

NPTEL

NPTEL ONLINE COURSE

Theory of Mechanisms

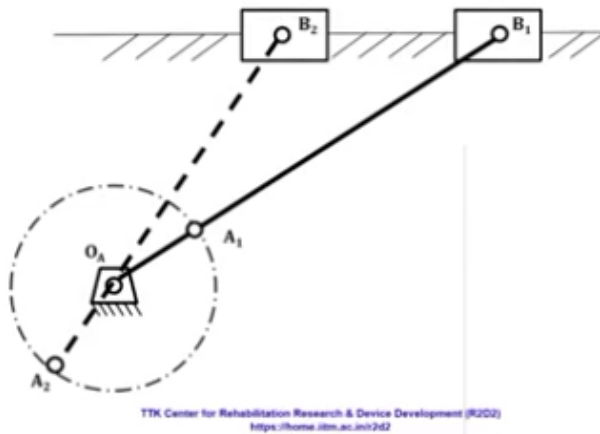
Quick-return synthesis-II, 3-position  
Motion Generation

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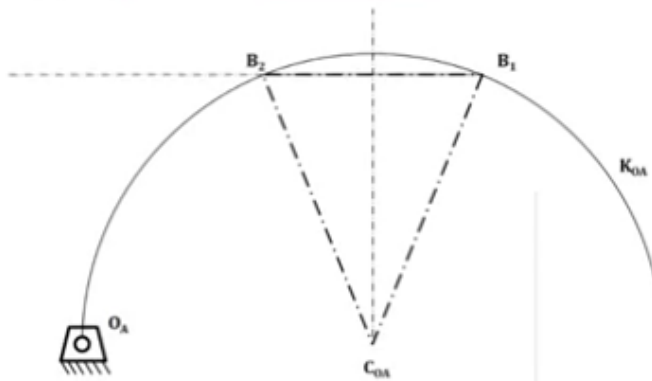
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Quick-return slider-crank synthesis



So last class we looked at the Quick-Return Synthesis, and for the Slider-Crank Synthesis, for a two position Slider-Crank Synthesis we saw that if you specify one of the three quantities either (Refer Slide Time: 00:36)

## Slider-crank synthesis: specified offset



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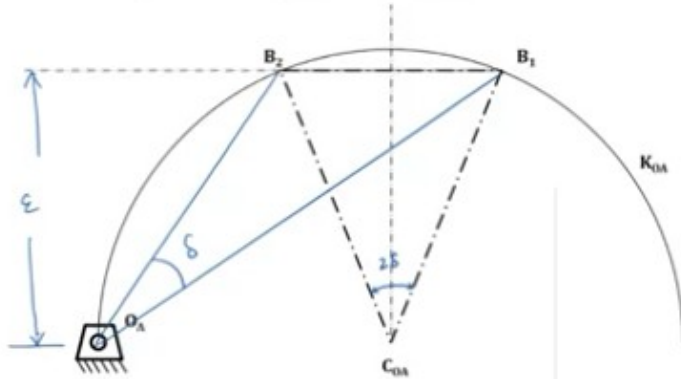


the offset or length of one of the links  $R_2$  or  $R_3$ , then you get a unique solution. So we looked at the case where the offset was specified, we looked at the other two cases today.

So essentially the fixed pivot lies on the circumference of a circle whose central angle is  $2\delta$ , where  $\delta$  is your construction angle based on the time ratio, based on the given time ratio, so then this called anywhere on the circumference of that circle, so this angle is  $\delta$ , and you can have any number of solutions but this, if this is the specified offset of the Slider-Crank, then you look at the point  $O_A$  in this manner, okay.

So today we will look at the, so from here I can now determine  $R_2$ ,  $R_3$ ,  
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### Slider-crank synthesis: specified offset



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✓  
 offset /  $r_2/r_3$

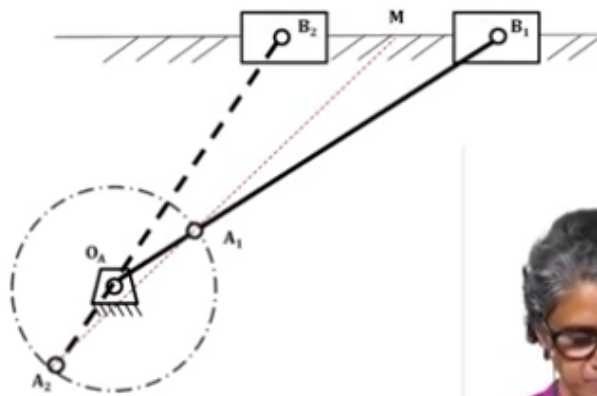
$\delta$  construction angle based on the given TR



so I again have  $OA B_1 = R_3 + R_2$ ,  $OA B_2 = R_3 - R_2$ , so I get a unique solution once I know, so what's given here is the stroke and the time ratio, stroke  $B_1 B_2$  and time ratio, and the offset is specified.

