Theory of Mechanisms
Lecture 19

Dyad Form Synthesis: Four Position Motion Generation

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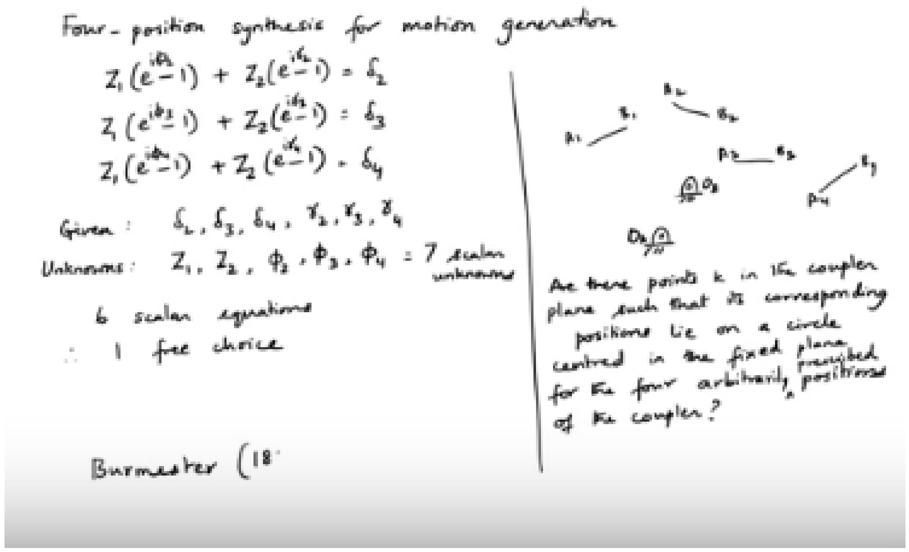

We look at, the four position Synthesis, for motion generation. Now, if you remember, in the graphical methods, we stopped with three. Right, because we ended up with a unique solution, so essentially the strategy that we used was. Okay? The moving pivot ,will move along the circular path, so up to three positions, I can fit, I can find the center of the circle, that passes through those three points and then I am, so but I ended up with the unique solution in that case. But, if you look at the table that we did, for you know when we looked at the diet synthesis method and you look at, the so if you look at a diet, keep r I, phi 2, minus 1, gamma 2 minus 1, equal to delta 2.

You have what's given, are the deltas and the gammas, because this is a motion generation problem and your unknowns are, Z 1, Z 2, then Phi 2, Phi 3, Phi 4. Okay? So equal to 7 scalar unknowns, you have 6 scalar equations above, 7 scalars in unknowns, therefore you still have, one free choice. Okay? Which means, theoretically you should have infinity of solutions. Right, but we saw with the graphical method, that we ended up with a unique solution, for the 3 position problem and we didn't explore the 4th position, although you know, some clue you have, like for instance, if you take the pole. Okay? If you take for two positions, if you take the pole say p12, as one of your moving pivots, now what happens, in position 1 and position 2, the location of that does not change. So I can actually, if I choose that as a moving pivot, I can actually reduce, the 4 position problem, to a three position problem, because one point does not change, between two positions ,ok so there are graphical methods, it's called point position reduction, okay there are graphical methods, but we didn't go into that ,essentially you do have. We saw that, you do have a free choice, the question is, how do we exercise that free choice, maybe not graphically ,but how do we solve this problem, so we should have an infinity of solutions ,so we saw that the strategy was. Okay? So if you take the fourth position problem. Okay? You take four positions of this thing in the plane, of the coupler and I find corresponding, you know say for the three positions, I find Oa and some OB, I'm just drawing it randomly, but you know, say I find the corresponding center points. Right, for these three positions, if I take say 1, 2, 3 and then I take 2 ,3, 4, the odds that OA and OB will, that a 4 will also, a 4 and b4 will also lie ,on the circle passing through and OB, you know which satisfies the first three position is very low. Okay? So that's not going to happen, so if I if I synthesize for three positions and add a third position, it's not likely that, the fourth one is going to be on that same circle. Right, so the question we want to ask is, are there certain points, on the coupler, for the moving plane, such that, you know, you can find a circle that passes through all four of them.

