**Engineering Mechanics**
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**Module – 01**
**Statics**
**Lecture – 14**
**Virtual Work – I**

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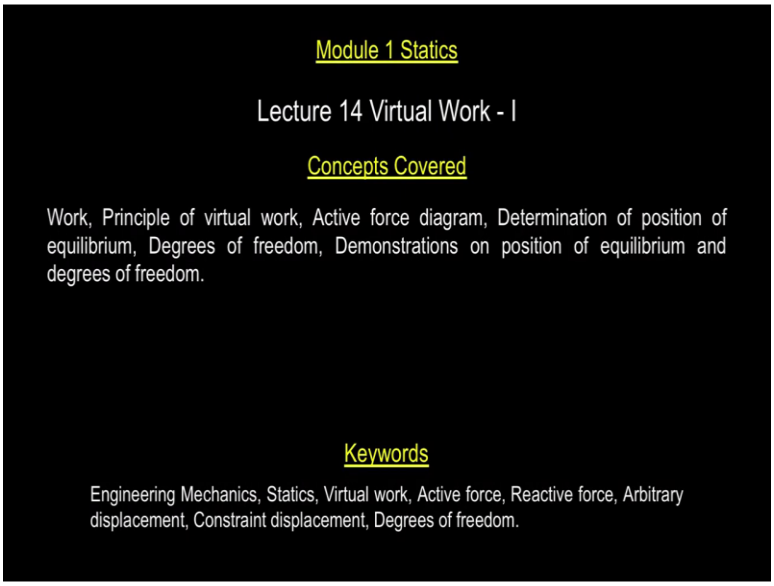


Let us move on to our next chapter on Virtual Work.

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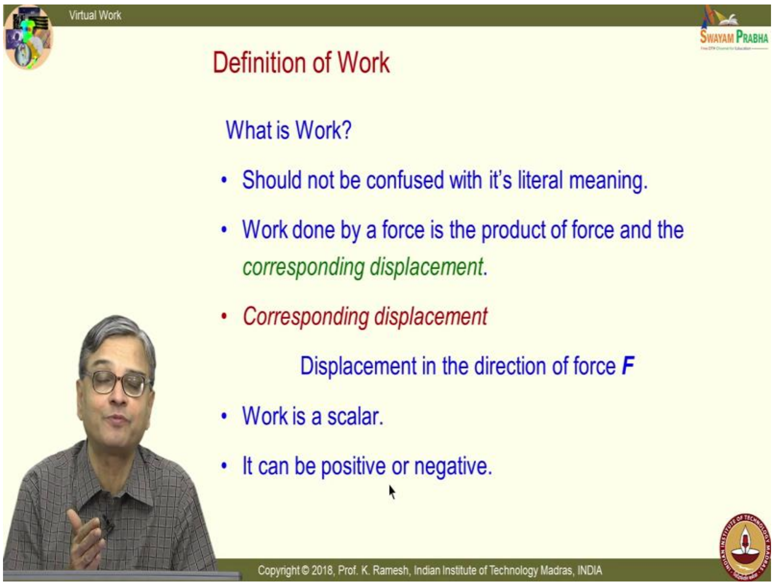
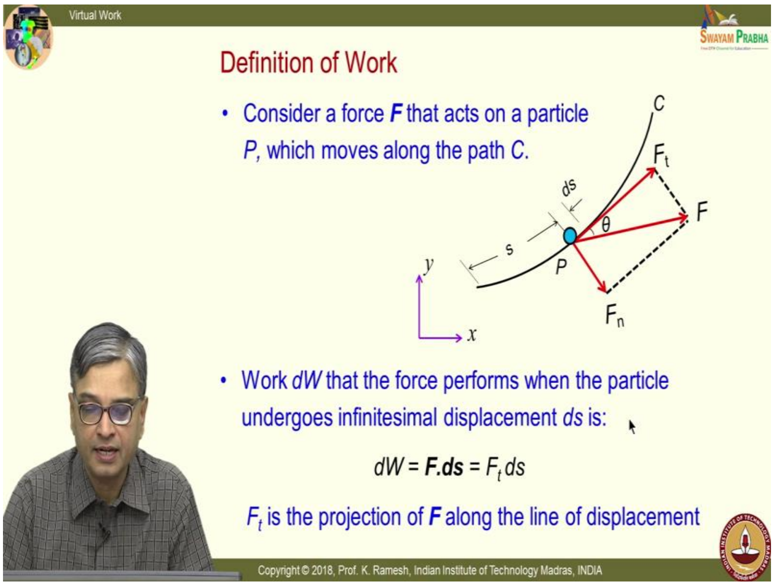



So, we have to understand what is the definition of work. See we have always seen what is the origin of the word truss, we have also looked at where do we use beam you use it for light beams, beaming smile, but in the context of engineering mechanics we call the member that supports the load transverse to its axis; we call it as a beam. Similarly, when you say work, we should also understand what is work? It should not be confused with its literal meaning that is quite obvious.

And work done by a force is the product of force and the corresponding displacement that is very important. So, you should look at which component of force contributes to work. So, you call that as a work absorbing component. And what is corresponding displacement? Displacement in the direction of force *F* that is very important.

And in contrast to force which is a vector work is a scalar. All of you know this. It is better that we revisit this whole the concepts work can be positive or negative and that is going to be very crucial when you are the working on principle of virtual work, we should identify when I have positive work when I have negative work.

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Now, let us consider a force F that acts on a particle P which moves along the path C you have an arbitrary path, you can make any path of your choice, I have a particle P which is subjected to a force F at this instant of time. Now we will have to find out which component of force contributes to work. The idea is the particle moves along this path. Let us say that it is at a distance s from a reference. Under the action of this force F, it has move in an infinitesimal distance ds and you should also look at the symbolism that I am using here. We are now discussing about work not about virtual work when we discuss about work, I put the infinitesimal quantities as simple d and s. We will use a different symbolism when I going for virtual work.

And when I have this force this could be split into two components tangential to the path and normal to the path then I can easily say what is the work done in the process I get this as $dW = \mathbf{F.ds} = F_t ds$. So, only the component $F_t$ contributes to work from moving the particle from s to ds and Ft is a projection of F along the line of displacement.

