


**Lecture – 20**

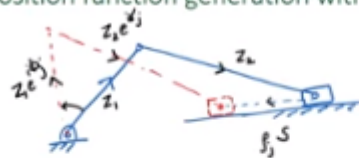
**Theory Of Mechanisms**

**Coupler Curves - I**

So, we looked at, the fourth position motion generation and we'll briefly look at, just for completeness.

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Four-position function generation with a slider-crank 



$\phi_j, \rho_j$  are specified

$$z_1(e^{i\phi_j} - 1) + z_2(e^{i\gamma_j} - 1) = \rho_j s$$

Compatibility eqn

$$\begin{vmatrix} e^{i\phi_2} - 1 & e^{i\gamma_2} - 1 & \rho_2 s \\ e^{i\phi_3} - 1 & e^{i\gamma_3} - 1 & \rho_3 s \\ e^{i\phi_4} - 1 & e^{i\gamma_4} - 1 & \rho_4 s \end{vmatrix} = 0$$

$$\Delta_1 + \Delta_2 e^{i\gamma_2} + \Delta_3 e^{i\gamma_3} + \Delta_4 e^{i\gamma_4} = 0$$

Solve  $z_1$  &  $z_2$

We'll look at the case of, a slider-crank, synthesizing that, for four positions, of the slider. Okay? So I can again use the dyad form. So say, this is. Okay, so you have  $z_1, z_2$  and if this is the, the path, is given by a vector  $s$ , I can specify the displacement of, the slider as,  $\rho_j$  of  $s$ . So this will be  $e^{i\phi_j}$ ,  $\rho_j s$  and then, this will be  $z_2 e^{i\gamma_j}$ . So again I can write my loop closure, my loop closure will take the form,  $z_1 e^{i\phi_j} + z_2 e^{i\gamma_j} - \rho_j s = 0$ . Okay? Where  $s$ ,  $s$  is the vector, specifying the path, the vector specifying the path, so I just need a scalar, to say how much it has, displaced along the path. Again, it's in the standard form, for the four positions, you will have to again solve the, compatibility equation, which we saw earlier. So you'll have, hmm, row  $j$ , is just a scalar, that specifies, how much along the path. So it's a factor, because you know this is, from, from the initial position, how much it has displaced, a scalar times, the vector, that designates the path. So I have the compatibility equation, just as we did earlier and this will be,  $\rho_2 s, \rho_3 s$ , instead of the  $\Delta$ s, that we had for the motion generation. Okay? This is equal to 0. So if it solves the compatibility equation, so for combinations, of  $\phi_2, \phi_3, \phi_4$ , so, if I choose  $\phi_2$ , as my free choice, solve for  $\phi_3$  and  $\phi_4$ , again you will get the same 4 bar loop closure, compatibility equation. You will get  $\Delta_1 + \Delta_2 e^{i\gamma_2} + \Delta_3 e^{i\gamma_3} + \Delta_4 e^{i\gamma_4} = 0$ . So if you solve this equation, then you will get, combinations, so you can again do the same thing that we did for this. Okay?

