Theory of Mechanisms

Lecture 35

Balancing of Mechanisms using Springs

So last class we looked at essentially balancing the effect of gravity on the mechanism, using counter weights, Right. So that because the system will tend to go to its position of minimum potential energy, Okay, so by making the CG, of the system constant you know that then there is it will stay at whatever position it is, put in. so that was the principle behind doing the balancing, that was the that was how we achieved the balancing by keeping the CG up. so we did that for a single link and then we looked at doing it for the four bar linkage. Now using counter weights is one option, but it also adds mass to the system, adds you know increases the size, it also so it is not as desirable always to you do it using counter weights and today we look at doing similar kind of balancing, but using Springs, because Springs give you some more flexibility they don't add as much mass to the overall system, so they have certain advantages, they also have certain disadvantages, which we will look at.

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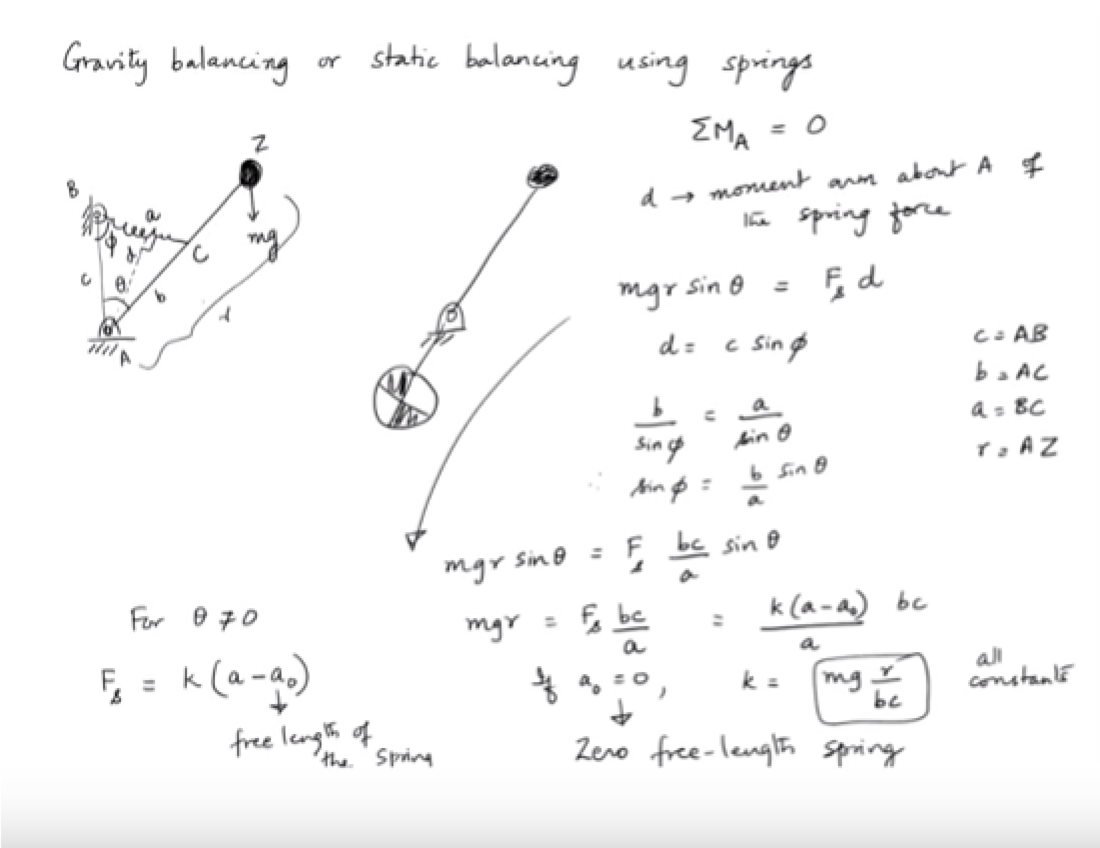