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### Slider-crank synthesis: specified $r_3$ (AB)



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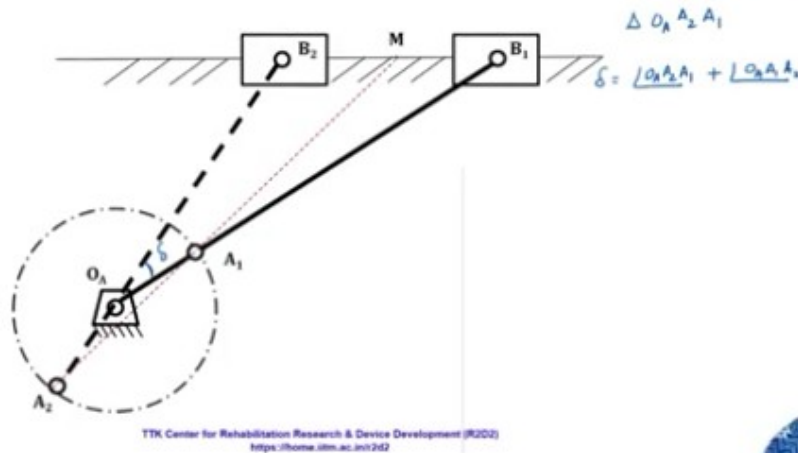
So let's look at the next case where you're given stroke, you've given the time ratio hence you know  $\delta$  and you're given  $R_3$  which is the length of the connecting rod, okay. If I look at the geometry, okay, I don't know the solution here but I know what it's going to look like in the two extreme positions, from the geometry I want to figure out how to construct this mechanism, so

if I join A2 A1 and extend it to meet B1 B2 at M, okay, I've joined A2 A1 the two extreme positions of the crank and extended it to meet this act M.

Now this angle, what is this angle? That angle is just delta, right, if this is OA, then this angle is delta. Now if I look at triangle OA A2 A1, okay, delta is my external angle, so delta = some of the interior opposite angles which are OA A2 A1 and OA A1, sorry, A2, (Refer Slide Time: 04:56)

Slider-crank synthesis: specified  $r_3$  (AB)

Stroke,  $TK(\delta)$ ,  $r_3$

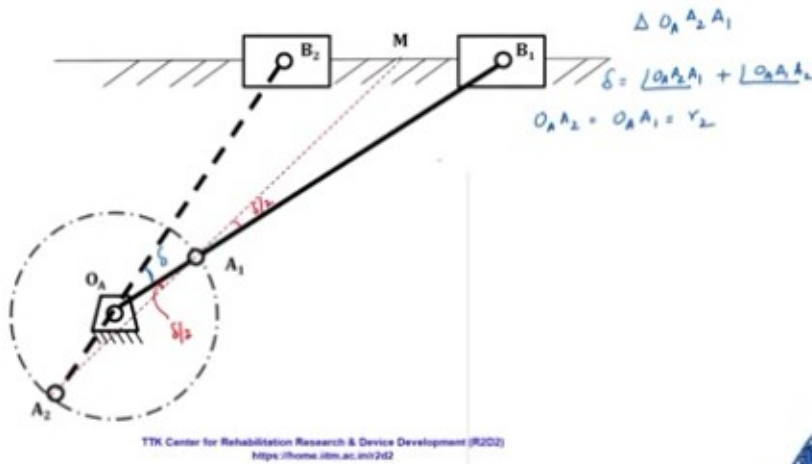


but obviously in this triangle you have  $OA A2 = OA A1 = R2$ , that's the radius of that triangle, therefore this two angles are equal, so each of these angles, so this angle here is delta/2 therefore this angle, vertically opposite, this is also delta/2, okay.

Now let me look at triangle M A1 B1, (Refer Slide Time: 05:41)

Slider-crank synthesis: specified  $r_3$  (AB)

