Engineering Mechanics
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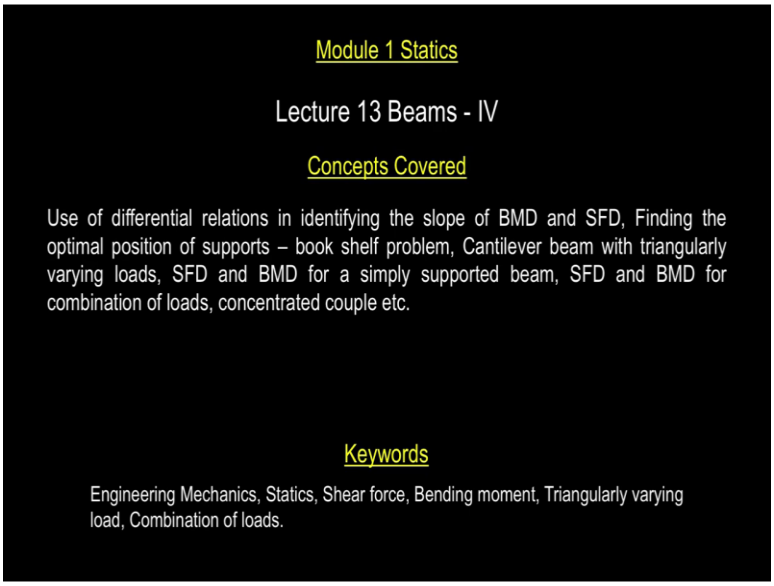
Module – 01
Statics
Lecture – 13
Beams – IV



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Let us continue our discussion on Beams.

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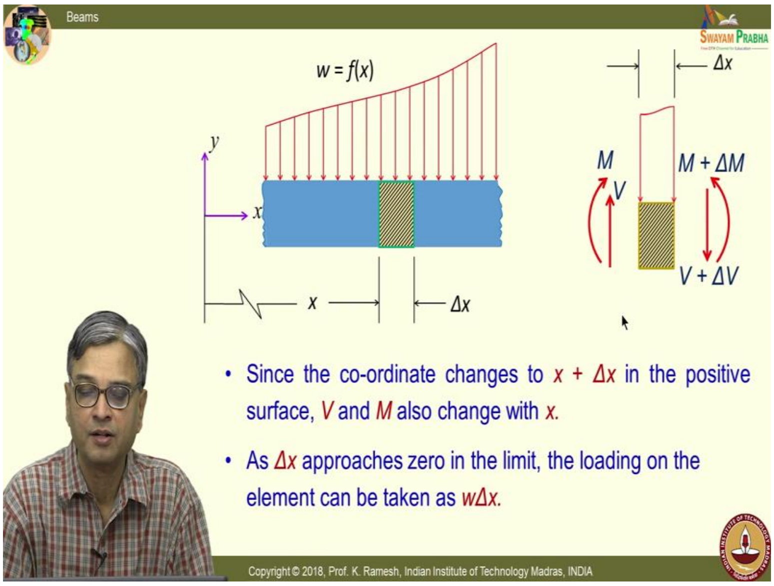
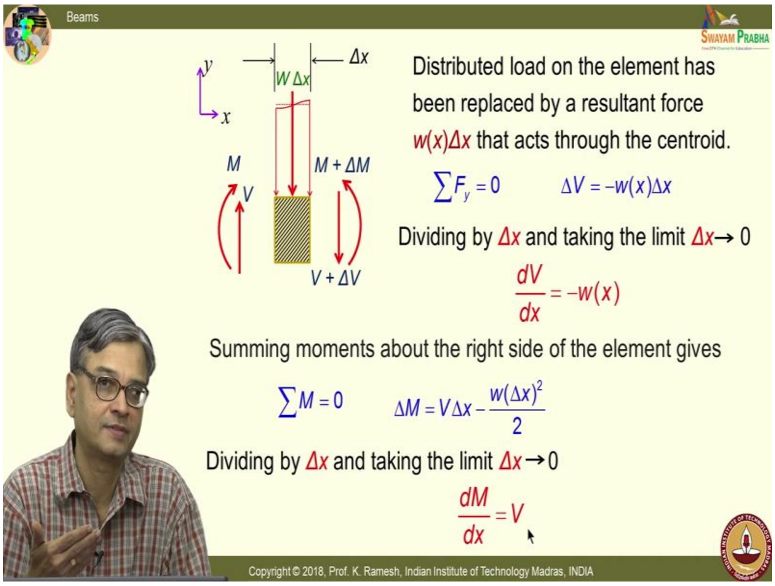
See in the last class we had looked at when there is a variable loading on the beam. We cut two sections; one at a distance x from the origin, other one at a distance delta x from it. So, we have looked at two sections cut on the beam and we have isolated this and have drawn the free body diagram for this. So, far you have seen that in general both shear force and bending moment varies from point to point along the length of the beam.



They do not remain constant, when they remain constant, they are exceptional situations we had seen when shear force is constant, we had also seen situation where bending moment is constant, they are special situations. In general, both shear force and bending moment varies along the length of the beam. So, when I have taken sections at x and x + Δx, I will have to write the shear force and bending moment on the negative surface and positive surface with an incremental change. You have to recognize when we wrote the sign convention for convenience, we had shown a finite length.

I said it is showing what is happening on the positive surface and what is happening on the negative surface for convenience you have provided a distance between them, it is not a block taken out from the beam. Whereas, here we cut at section x and section x + Δx and you have to write the incremental change in the shear force as well as the incremental change in the bending moment and it is easy to get the equations of equilibrium written.



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And what we have learnt was a very important interrelationship *dV/dx* = -*wx*. Please note we have got this expression when loading is acting downwards, we have got the expressions for that situation. And we have also been able to look at the moment equilibrium and finally got the interrelationship *dM /dx* = *V*. See I have all along been saying it is a good habit to plot loading diagram, shear force diagram and bending moment diagram one below the other. In fact, you would see the advantage in the problem that we are going to solve next.