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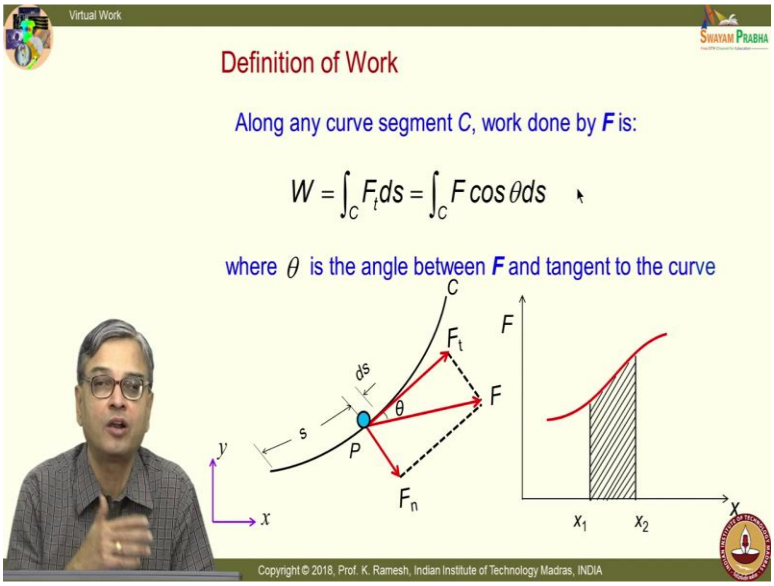


Suppose I want to know what is the work done when it has moved along the curve? I simply do the integration of this over the path and this also you have studied earlier. And you can find out work done as the area under the curve, when you go from one point to another point force is varying along the path displacement is also varying along the path you can always find out the work done in the process.

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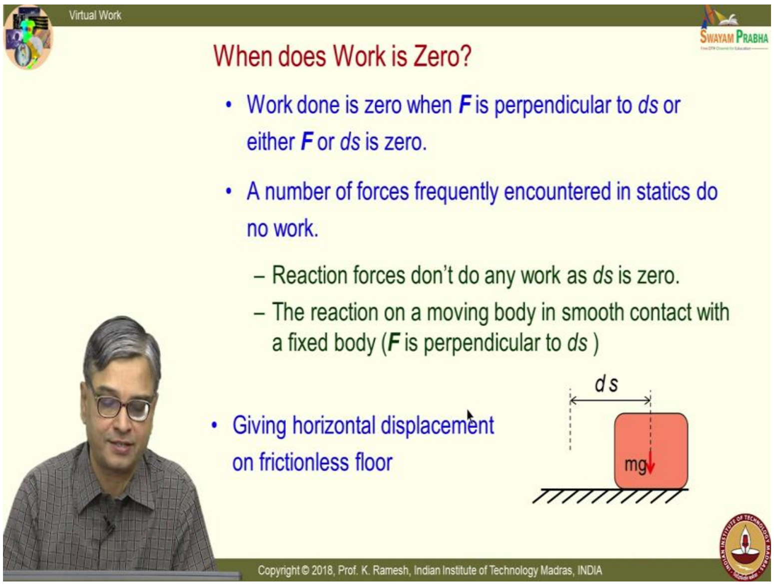
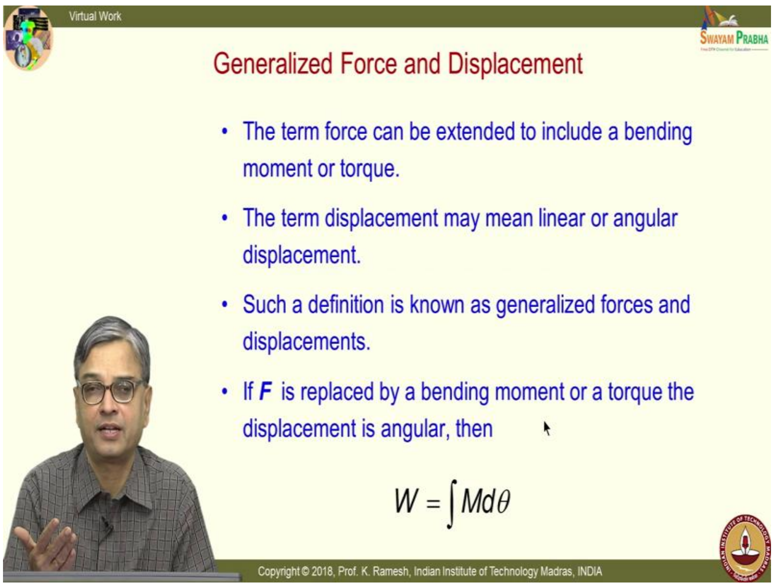



We will have to know when does work is zero? Work done is zero when the force is perpendicular to d s. We have already seen then normal component did not do any work; it is only the tangential component contributed to work. See these concepts may appear simple, but when you employed for a problem all these properties are useful. And it is also quite obvious since I find out *F* as the product of the force and the displacement if either force or ds is 0 then net work done is going to be zero.

So, in the context of engineering mechanics let us see what forces contributes 0 work and surprisingly a number of forces that are frequently encountered in statics do no work it is a great advantage. You in fact, use this advantage in solving problems. The whole approaches minimize the number of computations to arrive at the results. So, if I find a number of forces are not going to contribute to work, you do not have to consider them when you compute the work.

And we have seen the beam was supported on hinge and roller support truss was also supported like this and we saw that you will have reactive forces developed; because they are supports, they do not move, fine. You may argue when there is a thermal expansion the roller support allows that movement. So, that additional stresses are not introduced leaving that, even there you will have the reaction force will be perpendicular to it.

So, reaction forces do not do any work as ds is 0, when I have a roller support, I can say that the force is perpendicular to the displacement it does not contribute to work. So, when you recognize the reaction forces do not contribute to work in my approach in employing virtual work I do not have to worry about reactive forces. Let us look at giving horizontal displacement on a frictionless floor, you can also think that when I have a roller support a similar thing happens.




And I have this as *ds*, when I put this as simple *d*; we are talking about real work not virtual work. So, when I move this the force has moved, but the force and the displacement are mutually perpendicular to each other. So, it does not contribute to work you can also look at as a reaction force you can also look at what happens to the mass whichever way you do it there is no work done in the process.

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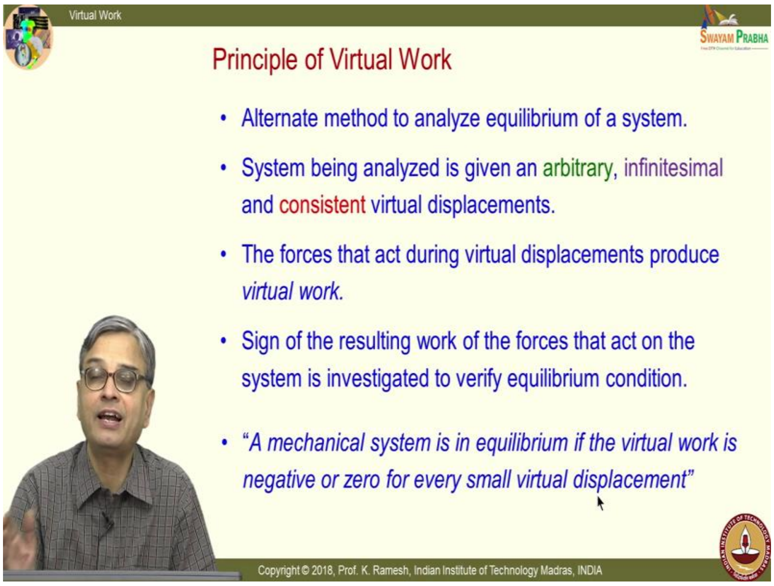
And once we have understood by a simple force and linear displacement, we would generalize the discussion, the terms force can be extended to include a bending moment or torque in such a case you would call these forces as generalized forces.