So that they will lie on the same circle, so is there points k, in the coupler plane, such that its corresponding four positions, lie on a circle centered in the fixed plane and centered in the fixed plane, for D for arbitrary positions of the coupler. For arbitrarily, arbitrarily prescribed that's my design condition right, i give four positions based on some other criteria, but can I now find, you know can I find points, on the coupler, such that they would still lie on a circle centered in the fixed plane. Okay? So that is the task that we have, so this Question, was asked by Burmister. So this theory is called Burmister theory in 1876 and he figured out the solution to this as well, so the Burmister theory was proposed in 1876 basically deals with the 4-position problem of motion generation. So now let's go back to the equations.

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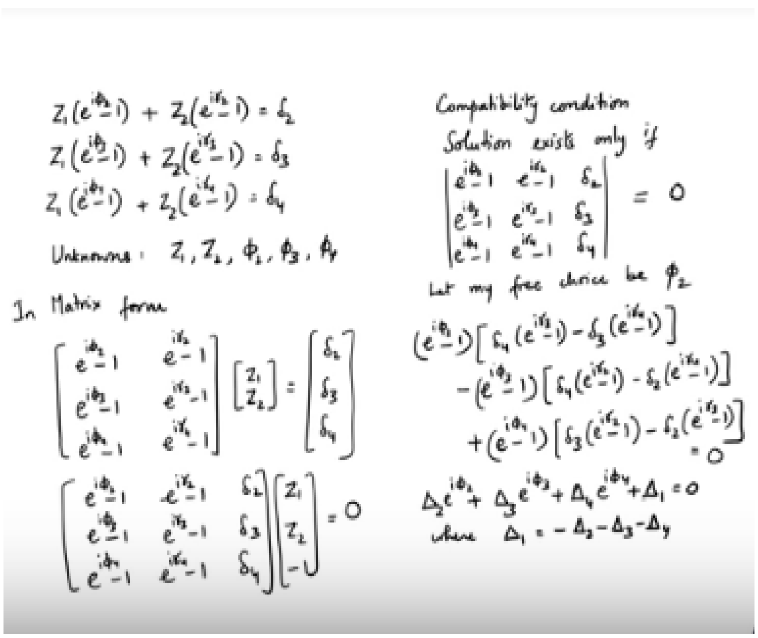

You should be able to write these equations in your sleep now, the standard form a power, I Phi 4 minus 1, Delta 4. Right so here the unknown angles. Okay? So the unknowns are Z 1, Z 2, Phi 3, Phi 4 and the unknown angles are in transcendental form. Okay? You don't have up to three positions; we saw, we got a nice set of linear equations that we could solve for the diet. Right now that's no longer the case. Okay? We saw that even with the, in the case of specifying the fixed pivot, even for the 3 position problem, if you specify the fixed pivot, then you actually get, you have to satisfy a condition, a compatibility condition, so that which is transcendental in form. Okay? And we did that, with the using Geometry. Right forming that triangle, we had that D 1, plus D 2, e to the power, Phi 2, plus D 3 e to the power Phi 3, ok. So you've already seen, that kind of a form, this is sort of an extension of that, so you'll see here that, these angles are in transcendental form and therefore these equations are not easily solvable, also you have only, you want to really solve for Z 1 and Z 2, those are what and you have only one free choice, but no matter what I pick as my free choice, the other angles are still unknown. Okay? So the three unknown, therefore you need some kind of a nonlinear technique, so I can write this in matrix form, I can write the set of equations as. Okay?

