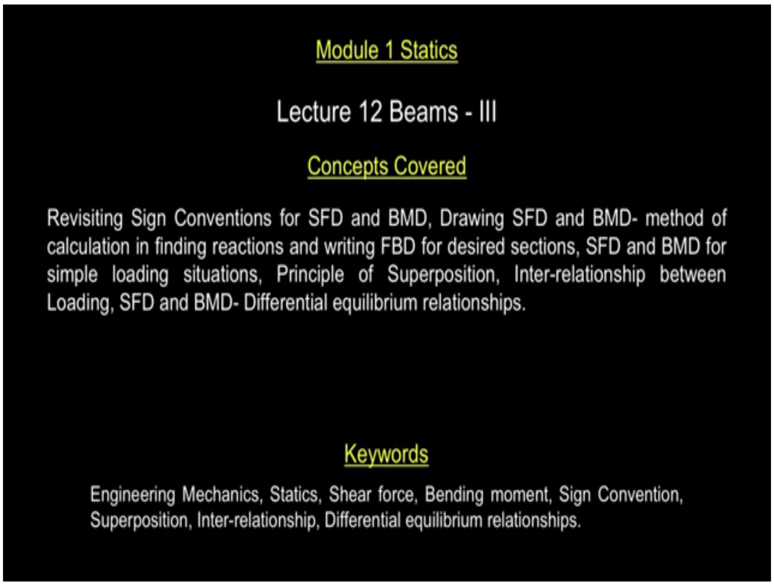
Engineering Mechanics
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module – 01
Statics
Lecture – 12
Beams – III



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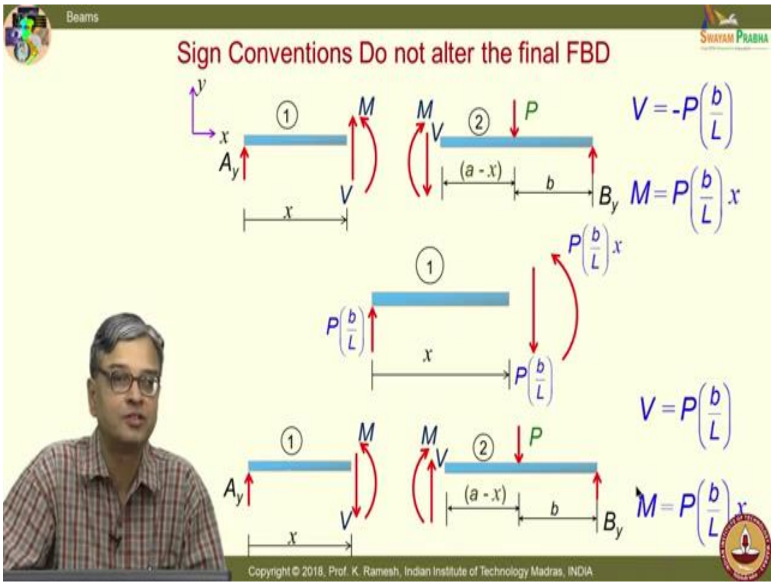
Let us continue our discussion on Beams. See in the last class towards again students came to me and then said, they have some difficulty in understanding the sign convention. And I thought that I would revisit sign convention from a different perspective, because you understand the concept better with a same concept is discussed from different perspectives.



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First let us look at what way we discuss the sign convention. Sign convention is required no matter when I analyze the beam on the left portion or the right portion. I should get one unique answer including the sign for the shear force and bending moment. More of a discipline; see in this specific case I have taken a section and analyzed the left portion and right portion. In an actual problem depending on the complexity you may solve for certain portions of the beam from the left, certain portions of the beam from the right; unless you follow a discipline, your answers will be different in the form of signs. So, it is more of a discipline.

So, the idea is once you assume in one section by Newton's third law automatically what you have to assume for the other section is fixed. Here you are looking the left portion and right portion of the same section. Suppose I take the right portion of this you would have put what are the unknown forces. And if you do not follow a discipline, you will not get one unique answer. And what we have learnt when we draw the free body diagram? I can start assuming unknown forces in any direction, if my direction assumed is wrong my mathematics will come back and tell me what is the correct direction?

So, here I have assumed V as positive in this manner my answer gives $V = -P / L$. So, when I write the actual free body with the correct direction, I would have the shear force like this and the bending moment you have taken anticlockwise as positive my mathematics also has given me the bending moment as possible. So, I do not need to change anything on the direction.

So, that is what you get when you interpret your answers after getting started with some directions. Now the question is, is these assumptions are unique, different books follow different conventions. And we have looked at that this was suggested by Crandall Dahl and Lardner and this is the symbol that you use in Meriam.

So, the difference is we assume the direction of shear force differently. But once I have assumed on one portion of the beam what way I should take the forces in the other section of the beam is fixed from Newton's third law that is very simple. When I solve the free body when I get the answers, I get shear force as $P\ b\ /L$ there is no change in the sign. So, when I go back and write the free body diagram, I will put the same force like this and moment is the same convention like random so there is no change.

So, the idea is when I isolate a section of a beam finally, what are the forces acting on that section is unique and there is only one answer. I can have multiple sign conventions to determine these quantities and also plot them and you should remember that, shear force diagram and bending moment diagram is only an intermediate step. This intermediate step appears differently because of different sign conventions.

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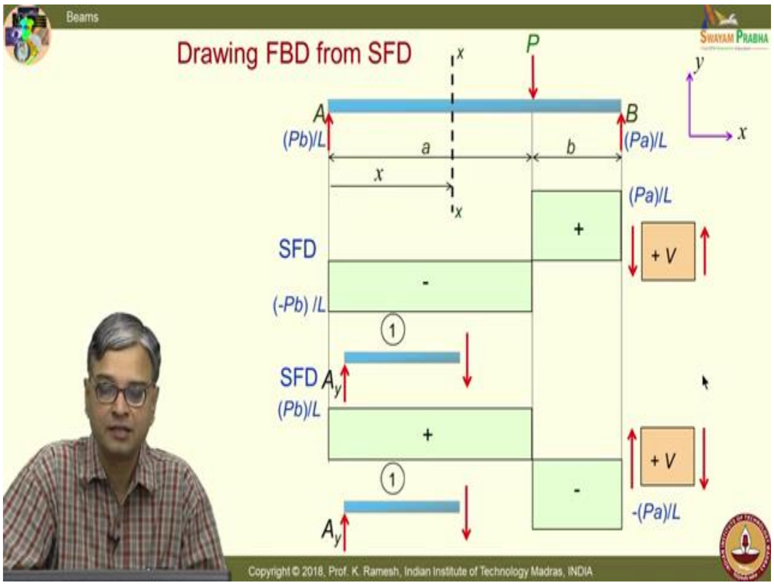