For a force *F* you have a linear displacement when I have a moment or torque, I would have angular displacement. So, if you have the force *F* is replaced by a bending moment or a torque and the corresponding displacement is angular, then I can also write work as $W = \int M d\theta$. You had $W = \int_C F_t ds$; *ds* is now replaced by $d\theta$ and you are simple force is replaced by a moment; it could be a bending moment or a twisting moment whichever the case may be.

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And what is the principle of virtual work? It is an alternate method to analyze equilibrium of a system. The system being analyze is given an arbitrary, infinitesimal and consistent virtual displacements. How many of you have climbed a simple ladder, what do you do when you go and climb on a ladder, first thing what you do. You just go and shake it and then see whether it is stable involuntarily nobody has taught you know you are very conscious of your security and if you have a simple ladder when it is leaned against of wall if you have to climb on it you slightly go and shake it.

We do precisely the same in a mathematical fashion in the principle of virtual work. You had actually given the motion to your ladder when you want to climb instead you are doing it virtually. And if you understand that then you will know I should give infinitesimal displacement and consistent with the constraints. When I have a system, the system is allowed to follow a particular path, whatever the constraint in the support you should honour it.

So, you should give virtual displacements. And whatever the forces that act during virtual displacements produce virtual work and we have already not talked the reaction forces we have said that reaction forces do not contribute to work. So, we also being in another terminology call active forces that contribute to virtual work. And you have to find out the total virtual work and you investigate whether the system is in equilibrium or not by the sign of the virtual work.

And you know the statement is like this "A mechanical system is in equilibrium if the virtual work is negative or zero for every small virtual displacement". And we have already qualified what where the virtual displacement should be should be infinitesimal and it should be consistent with the constraints of the system. And many books would simply say when the virtual work is zero the system is in equilibrium, we would generalize that statement virtual work can also be negative, we would see with as example when what sort of situations you can have negative virtual work.

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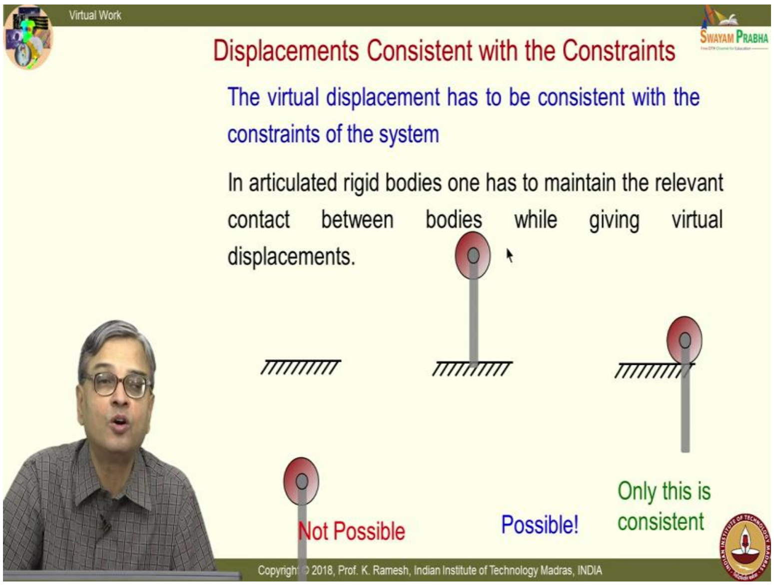
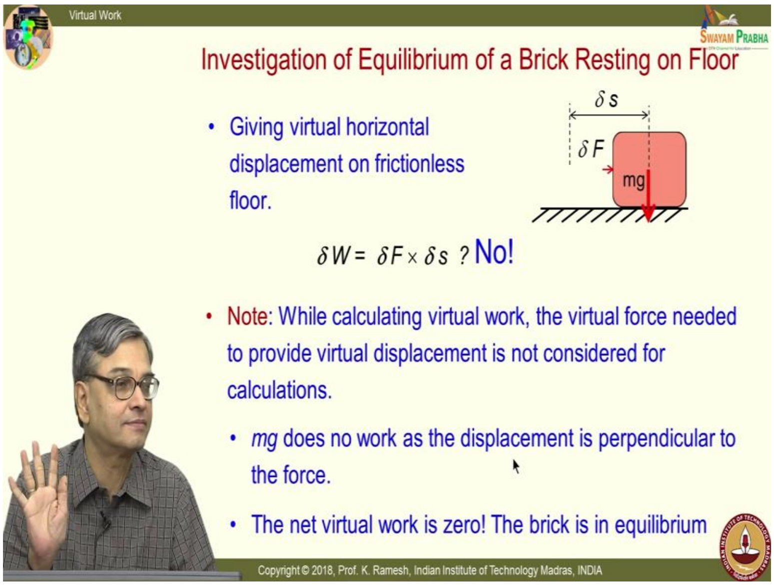



And before we proceed further let us understands what is the meaning of consistent with the constraint? See I have a floor and I have a roller on it and I would like to see what are the ways can I move it; can I move it down? It just not possible; so, you cannot have a virtual displacement where it is moving down and solve the problem you cannot do that, but definitely I can lift it up and I can also slide it along the floor both are possible of this, which is consistent with the constraints? Only this is consistent with the constraint of the system. Because other one you have to remove it from the whole floor it defeats the purpose of it. Suppose I have one more guideway I cannot remove it will just slide many of the appliances you will find that it will slide on a given cavity.

Particularly in articulated rigid bodies because we have already seen one rigid body is not going to be useful for any applications you need to have connect a rigid body one has to maintain the relevant contact between bodies while giving virtual displacements. So, you have to visualise the physics of the problem. So, in a sense you understand better engineering by solving problems in virtual work because it forces you to visualise what happens to the system.