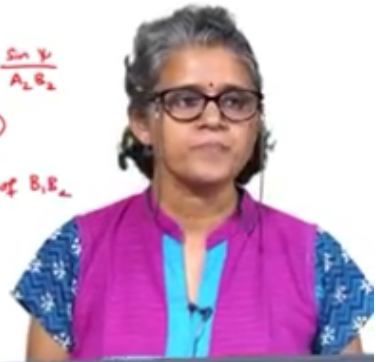
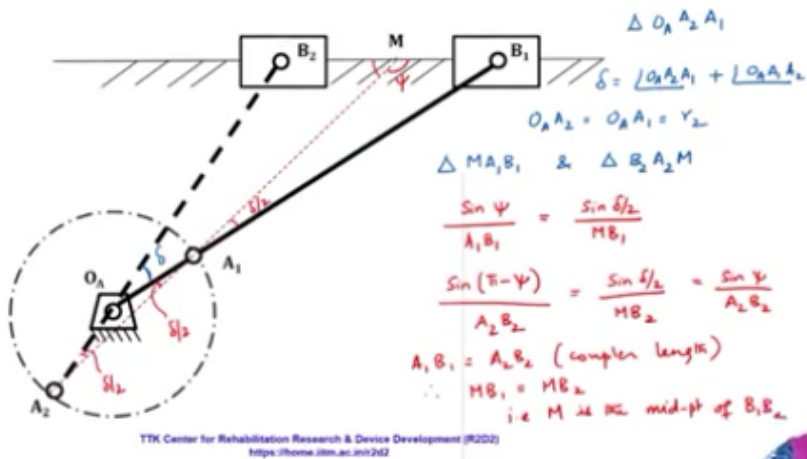
Stroke,  $TR(\delta)$ ,  $r_3$



and triangle  $B_2 A_2 M$ , this angle is also  $\delta/2$ , so let me call this angle  $\psi$ , okay, so if I say, if I look at these two triangles I can write  $\sin \psi / A_1 B_1 = \sin(\delta/2) / MB_1$ , and then from the other triangle I can write  $\sin(\pi - \psi) / A_2 B_2 = \sin(\delta/2) / MB_2$ , but  $\sin(\pi - \psi)$  is  $\sin \psi$ , so this is equal to  $\sin \psi / A_2 B_2$ , therefore so you have  $A_1 B_1$  is also equal to  $A_2 B_2$ , because that's the coupler length, okay. Therefore from these two I see that  $MB_1 = MB_2$ , okay, that is  $M$  is the midpoint of  $B_1 B_2$ , okay.  
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Slider-crank synthesis: specified  $r_3$  (AB)

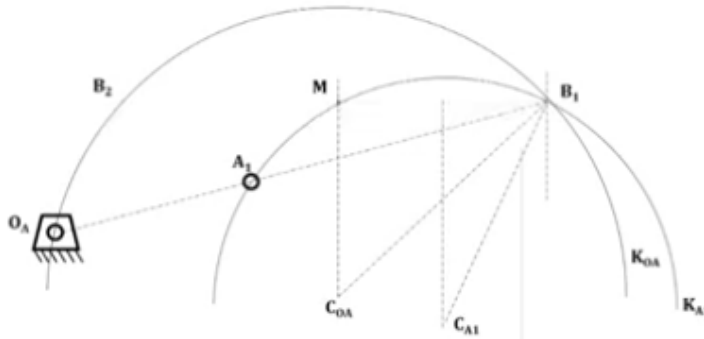
Stroke,  $TR(\delta)$ ,  $r_3$



So what is this tell me?  $A_1$ , okay, lies on a circle which makes a center, for which the chord  $MB$  makes a central angle of  $\delta$ , because  $MB_1$  makes  $\delta/2$ , okay.

Similar to what we did here, you have  $B_2$   $O_A$   $B_1$ , anyway  $B_2$   $B_1$  that's the chord  $OA$  lies on a circle whose central angle is  $2\delta$ , and now I have this additional information that  $A$  lies on a circle for which this chord  $MB_1$ ,  
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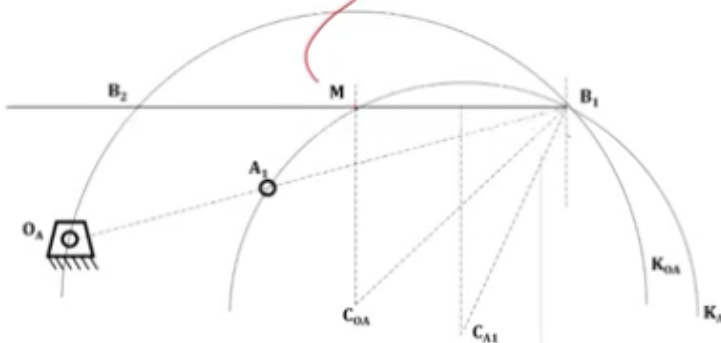
Slider-crank synthesis: specified  $r_3$



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Slider-crank synthesis: specified  $r_3$



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where  $M$  is the midpoint of  $B_1 B_2$  makes a central angle of  $\delta$ , okay, so I use these two to basically construct, so because I'm given  $R_3$ ,  $R_3$  is  $A_1 B_1$ , right, this is the given  $R_3$ , this is what I want to use, so let's do that.