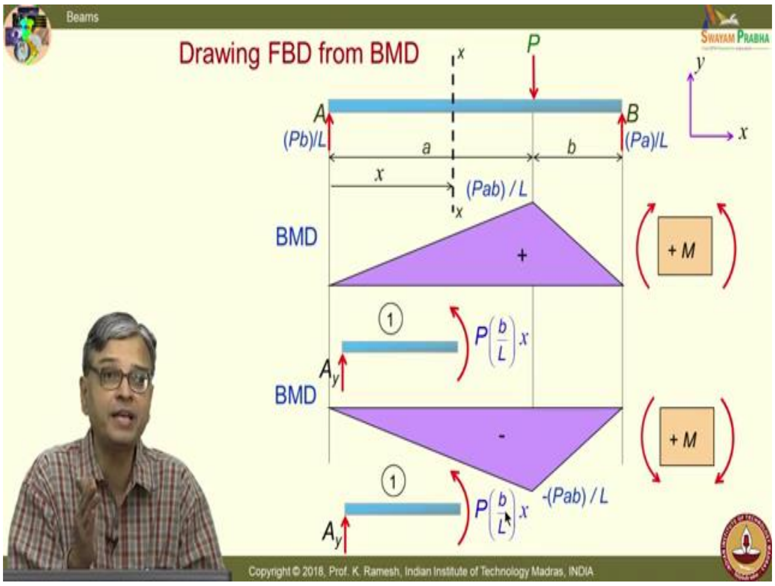
We will also look at it from another perspective and you should recognize when I write the sign convention like this; this is not a block. What it says is what is the sign convention I should use on the positive surface and what is the sign convention I should use on a negative surface? You have to imagine that this is collapsed as one plane, I look at the positive side, I look at the negative side for convenience we put it as a block it is a block of 0 length you can take it.

So, what I would write on the positive face and what I would write it on the negative face? Now let us look at what is the shear force diagram, when I get using this sign convention. I have this as positive and this as negative when I use this sign convention, I get this as positive and this as negative. You essentially have this as the base axis and write the quantities like this. Ultimate objective is what? I have to design a beam; I have to know at a given cross section what is the shear force and bending moment it transmits.

So, my idea is to draw the free body diagram from the SFD from different sign conventions. So, let me take the free body and let me interpret how do I understand what is the quantity mentioned in this free body diagram how do I write it here? I have taken a section in this portion here it is negative, and we have assumed what is the positive direction please have a note of it I will also draw the other beam.

So, I have one sign convention here another sign convention here and if I go and look at what I have to do? I have got this as negative; positive is like this so I should put the shear force transmitted in this direction. And in this case, it is in the positive direction and what is positive here? Pointing downwards so I would also get shear force acting downwards.

So, please understand an isolated portion of the beam is identical, no matter what sign convention I used, but definitely the appearance of shear force diagram is totally different. You must recognize this fact we would again repeat it for the convention for the bending moment.



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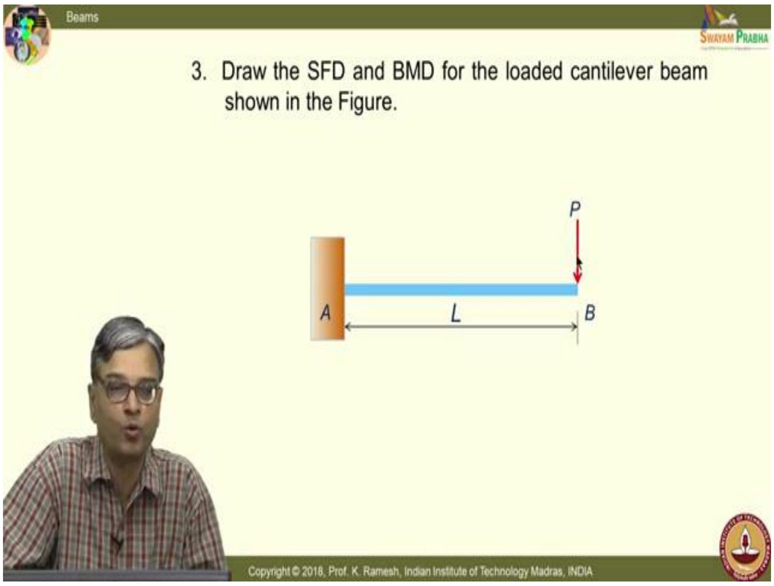
I have taken two opposite sign conventions; here we have taken anticlockwise moment is positive here we have taken clockwise moment as negative. So, when I solve the problem and draw the bending moment diagrams, they appear like this. So, one is positive and one is negative.

Again, I will take the section in the first portion of the beam. So, what I find here is I have to read the bending moment diagram I know the value is positive and this tells me how I should put the direction of the moment and we have interpreted what is positive? So, I would put the moment as anticlockwise.

If I interpret this bending moment diagram here, I read this value as negative. And we have said what is positive? Positive is clockwise when it is negative, I should interpret that this is anticlockwise. So, different sign conventions provide the same answer. But within a problem you cannot use different sign convention for the left portion and right portion that is completely dictated by Newton's third law.

So, sign convention once you adopt you should follow it systematically. It helps you to get consistent results whether you solve left portion of the beam or right portion of the beam whichever is simpler from computational point of view.



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Now, let us go on solve a series of problems with the level of difficulty slowly getting involved; let us take one of the simplest problems we have a cantilever with a tip load.