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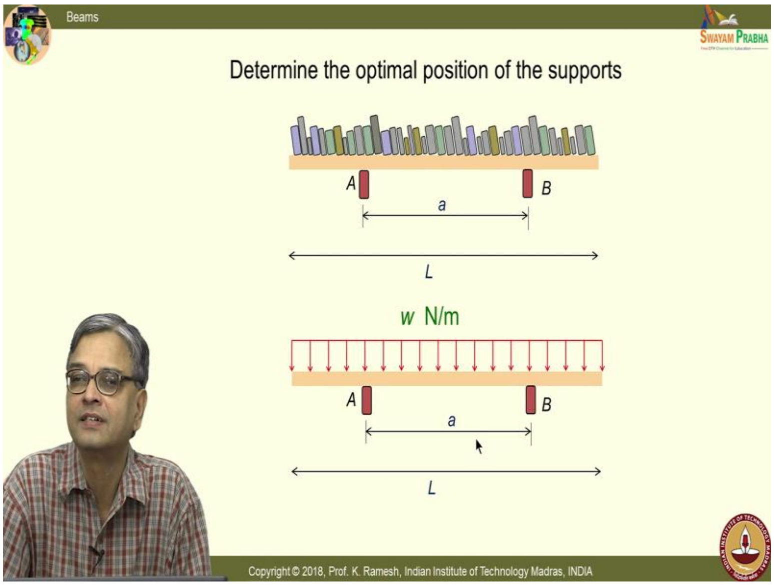
You know I have a bookshelf impromptu bookshelf you have a plank and you have two supports and books are stacked on top of it. You know I also thought that I should give some simple exercises on how engineers do idealization. And you know very well that books have different weights and they are stacked on top of it.

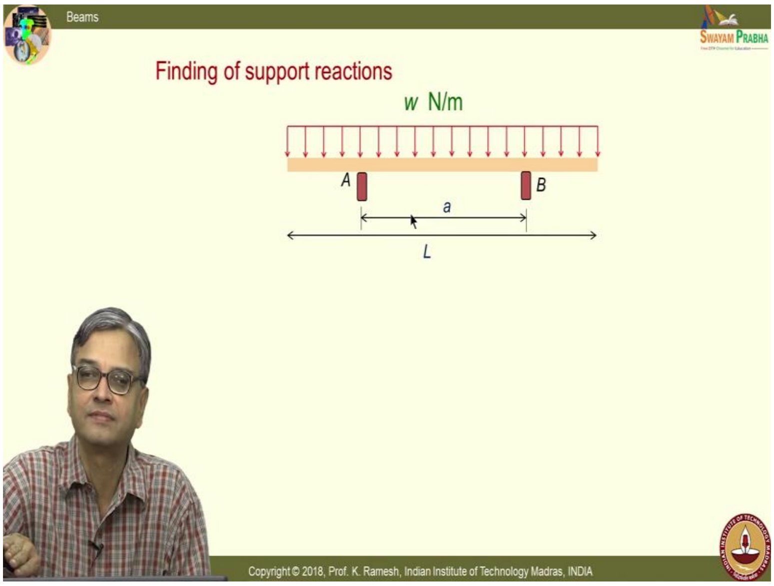


Suppose you want to find out what is the bending moment acting on the plank, what can be an idealization to start with? Can I take the weight of all these books to act at the central of the beam and start proceeding or can I do slightly better, yet simple to analyze? Can I have an idealization, what are the idealizations you have seen in terms of loading? When I said that you have to accommodate the weight of the beam in civil engineering constructions, how do they model the weight as a uniformly distributed load.

So, as a first approximation you can also replace this by a uniformly distributed load on the beam, can we do that? Reasonable approximation, you are not putting the entire weight on the center of the beam you have taken one step ahead. If you want to analyze it as such you may have to bring in the weight of each one of the books what is the length it occupies and put the variation. But finally, it can be averaged out to a uniform loading like this, this is a good approximation to start with.

See the question here is determine the optimal position of the supports. So, you have to understand how do I impose the condition of optimality. When do you say the supports are located at optimal positions? You know very well that bending moment is going to vary along the length of the beam and do you realize by adjusting the supports you can modify their variation of bending moment on the length of the beam. Can I put the supports at the ends of the plank will that be an optimal position; obviously, that is not an optimal position.

Intuitively can you say what should be the length *a*? Because we all say why do not you solve by intuition, what is the ball mark figure, what do you feel intuitively what you feel *L*/2. Now let us go and look at the mathematics and find out whether our intuition is correct, for the idealized loading, the moment I idealize the problem I do not worry about the actual problem. The answers will closely match with reality within small amount of error which is tolerable for a given design situation. Intuitively you feel that *a* should be equal to 0.5 *L*. We will see what way the mathematics is going to give us the answers.



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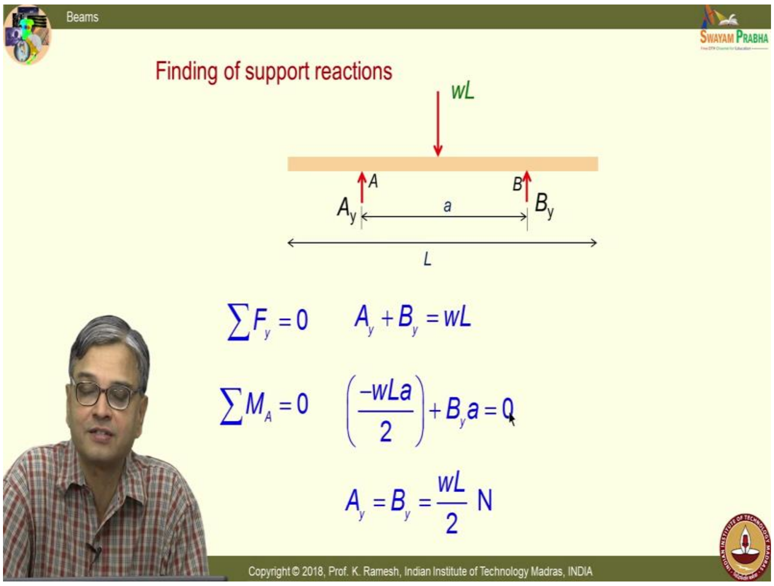
We have just taken some blocks and then put it below the plank. Suppose you want to keep your table clean and you have stacks of book, you have found out a plank and then put a plank and then put some support and what way can I idealize this support? I can simply say that arrays translation in a vertical direction, you can freely move the plank on top of it.