So I have  $B_1 B_2$ , I construct  $M$  is the midpoint of  $B_1 B_2$ , okay, so from  $B_1$  if I draw the perpendicular bisector of  $B_1 B_2$ , then if I construct this with  $\delta$ ,  $-\delta$  angle okay, this way then this gives me where it intersects the perpendicular bisector is gives me the center of the circle on which  $OA$  lies, because this is again going to make the angle  $\delta$ , central angle will be  $2\delta$ , okay, so this pivot, the fixed pivot for the crank lies on this circle, so I can construct a circle that passes through  $B_1 B_2$  with center at  $OA$ , okay, and I know, so all I know now is this is the circle on which  $OA$  lies, okay, then I also have this additional information that  $B_1 M$ , okay, I'm looking for the point  $A$ , right, that has to lie on a circle for which this chord makes a central angle of  $\delta$ , okay, so this is if I construct the perpendicular bisector of  $M B_1$ , okay, and then this angle is  $-\delta/2$ , so where it intersects this perpendicular bisector gives me the center of the circle on which  $A_1$  lies, this is from the previous slide the geometry.

So now I draw that circle, this is circle  $A_1$  on which the locus of the possible points for  $A_1$ , this is the circle on which all the possible  $OA$ 's lie, okay. So now once I construct the circle, I know the length  $R_3$ , so from  $B_1$  okay I cut  $A_1$  such that this distance is  $R_3$  that gives me the location of  $A_1$  lies on this circle.

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Slider-crank synthesis: specified  $r_3$

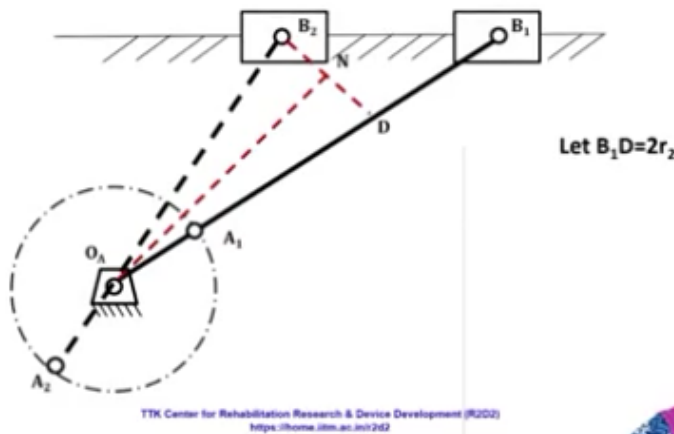
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Now I extend that because if you go back you see that  $B_1 A_1 O A$ , they lie on a line it's an extreme position, so the two links are collinear so I extend, I found  $A_1$  on the circle  $K_1$ , then I extend it to meet the circle  $OA$ , and that point gives me the point fixed pivot  $OA$ , okay, so this is sort of a zoomed in version of this, okay because I'm showing you the, so you see that  $OA$  still lies on a circle which makes a central angle of  $2\delta$ ,  $A_1$  lies on a circle with central angle of  $\delta$ , where  $M B_1$  makes a central angle of  $\delta$ , in the first case  $B_1 B_2$  makes the central angle  $\delta$ ,  $OA$  lies on that circle, okay, so using simple geometry I have now gotten a unique

solution, so I have a particular stroke length, I have a particular time ratio, I have a particular link length for the connecting rod, and then now once I do this how do I find R2? R2 is nothing but, once I do this construction R2 is OA A1, and my offset is this, so get R2 and offset from the, once you finish the construction, okay, so this is for the case where the length of the connecting rod is specified, so you had options for three things, the offset then the length of the connecting rod.

Now the last one that we look at is the length of R2, OAA,  
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Slider-crank synthesis: specified  $r_2$  ( $O_A A$ )



do you need me to write the steps for the previous one or you are okay, right? It's fairly straightforward.

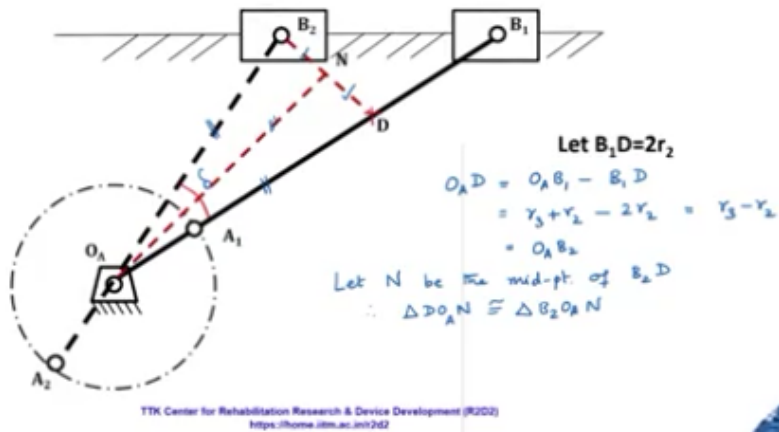
So let's look at case 3 where stroke is given, delta is given and R2, these are the given quantities, I need to determine R3 and the offset, so again let's look at the geometry, so first I construct the point D on this A1 B1 such that B1D equals twice the R2 that is given, so this angle is delta, so here I don't know, I'm taking a general case, I'm trying to figure out what my construction should be, okay, so let's say my B1D is 2R2, then what is my OAD? OAD is OAD1-D1D which is equal to R3+R2-2R2, because by construction I've constructed D in such a way that B1D is 2R2 so this is equal to R3-R2.