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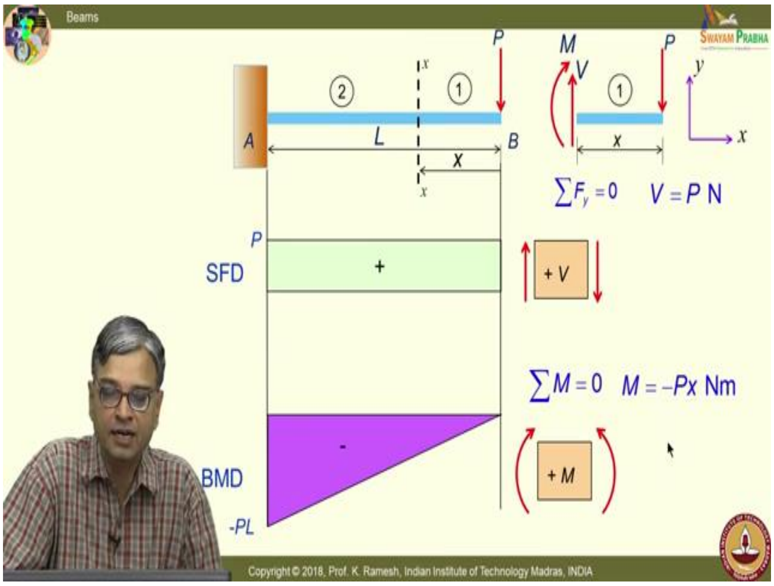
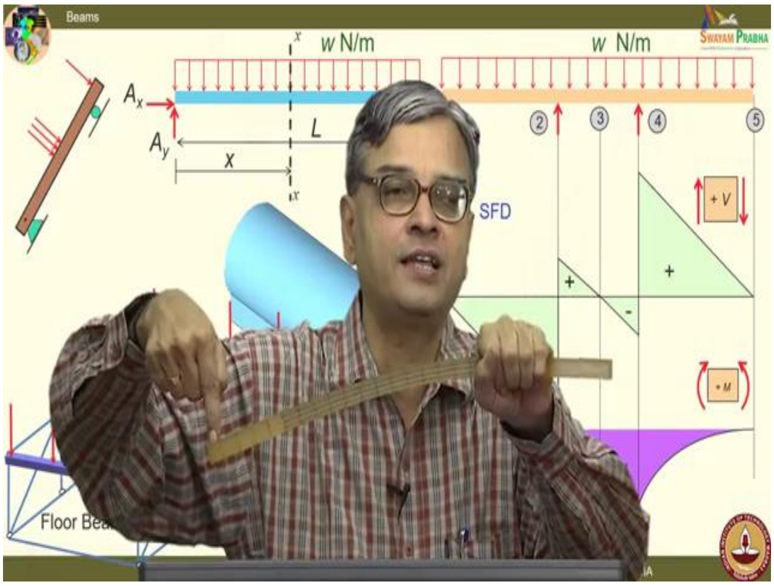
So, I have the cantilever represented here and I have put the reference axis x and y. And I have to get the variation of shear force and bending moment along the length of a beam I have a complete choice; I could take the portion 1 and solve the problem or I can take the portion 2 and solve the problem. In this problem if I have to take portion 2, I will have to remove the support determine the support reactions, only then I have to find out the shear force and bending moment



Instead if I analyze section 1, I can avoid finding out these reactions to start with so, my mathematics is simpler. So, I take it like this and then, I follow the sign convention I have already said we have adopted Meriam at IIT Madras. From now onwards, I would consistent to use Meriam sign convention, I would put the unknown forces anticlockwise moment is possible and on a negative surface positive direction of *V* is positive.

So, I have determined the I have written down the unknown forces, now I can apply the equation of equilibrium and find out this. So, when I put $\sum F_y = 0$, I would put this as positive $V - P = 0$; I am not writing that step I am directly writing the final answer. I write $V = P$ N. And similarly I can put $\sum M = 0$, I have drawn the shear force diagram see this remain constant I have taken a generic section, because it is constant I can easily write this; draw this shear force diagram as a rectangle like this and I have this shear force as value as P.

Now let me apply the other condition $\sum M = 0$. So, I get $M = -P\,x$; see I am missing out one intermediate step, you could write that, I would write this as anticlockwise; anticlockwise is positive moment this is clockwise here on the negative surface. So, it is clockwise moment I will put as $-M$ and then this also is $-Px$; $-M - P\,x = 0$. So, finally, I get $M = -P\,x$. please write the intermediate steps and also note down, as soon as I draw the shear force diagram I indicate the convention used or you could first put the convention then write the free body diagram and draw the shear force diagram.



So, that you do not make a mistake, the message here is I have planned my screen in such a manner I have put the loading diagram, I have space for doing the computations. I put the shear force below the loading diagram and I also put bending moment below the shear force diagram one below the other. It is a good discipline it would help you to find out if you have done any glaring calculation mistakes either in terms of magnitude or in terms of sign.

I would recommend you to plan your answer scripts and also while you take down the notes find out a different space for doing the computations, reserve the space below the loading diagram for drawing SFD and BMD.

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See, this is a very nice problem that you come across and this is like I have a you can consider this as a fixed end I apply a load here. So, when I apply a load what happens you could visibly see that there is maxima deflection at the free end, this is called a free end this is the fixed end you have the maxima deflection. But what we usually find is usually students study bending moment diagram and shear force diagram in the first semester or the second semester or so, by the time they come for the 4th year when they apply for MS or PhD.

When we ask the question where we have the maximum bending moment? We will always confuse between deflection and bending moment because always remind a puzzle, if they learn it correctly there is no way they can make a mistake. Invariably students will say in an interview they will say maximum bending moment occurs here where the bending moment is zero from your bending moment diagram and maximum bending moment occurs at the fixed end; maybe they have not really got the concepts clearly.

If you get the concepts clearly there is no way you can make a mistake in a simple problem like this. So, get your fundamentals clearly if you have difficulty please ask me, I can explain it again. So, in every case what I want to insist is I should have the sign convention for bending moment as well as shear I have also told you that this is not a square block it is a block of zero length.

It indicates what happens on the positive surface and what happens on the negative surface. See soon we will take out a block from the beam when there is distributed loading and identify what happens in the negative surface and positive surface, there the block will have a finite length of delta x when I write the quantities, I have to write the quantities carefully.

Since you are not seen that you do not get a confusion once you see this, then you will get confused on this and this is the symbolism used in higher studies also when you want to learn stress tensor and so on.

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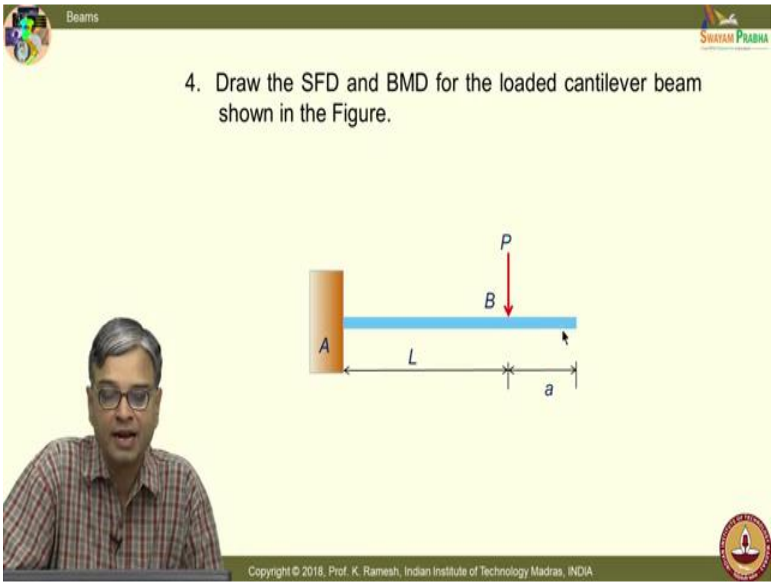

Let us move on to the next problem. I have a same cantilever beam and I have a load slightly inside.