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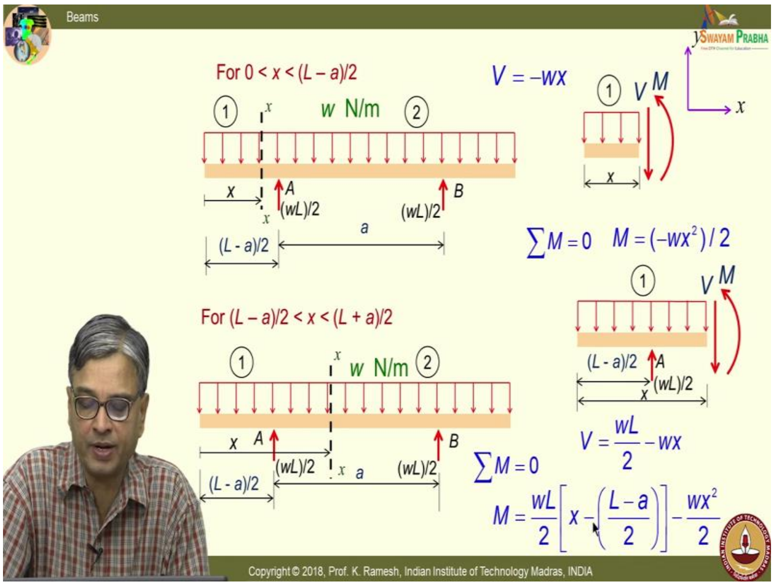
So, I can simply say that this is replace by a normal force. You get the idea how engineers start looking at problem, capture the essence of the problem, but simplify from mathematical analysis point of view. And we have also seen earlier for the purpose of finding out the reaction I can replace the distributed load by its resultant and this is the problem involving symmetry you should recognize that. Once I find out A_y, B_y is automatically fixed.

Even before solving you can guess what is the answer what is A_y and B_y, they support the load wL. So, it should be one half of wL. So, I have $\sum F_y = 0$ this gives me $A_y + B_y = wL$. And I have also told you when we solve problems when we write moment when I put the subscript the subscript denotes about which point, I take the moment.

So, I take the moment about point A. So, that gives me this gives the clockwise moment and this gives an anticlockwise moment about A. So, I have $(-wLa/2) + B_y a = 0$ and this gives me $A_y = B_y = wL/2$. In fact, when you solve problems after problems even by inspection you should be able to identify what should be the values for A_y and B_y. Nevertheless, you should write down the mathematical expressions you convince the teacher that you have really determined them not copied it from anybody else.



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Now, you have a problem like this and I have to find out the variation of shear force and bending moment along the length of a beam. You should know how many imaginary sections that I have to cut so, that I get the variation for the entire length that is also the decision that you have to take. See if you look at here when I move along this length, I have a load coming across here. Suppose I have this support at the end and this support at the end just one cut was sufficient, we have solved a problem which is simply supported uniformly varying load we just had one imaginary section that determine the variation along the beam.

So, one aspect which you have to keep in mind is whenever you have external loads you have one section like this and between these two supports I can I need to have one section and after this I need to have another section for this. Or you can also argue from symmetry if I determine for a section here and section here you can also write without calculation what should happen for the other section, but I would do for all the three sections. So, one of the decisions that you have to take when you have to draw the shear force and bending moment along the length of the beam, you should have a clear idea how many imaginary sections you need to take it. So, that you get the expression for the entire length of the beam.

So, I put a section at x and identify this as a section 1 and this is easy for me to analyze. So, I take out this and we are following the convention by medium so, I put it on the positive surface a negative direction *V* is positive and anti clockwise moment is possible. But the moment I put these forces as per the sign convention I will have to put all my quantities with respect to the reference axis when we employ the equation of statics do not make a mistake there. So, I get $V = -wx$ you know I am not solving it step by step I am missing steps I expect you to fill in the blanks. So, when I use $M = 0$ that gives me $M = -wx^2/2$.

So, then I move on to a section in between the supports A and B. So, I have a section like this, I take out this draw the free body diagram make a neat sketch of the free body, label all these values. See if you had numbers for a w and L you know the arithmetic will look simpler, but now this is written in a generic fashion. So, it looks clumsy in that sense and I have to complete the free body. So, I put the unknown forces shear force and bending moment like this. And when I apply the equations of statics, I get the shear force as $wL/2 - wx$ and I get my moment equation as

$$M = \frac{wL}{2}\left[x - \left(\frac{L-a}{2}\right)\right] - \frac{wx^2}{2}$$

Please verify this mathematics, check in your notes in your notebooks verify the calculation. So, I have determine for this portion, the portion 1 what is the expression for shear force and bending moment and in between the supports I had made a cut and I determine the shear force as well as bending moment and I would also make a cut at this portion of the beam and write down the equations.

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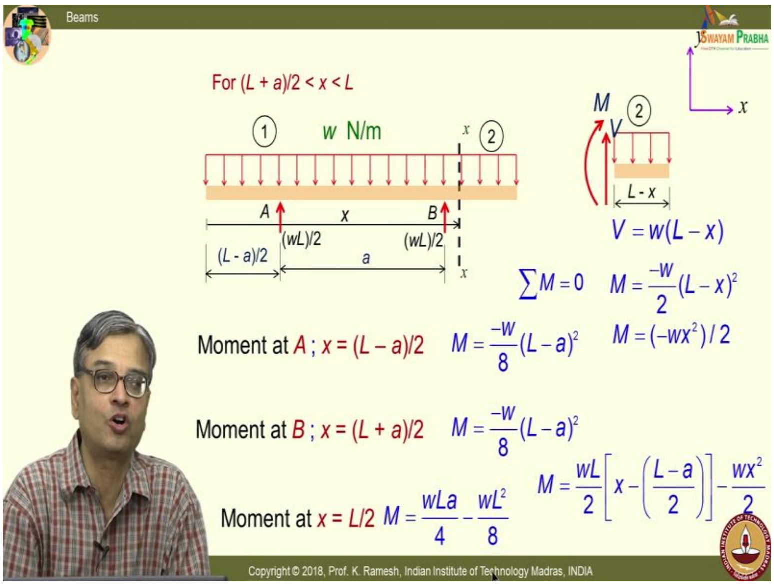

So, I would take a section like this and this is what I said you know you have a freedom to choose which portion of the imaginary cut you will take it for finding out the forces. In the earlier two cases we had taken the segment 1 for analysis in this the mathematics is lot simpler if I analyze the segment 2.

So, this is where your sign convention is important to get consistent results including sign for plotting shear force and bending moment. And I would put the shear force and bending moment I am looking at the negative surface. On the negative surface we have said the positive V is considered as positive, and we have taken the bending moment as clockwise. And this is again very easy to analyze I get the shear force as $V= w(L – x)$ and I get the moment variation as M = -w / 2 (L – x)^2. See what I need to do is I need to plot the variation of shear force and bending moment along the length of the beam.

So, I need to get the values at key points, which form the key points definitely the starting point of the beam is important. Whenever I have a load change I would like to know what is the value here; I would also like to know what is the value here; I would also like to know what is the value at the end; in addition I would also like to know what is the value at the mid span of the beam. See for shear force the expressions are simple and you can easily write down these values from the general expression. In order for me to easily plot the variation of bending moment let me get these values of bending moment from the earlier expressions.

We have already determined for the section ending at A, a generic expression for moment as $M = -wx^2/2$. So, if I need to find out what happens at point A; point A is located as $x = (L-a)/2$, when I substitute for x in this expression this simplifies to; please do the simplification yourself I will give you minute for you to do the simplification and you can verify your simplified expression with what I have in my slide. So, you have to go back to your notes and see what was the original expression we had for the segment ending at *A*, for the segment between *A* and *B*, for the segment after *B*.