So suppose we have a link, so a rotor, like you know, we so we have a link or a rotor say with the, this is the mass center. Okay? Now and it is making a certain angle, theta with the vertical, Okay. Now I want to balance this link, Okay using a spring, Okay. So I, so essentially, what I need to do is this, weight exerts a moment about A, I'm trying to balance that moment using a spring you know using the spring force but I want to be able to do it for all theta. So the idea is if it is balanced, then no matter what the theta is if it is perfectly balanced, it'll stay at that position, that's what we did. So when we did it last time with the counterweight we essentially did that, so if I had a link like this pivoted about A, I would just add attached a counterweight to it like this, Right such that the moment due to this would be balanced by the moment the clockwise moment would be balanced by the counter clockwise moment of the counterweight. so same thing we are now trying to do with the spring, Okay. So if I see here, so let's let me call this distance b, this distance a, you know like we do in triangles, Right, a,b equal to yeah. So and so if I look at the moment, I want to balance the moment about point a, should be zero, Okay, so and let me call this distance AZ as r, distances is r, Okay so I have this is the Monday so I have four stupid and see and I have the momentum of this force the spring force let's call that D so D is the momentum off the spring force. so I have the moment to be balanced is mg r sine-theta Okay due to the weight and that has to be balanced by the spring force times D, Okay the spring force will act along BC. and let me call this angle Phi. Now D Okay, so D equal to C sine Phi, C is this length ab, ab, B equal to AC, we call this a, a equal to b, c and d sorry R is a to C. Okay now if I look at this triangle, I have B by sine Phi, equal to triangle ABC, is equal to a by sine theta. Okay by the sine rule, therefore sine Phi, is B by a sine theta and therefore in terms of theta, this equation now becomes everything in terms of theta, I get MGR sine theta, is equal to F s and D s, BC by a sine theta, Okay. So for theta not equal to 0 because then I can't cancel sine theta, I get mgr equals fs, DC by a. Okay, now the force in the spring, will be the spring we'll have some regular Springs, we'll have some spring constant, and it will be proportional to the, so a naught, is say the free length of the spring, Right. So I can write this as, this is equal to K into, a minus a naught, by a, BC, Okay. Now if I look at the expression for K, Okay so I want to find what would be the spring I need to use in order to be able to perform this balancing. if I can make the spring proportional to the length, basically the spring is linear such that F equal to K a, that is if a naught is equal to Zero, Okay so the spring force is proportional to its length, not do its displacement from its free length, so the free length is zero basically, if a naught is equal to zero, then I get K equals mg what do I get for K? so k a and a will go, Right so I will get mg R by B C, Right.

Now if you look at all this, this is all constants, Right so I could use a spring of constant stiffness, equal to this, provided its free length is zero. So there are some the springs can be wound in such a way that the initial tension it's proportional to its free length, that's essentially what we're looking at Right. so if when the spring you know when is that its free length, if it applies a force, which is proportional to its free length, so if F equal to KS naught f naught is equal to KS naught, that's the force it applies when it's then you have this condition or in other words you can the such a spring where a naught equal to zero, is called a zero free length spring. So that is the challenging, challenge when it comes to using Springs for balancing. You can only use these special Springs. So there are ways to achieve the zero free lengths, I'll just show you now how you can use a regular spring and make it zero free lengths, but it's that that's what adds complexity to balancing using Springs. You can perform approximate balancing with regular Springs, by using optimization and trying to find where to attach it, so that over the range that you are looking at your potential energy is fairly constant, potential energy fairly constant, implying that you know it will not move from that position because otherwise the system will seek if it's only under the action of gravity it will seek the point of lowest potential energy, that's what you, so when you save you were making the CG constant, all these mean the same thing, essentially the CG remains constant, the potential energy remains constant. Okay? We're only talking about balancing the effects, beyond these you know external forces that you have those you don't have controllable you're talking about you know if the system masses and you know so that any you're trying to basically balance out the effects of gravity on the system, Okay.

So this is said such springs are called zero free lengths Springs and so you can use a zero free length springs which has this spring constant, to balance a mass M that's located at a distance-r. so for a single link you can perform the balancing in this manner. so now how do we achieve this zero free? so most so either like I said you could have special Springs made but that's not always an option, **so** another way to achieve this is, to construct the spring like this.

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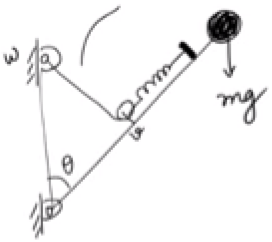
inextensible string attached to the spring



Choose a spring of stiffness $k = \frac{mgr}{bc}$

and place in outside the line wv

connecting the link & the fixed reference.