What else is R3-R2? OA B2, so this is equal to OA B2, I take N a point N such that it is the midpoint of B2D, so let N be the midpoint of B2D, then I've already shown this is equal to this, I've taken N as the midpoint, so this is equal to this, right, B2N = ND and this side is common, so the two triangles are congruent, therefore triangle DO2N is congruent to triangle B, sorry OAN, B2 OAN, those two triangles are congruent, okay.

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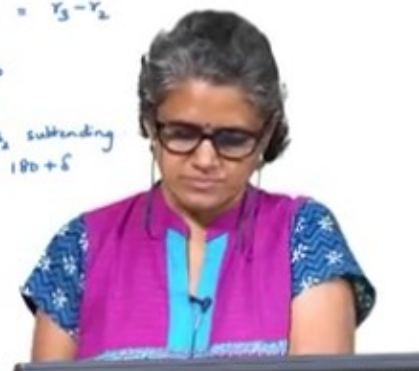
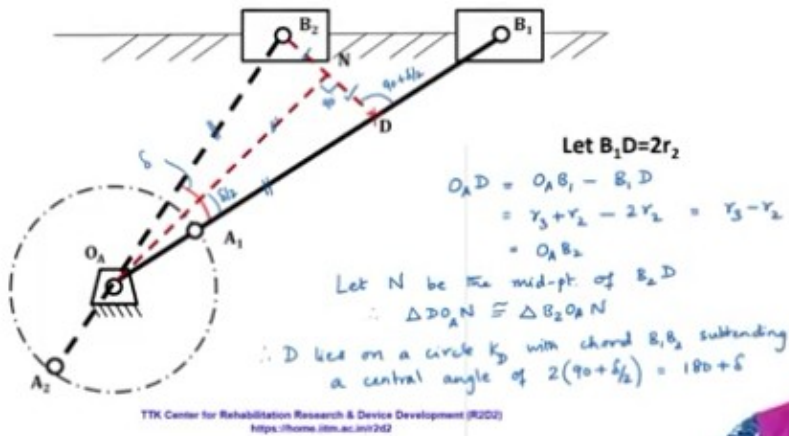
Slider-crank synthesis: specified  $r_2(O_A A)$  - stroke,  $\delta$ ,  $r_2$  are given



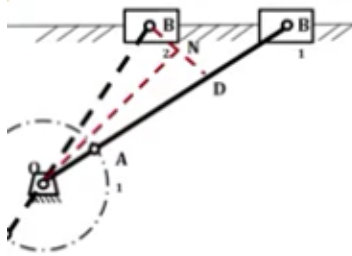
If I take this angle, so these two angles are half of each other, right, and this is a 90 degree angle, this is 90, this angle is  $\delta/2$ , so what is this angle?  $90 + \delta/2$ , therefore  $D$  is going to now lie on a circle, call it  $K_D$  with chord  $B_1 B_2$  subtending a central angle of  $2$  into  $90 + \delta/2$  which is equal to  $180 + \delta$ , okay.

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Slider-crank synthesis: specified  $r_2(O_A A)$  - stroke,  $\delta$ ,  $r_2$  are given



## Slider-crank synthesis: specified $r_2(O_A A)$



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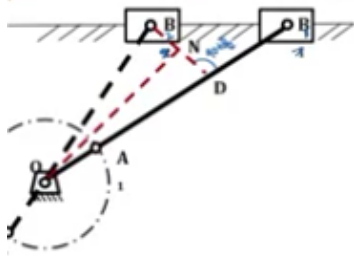


So let's do the construction, so I have  $B_1 B_2$ , this is to be  $B_2$ , this is  $B_1$ , I have, yes, why would it be  $90 - \delta$ ? So this would be on the other side, yes, this would be on the other side of the, it will be on the other side of the, other segment of the circle, because it's an obtuse angle, so it will be on the smaller segment of the circle, so you have  $B_1 B_2$ , okay, then I have, I construct the perpendicular bisector of this, okay, so my  $OA$  will still lie on the circle which makes a central angle of two  $\delta$ , so if I take  $-\delta$  from here this will give me the location of the center of the circle on which  $OA$  lies, just like earlier, okay.

Now to construct the other one I'll measure to find, say again  $B_1 B_2$  has to subtend an angle equal to  $90 + \delta/2$  on the circumference which means  $180 +$ , so I take this  $90 +$  this angle, okay, that's equal to  $90 + \delta/2$ , I construct it there, so this gives me the center of the circle on which point  $D$  lies, okay, so I can construct that circle with center at  $CD$  and this radius okay, and then I know, okay, so this is the circle  $KD$ , okay.