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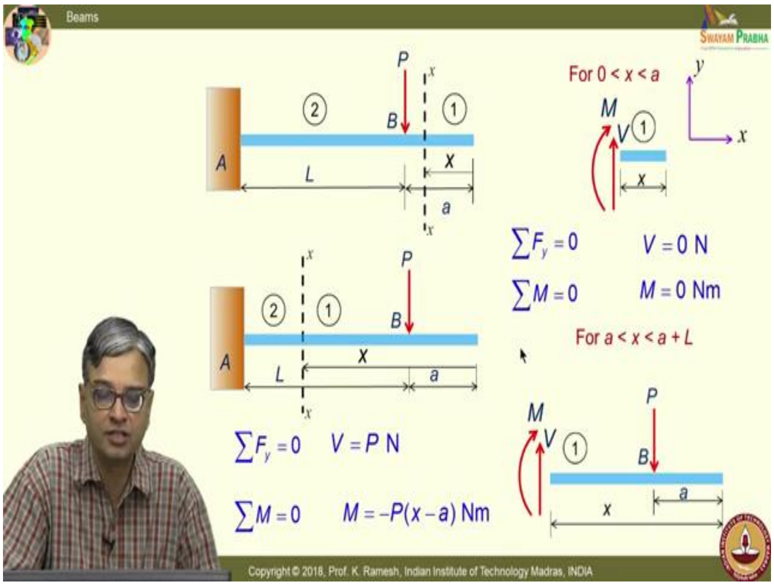
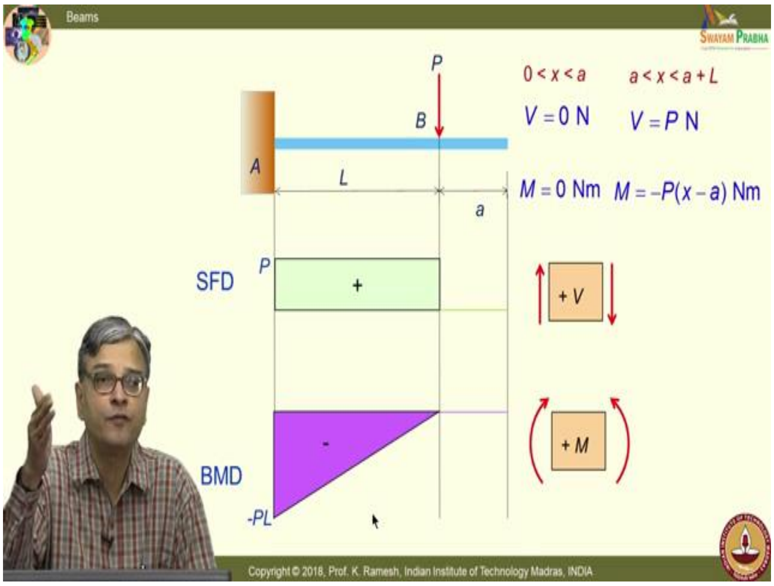
And one of the first question that you need to ask yourself is when I have a problem like this. How many sections I need for me to get the shear force and bending moment diagram along the length of the beam? See I have a portion like this. So, I should have an imaginary cut here and unless, I have another portion here is I have to put an imaginary section and cut. The behavior will change after every load, for a thumb rule is for every load you need to have when there is a transition you need to have a cut. When I have a distributed load in between you put one imaginary section and find out what happens will be a variation of the load in that section fine.



So, I will have to have two sections one section in this portion, another section in this portion and we have already seen taking the problem from the right is much simpler and I have this here I put the unknown forces based on the sign convention. And this is the clockwise bending moment and shear force is possible. And even before solving, what would I get for the shear force and bending moment? There is no force existing. So, I would get 0; 0 is also a valid answer $\sum F_y = 0$ gives me $V = 0$ and $\sum M = 0$ also gives M as 0 in this section of the beam between 0 and a. In this section of the beam I get shear force is 0 as well as bending moment is 0. You have to recognize 0 is also an answer we have seen zero force members in the case of a truss.

Now, I will take a section between B and a; here again I can analyze the right portion comfortably and put the unknown forces. Then you can write $\sum F_y = 0$ and $\sum M = 0$; I would appreciate that you fill in the intermediate steps which I miss in my slides. $\sum F_y = 0$ gives me V = P and $\sum M = 0$ gives me $M = - P$x - a.

Please fill in the intermediate step you should not make a mistake while you solve the problem in a quiz or an exam, please fill in the intermediate step. And also notice here I have used this slide completely for my calculations, I have not attempted to draw the shear force and bending moment diagram. So, this is what I mean that you should plan your notes as well as answer book.



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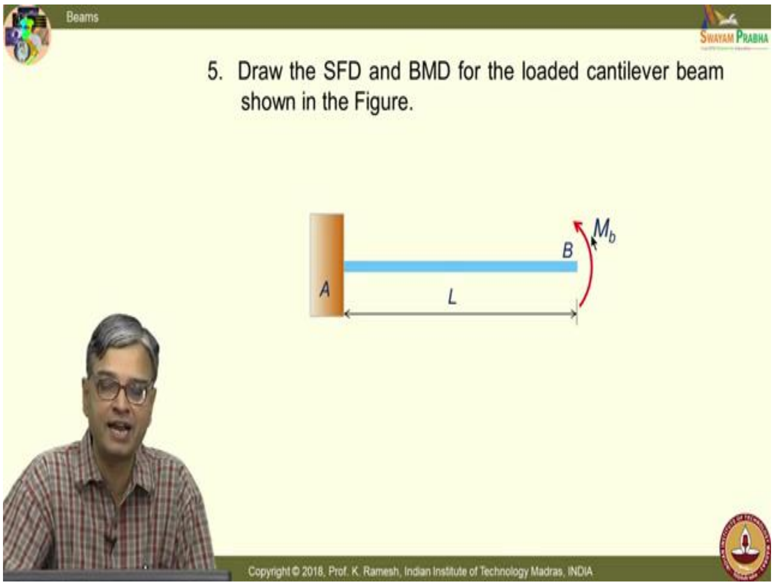
Now, I will draw a fresh I have a loading diagram, I have the expressions that we have found out for these portions what I have for the portion 0 to *a*, and from *a* to *a*+*L*. So, my shear force diagram is 0 in this section and it is a constant value in the section between *A* and *B*; and I put the sign convention used and we have the bending moment for the section 0 to *a* is 0 for the section *a* to *a* + *L*, *M* =- Px - a.

So, it varies linearly and it is 0 here, I also put the sign convention used for bending moment. So, now, it will be very clear why I need to put a sign convention? Because I can interpret what is the free body if I isolate a section how to read from here SFD and BMD, then only I can put the force interaction. So, I need to know the sign convention so it is always a good practice draw the loading diagram, shear force diagram and bending moment diagram one below the other and always put the sign convention even if some books do not follow this habit. What we discuss in the class is what I anticipate in your exam and this is the good practice.