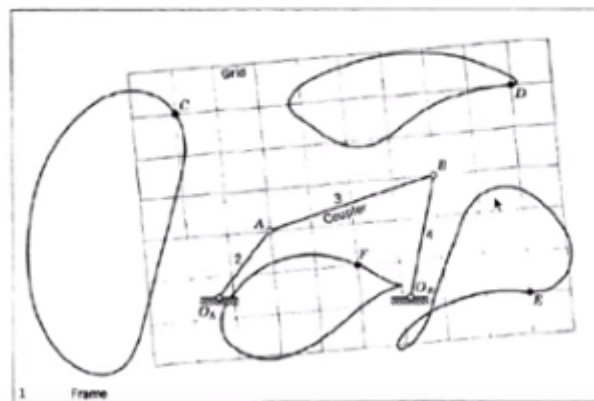
So, fourth position, function generation, with a slider-crank, is not, then.  $\gamma$ s are given, or in this case, actually the  $\phi$ 's may be given. So it depends, if you want to coordinate the input motion, either way, the form will be the same. Okay? In which case, you will take  $\gamma$  to solve, for  $\gamma_3$  and  $\gamma_4$  that may be more likely actually. Because usually, you will coordinate the slider displacement, so, yeah, in this case, okay, let's say,  $\phi_j$  and  $\rho_j$ , are specified, so you would actually, do this, in terms of,  $\gamma_2, \gamma_3$  and  $\gamma_4$ , yeah, that makes more sense. Okay? So you'll find the, what are the, angles of the, connecting rod or coupler, that will give you the, that will let you solve the, equations for  $z_1$  and  $z_2$ . Okay? So you'll solve, once you solve the compatibility equation, you solve for  $z_1$  and  $z_2$ . Okay? So this, so the dyad synthesis method, is very flexible, it can allow you to design, many kinds of mechanisms, including multi loop linkages, as we saw. So you can extend that method and use it for motion generation, function generation, path generation, prescribed timing, etc., lots of. And with an analytical method, you can build in other

things, into your design. For instance, for every mechanism that you choose, you can compute, the minimum transmission angle, for instance. Or if you want only a crank-rocker, you know, you can, when you have an analytical routine, you can go through, design choices, more quickly, setting it up takes a lot of time. But if you're going to do, you know, go through a number of these, then it makes more sense, to put in the initial effort, to use an analytical method, also it's going to be more accurate.

So if you're, if accuracy is something, that you're looking for, obviously the graphical methods are not very accurate and you're also limited, in the number of positions that you can synthesize for it. Again analytical methods also give you the, we saw with the blocks method, for instance, you can connect velocities, angular accelerations, etcetera. So if that was important, for their application, then obviously, an analytical method is what you want to go for. So as you saw, most of the analytical methods are based on the vector loop equation and the use of complex numbers makes that, very elegant in terms, because you can express rotations nicely, with complex numbers. So that kind of concludes what I wanted to do with, analytical synthesis. Now although we have also dealt with path generation, so path generation, also we saw that, it's, it's a matter of you know specifying, in fact, it's easier to do than motion generation and you can actually synthesize, for more precision points, because you are not worried about, the orientation of the coupler. But the next topic that we shall do is coupler curves. Okay? So coupler curves, are essentially you know, path generation.

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### Coupler curves



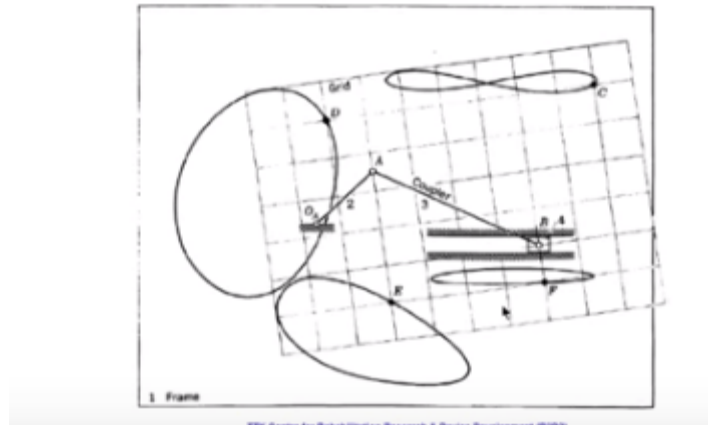
You are looking at the path of coupler points and how to use that, for your design. But they warrant, you know, separate treatment, because, you know, with the precision point synthesis, that we have done, we are only looking at specific points. But there are some characteristics of the curve itself, which can prove, quite useful, when you are looking at applications, and that's the reason we will look at coupler curves, in a little bit more detail. So if you look at the four bar, you have the crank, you have the rocker, there and you have the coupler, which is the floating link, the crank and a rocker are pivoted to the ground, but the coupler is, the one that is floating, we call it the floating link in the plane.