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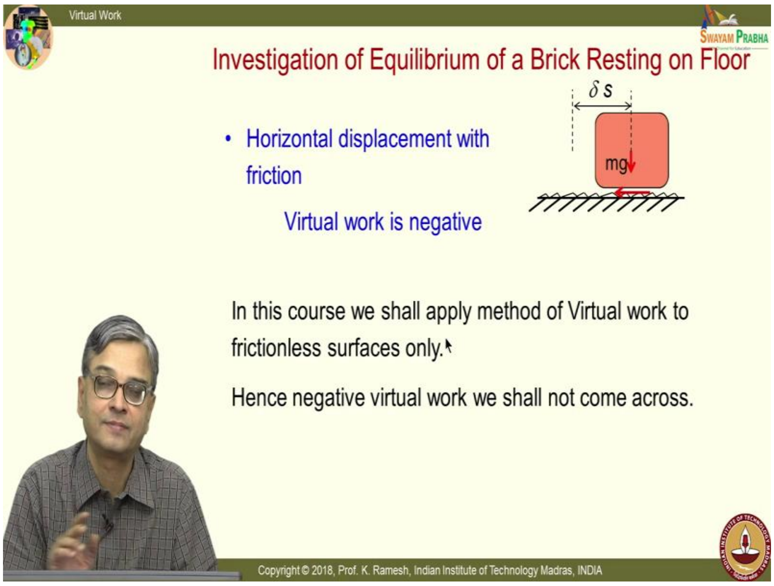
Now, let us look at equilibrium of a brick resting on floor. You know very well when I put the brick it is in equilibrium; we know the answer even before starting the investigation. What we do is? We give a virtual displacement. You know I have shown this displacement very large for clarity and note the symbolic difference, I have used a quantity delta it is not ds it is delta s.

Whenever I use this symbol it indicates we are talking about virtual displacements and not real displacements. Can the block move from this place to this place by itself? You have to give a small force, yes or no, how do I give virtual displacement? And this is again emphasised in the slide also the virtual force and the virtual displacements are exaggerated. Can I say the virtual work is $\delta W = \delta F \times \delta s$? That is not the way you to compute the virtual work.

Even though we give a virtual displacement we do not consider the force that is necessary to give the virtual displacement; we only give the virtual displacement you will never compute virtual work like this. When I have active forces on the system under the action of this virtual displacement what is the work generated by those forces this is what we are going to consider. So, this is what is emphasized here while calculating virtual work the virtual force needed to provide virtual displacement is not considered for calculations.

And you can see very well here mg does not do any work as a displacement is perpendicular to the force. The net virtual work is 0 so, the brick is in equilibrium fine, I have considered a smooth floor.

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Suppose I consider a rough floor where I have friction, when I apply this virtual displacement these are all highly exaggerated for clarity fine. So, you should keep that in mind. In fact, I have shown these delta s much larger than even d s which you saw when the particle was moving along a curved path. So, this is just for clarity. So, when I move this mg does not do any work what happens to the frictional force; frictional force does work; what kind of work does it do? We have seen work can have a sign it is positive or negative.

In this instance what is the direction of displacement and what is the direction of force they are opposite to each other. If the force direction and the displacement are opposite to each other then you assign the sign of work as negative, it is a very important and settle concept this is how we are going to solve the problems. And you should know how to assign the sign for the work whether it is a positive work or a negative work.

So, the virtual work is negative in this case and we would confine our attention only to frictionless surfaces. So, we will never come across negative virtual work. So, I can also simplify the statement as. If I compute the virtual work and we have virtual work is zero the system is in equilibrium.

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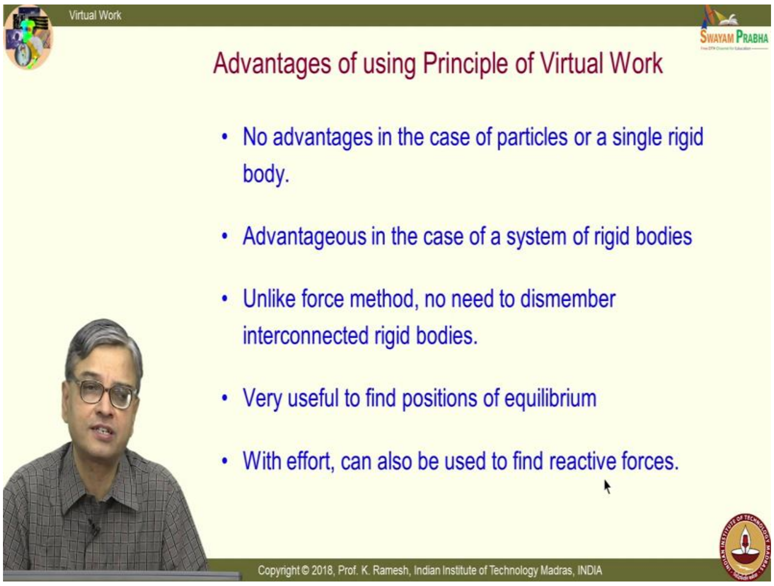
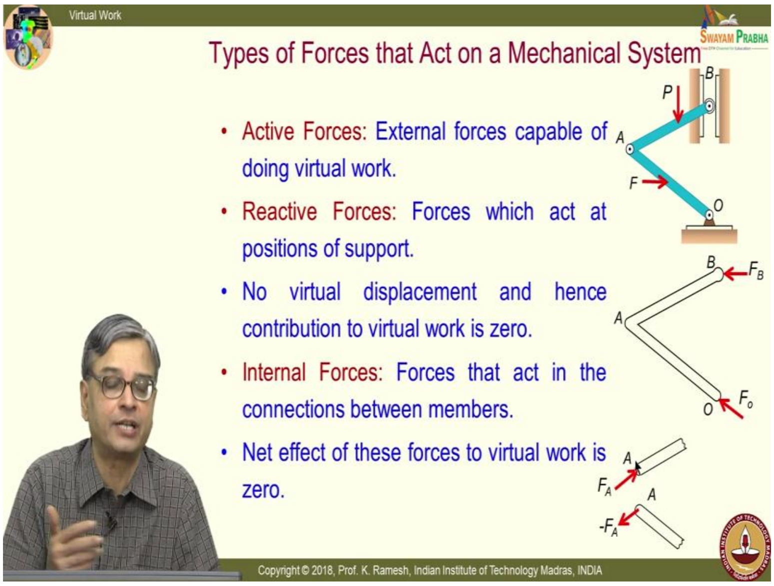



Because the books normally say virtual work is zero, but you should also recognise it can also be negative still the system is in equilibrium. Why are we learning the principle of virtual work? There must be definitely some advantages and we have seen the brick whether it was in equilibrium or not it is a single body, there is no advantage when I use it for a single rigid body. On the other hand, when I have a system of rigid bodies the method is very elegant to handle. So, one of the greatest advantages there is no need to dismember interconnected rigid bodies.