And you have symmetry in this problem. So, what I would get for segment 0 to *A* would be similar for *B* to the end of the beam, we would see that black and white in our slide fine. So, when I substitute for x this expression turns out to be $M = (- w /8) (L – a)^2$. Can you find out what is the value of the bending moment at mid span of the beam? You have the expression; you have the generic expression at mid span $x= L/ 2$. So, substitute $x = L/2$ and calculate at the mid span what is the value of the bending moment?

You know before even I do that I find out what is the moment at *B*, where x is defined as $x = (L + a) / 2$ and because of symmetry I get an identical answer; I get an identical answer at *A* and *B* that is what you anticipate. This is what I say you should have physical appreciation of the problem when there is symmetry the results what you get also show that symmetry that is one kind of check while you solve the problem whether you on the right track or not. And if I have to find out the bending moment at $x= L/ 2$, I will have to substitute $x= L/ 2$ in this expression.

The generic expression is

$$M = \frac{wL}{2}\left[x - \left(\frac{L-a}{2}\right)\right] - \frac{wx^2}{2}$$

When I substitute $x= L/ 2$, I am going to give the final simplified expression please verify whether you have also got

$$M = \frac{wLa}{4} - \frac{wL^2}{8}$$

some of you seems to have got identical answer.

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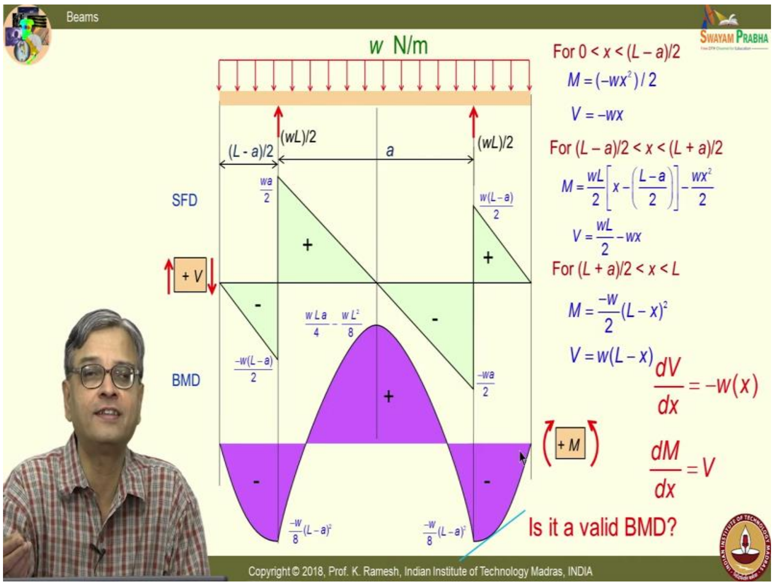


Now we are all set to plot the variation of shear force and bending moment along the length of a beam. And what is shown here is I am also giving you ideas how you should plan your notes and answer scripts. We have done all the calculations earlier and the final expressions are written down here to facilitate easy plotting of the shear force and bending moment diagram. So, if you follow that discipline it will help you.

Now I will plot the shear force and mind you this animation is plotted from left to right. We will revisit this again why I plot from left to right because I have also told you have to isolate the free body determine the forces acting at various sections of the beam fine. But you should also have another methodology to verify whether your final answers are correct for that if I move from left to right it is easy for me to plot the shear force when I use the convention of Merriam.

So, I have this expression this is $V = -wx$ and it varies linearly. And once I have an external support, this shear force increases at that point and the total value is equal to $w L/2$ and then it drops down touches 0 at mid span and goes like this and plotted like this. Please mind you this is only illustrative it is a highly an exaggerated picture of the shear force variation. You do not get confused that I plot shear force very clearly, I also plot the bending moment they are of different units also shear force is a force.

So, I write it in terms of Newton's, when I go to moment, I write it in terms of Newton meters, you should not conclude in my picture I have a very large shear force and it is going to introduce me large stress. In fact, in our practical problem usually the maximum stress developed by a shear stress would be at least 15 to 20 times less than the bending moment. The stresses introduced by bending moments are very significant and that truly dictates the design of a beam, shear is only an additional effect that you have to consider.

So, when I have to find out optimal support, I was look for what is the variation in bending moment to take a decision shear force is only an intermediate step. And since you are learning how to draw shear force diagram and bending moment diagram, we are also drawing the shear force diagram for this problem as well as bending moment diagram for this problem.

So, you have to keep in mind from stress analysis point of view the effect of bending moment is very significant on the beam, the stresses introduced are much higher than the stresses that is caused by shear force. So, do not get carried away by an illustrative sketch. Now let me plot the bending moment variation, I have expressions here see which shows that this is the quadratic expression, this is also a quadratic expression and this is also a quadratic expression.

And let me plot we have already determined the values at key positions. What would be the bending moment at the starting point at the free end? It is 0. So, you have that what is happening at support A, what is happening at mid span, what is happening at support B and what is the other free end all these quantities you have. And let me plot this, I have the bending moment diagram drawn like this.

The bending moment diagram is very colorful and this is again illustrative. See do not compare the peak magnitude of bending moment with peak magnitude of shear force they are shown very clearly to identify what are the variation we are not plotted to a scale. Now the question is you are drawn the diagram so nicely you should also investigate have you drawn it correctly? Is it a correct bending moment diagram? We have definitely followed this is the quadratic curve, this is the quadratic curve, this is also a quadratic curve.

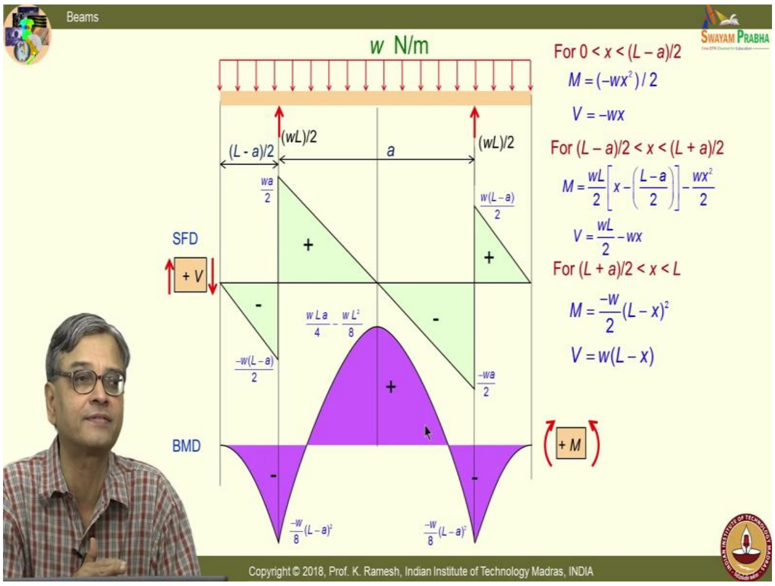
You have learnt few slides back what is the interrelationship between the bending moment and shear force. Can you use that knowledge and verify whether this bending moment diagram is correct? It should be if you want to accept as a final answer, it should verify the test, it should pass the test and you investigate whether the interrelationship what we have evaluated is valid here, shall we go and investigate now. I have drawn consistently in all my slides the sign convention for shear force and sign convention for bending moment. See I deliberately put it at different places, if you see always on the right side then you do not get the idea of what is a positive surface and negative surface clearly.