This is one way of realizing a

"zero free-length" spring

So if I have, I have the link like this and to a pulley. Okay? So now when it's folded, so when theta is close to zero, the string it's an inextensible string, attached to the spring, Okay it's an inextensible string attached to the spring, so what this sees is only the extension, s minus s naught, will be this distance, the a that we are talking about between those two points will be a minus a naught, of the spring that you are using it's only the extended part of the spring that comes into play, in this region. Okay but obviously it requires more effort to you know locate to attach a spring in this fashion, so that this is the mass, Okay so this is one way of achieving this zero free link condition using a regular spring. So this is an inextensible string attached do the string. Okay so this distance is the extension of this spring and the force will be proportional to that extension. Okay so this is one way of, so you choose a spring of stiffness, k equal to mg R, by BC, that is constant and place it outside the line let's call this WB, WB connecting the link and a fixed reference. So this is one way of realizing that a zero free length. Okay so you if you look at the literature there's a lot of literature on balancing, using springs and all that, exact balancing is only possible using zero free lengths spring. so in the literature on balancing you will come across this all the time, so in many cases they'll just assume that zero free links Springs are available or that you can create it, using some way of doing it, Okay there are one ones with you know you actually for more exact balancing you have to have like three pulleys to get it because of this angle that the rap angle so they'll take that into account and all that, so you can you can go if you're interested you can go look at how to achieve exact balancing using, but zero free length spring so in most literature on balancing they will only say the supplies with zero free length Springs and assume that all the springs that are being used are zero free length Springs. Okay so you will come across this in the, this is what zero free lengths Springs. Mean so it's a spring that's linear so it the it's proportional to its length the force that it produces is proportional to its length, not just its extension, Right Okay.

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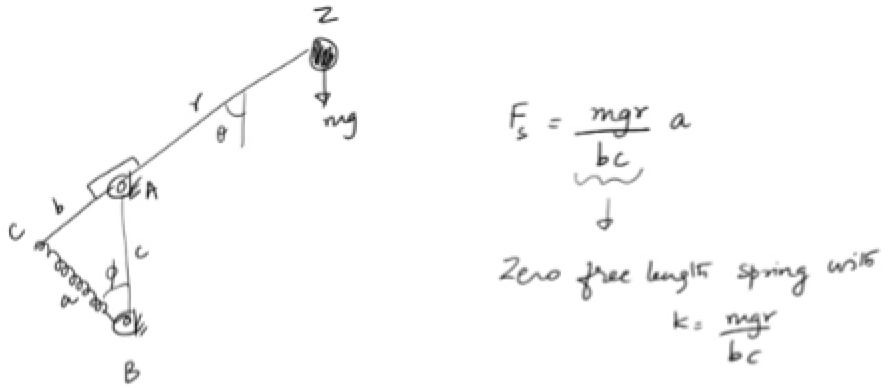

So, so you could achieve the same in, by other means as well, in the sense, you know, locating, single-link, AB, sorry CB, C. So this is R, this is B, this distance is C and this is A. Okay? And theta is the, angle with the, vertical. Of this, say, same thing, you'll get the same Result, if you do this, the reason, we'll see, how to extend it to, more than one link, with this. So some of the applications, of this kind of balancing, you'll see, any of you who have those desk lamps, which have the springs on them, so you set it at any position, it'll stay there? It'll have, you know, you, it'll have two degrees of freedom, generally. You can fold it, you can and you can set it at any ,you know there'll be, two degrees of freedom, for that and you can set it at an angle and it'll stay there it's quite easy to move, also that's an example of a balanced two link system. Similarly you know, this could be the case, for say, a microphone, where you're adjusting, and the position of that. Right? And it stays there, so those are all examples of where, spring balancing, is used. Okay? This kind of gravity balancing, so you change the position and it stays there, it doesn't just fall over. Okay? So again in this case, you, you can very easily see, you do the same moment balance, you'll find that, the force in the spring, should be mgr by bc into a. Which means, it should be proportional, to the length, a, which means, this should be the stiffness, it should be a zero free length spring, of this, with k equal to mgr by bc. Okay? So, you can extend this, you can extend this, so one way is, you know, you suppose, you have two lengths, so you have two degrees of freedom, you have a theta 1 and you have another link.