This also we will again look at when I have the members disjointed, I will have some internal forces whatever the work done by them will go to 0 net work done by them. So, I do not have the consider the internal forces, I do not have to consider the reactive forces so, I can only worry about the active forces, it greatly simplifies the computational effort. And more than investigating the equilibrium, it is very useful to find positions of equilibrium you need that kind of mechanisms for many engineering applications.




Though you can find out reactive forces you have to consciously make an effort to do this; generally, I would not recommend you to determine the reactive forces by the principle of virtual work unless it is asked to test whether you have understood that aspect of the course.

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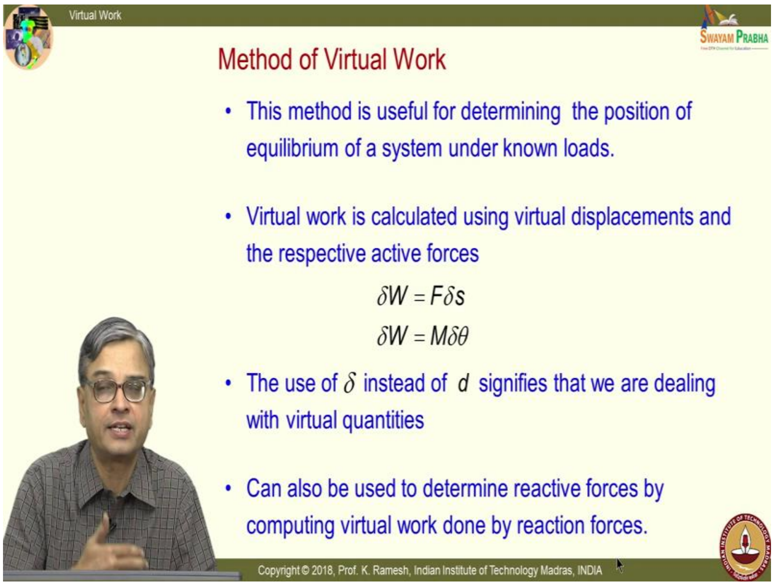
Now, let us understand what is the meaning of positions of equilibrium? I think before that I have a discussion on what is active forces see, I have a system shown here I have a constraint at point *B* it can slide up and down, and I have this has hinged support. And I have some force *P* acting on this link and some other force *F* acting on the link *AO*. Depending on the complexity of the problem you may consider the weight of the link or you may say I have a mass less link, these are all idealization you do to simplify your computational effort and the result you get out of this is reasonable acceptable within the level of accuracy you are looking forward to.

So, what I call as active forces they are the external forces capable of doing virtual work. See another important aspect you have to look at, see these forces when acting on the body and I give a virtual displacement when I compute the virtual work, I will not put this as one half of F into delta. When you had calculated work, you would have noticed the force is gradually applied there you will have to put one half, but here the force was in full and you have given the virtual displacement. So, I will always have force in to virtual displacement this is one difference that you have to keep in mind.

And I can also put the reactive forces fine because I know interactions, and the interaction in this is because it is a roller support, I can put this perpendicular to the reference, and I have an arbitrary force direction here and this is a force is put $F_0$ here. No virtual displacement at the supports and hence contribution to virtual work is zero. So, I do not have the concert the reactive forces. See in many problems when we solved, we have to first calculate reactive forces and then only proceed to find out.

Say for example, forces in a truss by the method of joints; in the method of sections depending on the problem contest you can escape out of finding out the reactive forces. So, one of the advantages in using virtual work approaches I do not have to cancel the reactive forces. Suppose I want to look at what happens at joint A I will have equal and opposite force as per Newton’s third law. So, if I compute the net virtual work contribution this also goes to zero; so, the greatest advantage is I need to consider only active forces when I employ the method of virtual work. We have looked at what is the free body diagram in the earlier chapters, in this chapter we would call what the diagram that we need as active force diagram.

I don’t need to find out the reactive forces, I don’t need to find out the internal forces, I can consider the system as a whole and vary only about the active forces make a neat sketch of this for you to appreciate the concepts because we will repeatedly talk about active forces. So, you should be able to recognise what are all the active forces. And by enlarge active forces are already known, they are the forces that are acting on the system.




Normally you use principle of statics to find out the reactive forces; so, active forces are usually known they are given in the problem statement and you are using it for computations.

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So, the method of virtual work is useful for determining the position of equilibrium of a system under known loads. And this is summarized again; so, when I put this I will put this as $\delta W = F\delta s$ ; the moment I put $\delta W$ I am really looking at the virtual work, if it is a simple force I will have delta d s, if it is a movement I will have $\delta W = M\delta\theta$ . The use of delta instead of d signifies that we are dealing with virtual quantity they are not real quantities; they are virtual quantities. And this is again summarized can also be used to determine reactive forces by computing virtual work done by reaction forces.

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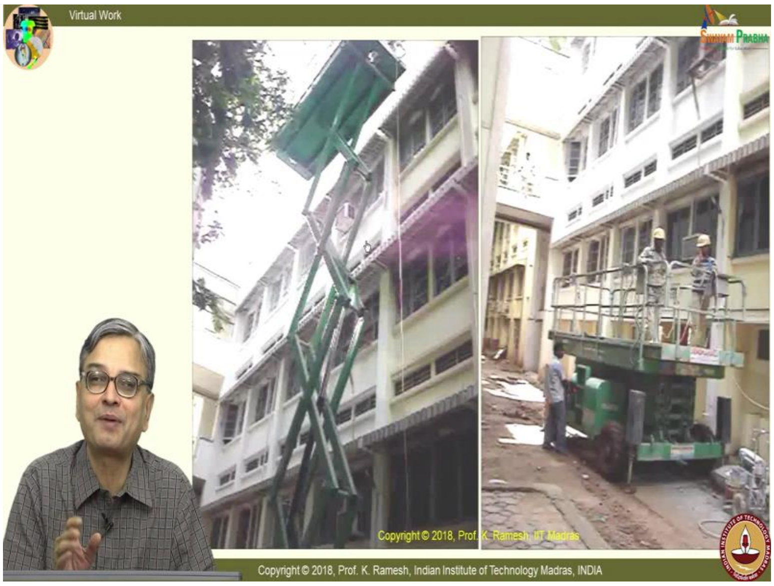
So, you have to remove the support put the reaction and then compute the virtual work and then get the reaction forces. And I said positions are equilibrium see they were painting the blocks that IIT Madras. So, I found an opportunity to record this not done by a professional cameraman to record it, that is why you see the shaking of it but it is very nice to see it quickly moves up in one and it comes down the other one.