So, when I write this as a block if I have the outward normal in the positive direction that is the positive surface, if I have my outward normal in the negative direction it is the negative surface. And we have look at two expressions; one is $dV/dx = -wx$ and what I get this is the slope is negative that is what it says. And obviously, yes, I have the slope of this diagram is this is the negative slope. So, the shear force diagram there are no major hiccups, shear force diagram is consistent with whatever the interrelationship that we have got.

Let us look at the interrelationship between bending moment and shear force. The slope of the bending moment diagram is related to the magnitude of *V*; *V* is positive as well as negative fine. So, I will have to look at the slope and verify am I on the right track. I have put a slope here and what is this slope here? This is a negative slope that is a tally with what is given in the *V*; *V* is negative so this slope is valid that is also fine. And as I come to this centre of the beam what happens to *V*? *V* goes to 0 and suppose I plot a tangent at the mid span I would also have a 0 slope.

So, this parabola is quite alright with the sign convention and also the interrelationship that what we have determined. So, there is no major hiccup in this parabola. Let us look at have you drawn this segment correctly. Can anybody identify whether this segment is also correct? It can also be correct; it can also be a correct I am not said it is wrong we have to establish why it is wrong; if we say that it is wrong. Let me again draw the slope; I draw the slope here my shear force is positive here, my slope is also positive apparently it looks I am not on the wrong side, but look at one more aspect when it is on the free end shear force is 0.

What I am getting here? I am getting a finite slope at the free end; it cannot be there. See suppose you have a plotter where you feel in this equation because people are becoming very easy now. You know you have symbolic computations have come no visualization you simply feed in the expression it itself plots beautifully. If we have that luxury then you will not make a mistake in plotting the either the shear force diagram or the bending moment diagram along the length of the beam. Because you do not have to apply your mind, you are going to determine the various y co ordinates based on the *x* co ordinate you can easily plot and connect it you will not make a mistake there. There is a possibility of making a mistake when you do by hand calculation and when you have this you have another clue that slope is 0 here.



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Suppose I replot this portion of the beam let us it satisfies all of it what we have learnt earlier. The, this is again a parabola secondary the curve, here again the slope is positive tallies with this, but the important clue what I have used is I have identified a point where the slope is 0, that guides me in finding out whether I should plot the curve like this or in the opposite way.

You can also think in one more direction see I can also draw a straight line and find out what is the value at the mid span. I can easily find out that is the just the average fine. And you can decide whether the average from your expression at this midpoint not the average the actual value at the midpoint is higher than this average or lower than this average. That will tell you whether I should plot the curve in this fashion or in this fashion.

So, now would you realize when I say that you have to plot the loading diagram, shear force diagram and bending on diagram one below the other at least proms you to think and find out whether your answer is correct. Whether you have drawn these variations correctly translated your expression correctly in the drawing. So, that is very important.

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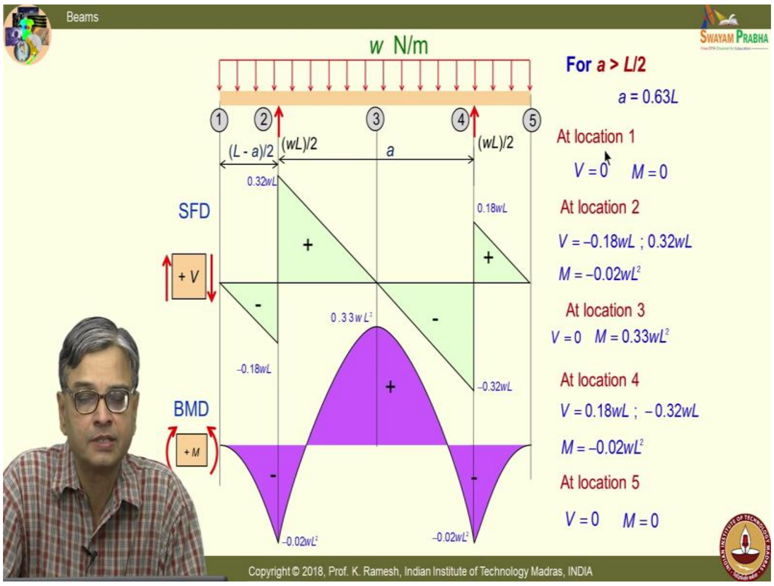
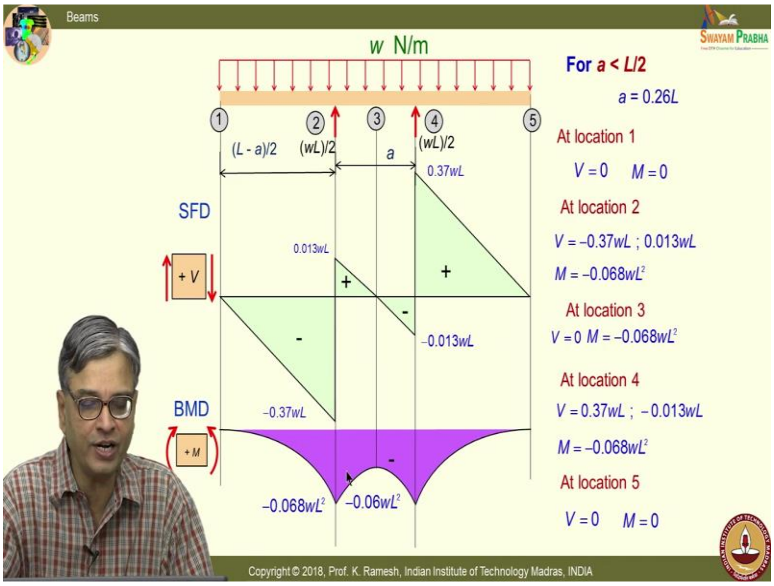


So, that is a reason why I want you to plot one below the other and I have also given these for numerical value of *a* = 0.63 L and I get these numbers like this so, this shows how the variation is. I have done it for two cases I have taken a case where a is greater than *L*/ 2. I have also taken a case a is less than *L* / 2.