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## Slider-crank synthesis: specified $r_2(O_A A)$



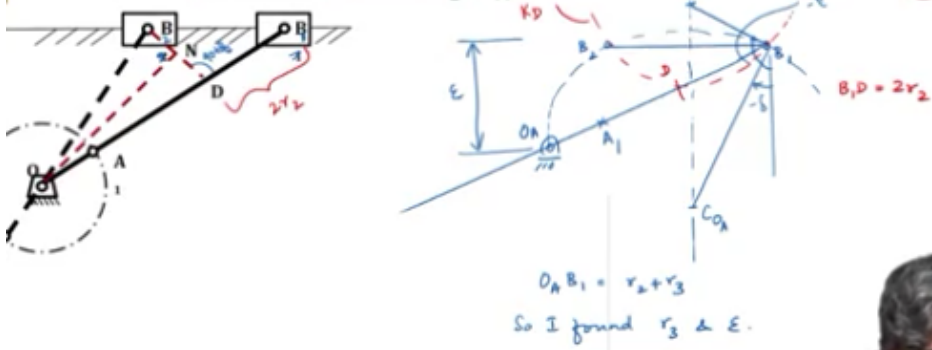
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Now I know that  $B_1$  to  $D$  is  $2R_2$ , so from  $B$  I cut an arc to find this point  $D$  such that  $B_1 D = 2R_2$ , okay, so and I still have to draw this circle, so I have, because I have to find  $O_A$ , right, so this point here will give me, okay the fixed pivot of that, okay.

So now I have located  $O_A$  from, so  $O_A$  to  $B_1$  is  $R_2 + R_3$ , okay, so I know  $R_2$ , so if I, this gives me, sorry  $A_1$ ,  $A_1 B_1$  will be  $R_3$ , okay, and my offset is this, so  $O_A B_1 = R_2 + R_3$  so you know, so I found  $R_3$  and the offset, okay, so if you remember this one figure the two extreme positions and this is geometry using congruent triangle are similar triangles to do this construction, (Refer Slide Time: 26:24)

Slider-crank synthesis: specified  $r_2 (O_A A)$



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so you don't really have to mug up the steps or anything, it's fairly straight-forward, okay, so this circle here is as it has been from the first construction that's a circle on which OA lies, okay, so this kind of raps up the Slider-Crank Synthesis for a given stroke length, if you know the two extreme positions that X center positions for the Slider-Crank then you can synthesis the Slider-Crank for a given time ratio and either the other things then you have you know, an infinity of choices but if you specify one more thing then your solution becomes unique, okay, otherwise you only have, how many choices? An infinity of choices, because if you have given the stroke length and the time ratio OA necessarily has to lie on the circle that passes through the two extreme positions and makes the central angle to delta, okay, so any point on that circumference of the circle, of the segment where it makes delta you can find the location of OA, but if you specify one more quantity then your solution becomes unique for that particular problem, okay.

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## 3-position Motion Generation

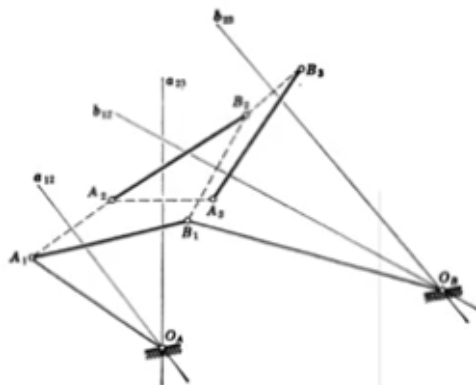


FIGURE 8-3 Problem 2, three positions of a link.

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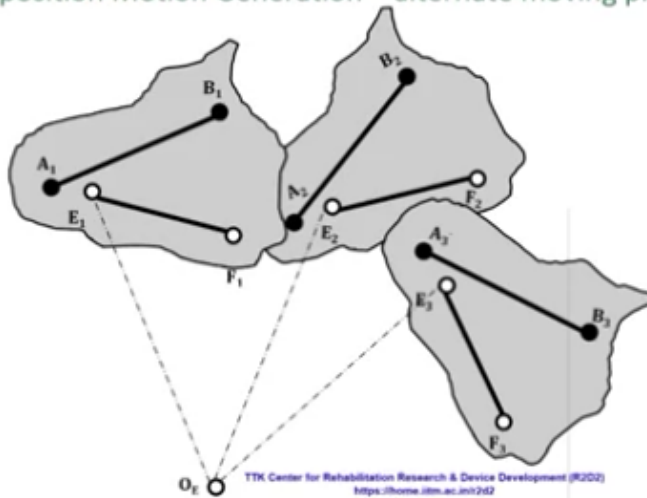
So now from two position motion generation and that center synthesis we move on to three position motion generation, so we saw that for two position motion generation you could have an infinity square of solutions, if you wanted a 4 bar, an infinity of locations for the pivot OA, infinity of locations for OB and then you could create, infinity square 4 bars to do that.