So in many cases, in robotic arms and all that. Right? You want; you don't want the actuators, to have to move the weight, of the arms as well. There will be some external load, but you want the actuators, to not have to do extra work, to actually deal with the, weights of the arms. So that's where you do the balancing, for the basic weights, of the links in the robot and then on top of that, you have actuators to, move them beyond that. You know, you may have external loads that you are trying to. So if we look at, this can be extended, so essentially here, you have a vertical fixed link. Okay? Or a link that stays vertical. Okay? Here and you have this fixed link and then you have a spring that comes from there, to balance this link. So now if you want to, balance more than one link, like this. Okay? So I may have a link here and then another link, So a two degree of freedom system. So should I do this?

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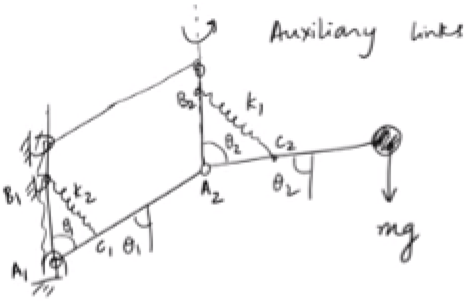

Moment to be balanced

$$= mg\,(r_2 \sin\theta_2 + r_1 \sin\theta_1)$$

$$F_1 \frac{b_1 c_1 \sin\theta_1}{a_1} + F_2 \frac{b_2 c_2 \sin\theta_2}{a_2}$$

$$= mg\,(r_1 \sin\theta_1 + r_2 \sin\theta_2)$$

This should hold for all θ_1 & θ_2

$$\therefore\; F_1 = \frac{mgr_1}{b_1 c_1} a_1 \quad \text{and} \quad F_2 = \frac{mgr_2}{b_2 c_2} a_2$$

Zero free length springs

So I have here, so say this is my fixed. Okay? I have a link like this and then, I have, something like that. Okay? And then, I have, so what I could do is, I balanced this link. Okay? Now I need another vertical link here. Okay? So one way to achieve that is to create a parallelogram. So I would do that. Okay? And then use another spring here, to balance this, so I have a degree of freedom here. Right? So this may be theta1, this may be theta 2. Okay? And so I can, so these are called, ‘Auxiliary Links’. So I attach these other Links, to create this kind of a parallelogram arrangement, so that, because this will always remain vertical, this link. Right?

Because it is, parallel to this, fixed link, so I can have. And I can also have, a degree of freedom, so I could do this, you know, I can and in each case, you know, start from the end and start, sort of find the, K for each of the springs, depending on what is beyond that. But I could also do, I could add a degree of freedom, in that direction. I can have rotation and it will still not change the balancing. So I have, say I have the stand, with these two links, the additional degree of freedom that I have, about this longitudinal axis, will not affect the balancing. Okay? So I can have additional degrees of freedom, in that direction, without affecting. So I can move this entire, so this is where, you know, robots etc, use it. Because they have, they'll have a degree of freedom, about this axis and then they will have manipulator arms, with the rotations about this axis. So this is in the plane and then I could rotate, that as well, perpendicular to this, about this axis and still maintain the balancing. Okay? So that’s, so like I said, there's a lot of literature on balancing, especially with the springs. So, those who are interested, can, look that up. Okay. So, so that's essentially what you would do. So if I have mg here, so this would be my, let's just call this B 2, C 2, E 2, this is my original a1, this is my B 1, and this is my C 1. Okay? So, say that so, this is Theta 1, this is Theta 2. Okay? What is the moment I'm trying to balance here? The same thing. So I would use, take this, find this spring. Okay? That would balance the moment, of this about A2. Okay? And then I balance the moments about A1.

So if I look at, the, what is my momentum of spring, about? find the moments about? Mg, so the moment to be balanced is, what is that equal to? mg, r 2, sine theta 2, plus, r 1, sine theta 1. Right? If I take? Where else have you seen practical Applications, of using Springs for balancing? Have you seen a car hatchback? Why is it so easy to lift? You'll see a gas spring, there? Okay? Otherwise it's, it's a lot of metal, for you to lift. Right? Reason it's easy to lift, is because of. So you'll see lots of applications, of balancing. What would be K 1 and K 2? F 1, b 1, sine theta 1, by a 1, plus F 2, using the same Notation, mg, r 1, sine theta 1, plus r 2, sine theta 2. So if this has to hold, for all theta 1 and theta 2. Okay? You want if you want balancing, you want this relationship, should hold, for all theta 1 and theta 2. Therefore, I can just say that, the sine theta term, sine theta 1 term should be equal and the sine theta 2 terms should be equal. So if I equate those, then it will hold for, all. F 1 should be equal to, mg, r 1 by b 1, c 1, a 1 and a2. Again, zero free length springs, directly proportional to, a1, directly proportional to the total length, a2, in each case. Okay? So you would use, zero free length springs. Instead of doing this, you could also, if you have seen the lamp, you will see that, both the springs are actually attached to the base. So this is, this is a bit cumbersome, because you know, you're attaching the spring to, moving links, ideally if you can attach, everything to the, base. Okay? That's better. So you will have configurations like,

