Theory of Mechanisms

Lecture – 2

Links, Pairs, Kinematic Chain; Planar Mobility Criterion

Last class we looked at kinematic pairs and we'll continue with, so kinematic chain is made up of links connected by kinematic pairs, essentially.

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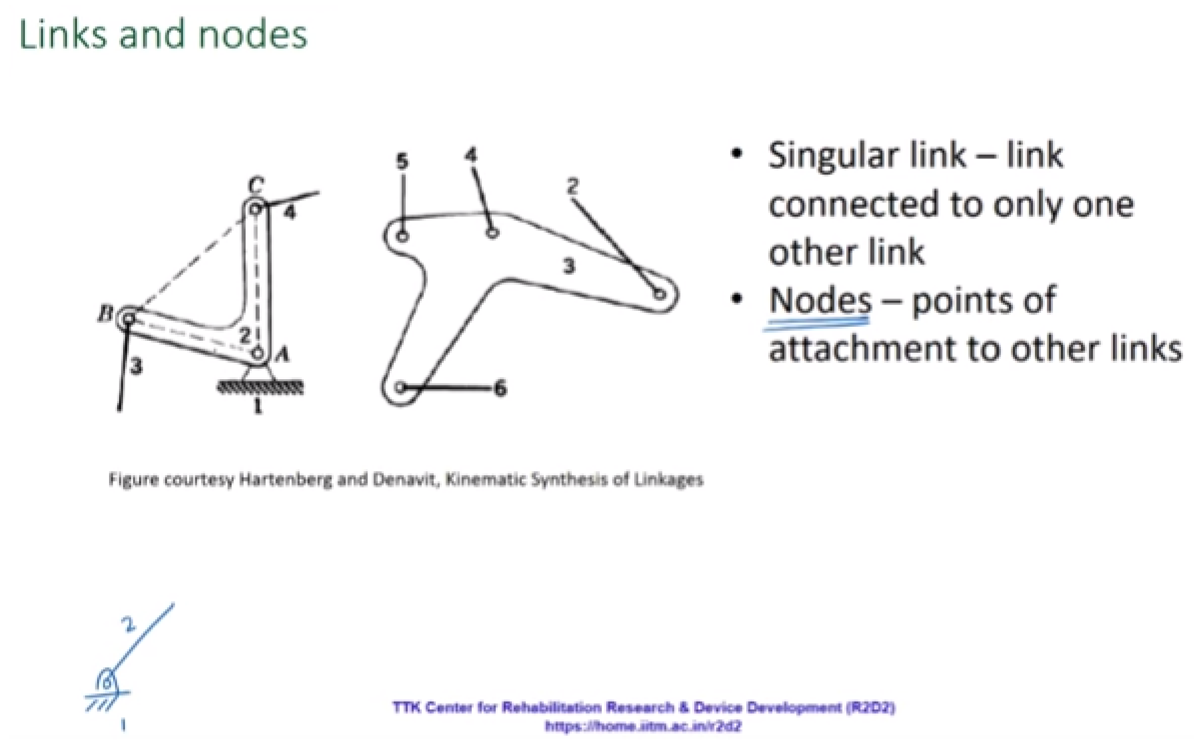

So a link, links are the rigid bodies and the points of attachment to other links, are called nodes so nodes. So nodes are the points of attachment to other links. If a link is only connected to one other link, so say, you have a link, that's only connected, so link to, is an example of a singular link, it's at attached to only one other link. Okay? So, here you can see, here is a link, which has four attachments, so it has four nodes, this link has three nodes. So links can also be classified based on the attachments to the other links.

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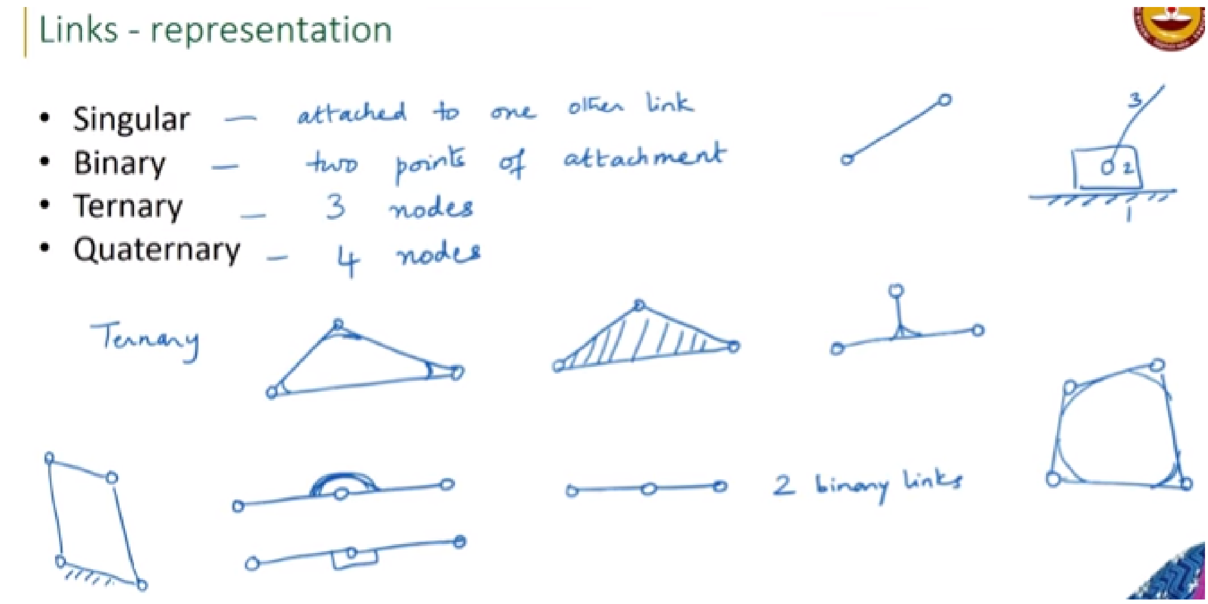