The same equations can be written in this form. Okay? Same thing, so this is, if I want this system, to have a solution for z1 and z2. Okay? Then the determinant of my augmented matrix has to be zero, so that is an additional condition that I have to satisfy. Right, so that means, two of these equations, should be linearly dependent. Right, i can have, only two sets of linearly independent equations out of the three, so the coefficients must satisfy certain what we call compatibility conditions. So the compatibility condition for this becomes, basically you're saying the rank must be two. Right, so the Augmented, the determinant of the augmented matrix should be two, so I can, I can first write this as. Okay? So the solution to that exists only if, this determinant, - 1 e Para gamma 4 ,minus 1 Delta 4 equal to 0, so this is the determinant of that matrix, so I have only one free choice. Okay? Let’s say, I take, let my free choice be Phi – 2, so in this the second column, everything is known because, gamma 2, gamma 3, gamma 4, are specified. Again third column, Delta 2, Delta 3, Delta 4 are specified, so let my free choice be Phi 2, so I will expand about the first column. Okay? So ,I can say, Delta gamma 3 minus 1, minus Delta 3, remember Delta all these are complex numbers, the deltas and so are the, e to the power I, gamma 2, gamma 3 Et cetera. Okay? Minus ill expand about the first column, because that's where the unknowns are, plus e to the power gamma 2. If I expand these and simplify, I get values Delta 2, E power I Phi 2, plus Delta 3, e power I Phi 3, plus Delta 4, e power I Phi 4, plus Delta 1, equal to 0.

Where, Delta 1, is based on the first position, minus Delta 2, minus Delta 3, minus Delta 4, this is what I get, if I expand this. Okay? I get this in this form, what does this form remind you of, remember these are all complex numbers, does that look familiar, if this looks like the loop closure equation, for a four bar, with fixed length Delta one and the other three links, Delta 2 ,Delta 3, Delta 4. Okay? So if I take Phi 2, as my free choice that becomes the input, to this 4 bar, do you see that, so this is ,so in the first position it is Delta 1 plus, Delta 2 plus, Delta 3 plus, Delta 4, equal to 0. That’s my loop closure equation, then I give everything a rotation, Delta 2, I rotate by Phi 2, Delta 3 then rotates by Phi 3 and Delta 4 rotates by Phi 4 and still the loop is closed, which is just like your four bar, this is a different four bar, this is your four bar and therefore we are not talking about the 4 bar that we are trying to synthesize. Okay? So this is, basically the compatibility condition, can be thought of as a four bar, satisfying this loop closure equation, where the lengths are basically Delta 1, Delta 2, Delta 3 and Delta 4 and all these Delta's are based on known quantities, so let's write that.

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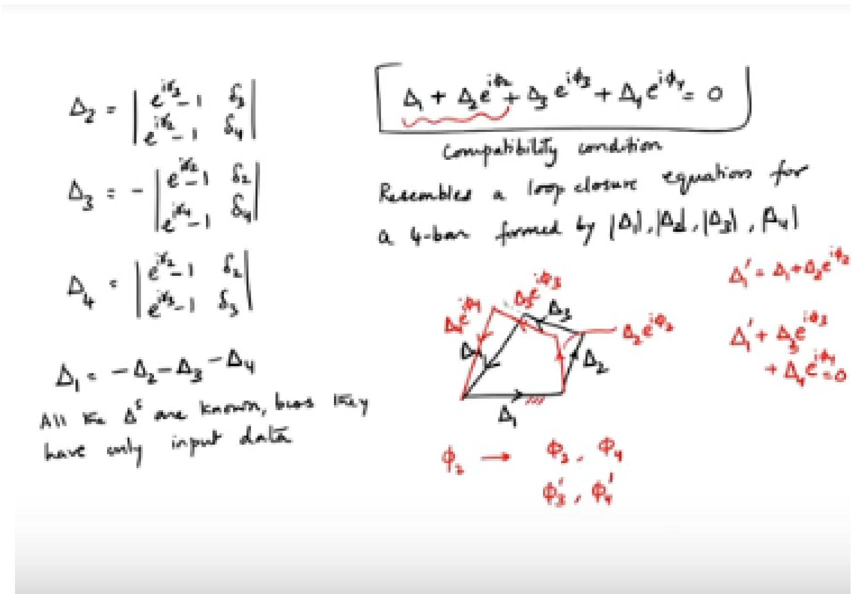