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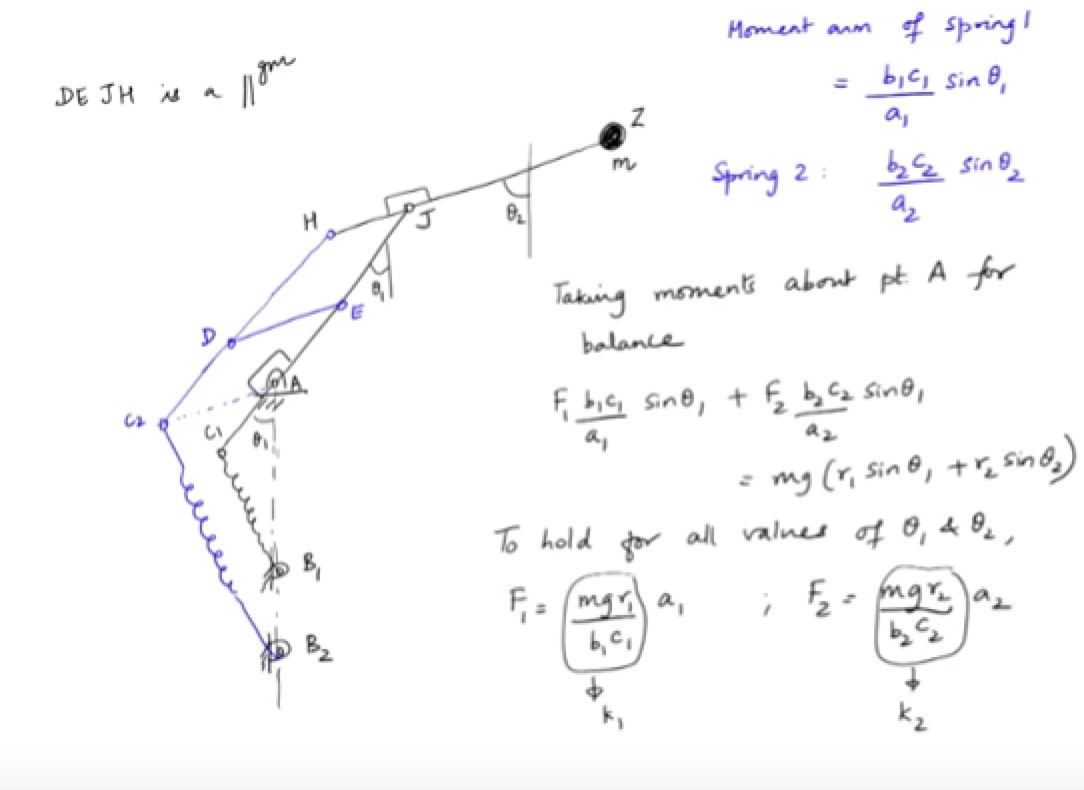

so there, now you would draw the, so I have my a, so I have these two links, and this is, b, let's call this b1 and b2. Okay? And here Okay? Such that, this would be, parallel to that.

So again, spring one, equals, b1, c1, by r1 sine theta 1, plus r2. Sine theta 2 and to hold for all values of theta 1 and theta 2, a2, b2. Okay? So this we'll be, k1, this will K2. Okay? Of the. Okay? Having to use zero free length springs, can make a design, quite complex. Because if each of these, you imagine, you have pulleys and springs, it can make the design quite, complicated. So, the usual way to deal with it, is to use, regular springs and then you get approximate balancing, so not perfect balancing, you cannot get perfect balancing, with regular springs. Okay? So you may not have perfect balancing, but you can get pretty good approximate balancing, with regular springs. Gas Springs also have, certain advantages, over extension Springs and mostly what we have used here, are all extension Springs, in this particular example. So gas Springs have, certain advantages, because they're very compact, there, but you will not get perfect balancing, with them either. So it's, but it's, you can get a pretty good approximation. Okay?