So singular link attached to one other link, a binary link, which you'll find to be one of the most common, is attached to, it has two points of attachment. So links may be shaped, you know, in real life, links may have complex shape, but as far as the kinematics are concerned, we are only concerned with, how they are attached to other links. So a binary link would typically be represented like this, so, binary link, which has two, which is connected to two other, links, by revolute joints, would be represented like this. So it has two nodes, two points of. A ternary link similarly has three nodes, quaternary four nodes and so on. So if you have a block, that is sliding, with respect to one link and is attached to, another link, through a revolute joint, the block is still called a, 'Binary link'. It has two points of attachment. 1 is the prismatic pair, one is the revolute pair.

So two, is still a binary link. So in a kinematic diagram, this would be a binary link. A ternary link would be represented in the following ways. So you could have, this and you have typically have to show, when you have, when you want to indicate it's a single link, you either show it in terms of welds or you shade the entire link. We will see later, why it's not always necessary, in the case of a triangular link like this. But this is especially important, if you have a ternary link like. You would normally interpret this as two binary links, unless it has this weld. In that case it could be like this or like this. So it has to have that weld to show that it is part, the nodes could be on the same line. Okay? This is different from, this is true binary links, if the weld isn't present, present, it's interpreted as two binary links. These kind of links a ternary link, without, all three nodes on the same line can also be represented like this. Okay? So these are simplified representations of links for the purpose of kinematic diagrams. Similarly a quaternary link, may be, if I want to show a quaternary link, I would have to show. In this case, it's especially important, because otherwise it could be represented, it could be mistaken for a four bar linkage, linkage. Okay?

So these are the link representations that we will use, in simplified diagrams. Actual shapes of links, could be, Considerably, different. So you can see here, this is a mechanism. For first of all, a lot of the things that we are doing initially, in the first part, of the course or in the majority of the course, will be planar mechanisms, Where all the points on the mechanisms move, in parallel planes. So I can have most real-life mechanisms, will have a 3D configuration. Okay? because, you need to be able to take loads in all directions. So you will rarely find mechanisms that are in a single plane. Even if you look at a folding chair, you will have the same mechanism, duplicated on the other side. Right? With a common joint, So here if you look at this, all these links, this link, this link, this link and this link, so there are four links here. They're all what kind of links? Binary links, you can see, that this one has, two nodes, similarly this. But their shapes are considerably different. But if I draw a simplified representation, all I would do is, just use the nodes, basically and draw it as in this case, this will be my mechanism; say this is the fixed link. Okay? So that would be the representation of this mechanism, But every point, regardless of its shape. Okay, on the link, every point, on every link, will move in parallel planes, as the mechanism moves, that's what makes this a planar mechanisms. And therefore you will only count, the links in the plane, you're not going to be counting these, extra links, because that will. So the only thing responsible for the kinematics of this mechanism in the plane, is, this set of links. So you only call this, a four bar.

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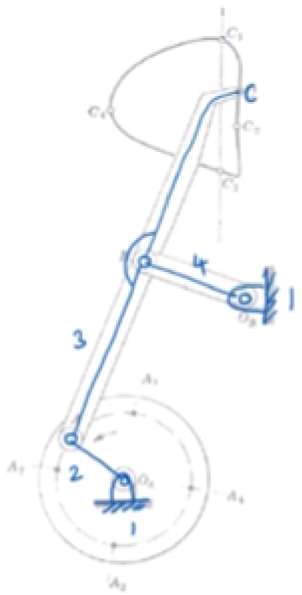
Draw the kinematic diagram



igure courtesy Hartenberg and Denavit, Kinematic Synthesis of Linkages

So the kinematic diagram is basically a simplified representation. So here is, you know, it's similar to what I showed you here, something a real-life mechanism, that has, links of various shapes and so if I want to draw the kinematic diagram of this, so in this case, since you should all be familiar with this, I'll just go ahead and do it .So you will represent the fixed pivot, like this, then my first link ,even though it's a circle here, the node, it's a binary link, the node is here, so I would just represent it in that fashion and then the next node is here, so, this is my link and then, this is another fixed pivot and this is my, fourth link. Okay? The fixed link is designated one, usually. So one, two, three, four. I have four links. And typically, if you have a point of interest. Okay? If you are interested, so for instance this mechanism, is used to, advance a film strip. So you, it's a four bar linkage, but really the point of interest, this point, C. Okay? Because you are interested in the path, that it follows. Okay? In this case, what it does is, you have the, the earlier movies, used to have these film strips, which will have the holes. Okay? So this mechanism, as this rotates, it will latch on to the hole here, pull it down and come out of it and then go in. So it will advance the mechanism, in that, it'll advance the film strip, in that fashion. Okay? So since this is a point of interest, typically, in the, kinematic diagram, you would also show the point of interest, but to indicate that, it's not a link, again, you will show that, it is part of, link three. You will show it in that fashion. And you don't, you also don't, put an open circle there, because that would indicate, a node for a revolute joint. So this is the, proper way to draw kinematic diagram, so you draw the revolute joints. Okay? So.