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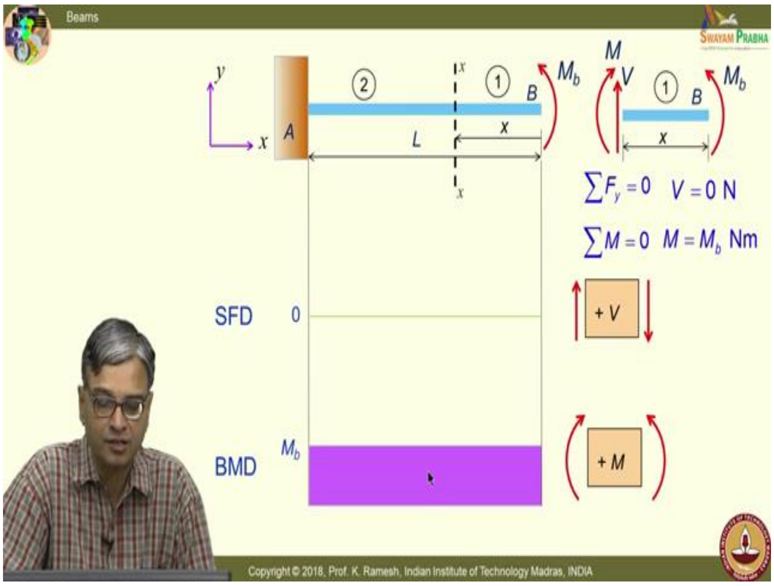
Now, let us take another loading I have only a bending moment appear; we will go and try to solve the how do I get the shear force and bending moment diagram. You know when you solve problems of a problem, you would be able to visualize things clearer and one of the aspects is I would like to visualize, how can I apply a bending moment at the end? Because we have seen, if I have to apply a concentrated force, we have to put a pin and put a fork and then put a apply a weight we have looked at elaborately; how a simple concentrated force is applied.



And another aspect what I have always been saying is when I put the bending moment, we always understand that this bending moment is applied gradually from 0, it is not that at time t_0 there was nothing, infinitesimal time after that you have the full bending moment applied. And we analyze the problem, after the bending moment is completely applied, we analyze only after that.

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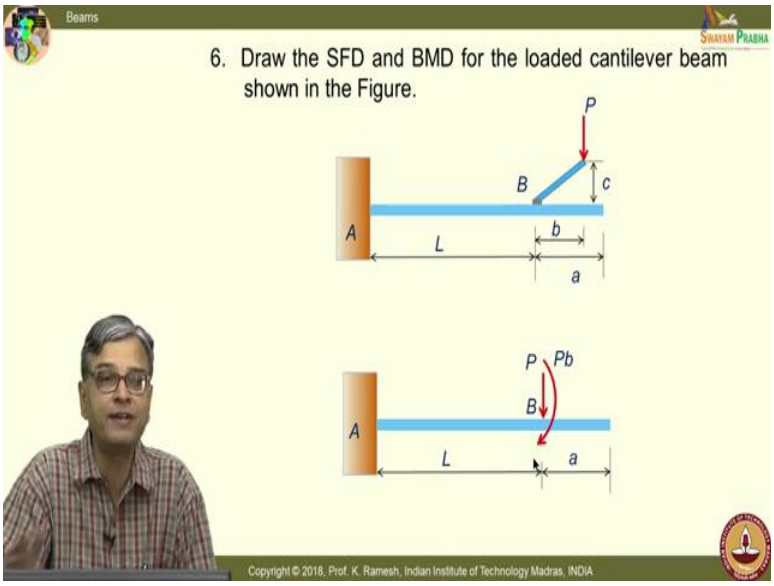
So, these ideas you should keep in mind while solving. How many sections do I required to find out the variation of shear force and bending moment along the length of the beam? That is one section is away. So, I take a section from right I put this as 1, I take this out like this, the next step is to put the unknown forces following the adopted sign convention. So, we have adopted this sign convention and this is the unknown moment and unknown shear force. And I can put $\sum F_y = 0$, I will have the first answer is there is no shear force; shear force is 0 and I have only bending moment, $M = M_b$. So, can I go on plot this?



In the earlier problem we found only 1 section of the beam at 0 shear force. So, in this problem the entire beam is not transmitting any shear force, because the loading is different. And what we find for the bending moment diagram? Bending moment diagram is constant fine and this is we labeled as positive.

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Then we move on to the next problem make a neat sketch of it. These problems are of graded complexity. If they solved a variety of problems you get the idea behind how to draw a shear force and bending moment diagram. And here I would like you to visualize what way should I analyze the beam? See I have a protrusion and this is firmly connected to the beam by welding I apply a load on this member.

So, the first step is to find out how to understand this force what way it influences the loading of the beam, that is the first step. We have already seen how to translate a force from one line of action to another line of action. Resolution of a force into a force and a couple you have to apply that concept here. While we discuss that we say the external effect produced by the original force need to be present. So, now, the first step what I have to do is I have to translate this force to the point *B*. So, when I translate this force to point B what will be the force interaction at point *B*?

You have to put a force and then cancel that force; recognize the other two forces forming a couple. So, I would have a force as well as a couple; is the idea clear, this you have to recognize when I have any protrusion where I apply the load, the loading is little involved you have to interpret the load first. So, I have this as a load *P* acting on the beam as well as the couple *Pb* do not make a mistake on the direction.

You should recognize what direction; we are only looking at what are the action on the beam? It is producing a clockwise moment people can make mistakes in all this, we are only looking at in a shear force diagram and bending moment diagram what is the action at every section? This is what we plot. And you should correctly interpret a force applied on a member which is firmly connected; and this is the member which behaves like a beam and on this member, you are asked to find out what is the variation of the shear force and bending moment. You could also draw it for this section that we are not really worried about fine.

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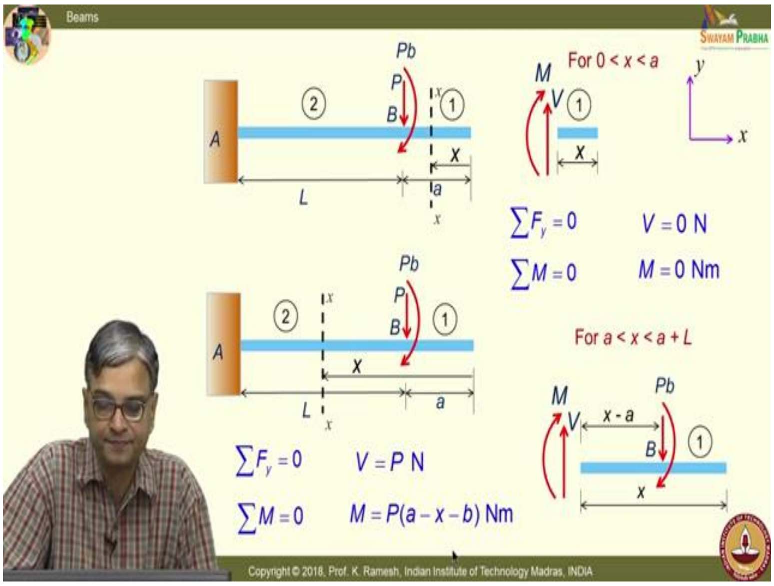

So, first you should interpret this and once I have this you have independently solved for an end load and independently solved for a moment. So, it is nothing but combination of these two we will have a look at. So, here how many sections would I require to plot the bending moment and shear force diagram? I need one section here; I need another section here. Even without calculation by inspection you can say the answer, because now you are told that you are sensitized that you can have 0 as correct answer ok. So, in this section of the beam there is no load acting nevertheless we draw the free body show the unknown forces and write the equilibrium equations then establish that $V = 0$ and $M = 0$.

