_2(e^{i\gamma_3}-1) = \delta_3$$

Possible free choices Z_1, Z_2, ϕ_2, ϕ_3

Linear system of eqns in Z_1 & Z_2

So let's look at two position motion synthesis, motion generation. So I have given Delta 2 and gamma 2, Delta 2 is a vector okay? Maybe I'll use the arrow like that so and I have my equation for the dyad, and we'll keep using that. So I'll write it as Z1 e power i, when Delta 2 and gamma 2. I have the equation would be delta 2 , so what could be my, I know I have three free choices, to solve these equations for z1 and z2 okay? That, that's my design right? We were finding the dyad is what I am trying to do, so to get that I can say I have to pick three free choices. Now I have only these two equations. I have five unknowns; unknowns are Z1, Z2, 2 and Phi 2, okay? So total 5 scalar unknowns, but I have only two equations. So what I could do is I could pick one vector, have a question, second equation they are real and complex. It's a vector equation right? So there are two scalar equations. Ultimately I have to solve for five scalar unknowns, the two components of vector Z, one, two components of Z2 and Phi 2 yeah, so yeah, just till we get used to it, let me just write out that arrow on top of that okay? so Z1, Z2 and Phi 2, so now I can choose as my free choices. What are the possibilities? I have to choose how many free choices I have to choose, 3 okay, so what could I do? The easiest way would be to either choose Z1 and Phi 2 or Z2 and Phi 2, so let's say I choose, say my free choices are, it's more difficult if I say I will choose z1 and one component of z2 right? Solving the equation will be more difficult so free choices are this then I get Z1 equal to Delta 2 minus Z2 into e power i, gamma 2 minus 1, by e power i, Phi 2 minus. So this defines my left dyad. I can do the same thing for the right dyad and come up with the solution for the 4 bar for motion generation okay? So this is 2 position motion generation 3 infinities of solutions with these two the choices for Z2 and Phi 2. Three-position motion generation, I am given Delta 2, Delta 3, gamma 2, and gamma 3. So I get and Phi3-1, delta 3. If I go back to my table how many free choices do I have? I have two free choices. Two free choices, if I do that then what are my possible, possible free choices? Are they 1 or Z2 or Phi 2, Phi 3? To solve these equations what would be the easiest choices? Phi 2 and Phi 3 right, because if I choose Phi 2 and Phi 3 it, this just becomes a linear system of equations okay? If I choose Z1 or Z2, one of the two because I'll need two components there right? That would mean that I am solving the equations becomes more than difficult because I have all these sine Phi, cos Phi, term Souter, so analytically it's easy to choose these two angles but imagine in the graphical method choosing a point was easier which essentially meant I was actually picking, yeah. So I I was taking two free choices in that manner okay, when I picked a point for the moving pivot because when I pick a point for the moving pivot that kind of defines my Z okay? As you mean I start off at the origin so I can say that that is my Z one okay? So that again number of free choices does not change because of the method that you use but what you can pick as your free choices. This is not easy to do, picking the angles and then trying to come up with a graphical solution to find the, to synthesize the linkage so do you see the difference between the graph fundamentally? They are the same. You have the same number of solutions, same number of, same number of free choices. What you can pick as your free choice varies with the graphical lengthy analytical solutions. So once I do this I can solve, it's a linear system of equations, in Z1 and Z2 and MATLAB can easily solve it for me okay, and I can find the two vectors. I repeat that for the other side of the 4 bar and I end up with the whole.

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4-position motion generation

$$Z_1(e^{i\phi_2}-1) + Z_2(e^{i\gamma_2}-1) = \delta_2$$
$$Z_1(e^{i\phi_3}-1) + Z_2(e^{i\gamma_3}-1) = \delta_3$$
$$Z_1(e^{i\phi_4}-1) + Z_2(e^{i\gamma_4}-1) = \delta_4$$

Knowns : $\delta_2 \ldots \delta_4$
$\gamma_2 \ldots \gamma_4$

Unknowns : $\vec{Z}_1, \vec{Z}_2, \phi_2, \phi_3, \phi_4$ (7 unknowns)

One free choice

System of non-linear equations bcos the unknown angles are in transcendental form.

For motion gen : 3 positions are the limit for a linear solution

Four position motion generation. We didn't look at this graphically, but let's say analytically we should be able to solve, Phi 3 minus 1, plus Z2 Phi four minus one okay? This is my fourth position problem, so the known’s for motion generation would be Delta 2, Delta 4 and gamma 2, 2 gamma 4 for this problem and therefore there are six equations, seven unknowns. Unknowns are Z1, Z2, Phi 2, Phi 3, Phi 4, 7 unknowns, 6 equations, 7 unknowns, one free choice, say you see here I could pick one of the angles but it will still no longer be a linear system of equations. So up to the 3 position problem the solution is fairly straightforward because you can pick your free choices in such a manner that you only have a linear system of equations to solve. From the 4 position problem onwards It becomes a little bit trickier, you still have an infinity of solutions, so I can pick one of the angles Phi 2, Phi 3, or 5 4 but it's all in transcendental form in the equation, because I can pick Phi 2, but they'll still be cos theta cos Phi 3 cos Phi 4 sine. So if it's not an easy system of equations to solve we look at a method later where we will do that a little bit later, where we will solve this problem for the four-position case. You can do that, maybe the five positions also will do with time permits, but beyond three positions you will not get a linear system of equations. The system of equations becomes a nonlinear system of equations to be solved. So this is the system, there goes r in transcendental form okay? Okay, so for motion generation, three positions are the limit for a linear solution. So for the 4-bar once you solve both dyads you get the coupler and D, so you could have the same routine. Basically we're in MATLAB, you would write a problem to solve the dyad swab for say two position or three position syntheses. Use the call, the same routine again to solve the right side dyad and then put together the four bar, for your solution.