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Closed kinematic chain

- Kinematic chain- series of links connected by kinematic pairs
- Closed chain – every link is connected to at least two other links
- Chain consisting of only binary links – simple chain

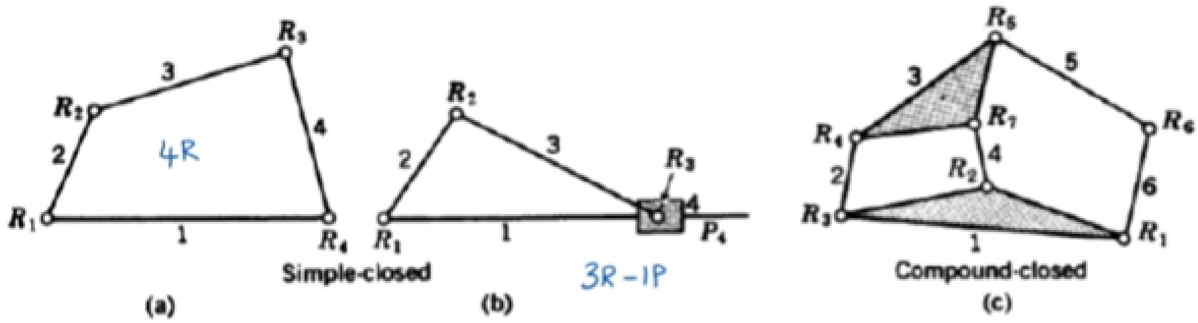

FIGURE 2-26 Movable closed chains.

So basically kinematic chains, are constructed by connecting links, with the kinematic pairs and you could have an open kinematic link. Okay? Or a closed kinematic, sorry, open kinematic chain or a closed kinematic chain. Closed kinematic chain, every link will be connected to, at least two other links. Okay? That's what distinct. So there will be no singular link. In an open chain, as we will see here. You will see that, there are two singular links here. Okay? So if I take apart a four bar and just leave it, that would become a, an open kinematic chain. So a closed kinematic chain will have, every link. So there will be no singular links, in a closed kinematic chain. Okay?
So, and for a lot of Analysis, if you consider a chain, that contains only binary links, we call it a simple chain. Okay? If, it has. So for a, slider-crank, so this is your typical four bar kinematic chain and a closed kinematic chain, becomes a mechanism, when you fix one of the links in the kinematic chain, when you fix one of the links, then, you can constrict, then it becomes constrained motion, between some input and some output one or more inputs, can be more than one, for a single output. Okay? So for a, so this is called a four-hour kinematic chain. You have four revolute joints, in this kinematic chain. This would be called a three r,1p, there are three revolute joints and one prismatic joint. In a compound kinematic chain, closed kinematic chain, you would have links, that are, higher than binary links, as well. If it's only binary links, if it's a singular, I mean, if it's a kinematic chain with a singular link, it's not even close to kinematic chain. Only binary links, it's a simple chain. More than binary links, it becomes a compound closed chain. So the third example, there is a compound closed chain.

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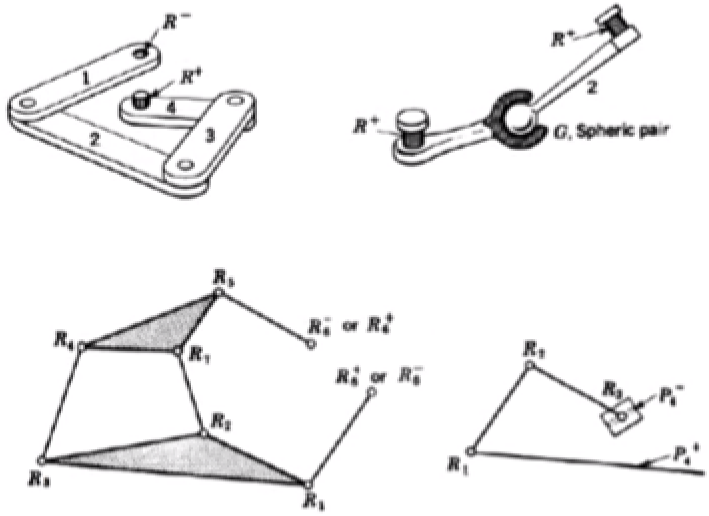
Open kinematic chain



This is your, these are examples of, open kinematic chains.