You know you have to do this in your exams because you are learning the subject for the first time, do not miss out any steps. You have to draw the free body of each section, apply the equilibrium condition and systematically establish the values. You may use at the other tricks that I have told you to verify your answer; you cannot afford to write without steps in solving problems in the examination.

You have to write the steps at this level of the course, that is what is expected out of you. And I also find out what are these variations for this section from a to a + L, I take an imaginary cut and I take out this portion label the values as x-a is the distance; and I put the unknown forces and I have the reference axis. See for all my analysis of this free body you have to go by the reference axis. So, when I recognize the sign for V, I will put that as positive.

And I am missing that step I am not writing systematically the complete steps I am directly solving it and give the values of F_y, but I would urge you to write the intermediate step yourself and verify your answer with my final answer ok. If you do not do that you may make mistakes later, so when I use this equation, I get the shear force as simply P in this section.

A similar exercise I would like you to do for the moment equation $\sum M = 0$, please write for all the forces shown in this and when I solve that I get the moment as

$$M = P(a - x - b) \text{ Nm}$$

If I solve for this completely, I get it like this please verify whether my final answer is alright. Now we go and draw the shear force in bending moment diagram. See I am consistently showing a place where you do the computation and reserve a portion of your answer script or your notes to draw the shear force and bending moment diagram one below the other.

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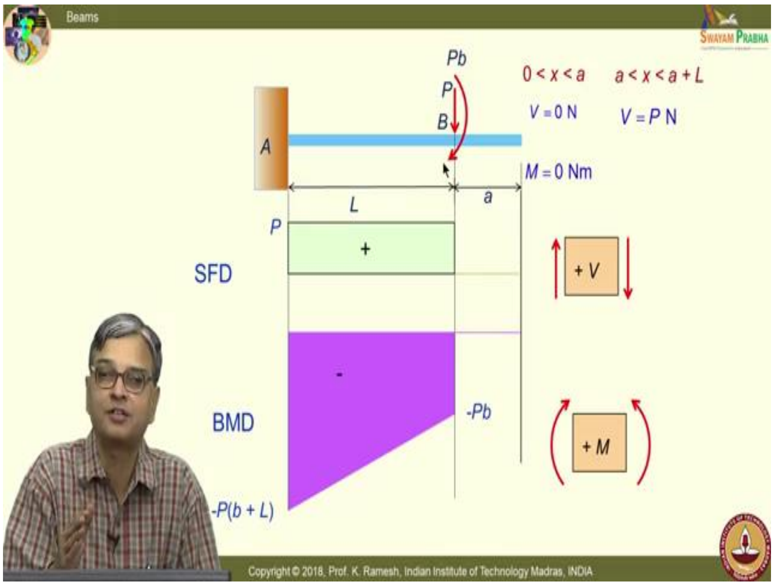
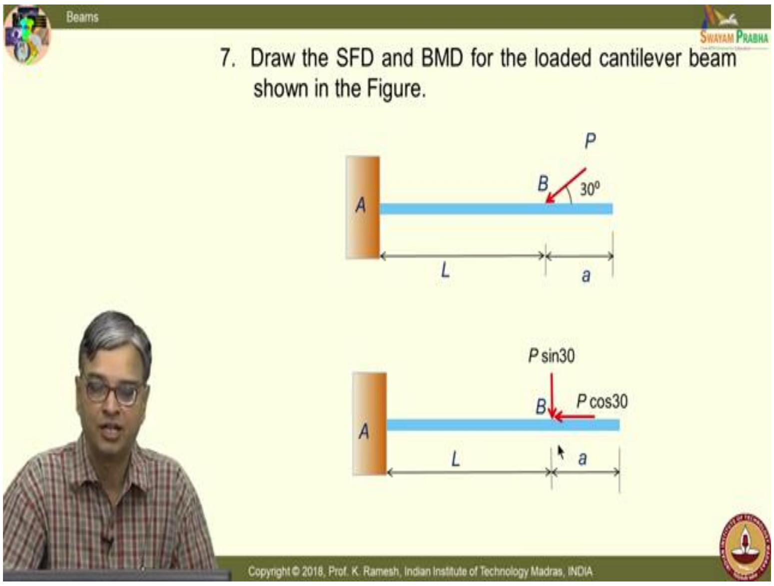
So, I have this I have the values from B computation over here if I get this like this and I draw the sign convention for the shear force; I also draw the bending moment diagram. So, I have a step change we saw when I have only a bending moment for the entire beam the bending moment remain constant and if I had only a shear force it was giving me linear variation of the bending moment. So, what I see here is this is nothing but addition of these two. See I said earlier we also make in our idealization systems behave in a linear fashion, when a system is linear, I can also analyze the problem with one load separately, analyze the problem with another load separately and add these two answers and this is labeled as a principle called principle of superposition ok.



And which is repeatedly used even in your higher-level courses as long as you assume the system behavior as linear. This is one of the reasons why even though a system may behave slightly non-linear model it as linear make sure computation extremely simple. And it is a very nice example to see here I could have analyze this as two problems take a beam with only load *P*, you draw the shear force and bending moment diagram take the beam with load only P_b as a bending moment analyze the problem and add these answers you would get the same answer.

So, this is repeatedly used in higher studies with a labeling principle of superposition is a very useful concept. So, I can also get this shear force and bending moment diagram by individually solving for only the shear force or only for the bending moment and then add those two answers. I have also labeled the key points -*Pb* and -P (b + L), let us move on to another problem.



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So, in this case I have a load which is at an angle. See we have already seen a slender member can transmit in general moments in three directions, forces in three directions, so it can transmit any of the six quantities. In this chapter we are more concerned about drawing the shear force and bending moment diagram, when I have a force like this, I should recognize how do I handle this force.