So, all these intermediate positions are; positions are equilibrium. So, you need this see if you have travel in an aircraft when the aircraft is about to take off you know they all load all the cargo in to the plane they would have platform like this to load the cargo if you have noticed it you can see that they can be deployed quickly. And you know even some of the toys that they have it like this to scare some of you to play a prank.

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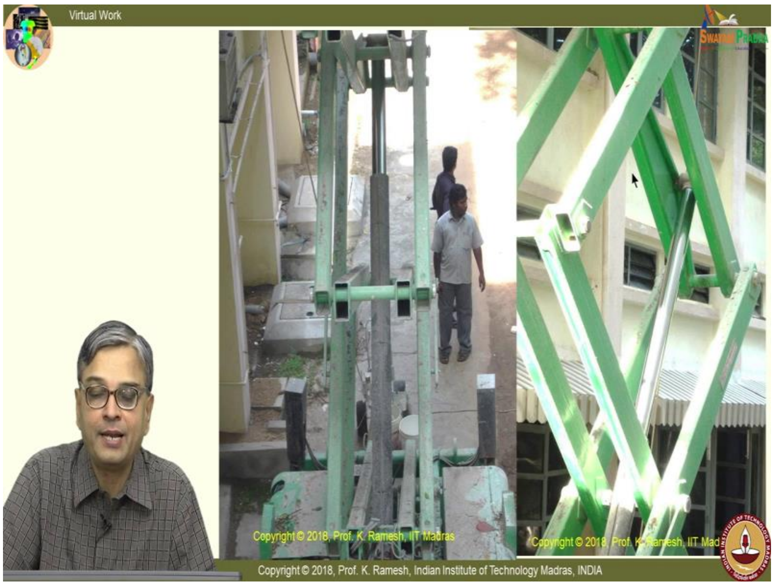
It is you have to appreciate the toy industry has learnt from whatever that is available mechanics and turn that in to your toy, it gives you joy they have use principle of solid mechanics and then device the toy that is also used. So, in one case that is moving up other case it has come down.

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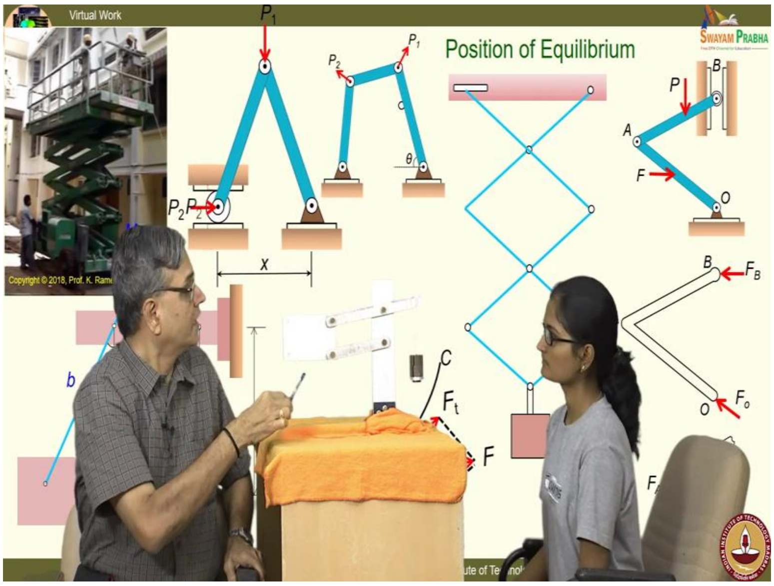
And if you have a closer look what makes it to move up or come down you have a hydraulic cylinder you have this piston is coming out, you can see this stroke this is only shortly linked at one position and this has come out fully this is a fully replied position. And you could see that these links are very heavy when I want to analyze a situation like this, I may have to consider weight of the link, if I want a reasonably a good answer, but usually in many problems we also make the idealisation that I have mass less links ok. So, you have a hydraulic cylinder which does this stroke you can see the stroke is adjusted and you are able to deploy this so, they could move up and down.

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Let us understand the meaning of position of equilibrium by a simple demonstration. Here you have a system shown this system is supported by a weight here for this weight the configuration of the system is I have shown here. Now let me introduce a small weight to that, the configuration of the system has changed I just come to this position which is also an equilibrium position for this weight this is the corresponding equilibrium position. See we have seen in these the fork lift that it is rising up slowly by a hydraulic jack. For each one of those positions of the hydraulic jack the system is in equilibrium.

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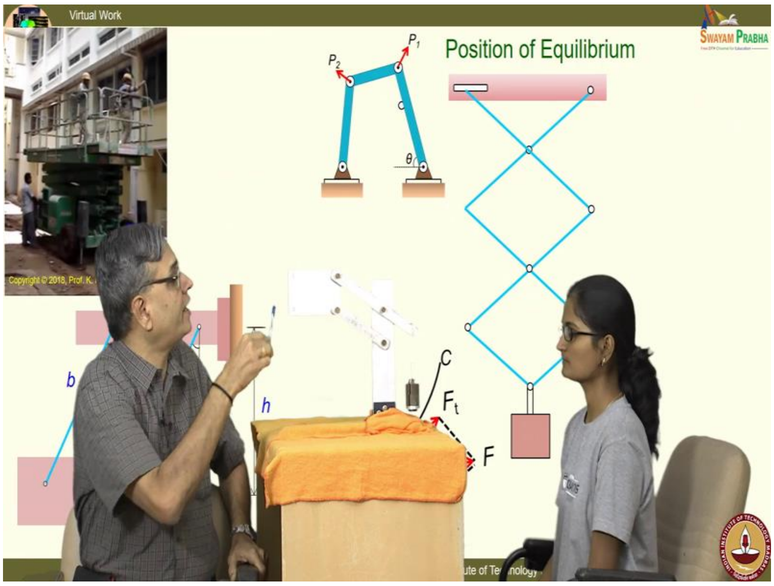


And let me introduce one more bite the configuration changes and this is the final configuration. So, what we learnt here is for a given weight there is a corresponding position of equilibrium for a given system. So, in this case by employing virtual work you would be able to find out one independent variable defining this system for a given weight. So, this way method of virtual work is useful to find out positions of equilibrium and this demonstration clearly brings out for a given weight there is an associated configuration of the system.