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Mechanisms

Closed

- Kinematic chain with one link fixed
- Mechanism with only lower pairs is called a linkage

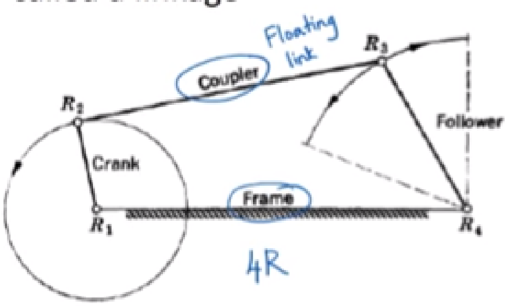
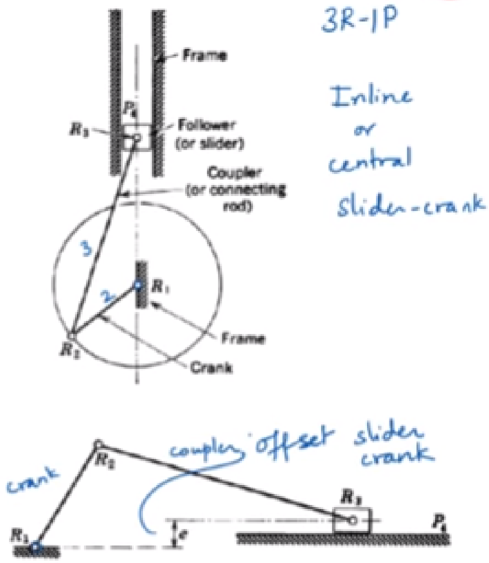


Figure courtesy Hartenberg and Denavit, Kinematic Synthesis of Linkages



So a closed kinematic chain, with one link fixed, forms a mechanism. So in a 4-bar mechanism, which is one of the most common mechanisms, we'll encounter. The fixed link, is called the, frame, your input is typically called the crank. So, the input can be, so the crank can sometimes be used, to refer to a link, that can make a complete revolution or sometimes it's used to refer to the, input link as well. Okay? Whether or not, it can make a complete, term. And the output is usually called the, follower and the link, so the crank and the follower, are both, fixed, pivoting about points fixed to the ground. Okay? In the case of, the coupler, this is a floating link, we call this a floating link and it is the link, that connects the, crank and the follower, the input link, in the forebar. Well I shouldn't say input link, because sometimes you could have, a mechanism, that has the coupler as the, input link. This mechanism here, it is what, what is, what is known as, an inline or a central slider crank. So this is a, so this is the, four bar is typically referred to as a 4r mechanism, 3 R, 1 P is your slider Crank. Okay? So again you have the frame, 1 link which is fixed, then you have a crank, you have a coupler or connecting rod and you have the follower, in the case of the slider crank, is your slider block. Okay? And if the rotation, of the club crank, is in line with the, path of the slider, then it's called, 'An inline or a central slider crank'. If the two are offset, if the part of the, slider is offset, from the point of rotation of the crank, then, this is called a, 'An offset slider crank, offset or eccentric slider crank'. Okay? Again this is, the crank coupler and the slider block.

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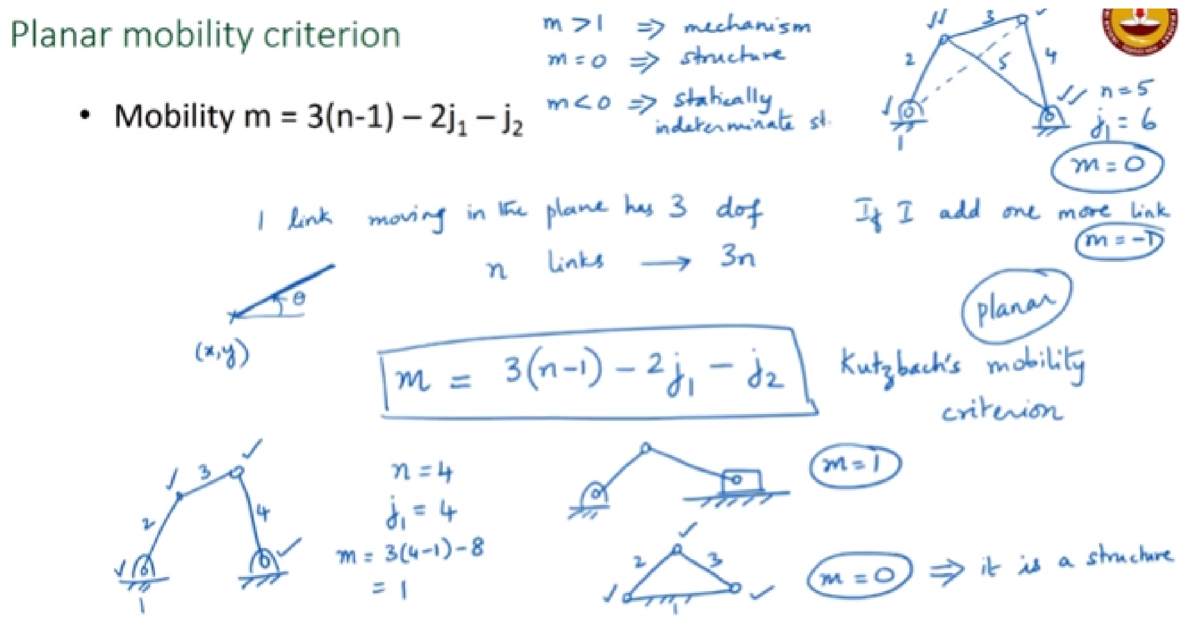

We will move on to the, mobility criterion. So, you have a set of links, connected by joints. Okay? You know what, the connectivity of the individual joints are, for this system, for this kinematic chain, that you have created, for the mechanism that you have created, by fixing one link in the kinematic chain, how do we predict the mobility of the mechanism? So let's look at the planar mobility criterion. If I have, one link moving in the plane? Okay?