Now we come to 3 position where again we use the fact that if I want to move the coupler from  $A_1 B_1$  to  $A_2 B_2$  to  $A_3 B_3$ , okay, that's my motion generation, position and orientation of the coupler in the plane changes, then essentially I'm looking at finding a circle that passes through  $A_1 A_2 A_3$ , and another circle that passes through  $B_1 B_2 B_3$ , so  $A_1 A_2$  perpendicular bisector,  $A_2 A_3$  perpendicular bisector they will only intersect at one point, so now this becomes a unique solution.

Similarly for  $B_1 B_2 B_3$ , so if you specify one more position for the coupler, you have gone from so many solution options to a single solution, okay, but that is assuming one thing, what am I assuming here? So that seems really restrictive right, that seems really restrictive that I have, but I don't have a choice anymore, but it's not really true, why is that? They're not the limit positions that I can make them the limit positions but what am I fixing here that I need not fix perhaps, the points  $A_1 B_1$ , say I can still have a body, okay, for which these are my reference points, I'm looking at the orientation of  $A_1 B_1$ ,  $A_2 B_2$ ,  $A_3 B_3$  to define the position and orientation of that rigid body, but I may have flexibility in, do I have to make  $A_1$  my pivot, my moving pivot, a circle point? No, I may not have to do that, in some cases I may have to in which case it becomes severely restrictive, but otherwise I actually have how many more solutions that are possible, I can choose any A on this body, so infinity square for moving pivot, anywhere on that plane, similarly moving pivot 1, and infinity square for moving pivot 2, so I need not pick  $A_1$  and  $B_1$ , so if this picking  $A_1$  and  $B_1$  gives me a solution that I'm not happy with, I do have options, that's the point I'm trying to make here.

For A1 B1, if I choose A1 and D1 as my moving pivots, I only have one solution, but I could choose something else as my moving pivots even though my reference for the motion generation is A1 B1, A2 B2, A3 B3, okay, because essentially what will I do? I could have like this A1 B1 moves to A2 B2 moves to A3 B3, (Refer Slide Time: 33:11)

### 3-position Motion Generation – alternate moving pivots



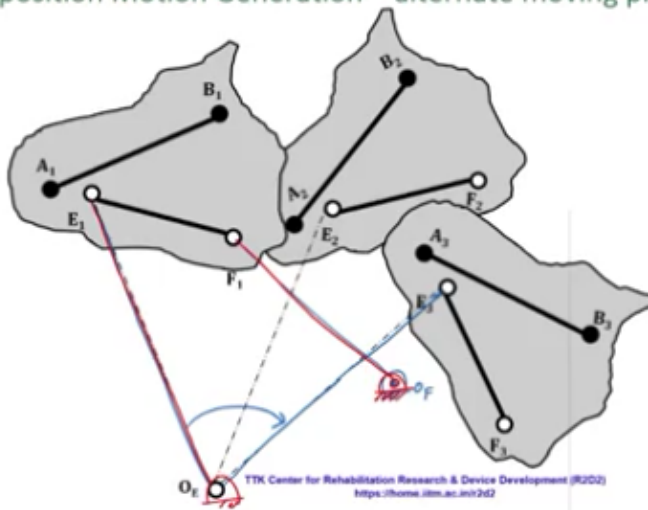
but I can pick some other point E and F on that body and what would be the relationship of E and F with A, B in each position? They would be, it's a rigid body there are all points on the same rigid body, so they are invariant with respect to A, B, so I just construct, I just change my problem from A1 B1 moves to A2 B2 moves to A3 B3, if I pick E, F, as my moving pivots then I say E1 F1 moves to E2 F2 moves to E3 F3, once I do that then my E, OE and OF maybe in more suitable locations or if the first case, if A1 B1 if that one gave me you know toggle positions in between or was unsuitable for other reasons, now I have more options, okay.

Again with the graphical method it's sort of a trial and error method that you are using here, so in each case you'd have to either construct a module or simulate to make sure that it moves from A1 B1 to A2 B2 to A3 B3 without encountering a toggle position, okay, which may or may not happen, okay, but intuitively it's very easy to understand those 3 points lie on a circle, and these are the choices that you happen, so these are, this is the case where you can choose alternate moving pivots or alternate circle points for this synthesis. If I wanted a limited to these positions, what do I do? And if I can, if I ensure that it is moving in the right order 1 to 2 to 3, than I can just connect a drive diagram, so I can say OEE, E1 has to move to OE E3, okay I keep this part of the mechanism the same, you know I may have OF I have not drawn it here, but I'll find OF, and OF maybe somewhere here, so I can say that this linkage OE E1, F1 OF so this is my fixed pivot E1 F1 OF, this 4 bar is capable of moving from, moving this rigid body from the, through the required positions A1 B1, A2 B2 and A3 and B3, okay.