So Delta 2 is, these are basically the cofactors, so Delta 4, Delta 3 equals and Delta 3, sorry, Delta 4, and Delta 1 equal to minus, Delta 2, minus Delta 3, minus Delta 4. All the deltas are known, why, because they contain only input data, all the gammas and deltas are known, so all four deltas are known, so this condition, this Delta 1, plus Delta 2, e power I Phi 2, plus Delta 3, e power I Phi 3, plus Delta 4, e power I Phi 4, equal to 0, is the compatibility condition, what that says is. I can choose this Phi 2, Phi 3, Phi 4, have to satisfy this condition, I can choose my Phi 2 and then my Phi 3 and Phi 4, get determined by this condition, so I cannot use, any random group of Phi 2, Phi 3, Phi 4, for my solutions. Right, for a particular Phi 2, Phi 3 and Phi 4, should satisfy this Condition, in order for me to have a solution for Z 1 and Z 2, that's what this essentially tells me. Okay? So, this resembles, a loop closure equation, for a four-bar, formed by, Delta 1, Delta 2, I can, you know mod of, if I take those as the link lengths. Right, so in the first position, I have Delta 1, plus some Delta 2, plus Delta 3, Delta 4, that is equal to 0, ok now I rotate, if this factor I rotate, by some amount. Okay? Then this vector becomes, Delta 1 remains the same, it's the fixed link, of this 4 bar, you see there is no fiber, you know there's no angle ,associated with that, it's depends on, I can think of that as a constant and then, this can be thought of as. Okay?

So this would be, Delta 3, e power I, Phi 3, this would be, Delta 2, E power I, Phi 2 and this would be Delta 4, E power I Phi 4, where the angle is from the hood. Okay? So, I can think of it in this way, so I can use this, so it's essentially solving for the other two angles in the 4 bar. Okay? By now, if you don't have that code, you probably want to create that code, for this is the position analysis code for a 4 bar, to find those angles and to draw the 4 bar in that position. Okay? Because, that's the reason, you're going to use it again and again this code. Okay? For this, you can also look at this is, because Phi 2 is your input, your free choice, I can also use the same code that I wrote for my other one, that d1, plus d2, plus d3, for the fixed pivot. Because this vector, I can take it and I can call it repeatedly, I can check compatibility by doing that, because this then becomes a known quantity. Right, so I can look at this as, Delta 1 - ,equal to Delta 1, plus Delta 2, E power, I Phi 2, so I can say Delta 1 - Plus, Delta 2 ,E power I, Phi 3, plus sorry, Delta 3 plus, Delta 4, E power, I Phi 4 equal to zero, which is what ,I used for the previous, for my, where I specified the fixed pivot ,remember I had to solve a transcendental, transcendental equation there. So, you can repeatedly call that for each Phi 2, that's another way you could do that, but I think it's useful to have this 4 bar position analysis code. It's useful for you to do that. And the serksa command makes it really easy to write, so you solve this compatibility condition and then so for each Phi 2, I'll get what? I will get a Phi 3, Phi 4 and Phi 3 -, Phi 4 -. Why? Because of my two assembly modes, for the linkage ok, because of the two assembly modes for the linkage I'll get these sets ok, so if assembly mode 1, if it's 5, 3, 5, 4. Assembly mode 2, I get 5 3 -, 5 4 -. Okay? So, then once I get this, I can use, two of these equations, two out of the three equations, because now I'll have this specific, phi2, phi3, combinations that will work, that will satisfy the compatibility condition. So I can take any two of these equations, plug in D, Phi 2, Phi 3, or depending on which equations I choose and find the z1 and z2.

Then it becomes just a linear system, the earlier, we were able to look for the two position problem. Right, we were able to pick Phi 2 and Phi 3, as free choices and solve for Z 1 and Z 2 sorry for the 3 position problem. Okay? For the 3 position problem, we picked Phi 2 and Phi 3, as free choices, made this a linear system of equations, in Z 1 and Z 2. Now I can't choose arbitrary Phi 2 and Phi 3, I can choose Phi 2 and Phi 3, such that it satisfies that compatibility condition. Okay? This means essentially solving for that for that, so specific values, says specific combinations, will give me solutions for this, Phi 2 how many choices do I have? I have infinity of choices, so again, I can technically solve the fourth position problem, I do have infinity of choices, so I should have infinity of solutions, so I will just show you.