Two, completely specify the motion of that link. How many degrees of freedom do I need? One link moving in the plane, I have a link moving in the plane. I need how many degrees of freedom? Three degrees of freedom. Because I need the x and y coordinates of one point and the orientation of, one link, in the plane, typically that's what I need, for that. What if I take, these two coordinates, x1 y1, x2 y2? There is a constraint. So I have four Quantities, but they the distance between those two points remains the same, that's a constraint, on the link. So again, I only need, three independent quantities, to specify the, location of the link. So one link moving in the plane, has three degrees of freedom. Now my kinematic chain, okay. First before I make the chain even, I take n links. Okay? And independently, if I have these end links, moving in the plane, how many degrees of freedom will I have? 3n. For my system, I have 3n degrees of freedom. Now I start connecting the links by joints. Okay? So suppose, I have, I connect the link by a joint, which has, which is a single degree of freedom joint. Okay? The joint allows only one degree of freedom, which means, it's taken away , two, okay. Before, before that, in a mechanism, I have to fix one link, so once that link is fixed, it loses all its degrees of freedom. So if I start looking at the mobility of the mechanism, I had 3n, now, because I've fixed one link, I have 3n2, n minus one. Okay?

Now, for every joint, that I had, every single degree of freedom joint, that I add to the mechanism, I am taking away, two degrees of freedom. Okay? Because the joint will only allow, one degree of freedom, so that's how I get the 2j one, single degree of freedom joints. And then if I add some two degree of freedom joints, like say, a cam joint. Okay? Then, that will take away only, one degree of freedom, because the cam joint allows, two degrees of freedom, I can have sliding and rolling. So this is how I get the mobility criterion, which is called the, 'Kutzbach's Mobility, Criterion'. And this is the, planar mobility Criterion. We'll come to the spatial Criterion, later. Because we're assuming, so because we are starting off by assuming, that it has only three degrees of freedom, in the plane. Okay? So now if we look at some, common examples. First let's start off with the, four bar. Okay? So I have the fixed link, as one, I do the kinematic diagram. Three and four, one, two three and four, four links. Now, each one degree of freedom joint, I designate, with a say, tick mark. So there's one between 1 & 2, 2 & 3, 3 & 4, & 4 & 1. So my N equals 4, J1 equals 4, so my mobility is, equal to 1. Similarly, for a slider crank. It's no different, because, the fourth joint, is still, say a freedom joint, the planar joint. So I still have, m equal to 1, for the, slider crank. Suppose I only connect 3 links? 1, 2, 3, say 1 is fixed, I have, what do I get for this mobility? I get 0. Mobility equal to 0, for this, for this mechanism, implies that, it is a, it is a, structure. So it cannot, the links cannot move, with respect to one another. Okay? So that is why, for a ternary link, even if I just indicate, as 3 links, it does not matter . Because it's already a structure, there can be no relative motion, between the links. But it's good practice, for you to either, shade it or use the welds. Okay?

Same four bar, if I add another, linked like this? Okay? I have one, two, three, four, I had a fifth link there. What happens to It? I have n equal to five, joints, one, here how many joints do I count? There's a join between two and three, and between two and five. So I have to count, make sure I count, two joints there. Here there's one, here again I have two joints. So if you have P links, meeting at a single joint, you would count, P minus one, joints there. Okay? So you have to make sure, that you get the right amount of joints. So J 1 is, 1 2 3 4 5 6 and you get mobility equal to 0. If I add one more link? Okay? Then I will get, I will actually get, mobility equal to minus one. So say I add, another link like this, I will get mobility is minus one. So this indicates that it is a statically indeterminate structure. So mobility greater than one, implies it's a mechanism, mobility equal to 0, implies it's a structure, mobility less than zero, implies, it's a, statically indeterminate structure.

So you need more than the, static equations of equilibrium, to solve for, such a system.

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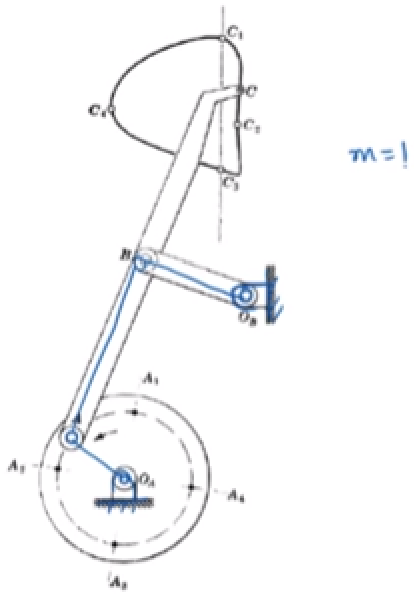
Find the mobility – film forward mechanism



So again, all of these, you would first do the, kinematic diagram, and this is nothing but you can see it's only a four bar, so obviously, its mobility will be, one. Okay?