I can always split this force in its components one acting vertical and one acting horizontal. So, I would replace this by two forces using its components; which one of these forces will provide you bending of the beam? It is only P sin 30 and what way P cos 30 will affect the beam? It will apply a compression. So, if you draw an axial force diagram of the member the slender member, I can write what way the axial force changes along the length of the member.

Because this member supports an axial force as well as bends. So, whenever any external loading is happening you should interpret what are the forces that cause shear force and bending moment in which plane, say I have discussed with you the principle of superposition, we have applied the forces only on one plane. I could also apply the forces on another plane and cause the beam to bend like this, there again you can use the principle of superposition.

I can analyze the problem independently and get the shear force and bending moment diagram in one plane, shear force and bending moment diagram in another plane. So, it is versatile; principle of superposition is versatile to handle different situations.

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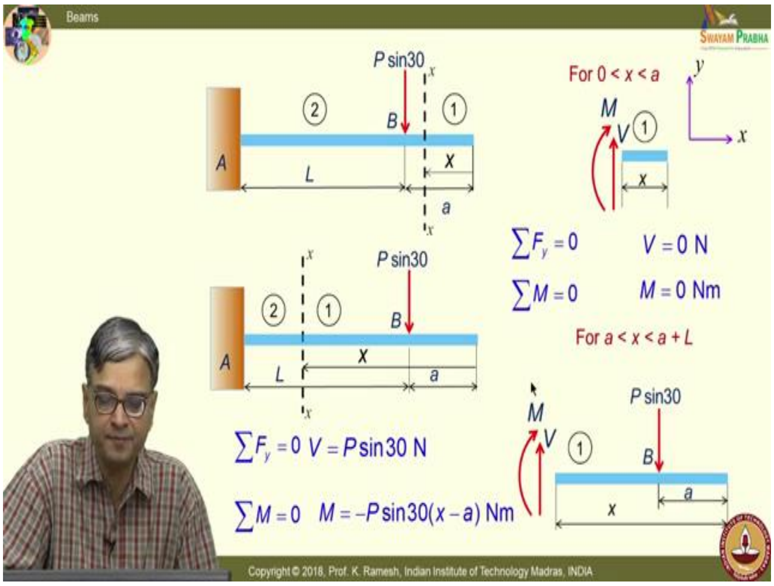
So, now I have this and you already have answers for all of it, earlier we have had this force as simply P and now I have only a small difference in terms of its magnitude. So, your appearance of shear force and bending moment diagram will be similar. See my idea is to bring in a variety of concepts so simple examples. See in all the cases I have taken a cantilever beam you could also solve problems for a simply supported beam. We have done it for a uniformly distributed load a simply supported beam we have taken and then we have determined what is the shear force and bending moment diagram.



So, the idea is how do I interpret the loading of external forces on the beam? If I have a protrusion how do I handle it, if I have an incline load how do I handle it and I would also urge you to visualize how can I apply these loads physically? Say I have shown the beam is subjected to a moment only what way you can visualize; I can add a bar at the end of the beam have two handles fine in one handle you pull it another handle you push it so you apply a couple.

It provides only rotation see we have not worried about how the loading is actually applied, but if we visualize it makes you a better engineer. As engineers I would like you to visualize some of these actions. So, I get $V = 0$ and $M = 0$ and I get this shear force and bending moment diagram. It is very simple I do not think, I need to spend more time on it, I will just show this and you can verify these final answers. I get V = P sin 30 and M = -P sin 30 (x - a) and I draw the shear force and bending moment diagram one below the other.

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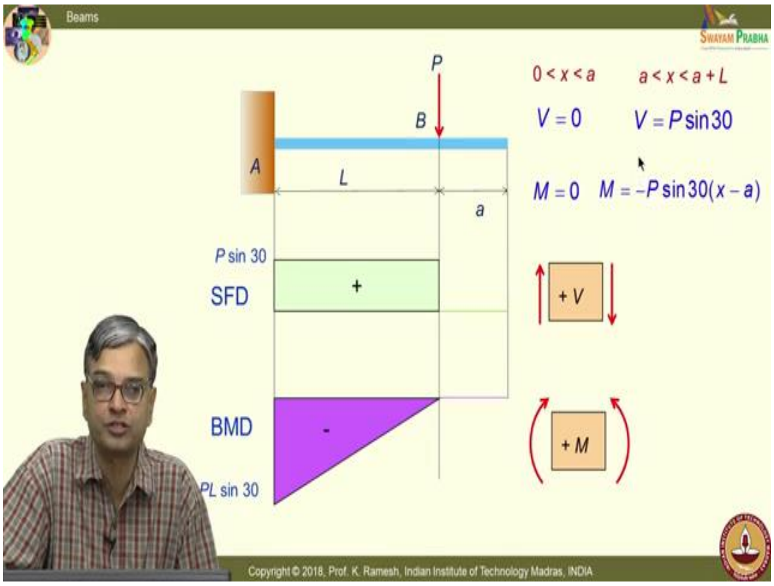

So, I have this; and I have repeatedly shown draw the loading, shear force, bending moment one below the other always along with the sign convention. So, you can summarize your final results in a portion and then do it.

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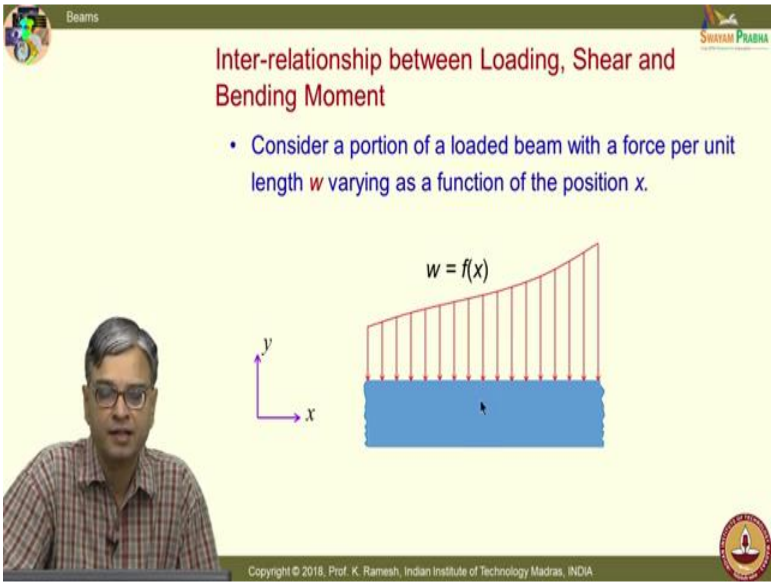

Now, let us move on to a very important aspect, suppose I have a variable loading I have already pointed this out to you there is some sort of an interrelationship between the appearance of a shear force diagram and bending moment diagram.

Now, we will formalize that interrelationship; and what I am going to do is I will take a beam which is very long, I take a section of a beam. And I have also shown whatever the loading is a function of x, which is varying in an arbitrary fashion like this. We do not have to mimic this exactly something arbitrary, we have a continuous loading acting on the beam fine.