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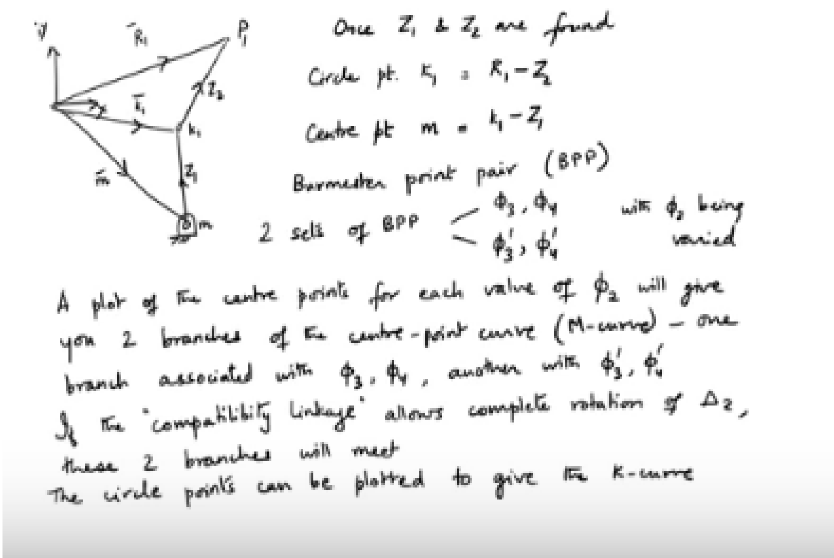

So this is my coordinated system, I have this point be, I have solved for Z 1 and Z 2, I can basically find, so I'll call this Point, circle point in the first position, so I will call it K 1, this is the fixed point, remember M for metal Punk, so it is the middle Center point. Okay? So then k, is the circle point, so I can, so this, vector M will define that and k1. Okay? So I can, my circle point, this is our r1 write, which is known, my first position, my first precision point. This should be p1, it's a circle point K 1, is defined as R 1 minus Z 2. Okay? R 1, minus Z 2, is K 1 and then my center point, moving pivoted, my center point or fixed pivot, M is, K 1 minus Z, all these are vectors serviced. Okay? So this set this combination of K and M, it's called a Burmister point pair. Okay? So VPP, Burmister Point because, it says that, for a specific server, so if I find the locus, of all the points k1. Okay? So see I found out, z1 and z2, I have solved it, for different values of Phi 2. Okay? I have taken Phi 2 as my free Choice, absorbed the compatibility condition, so I have sets, Phi 2, Phi 3, Phi 4, I have several sets of those, using that, for each set, I can find a z1 and z2. Correct, so from that, if I find, so now I have the locus, if I, if I plot all those points. Okay? I get a locus of all the possible circle points. Okay? For which I can hit these four positions and for each of those circle points there's a specific center point, which is determined by this relationship, so I can choose my circle point, I can, so once I draw the curve of all possible circle points, once I choose a circle point. Which means, I specify my K 1, then M becomes completely determined, because M equals, K 1 minus z1, have solved for Z 1 and Z 2 Okay? So for each circle point, there's a corresponding center point, or the other way around also, I can choose, so I can choose, my fixed pivot instead, I can choose my center point in the fixed plane, in which case, I will know exactly where my circle point is going to be, once I plot the locus of this point so. So I'll just show you, so a plot of the. Okay? Let me just write this down and then. Okay? So you get two sets, of the Burmister point pairs. Why two sets? Because again 5, 3, 5, 4, 5, 3 - ,5 , 4 -, so for two sets, one for Phi 3, Phi 4, 1, 4 Phi 3 minus, 5, 4 minus, with Phi 2, being varied. So for each Phi 2, you will get two sets of Burmister point pairs. So a plot of the center points, for each value of Phi 2, will give you, two branches of the center point curve, one for, you know, the compatibility equation in mode one, the other for the compatibility equation in mode 2, Center point curve is called the M curve. Because, one branch is associated with, 5, 3, 5, 4, another with, 5, 3 - ,5 ,4 -, so if the compatibility linkage of that, allows complete rotation of the Delta two, then the two branches will meet. Okay? And I'll show you that; don't confuse the compatibility linkage, with the linkage that you are trying to design. Okay? So and similarly, you can plot thee, the circle points can also be plotted, to give the circle point curve, k curve. Okay?