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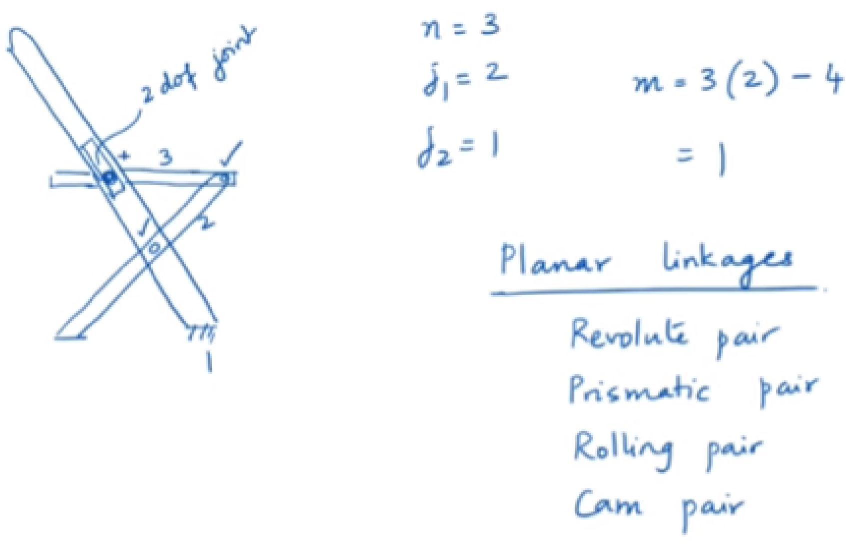
Find the mobility – folding chair



It's not very sensitive, so excuse my drawing. Because it's not letting me raise part of the, line, this. But if I have a folding chair, which has a pin that's attached to the seat. Okay? That can slide in the slot, in this back, this backrest link. Okay? What would be my mobility? So let's consider, say this link, I fixed this link. Okay? Usually when you hold the chair and then, you try to fold it. So I would have, this would be link 1, link 2, Link 3. Okay? The joints, this would be a revolute joint, revolute joint and this would be, what kind of a joint? 1 degree of freedom or 2 degrees of freedom? It's a 2 degree of freedom, it it's the pin in a slot, it can both, slide, it can rotate, as well as slide. It can be replaced, you can add another link, to make it all lower pairs. But the way it stands, this is a two degree of freedom joint. So say, you denote that by. an X, so now you have n equal to 3, j 1 equal to 2, J 2 equal to 1. So what is my mobility? So I get a, single degree of freedom, mechanism. Okay? So you need to be able to recognize, the connectivity of the joint correctly. The pin in the slot, is something that, throws most people off. That's why, I did this example. Okay? That's a joint with two degrees of freedom. For most, planar linkages, the joints that you are going to encounter are, your revolute pair, prismatic pair, rolling or a Camper. Okay? Planar you can think of as, Prismatic, plus rotation, like in your drafters, right? There, these are the joints that you will encounter in, planar mechanisms. Because for instance, a screw pair, the rotation is happening about one axis, but the translation happens about, if is observable from a different plane. Okay? So if I place the screw joint like this, I'm not going to be able to see the, Translation, I can only see the rotation. Similarly if I am looking at only the translation, I can't see the rotation, from that same plane of observation, are not part of, a planar mechanism. you will find screw joints in many applications, but they will be used for, things like a one-time adjustment.

So if you look at a pair of pliers, you know cutting pliers, right? Many of them will have a screw joint at, so the pliers if you look at it, it's essentially a four-bar linkage. But you will have one, screw joint here, which will basically adjust, one of the fixed pivots. So it can change, how it grips, the gripping. So that screw joint, you would not consider, as part of the mechanism itself. Okay? That's a separate thing, that is used only to adjust this, planar mechanism. So even though the joint exists, in the planar mechanism, it's not part of the kinematics, of that four bar. So you have to be able to differentiate, between, which joins belong to, the mechanism that you are analyzing. Okay? So now let's look at, okay, exceptions to the mobility criteria. So as with any rule, there are exceptions.

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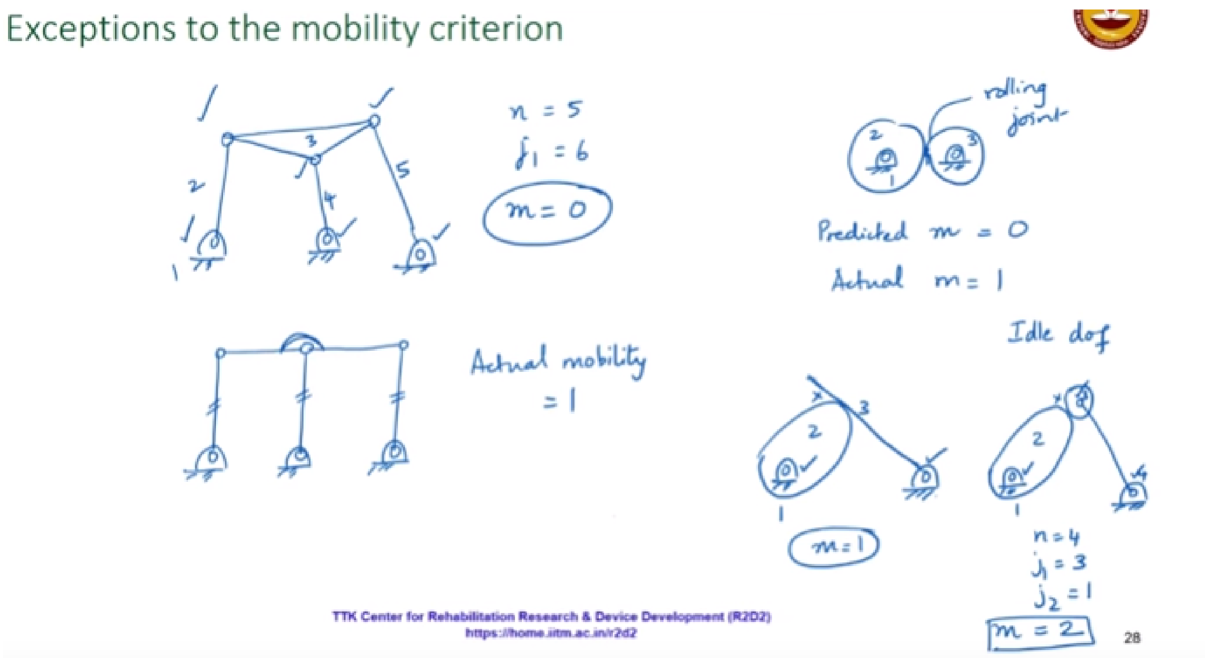