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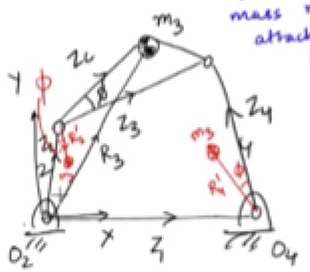
$$\therefore U_3 = m_3\left[Z_2 + \frac{r_c}{r_3}e^{i\phi}\left(-Z_2+Z_1+Z_4\right)\right]$$

$$= m_3\left[Z_2\left(1-\frac{r_c}{r_3}e^{i\phi}\right)\right] + m_3 Z_1\frac{r_c}{r_3}e^{i\phi} + m_3 Z_4\frac{r_c}{r_3}e^{i\phi} + \underbrace{m_3 Z_1 - m_3 Z_1}_{\text{add \& subtract}}$$

$$= \underbrace{m_3\left[Z_2\left(1-\frac{r_c}{r_3}e^{i\phi}\right)\right]}_{\text{equiv. to a mass } m_3 \text{ attached to link 2}} + \underbrace{m_3\left[Z_1 + Z_4\frac{r_c}{r_3}e^{i\phi}\right]}_{\text{equiv. to a mass } m_3 \text{ attached to link 4}} + \underbrace{m_3 Z_1\left(\frac{r_c}{r_3}e^{i\phi}-1\right)}_{\text{unbalance due to a non-moving mass}}$$



$$= m_3(Z_2+R_2') + m_3(Z_1+R_4') + \text{constant term}$$

So now you know how to balance a single link. Right? Yesterday what did we do with the 4 bar? We showed that, the unbalance, in the 4-bar, you know, due to the, the, the coupler, which is the floating link, its effect can be captured, in terms of, masses that are attached, to link 2 and link 4. Right? So I can attach a mass link, sorry, mass m3, at a specific location, to link 2, similarly. So I can capture the, effect of the coupler. The effect of the coupler is equivalent, to balancing a mass m3, that is located and moving with link 2 and another mass, moving with link 4. Now again, link 2 and Link 4, are Rotors, moving about fixed pivots. So I can do the same thing, I can now instead of, for this mass, so now I would have to take into account, the mass m 2 of link 2, in addition to, the contribution from link 3. The m3 located, find the combined CG, of those two, on link 2 and now that's just a single link, with a mass. So I can use a spring, to balance that. So to balance this 4 bar, I would attach a spring, to link 2 and a spring to link 4, zero free link springs, I can attach To, link 2 and link 4, to balance the Effects, of the motion and still get the same constant C G, so instead of using counterweight. So you can, what we have essentially done is, shown that, okay if I can balance a rotor m using counter weights m I can do that m using springs too. And for the 4 bar, essentially we've reduced the balance, in the unbalanced, is due to, some equivalent masses, on link 2 and link 4. Okay? The rest of is, it is of course, constant; I'm looking and balancing the moving masses. So if I do that, then, I can do the same thing. So I can attach a spring to, link 2, I can attach a spring to, link 4. Such that, it will balance this, combined mass.

So for link 2, it will be m2, plus, m2 located at whatever its CG is, just of link 2, plus this additional mass, m3 located at this. R 2 dash, at z 2, plus R 2 dash. Right? And for, link 4 again, I will take into account, its mass, m 4 plus this m 3 located at, z 1 plus, R 1 dash, R 4 dash, that I had here. So for each of those links, I would take those combined masses and then balance them, using a spring, like we did today, using a zero free length spring. In that case, then for the motion, of course, practical considerations, you know, if this thing is moving 360 degrees, how do you, you know, all those are there, so it is in the range, that you are interested in. Okay? So today when we looked at the single link, we were looking at theta, from 0 to 180, essentially. So there will be practical considerations, in every problem, which will, which is why, optimization is probably the, reasonable approach to take. The better approach to take, because, then you can put in all your constraints, with regard, you, you may not always, the spring that you find out, that you determine, is necessary for exact balancing, may not always be practical. Okay? In which case, you have to look at other solutions. Okay? So that's what, I wanted to, cover with respect to, gravity balancing, of mechanisms. Okay?