So every point, on the center curve, or the M curve, is a potential fixed pivot. Right, every point on the M curve, is a potential fixed pivot, therefore now you have an infinity of solutions, as long as it lies on that curve, you pick a point on that curve ,there are an infinity of points on a curve. So any point, that you pick, can be, is a potential solution for the fixed pivot, once you do that, the corresponding circle point gets fixed, so you don't you cannot pick, any point on the center curve and any point on the circle point curve, you can choose one or the other, because, only then your compatibility condition will be solved. Okay? So then you can form diet, for the diet for one side, similarly choose another center point on the curve, find the corresponding circle point ,you have a second dial, you can form your 4 bar. Okay? So let me just show you.

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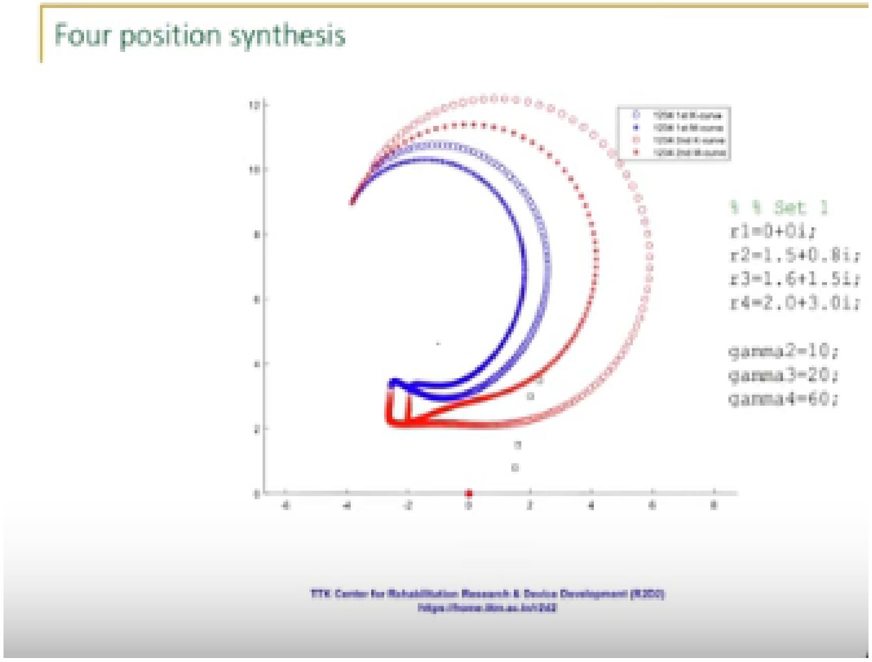

So you can write the code for this, but this is, I have four positions. Okay? 1, 2, 3, 4 and therefore the corresponding rotations Delta 2, Delta 3, sorry, gamma 2, gamma 3 and gamma 4, this one with the circles is my k-cup, that's my circle point curve, that's why I put circles, I wouldn't forget, the other one, with the stars is my, is the locus of my fixed pivots, this is only for, one mold of the compatibility linkage. Okay? Let me see if I have the other one, this is for the other mood ,so this is the second M curve, second k-cup, this is the first m curve and first k-cup. So what, these corresponds to 5, 3, 5, 4. The m curve and k curve, this is for fight, so this is for 5, 3, 5, 4, in the compatibility equation. This is for Phi 3 -, Phi 4 -, same Phi 2, Phi 3 - , Phi 4 - and the compatibility equation and if you look at it, if I plot both on the same, you can see that they meet, because this particular linkage is capable of full rotation. Okay? Now to create my linkage, I can't do it just with the, what I have here, because I don't know which point corresponds to what, but essentially I could pick, one of the one of these is the fixed pivot, but I'll have to look, to see which one of these circle points corresponds to that fixed pivot, so I can choose anything on this, the one with the Stars right, that gives me all the choices for the fixed pivot .okay? So for one diet, I can choose a fixed pivot on this doesn't matter, I can choose anything on this, find the corresponding circle point. Okay? And then if I connect it to the first position that gives me my first dial, right so I think I have, so here.