And if I have ensured that it is moving it in the same order,

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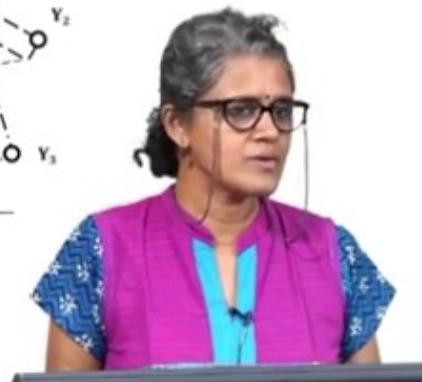
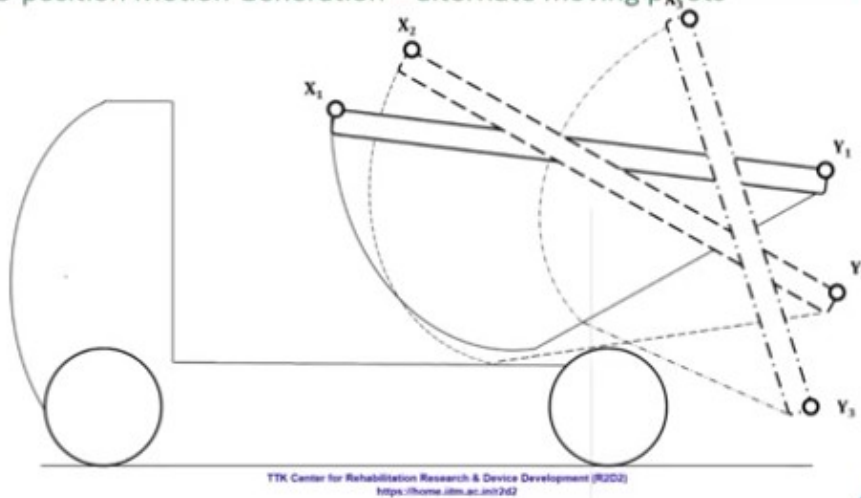
### 3-position Motion Generation – alternate moving pivots



if that's what I want then I can just look at the two extreme positions, I can make OE E1 and OE E3 my two extreme positions, so then I can say that's my rocker which has to move through this angle theta, and then I can connect a driver dyad to it. I can choose some point somewhere again on this body and connect a driver dyad to it, so when I do that now I have accomplished the three position motion generation as well as limiting it to the extreme positions, so that it's not going to same A3 B3, A4 B4 that I don't want, okay, I'm limiting it to go from A1 B1 to A3 B3 and back to A1 B1, and additionally with the driver dyad I can now give a continuous input, okay, so using simple geometry I've been able to accomplish all this synthesis objectives, okay, so that's the beauty of the graphical methods, it's very simple and very intuitive, okay.

Here is a real life example, so say you have a dump track, okay,  
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### 3-position Motion Generation – alternate moving pivots



and essentially you see all this construction trucks everywhere right, so you want a dump, so for which this bin has to move, that's your rigid body now, it has to move from  $X_1 Y_1$  to  $X_2 Y_2$  to  $X_3 Y_3$ , okay. I may or may not need to specify  $X_2 Y_2$ , it just depends on you know how it's going, but I may wanted to move gradually, so I may wanted to hit that position and so, but I may not be able to locate my moving pivots or if I locate my moving pivots at  $X_1 Y_1$  I may get a solution that's of the bed of the truck, okay, so in that case now I have to find alternate moving pivots, so alternate moving pivots that can still, so alternate moving pivots that will still give me a solution that lies within the space that I want, okay, so this is 3 position motion generation, and we've now looked at three position motion generation with alternate moving pivots as well, okay, but here again it's still hit or miss, I may have to try, so I may have to try, so I may have to pick some other points A, B, see if that lies within the bed of the truck, okay.

In real life situations you may be more constrained by where the fixed pivots are located, okay, I may have only a certain space where I can locate the fixed pivots for the linkage, so in the next class we will see the case where if the fixed pivots are given, how do we design the linkage to still accomplish the same motion generation for the coupler, okay, and that's where we will use inversion, okay.

### Online Editing and Post Production

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Jagadeeshwaran  
Jayanthi  
Kamala  
Lakshmipriya  
Libin  
Madhu  
Maria Neeta  
Mohana  
Mohana Sundari  
Muralikrishnan  
Nivetha  
Parkavi  
Poonkuzhale  
Poornika  
Premkumar  
Ragavi  
Renuka  
Saravanan  
Sathya  
Shirley  
Sorna  
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