And so, that is really the most interesting link, in the 4 bar. Because the crank and the rocker, all points on them, trace only circular paths. Okay So there's nothing exotic there. But if you look at, so if you take, this, so if I have a 4 bar, like this,  $O_1a$ ,  $a$ ,  $b$ ,  $O_2B$  and I assume the coupler is, you know, this grid that is attached to a B. Okay? So if I take different points on that grid, as the 4 bar moves, they can trace some very interesting curves. So if I take this point C. Okay? I get like a sort of a kidney

bean shaped curve, something like this, little fancier, then I can do like a teardrop. So this point F, on this grid, as their linkage moves, trace the sky, trace is this kind of a teardrop and then I can have sort of a figure 8, with this point E, So you can have some very interesting, paths and you can use portions, of these paths, for very interesting applications. So that is the nice thing, about coupler curves.

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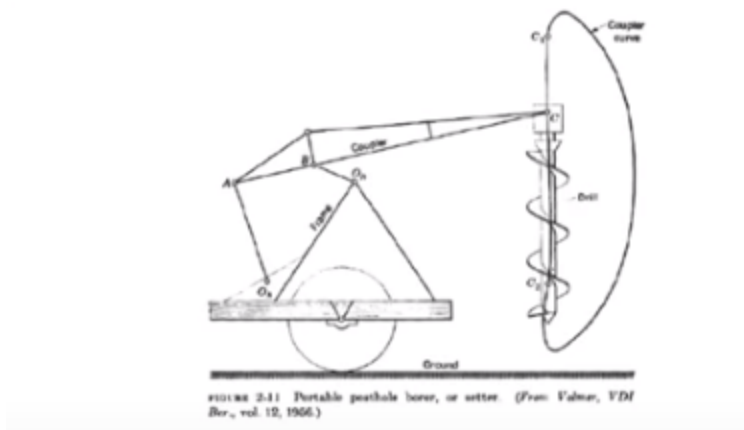
Coupler curves



Slider-crank, again, if you look at the Coupler, same thing, you have the grid, different points, you again have a figure eight, you can trace different paths.

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Coupler curves

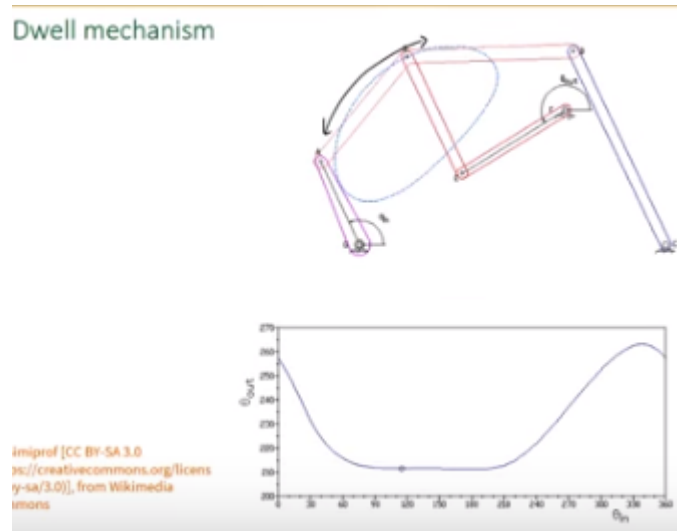


In many cases, the use of coupler curves, can enable you, there's a very special application of coupler curves, many cases they trace, approximate straight line paths. Okay? So in places where you may have had to use, see the problem with, a prismatic joint, is, the maintenance of it. First manufacturing it, to a certain level of precision, is difficult and then maintaining it, because if anything gets into it, the prismatic joint can really stick. Okay? Revolute joints are a lot easier to maintain. So if you can do

something with a linkage, with only revolute joints, that's a better option, than going to, a linkage with prismatic joints.