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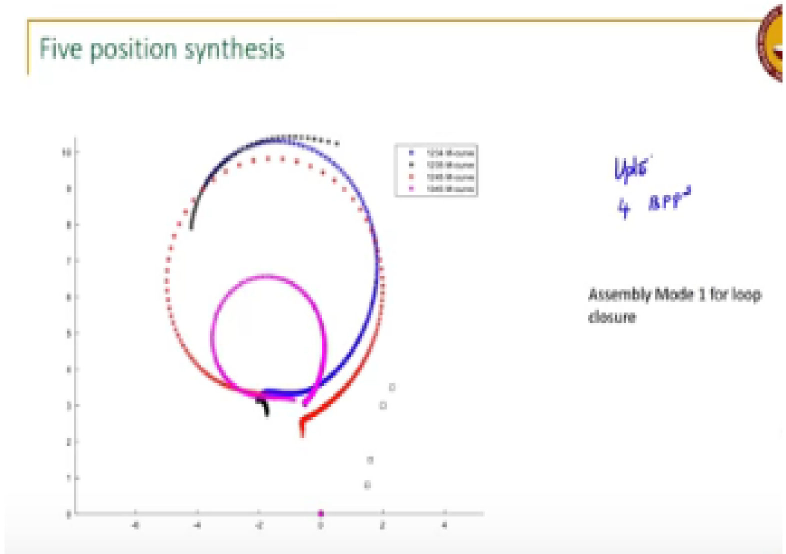

I pick, this is my one fixed pivot, I found this is the corresponding circle point, this is my first position so these squares are, P 1, P 2, P 3 and P 4, corresponding to R 1, R 2, R 3, R 4, ok so these are the precision points, p1, p2, p3, p4. Okay? So I can pick one diet, then that becomes my, so this becomes my Z 1, this becomes my Z 2. Remember it; your diets go from the fixed Pivot to the coupler point. So what is my coupler here? This link is my coupler link, so the other diet that I have chosen is this one. Okay? So I have chosen another fixed pivot here, its corresponding circle point is here. Okay? So this becomes my second diet, so this could be Z 3, Z 4 and so this becomes my linkage, this is the, so this, this triangle will be the coupler link. Okay? Maybe I should make this also blue. Let's do that, this, this and this. Okay?

So this is how I get an infinity of Solutions, yes No, so you can choose, so here, so I can choose my deities anywhere, I, I can choose one diet from because, it's only the corresponding circle and center point that matters. So for the 4-bar, for one died, you have an infinity of solutions, therefore you have an infinity square for the four bar. Okay? So this is the, the coding takes a While, so it goes by in a flash in class, but so, anyway you can do, that yeah, so that is your fourth position, so this can give you a clue, so if you if I want to do five position synthesis. Okay? I essentially have no free choices, but this should give you a clue, so if I take positions one, so I'd have five positions, 1, 2, 3, 4, 5 and if I look at, I take them four at a time, say 1, 2, 3, 4 or 1, 3, 4, 5 ok and I plot this and if I can find an intersection, of the M and K curves, then that is a potential solution. Okay? There's a lot of mathematics and this thing, I've just plotted some, so no this is just both of, this is for some other set of conditions, so they can look very different, so the first one was for certain, this is set 1 and this is just for a different precision points and rotations of the coupler plane, so you can see here in this one, they don't meet. Okay? Which means the compatibility linkage does not close for certain positions, so gamma that Delta 2 may not make a full rotation, in the compatibility linkage you can't close it for all values of Phi 2. Okay? So there will be there's a gap ,in the previous case, where it was able to make a full rotation those two curves met, they cover, in this case that's not done and again this is another set of diets ,so here so you can get some funky looking, that'll be this thing and this will be the coupler play, this triangle is the coupler play, so five position synthesis essentially, you can take so I can take the 1, 2, 3, 4, 1, 2, 3. So I can try to find intersections and see if you know I can, so you can get a maximum of four Burmister point pairs, so four real solutions for diets, so those diets can be put together in 4c ,2 ,so six linkages you can get by up to, up to four Burmister point pairs. Because when you solve for the beaters, you will get like a quadratic equation and he can have four real roots maximum, so you will have 0, 2, or 4, solutions for the five-position problem. Okay? So you can get four Burmister point pairs, for this resolving this five, with five position synthesis okay.