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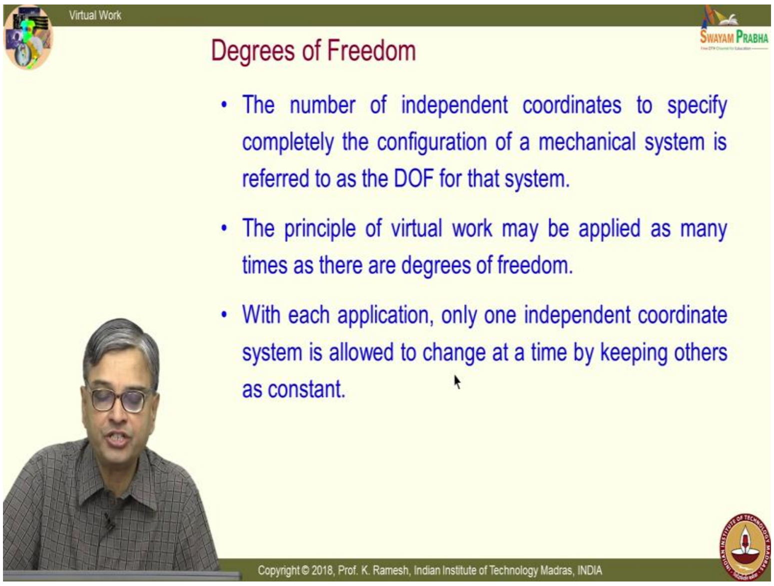
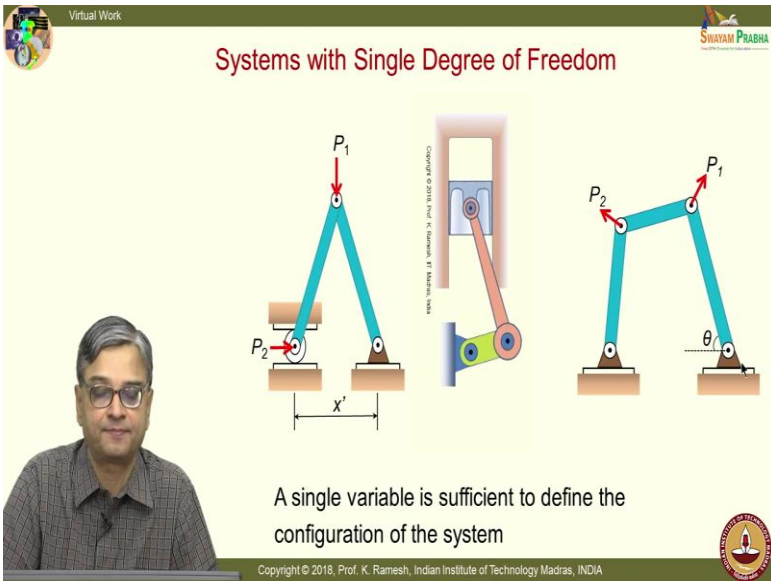



And another important concept that is usually introduced when we discuss virtual work is degrees of freedom, we may not use much as such in the solving of problems, but it is a very important engineering concept it is better that you learn it thoroughly. The number of independent coordinates to specify completely the configuration of a mechanical system is referred to as the degree of freedom for that system. And when you are employing principle of virtual work, the principle may be applied as many times as there are degrees of freedom. With each application only one independent coordinate system is allowed to change at a time by keeping others as constant.

Please write down these statements for your notes we would discuss the concept of degrees of freedom with animations first followed by actual demonstration to get the idea of degrees of freedom better. In this course we would confine our attention to single degree of freedom system. But nevertheless, degree of freedom is a very important concept in engineering and this is the time that is introduced in your whole array of courses, it is better that you understand degree of freedom.

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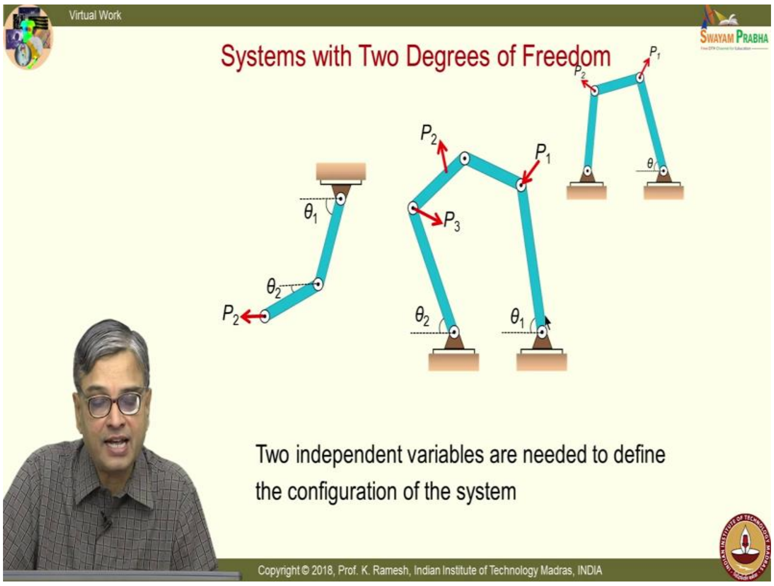
Let me now take a single degree of freedom system. See I have link like this and this roller support is guided on both sides. So, that is what you call this is constraint move in the horizontal direction it cannot move anything else. So, you have to also understand what is the meaning of constrained supports ok, these are all the terminologies used in engineering parlance. And you also have a reciprocating engine where I have this and this also has a single degree of freedom this translates here and this crank rotates.

And once you specify the position of the crank the position of the other parts of the system is completely fixed fine. And all of this come from what is known as four bar mechanism; I have 1 2 3 and this fixed end is called the 4th link and this is called the four-bar mechanism this is also for four bar mechanism this is also a four-bar mechanism. Actually, in engineering many useful machines can be traced back to some modification of the four-bar mechanism.

And in all this I need only one variable I have theta here and, in this case, I have this distance between it. I can use any angular depiction, see if I want to do this, I can have this angle specified or any one of the angles specified is possible. I have shown this as theta here I can also take this as beta and then define the system only one independent variable is required, but it is not quite clear as of now we will go to the demonstration and see.