So early on, the fact that, some coupler curves, can trace approximate straight line paths, was used extensively, for designing linkages, where you wanted, you know, like for instance, this is a portable drill and this can actually, so it, this coupler point can actually, trace that ,straight line path. So they're using, a four bar, to do this. So this is the four bar, o a, OB, O a, a b, O,b and this is the point, that traces, an approximate straight line path.

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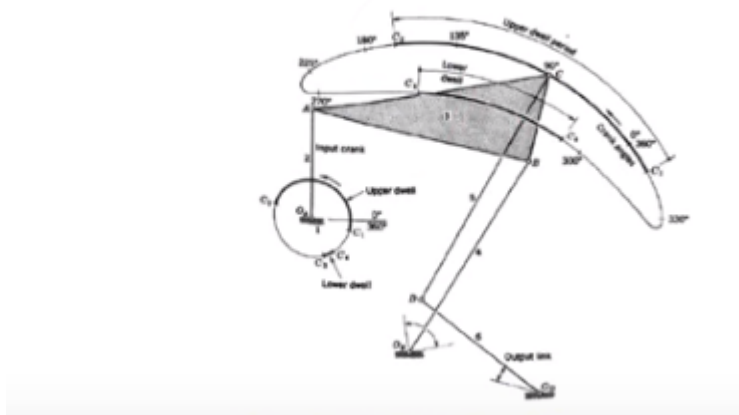
Then dwell mechanisms are, another common application, for coupler curves. So what is a dwell? Do you remember what a dwell is? Right, so the follower, you have, you have an input and an output, so, even with a continuous input, the follower, the output remains stationary, for a period of time. This is very useful, in assembly, your packaging or those sort of applications. Where you know, something comes on the assembly line, it stays for a little while, where some, some operation is done to it. Okay? You may have boxes moving and when the boxes at a particular location, they need to slap a label on to it. Okay? The dwell in the mechanism, gives you the time. So it's still, the bit, the boxes are continuously moving. Okay? You have a continuous input, the box comes, you know, at an instant, where there's a dwell in the mechanism, you put the label onto it, then it moves away and then it, you know. So it's, it's very useful for, those sort of applications, in the assembly line. And here is a nice animation of this. So we can use, the coupler curve or the coupler point, the coupler curve to design, these kind of dwell mechanisms. So here look at, you have theta in. Okay? And you have theta out. Okay? You see that, for a certain period of time, theta out remains more or less the same. Okay? This is, so if you look here, you can see this link, it doesn't move. When does that happen? Okay? You see here, that this link, this point is tracing the coupler curve. Okay? So D is a point, so this whole thing is a link, abd, is one link, as you can see, they're not two different links. Okay?

You have to be careful, that's one link. So D is a coupler point, on a B. Okay? So and B is tracing this path and it so happens, that this portion, this portion, I don't know if I can write, oh, okay, okay, this portion approximates a circular arc. So what happens? It means, it has a constant radius of curvature. So if I pin a link, this link. Okay? With a length equal to that radius of curvature, then when it comes, to that point. Right? This point E does not move and if E is connected to the output link, then the

output link remains stationary, as this point D is traversing that circular arc. Now we would have seen, dwell linkages, many designs based on cams, cams are a common way and it's a very easy way also, to introduce dwells into mechanisms. Again cams have the problem of, because it's a higher pair. Right? Maintaining lubrication in the joint is difficult. So if you have, you know and so wear can be high, in cams and which of course changes your output, for the mechanism. So if you can do it with a pin jointed mechanism, then, it's. Of course, cams are of course more compact. So it again depends on your, application. That's really where the type synthesis that we first talked about comes in. Because you have to consider, a lot more factors, when you're designing something. This will occupy a lot more space, obviously a linkage. But if, if you can use it, then, the coupler curve, gives you a nice way of, going about designing a dwell in the linkage.

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### Double Dwell linkage



Here's a coupler curve that actually can, do two dwells, it's a double dwell linkage. Same thing in this case, you have two portions. Okay? of approximately the same radius of curvature.