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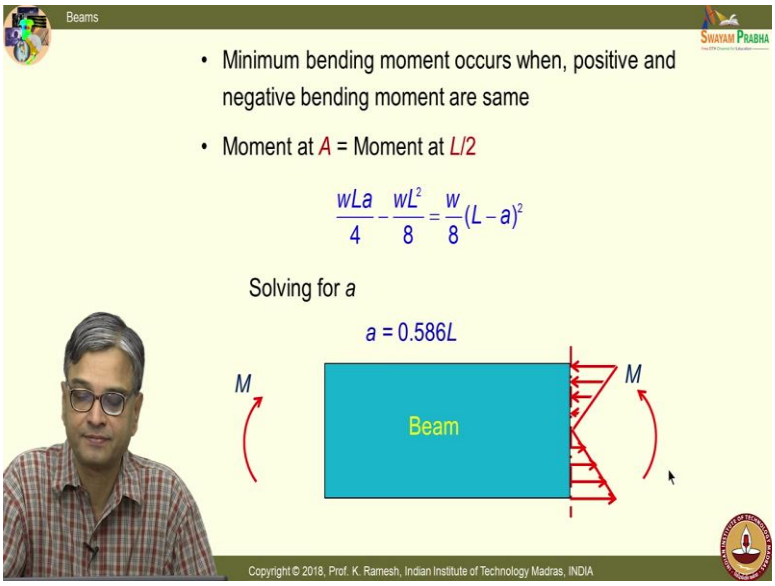
Just to illustrate by changing the position of the supports you can have totally a different variation of bending moment along the length of the beam. The bending moment variation is totally different here, you can see this in the previous slide it is like this and mind you that these are not plotted to scale I am only interested in the variation. In this case bending moment is both negative as well as positive, in the other case bending moment remains negative for the entire length of the beam.

And I go back to the generic solution which is given in terms of a, now we have to go back and answer the question what is the optimal position? The intuition said it is 0.5 we have to verify whether it is 0.5. How do I put the condition? See if I have to apply the optimal condition, I must also know how to bring in that in mathematics. Along the length of the beam the bending moments varies from negative to positive and one case when I brought them very close, I had that as on the negative side completely.

So, by placing the support I can vary the variation of bending moment along the length of the beam. Can I say that I would determine these supports in such a manner that whatever is the maximum negative bending moment is equal to the maximum positive bending moment. That means you are limiting the magnitude of maximum bending moment possible on the beam.

Even at the start of this discussion I said we are not really worried about the sign of the bending moment I will provide you a reason from strength a material a little while later we will revisit it again. Do you get the idea if I have to get the optimal position, even though one or done a second level course I have always been saying this is only feeding information to a second level course and I thought can't we have one problem where, with your knowledge you can also do some optimization and get an optimal solution.

So, from that perspective this is the very nice problem. So, what we are really trying to say is if I find out length a such that the bending moment in the negative side and positive side all equal, let me put that condition.



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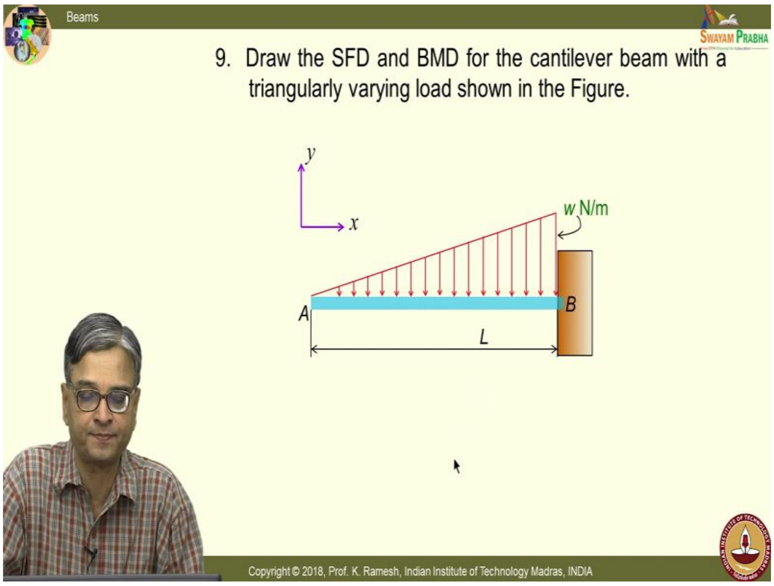
So, when I equate these two expressions, I get my a as 0.586 L. You can argue either way my intuition was correct I have got 0.5, I can also argue you are not correct you have got 0.586 this debate can continue for any length of time ok. But what you will have to appreciate is, when I have a beam suppose I have the bending moment is in one direction I am having this as a clock wise bending moment.

And when I see the resistance offered by the beam, it is tensile on the top and compression at the bottom and mind you the tensile magnitude and maximum compression magnitude are identical fine. And when I reverse the moment direction the top fiber gets compress and bottom fiber gets elongated, so this is the reason why I said when I want to find out the bending moment. I am more interested in getting the magnitude along the length of the beam either way the member is going to experience similar loading either on the top fiber or bottom fiber is going to experience a similar loading. So, this also gives a convincing answer that for optimal condition you equate the maximum negative moment to maximum positive bending moment. So, that you find out the position where the maximum magnitude is minimized. So, that is what we have done.



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The next problem is; draw the shear force diagram and bending moment diagram for the cantilever beam with a triangularly varying load as shown in the picture. See this is one of the common variations of load you come across in engineering application. The other common variation what we usually come across is uniformly distributed load. Usually when you are analyzing beams in civil engineering the self write is so important you cannot ignore that that is modeled as uniformly distributed load.

Another load with that you come across very commonly is the triangular loading; one example is when I have a fluid which is exact in pressure that could be modeled as a triangularly varying load.