So if you look at, say, I construct a mechanism like this. Can you tell me what its mobility will be? I will get a mobility of, n equal to 5, J 1 equal to 6 and get a mobility of, 0. Okay? That means it's a structure. Now suppose I change the form? So I keep, I make these links, equal. Okay? I change the form of the mechanism, I make these links equal, this would be one link. Now what happens? I can actually move this mechanism, right? The mobility equation still tells me, that the mobility of this is, zero. But actual mobility, equals one, for this mechanism. Okay? The reason for that is, the mobility criterion does not and that is possible, with this mechanism, a mechanism of mobile, sorry, a mobility of one, is possible, only because of the geometry, the special geometry of this mechanism. Because, all the links are equal. Okay?

So if there are special geometry considerations, you will find that, sometimes the mobility criterion will not apply. Okay? So, you will have, similarly you have two friction discs. Okay? You have say, a rolling contact. Okay? Here. If you do, if you use the mobility Equation, what happens? What does it predict the mobility today? Predicted M, will be zero. But you know that, this friction, can transmit motion to the, other disc. This is, the actual mobility is one. Again it doesn't take into, the fact that, the way these disks are located, the geometry, is what is responsible for, the actual mobility, so actual mobility equal to one. So there may be exceptions to the mobility criterion. Sometimes, sometimes, you could have a predicted mobility, that is greater than one, so say you have a camp. Okay? And say, I put a follower, a rocking follower, like this. Okay? Then I have, one, two, three. So I have a camper here, I get a mobility of, one, for this. Suppose the same thing, I change the structure, so you know, I don't want, too much wear on, just one part of this, follower. Okay? So I modify that, to say, that I'm going to have a, roller here, instead of just this link, directly in contact with the camp. Okay? Because the roller will keep spinning and I will have different points of contact. So practically, that's a better way to handle wear.

So in this case, one, two, three, four, I have one, I have that, this, and this, what is my mobility here? I have n equal to 4, J 1 equal to 3, J 2 equal to 1, gives me a mobility of, two. Just plug it in. 9 minus 6, minus 1, I get a mobility of 2. It's pretty much, to say. If you look at the input and output, you'll get the same, result, in the output. So if I move this camp, this, this predictable motion, of the output link, here. So what is this additional mobility that we have here? It is the angle of the roller, above that. Which for your practical application does not matter. But it's an existing, degree of freedom, in the mechanism. So mobility basically tells you, how many inputs do you need, to get your predictable output. So if a mechanism has, mobility equal to 1, it means, one input, will give you, a predictable output. Here this mobility equal to 2, tells me that I have to give 2 inputs, but it turns out that's, not really true. I don't need to give, an input to the roller. Right? Because it doesn't matter, I mean, the, the roller has a degree. So this is still, the actual mobility of this mechanism, is still 1. With one input to the cam, I get a predictable output, of the follower, the additional degree of freedom, does not really matter to me. Okay? So when you have degrees of freedom like that, which is, encountered typically, with rollers and things like that, those are called, 'Idle Degrees, of Freedom'. Because, they don't contribute to the input-output, relationship, they don't affect, the input output relationship. So any degrees of freedom, that don't affect the input-output relationship, are called, 'Idle degrees of freedom', and they will give you they will over predict the mobility of the mechanism.

So in some cases, in this case, the mobility was under predicted, for this. Because the actual mobility is 1, the equation tells you it's 0. In this case, the equation tells you it's 2, when the actual mobility is still 1. So you have to be careful, to see actually, what's happening with the mechanism. So the mobility criterion, is a first check, on how things are going to move, but then you would have to actually, look at how things are.

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Grübler's criterion

- Any linkage can be represented as a kinematic chain composed of links and simple hinged joints
- Simple hinge – a revolute joint connecting only two links

$$m = 3(n-1) - 2j_1$$

n total no. of links

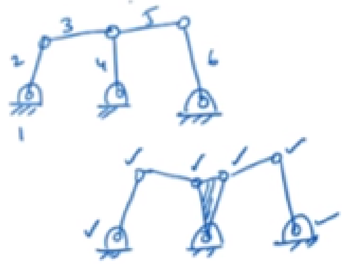
n_2 be the no. of binary links

n_3 be the no. of ternary links

⋮

n_i → no. of links with i nodes (hinge joints)