So with the, the dwell with this kind of linkages will not be like a perfect dwell. So there'll be some jitter, possibly, in the output link. It will not be perfectly stationary, but if that's acceptable for the application, then it doesn't matter, again depends on what the need is. Okay? So this is another good example of, using coupler curves. And this is something that, I mean, you could design, you know, you could, specify points, that would fall on, but you can't really say. See with the, if you do the precision point synthesis, I can specify points, but I can't really say, how the linkage will behave, as it moves between those precision points. Coupler curves give me a little more control, over that. Because the curve is a, continuous curve, so I'd, I have an idea of, how that is going to be, traversed.

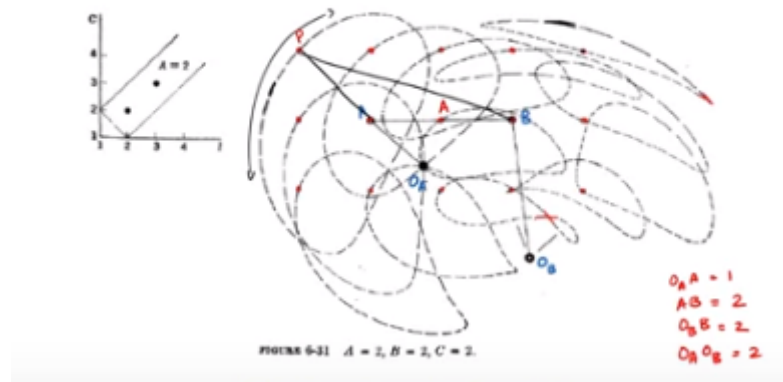
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follower length and  $c$  is your fixed, fixed link. So based on this, you can find, so the atlas will be something, a page in the atlas will be, something like this.

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### Hrones and Nelson Atlas of Coupler Curves



Okay? So you have to imagine, so if you look here. It's a little confusing. But, this is your, so when we talk about coupler curves, we refer to a B, as the coupler base. Because you are looking at other points, you are really interested in, so we don't talk of a B, alone, as the coupler, we call it specifically, the coupler base, because the point that you take, may be something. So if you look here, you can see a grid. So you can see equally spaced points. Okay? And then, it tells you, what kind of curve, that point, is going to trace, as the crank rotates, in the mechanism. Okay? So that's what.

So if I look at, if I choose, a and B, it's obvious, circular path. So they're not, the coupler curve for that is not marked. If I choose this point? Okay? And I construct my linkage, so I connect, if that's the coupler point I choose? Then what this atlas tells me is, as this linkage as the crank moves, this point is going to trace, this curve. Okay? And the dashes. Okay? You have these dashes that give you an indication, of the crank rotations, between the points. So if, so for, for equal crank rotation, Okay, so I think 10 degrees, yeah, so there for every 10 degrees of crank rotation, this would be the distance traversed by the coupler point. Okay? So that means, if I look at this curve, so if I look at this coupler point and I look at this curve, where is it moving, on which part of the curve is the point moving faster? On the top part, because it's traversing, more distance, for every 10 degrees of, crank rotation. So you also get an idea of, what the velocity, of that point is going to be. Okay? So let me show you. So there's a nice interactive Atlas, which is what I was, trying to download and install. Let's hope, it's done. Okay, so this tells you, that this is, these are the coupler curves, the crank is always 1. So OAA is 1 unit, A which is the coupler or the coupler base, AB, equal to 2 units, then OBB, equal to 2 and Oa, OB, is also equal to 2. You can say this is a Grashof linkage and the crank is the, smallest link, so it's a crank-rocker. So I can now try to scale it, so I can now measure this distance, measure the location of this coupler point P, from this scale it up accordingly. Okay? It'll be an approximation, but you can get, you get a pretty close, the coupler curve that you will get will be close to that. Now there are some special curves here, as you can see, some of them which have these kind of sharp, these are called, 'Cusps' or if it traces a figure 8 here, this is called a, 'Crew Node', that point, where it's crossing over itself. Right? That's called, so these are called, these are special points, called, 'Cusps and Crew Nodes'.