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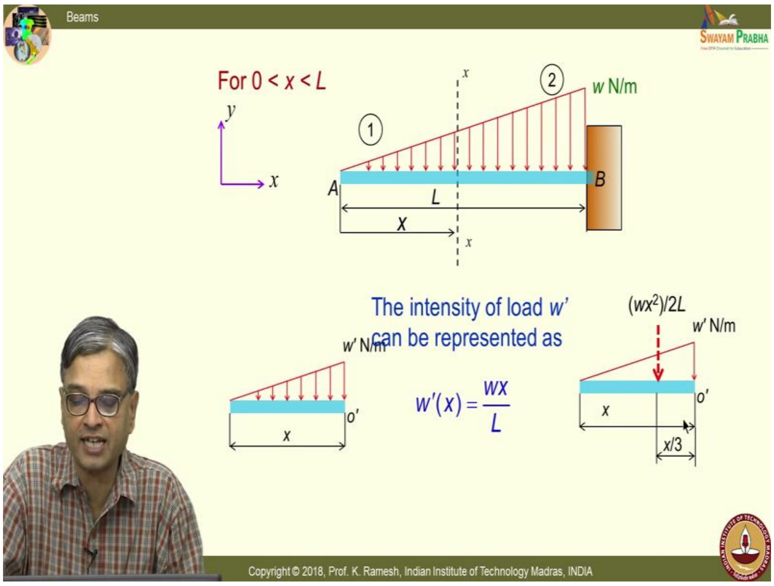
And here again to plot the shear force and bending moment diagram take a section at *x* isolated since you have complete knowledge of what happens in a triangle you know how to find out the centroid as well as the area, one can easily find out the resultant and that acts at the centroid which is at a distance *x* /3 from the base.



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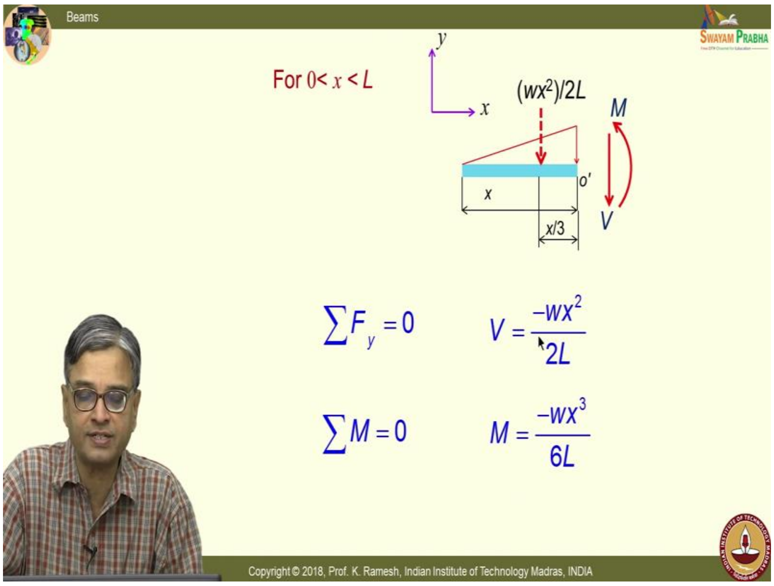
And using this one can apply the equations of statics, and

$$\sum F_y = 0 \qquad V = \frac{-wx^2}{2L}$$



This is the first occasion that you come across that *V* varies as a quadratic expression all along you have seen *V* remaining constant or varying as a linear load. Whereas, here because the loading itself is varying linearly I get a quadratic expression for *V* and a cubic expression for *M*;

$$\sum M = 0 \qquad M = \frac{-wx^3}{6L}$$

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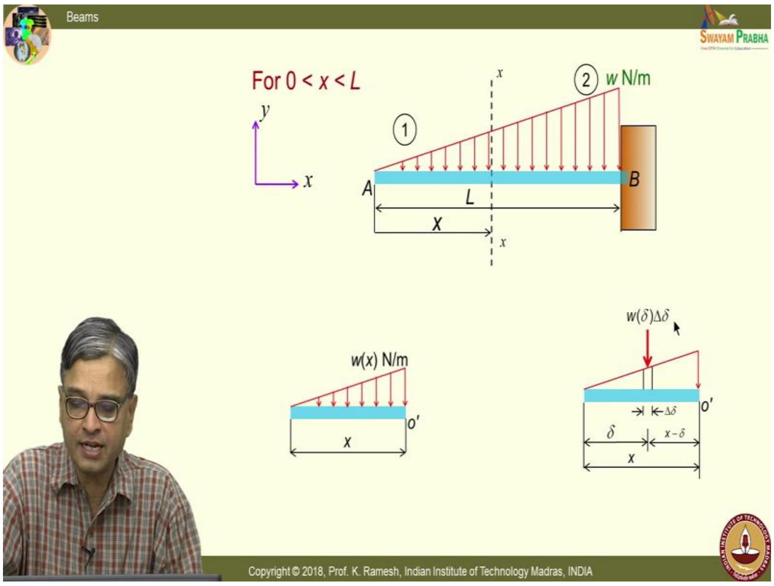

One can plot using these expressions, but we will also see suppose I have a load variation not as a triangular but it is given in a very generic fashion what is the kind of mathematics that I will have to use? Here again I take out this and I write it in a general fashion and within this load variation I have another variable delta, and go from fundamentals how to find out the centroids there again you use a similar expressions same thing is mimic here. So, I find out what is the resultant force and the moment at because of this loading from first principles.

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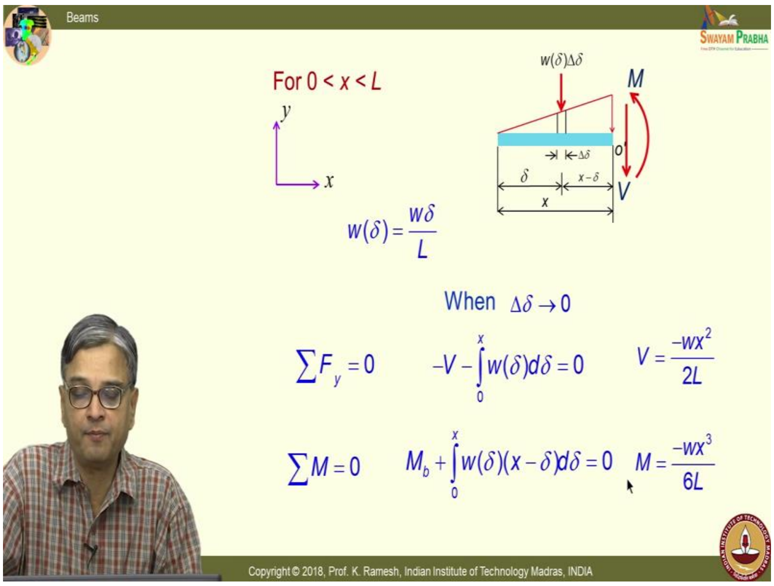

So, that is what is written down here the free body diagram is written down and the function w as a function of delta is written as w delta by L.

$$\sum F_y = 0 \qquad -V - \int_0^x w(\delta)d\delta = 0 \qquad V = \frac{-wx^2}{2L}$$

$$\sum M = 0 \qquad M_b + \int_0^x w(\delta)(x-\delta)d\delta = 0 \qquad M = \frac{-wx^3}{6L}$$