$$j_1 = \frac{1}{2}(2n_2 + 3n_3 + \cdots + in_i)$$



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So now let's do a few things with the mobility equation. Okay? So you have, you can represent, any linkage. So if, if a mechanism has only, lower pairs, we typically call it a linkage. We call it a mechanism, it's called the more general, more generally it's referred to as a mechanism, if it also has, higher pairs, in the kinematic chain. But we are only using lower pairs, we tend to call it a linkage. And even in a linkage, you can always, represent any linkage, as a kinematic chain, which has only, links and simple hinge joint. So a simple hinge joint, is a revolute joint, that connects, only two links. Okay? So it doesn't say, that the links have to be binary. Okay? It only tells you that, so it, so for instance, instead of creating a mechanism, if I have a mechanism like this, if I want to represent this only with simple hinge joint, so this is a six bar mechanism. Okay? I haven't put a weld here. Okay? So that is, these are separate links. One, two, three, four, five, six, you will find that it has mobility, equal to one. This is a six bar mechanism. I want to represent this, with only simple hinge joints, then what I would have to do? See here I have, what is known as a, this is a multiple joint, I should circle this, I have three links coming together at this joint. So when I do my mobility calculation, I would count two links here. Okay? I don't want to do that, I want it to have, only simple hinge joints, in which case, I would redo this mechanism as, this is what I would do. I would convert this into a, ternary link. So that now, all my hinge joints, are simple hinge joints. Okay? So I convert, out of these three binary links, I convert, one into a ternary link, so that I have only simple hinge joints. The reason I'm doing this is, because, we will now, do some things with the mobility equation, where we make the assumption that, we are, converting any mechanism, in any linkage, into something, with only simple hinge joints. Okay? So we will do that.

So we take the mobility criterion, we see that. So if, if I'm only dealing with simple hinge Joints, I can write the mobility criterion as, 3 into n minus 1, minus 2 J 1. Okay? I am saying, I don't have any two degree of freedom joints, in the mechanism. Now, let's say that, so if n is my total number of links, let n2, it's a closed chain, so I don't have any links, that are singular links. So n2 be the number of, binary links, n3 be the number of, ternary links and so on. So n i is, number of links, with i nodes. Okay? Each of those links, has i nodes. Okay? And I am considering, that all these nodes have, hinge joints. Okay? Let's say, they have, simple hinge joints. So I can write my mobility equation, as, so if, if this is the case, then, what is my j1? I can express j1, each simple hinge connects, two links. Okay? So my j1 will be, half of, 2n2 + 3n3 + ini. Okay? Because, binary link, will connect the two other links, so if I account the joints, I will get, this will be the total number of joints.

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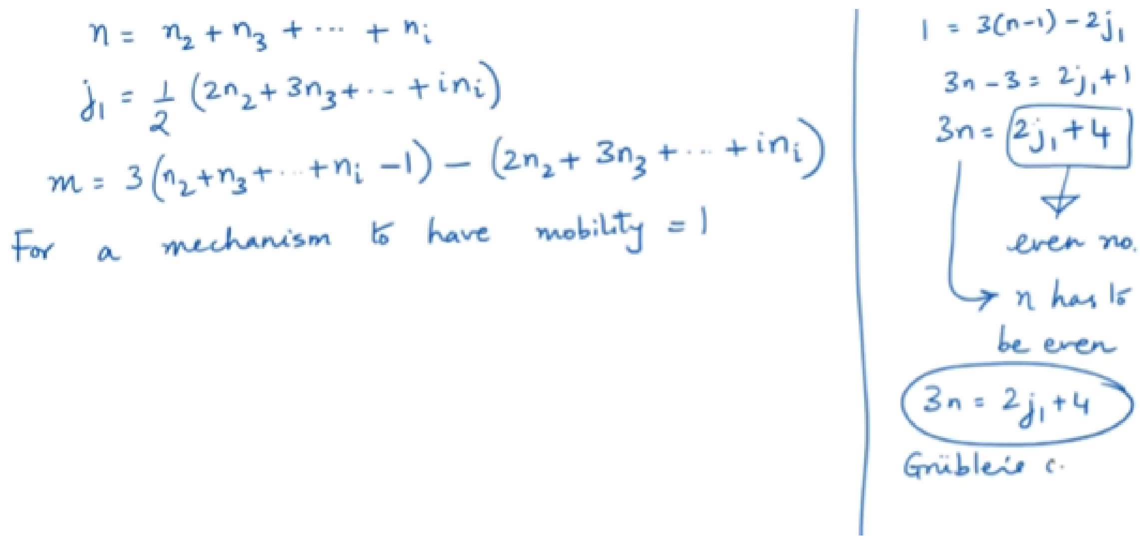

So now, let me just, n equal to n2, plus n3, plus ni, I have J1, equal to 1/2 of, 2n2 + 3n3 + ini. Okay? So this gives me, m equal to, you know I can write, in this form. 3 into - 1, - 2 J 1, so I get, 2n2 + 3n3 + ini. So what do I get? Okay, o this is them. Now for a mechanism, to have mobility one, which is the most practical application, you have one input, you want a predictable output. I can write the original mobility equation as, 1 equal to, 3 into n minus 1, minus 2 J 1. So I get, 3 n, equal to, 4 plus 2 J 1. Let us do that right now. 3n minus 3, yeah, so 3 n equal to, 2 J 1, plus 4. So first off, this tells me, J 1 is an integer. Okay? So this tells me, that so what kind of a number will this be? If it's a valid linkage, this is going to be an even number. 2 J 1 plus 4, is an even number, which means, n has to be, even. Okay? This 3 n equal to 2 J 1, plus 4, is called, ‘Gruebler's Criteria’. So the first thing that we see from this, Gruebler’s criterion is, if you want a mechanism with mobility 1, the number of links have to be even. Okay?