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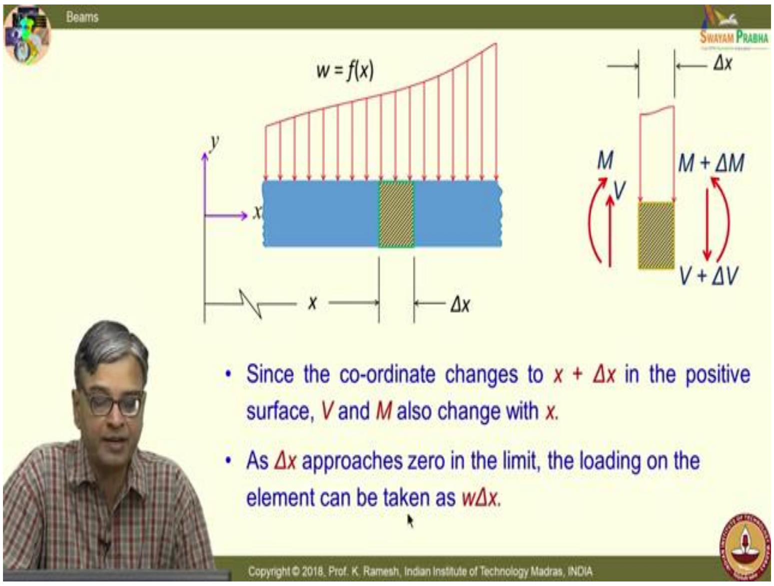
And what we would do is? We would identify a section at a distance x from the reference axis and isolate a small element of width Δ*x*. So, it is fundamentally different then we have shown this sign convention I do that as a block, I put something on the positive phase something on the negative phase; and I cautioned you that this is the block of length 0. But here I cut a section at *x*, I cut another section at x + Δx from the appearance of the diagrams they look of similar dimensions, but here I deliberately label it as distance delta x.



So, now, following the sign convention you should put the shear force and bending moment acting on this surface, shear force and bending moment acting on this surface. You have already looked at shear force and bending moment diagram do they remain constant depending on the application of the load, they can remain constant, but in general they all vary from point to point; and I have deliberately shown a variable loading.

So, when I have a variable loading you can anticipate shear varies in general from point to point, bending moment varies in general from point to point. So, if I put what are the shear force acting on this surface, when I go to the surface here I have to put an incremental change on the shear force is the idea clear; you have to catch this idea, because I have taken a section at x and cut another section at x + Δx. I am removing this block and I want to find out the free body diagram of this block, it is not then I pass only 1 section, I pass 2 sections at a distance delta x and want to find out under what forces this block is in equilibrium.

So, I will have to put this as $V + \Delta V$ I am looking at the positive direction. So, I put this as $+\Delta V$ by mathematics will tell me whether it is plus or minus. And when I put bending moment like this, you could also various $M + \Delta M$ this idea you have to catch. I have taken a block of length Δx the height is equivalent to the height of the beam.

So, there is a variation of the shear force as well as bending moment, when I look at the negative surface as well as the positive surface of this rest is then straightforward. I have to write the equilibrium of this block and establish a relationship between variation of shear force and bending moment and this variation with respect to the applied load. This is the key aspect in current discussion see you have understood this rest of it is near application of your equilibrium conditions.

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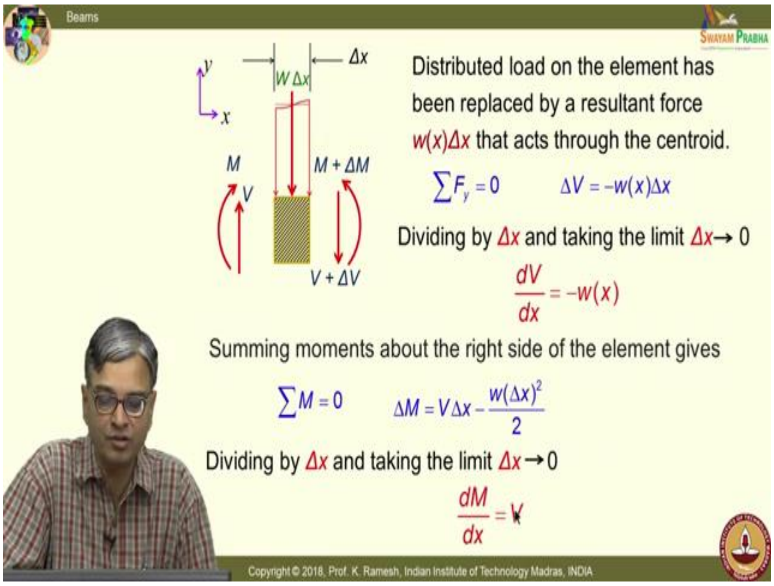

And as Δx suppose to zero in the limit, the loading on the element can be taken as $W\Delta x$ that is what I am going to do and I take this diagram. So, I replace this by a concentrated force which we have always learnt, I can replace a distribution by the principle of composition and then put that as a resultant passing through the centroid.

Then write the sigma $\sum F = 0$, $\sum M = 0$ for this block rarely straightforward. $\sum F_y = 0$ gives me $\Delta V = -w\ x\ \Delta x$. And dividing by Δx and taking the limit $\Delta x = 0$ gives me a very important relationship $dV/dx = -\ wx$. So, similarly I sum the moments please verify these calculations yourself independently and finally, writing the expressions. And we have already said delta x is very small and what way would I keep Δx^2 I would make it as 0 ok.

So, dividing by Δx and taking the limit Δx tends to 0, I get this $dM / dx = V$. So, in this class we have revisited the sign convention, I said sign convention is a discipline that you need to follow because depending on the problem you may want to solve the left portion or the right portion whichever the mathematical steps are less, but you should get one unique answer on the value of bending moment as well as its sign value of shear force as well as its sign.

But you could have different sign conventions there is no one said sign convention like we have keep left in India and keep right in United States where you have the road rules. So, whatever the rule that you adopt consistently follow that. Then we have solved the variety of problems with graded degree of complexity; the idea is to recognize what way we interpret the loading on the actual beam.

The loading could be applied by attaching a member to the beam you apply the load or you apply the load which produces axial compression or bending your interest is to plot the shear force and bending moment diagram recognize the appropriate force and then do it. Then we also discuss a very important concept of principle of superposition. And finally, we also determine the interrelationship between shear force and bending moment, shear force and the load applied; and I have always recommended you must plan your answer script as well as your notes.

So, that you always write the loading diagram, shear force diagram and bending moment diagram one below the other. Its utility you will find for difficult problems, where you may miss the plotting of the bending moment diagram, it may be difficult where you can take advantage of this interrelationship.

Thank you.