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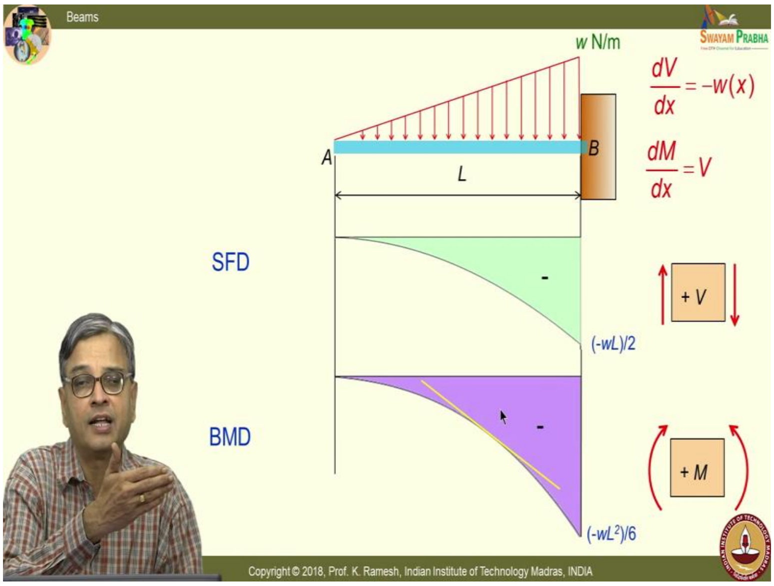
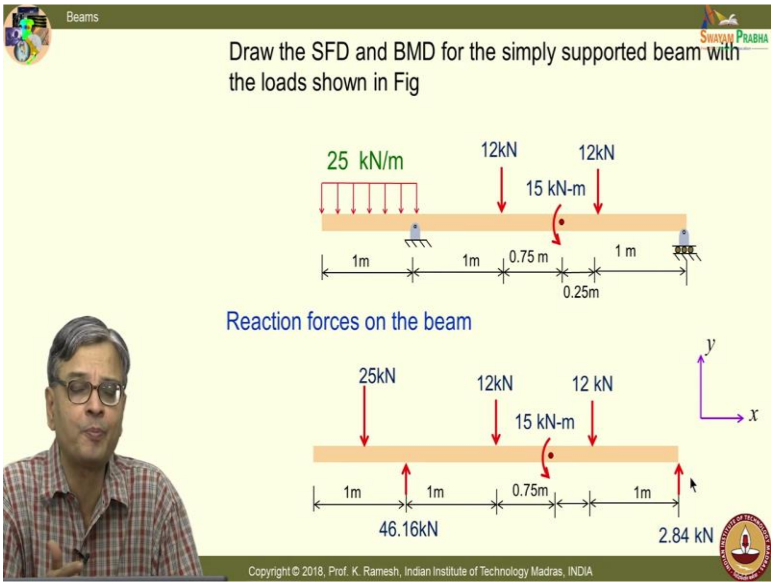
And one can plot the variation along the length of the beam. So, you plot this, this is a quadratic curve and you plot this you have to take it as a cubic curve. While doing hand plotting it very difficult to show a subtle difference between quadratic curve and a cubic curve definitely you can draw it and show distinctly that it is not a straight line. And we can also verify whether the interrelationships are satisfied I have *dV/dx* = -*w*.



So, the slope is negative and when I plot the slope; the slope is negative. So, this is ok when I look at the plotting of bending moment diagram, the interrelationship is *dM/dx* =*V* and *V* is negative. So, I expect the slope to be negative. So, that gives a verification that we are on the right track and to plot the curve whether this way or the other way. It is better that you find out what happens at the centre you can imagine a straight line and find out the average and find out what is the value.



And from the expression for bending moment find out the value at the center, compare these two values that will guide you better to plot the curve in this fashion or the opposite fashion.

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You know we have so far seen a variety of problems, where the focus was how to handle an external load? The focus was not on repeatedly learning different problems. So, that you hit one problem for you to answer the question in the final exam more on a conceptual level we have looked at it. And it is not difficult for us to complicate the whole development of loading you can have multiple loading on a beam, I have a distributed loading, I have a concentrated moment, I have concentrated loads acting on it and also, I have an over hug beam. And you can also find out what is the shear force and bending moment variation on the beam.

And when I have this you have know how many sections I have to take. So, I have to take one section here; I have to take another section here; I have to take another section here; another section here; another section here. So, in all I have 1 2 3 4 5 sections I have is it all right 5 sections I have to do. So, I have to do the calculations 5 times then only I will be able to get enough quantities for way to plot the variation of shear force and bending moment.

And if I have to draw any one of this my first step is to find out the reactions. I have to get the reactions without the reactions I will not be in a position to do this computation, but you know how to handle a distributed load, how to handle a concentrated bending moment individually you have solved the problems. So, when they come in an actual beam it is not difficult for you to accommodate that in your calculation. I would appreciate you do this as a homework and bring it out then we will have a discussion, I would expect you to identify different free bodies put the quantities the way we have learnt in the class. And determine the values and plot the shear force and bending moment diagram and for you to compare and facilitate your answers I have put the reactions. And the reactions are 46.16 kilo Newton on this end and 2.84 kilo Newton on this end.

So, it is not at all difficult for you to draw the shear force and bending moment diagram when multiple loads are given in the beam; the only difference is its going to take a little more time for you to accomplish this. So, in this class we have determined what is the optimal position of supports for a simple problem of arranging books on a plank. In the process we learnt the sign of the bending moment is not of that concern only the magnitude is very important that knowledge you derive from what you see in chunk of materials.

And we have used that as a criterion to write the optimal condition and determine the distance between the supports. In a sense it tallied very closely to your intuition, but from puritan point of view it is not exactly intuition you need the elaborate theory of bending of beams is necessary for you to identify an optimal location correctly. So, that is another way of looking at it.

And in all these shear force and bending moment diagram, I re-emphasized what is the role of interrelationships. How do you interpret the interrelationships to verify your final plotting of shear force diagram and bending moment diagram? See the focus here is it is not that you have got an answer and you have plotted it in one way without verifying whether this is the correct way of plotting or not.

As engineers you have to be very definitive in your final answers and statements. So, need to use every possible auxiliary method to verify your answers, at least whether it is in the range of your expectations, some such visualization is very essential for you to become a good engineer and we have also looked at another problem where there is a triangular load variation. And finally, we looked up a beam with multiple loadings for you to find out the shear force variation and bending moment, I wanted to do it the way that we have done in the class. And in my review, I will solve it from a different perspective at least give you an idea how to verify your answers.

Thank you.