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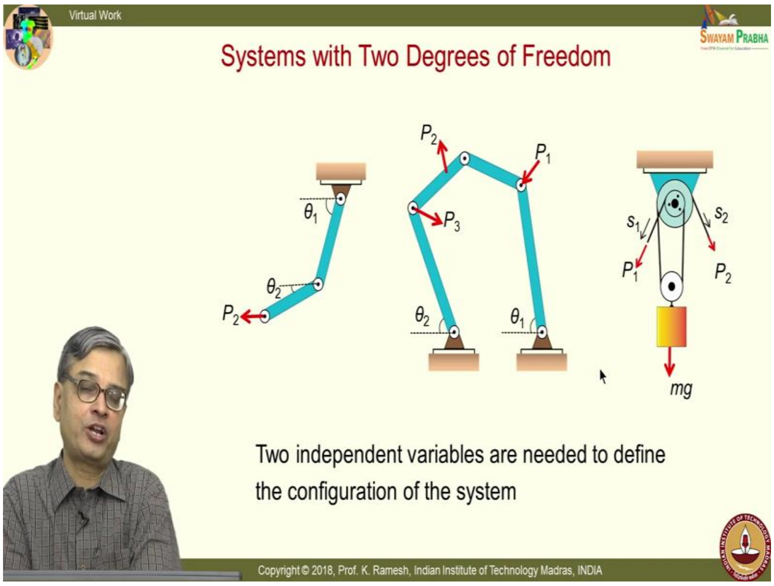
You may slightly understand it better if I look at what are two degrees of freedom systems? See in the earlier case I had two links, one of the ends was constraint move in a straight line. Now I have removed that constraint, if I remove that constraint this system cannot be defined only by one independent variable, this link will move freely because this is pin jointed here and there is no constraint on this end this link also can move independent of this.

So, I need two independent variables to define the configuration of the system when I need two independent variables, I call that as two degrees of freedom system. I have one example like this, I have another example is I do not have four bar linkage I have introduce one more linkage, the four-bar linkage was like this it is enough that I specify theta the configuration of the system is completely defined.




On the other hand, in this case because I have an extra link here, I need to have two independent quantities theta 1 as well as theta 2. The ideas will become clear when we go and discuss the demonstrations, we will see that short by later.

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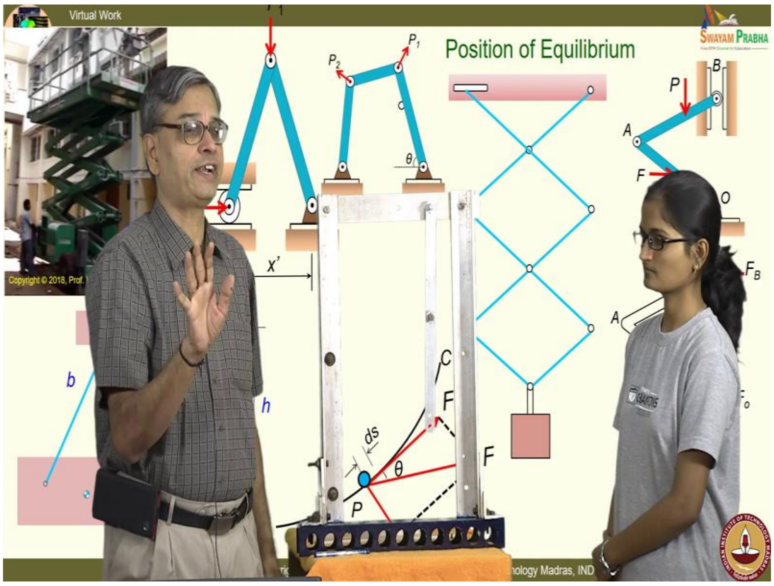
And you could also have another example of a compound pulley. I can independently move this pulley and this pulley and that is how you have the rope arrangement this (Refer Time: 36:56). Although we discussed two degrees of freedom system from the point of view of developing the subject, we would confine our attention to single degree freedom system in this course.

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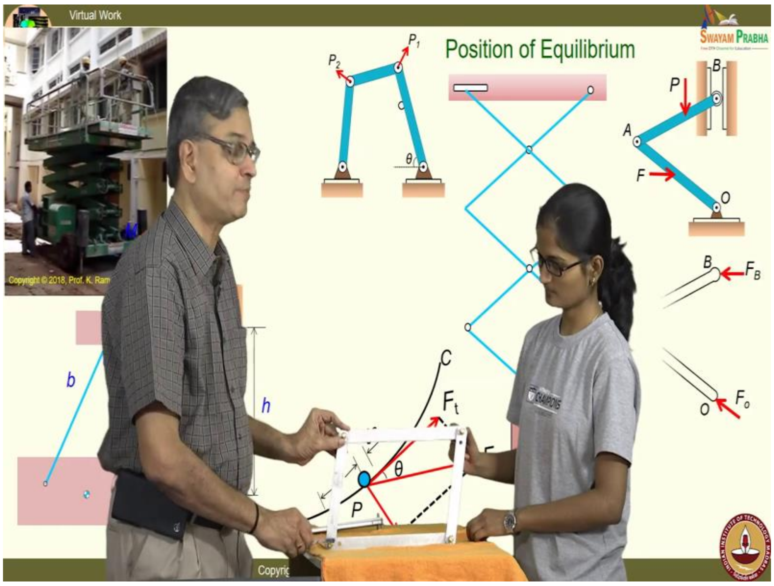
See we have seen in the animation what is the meaning of degree of freedom? And we will see with the practical demonstration what we understand. I have just a link this is on a pin joint here I can freely rotate this no problem ok. Suppose I put a link at this end this is again a pin joint I can rotate this independently; I can also rotate this independently both of them can rotate independently. So, if I have to define the configuration of this system, I need one variable to define this position link, I need another variable to define this position this is the simplest two-degree freedom system fine.

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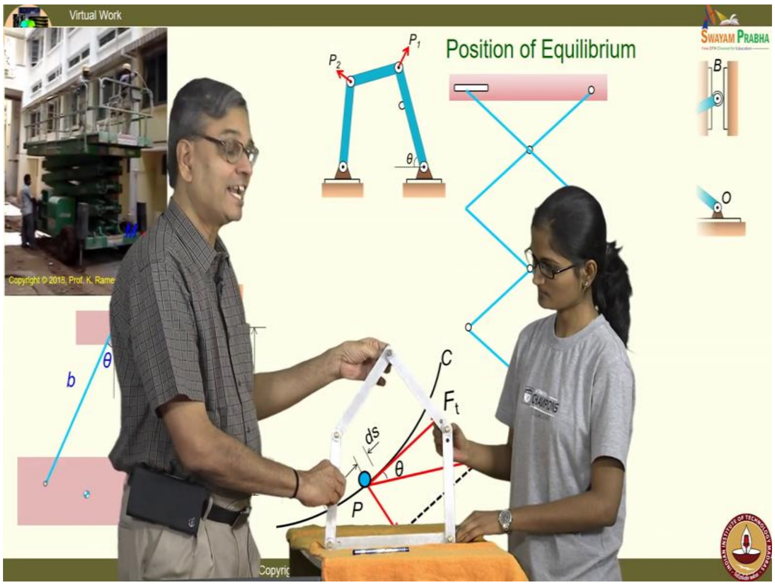
Now, we will move on to a four-bar mechanism, I have pin joint here; pin joint here; pin joint here; pin joint here and when I rotate this also moves. So, I need only one coordinate either this angle or this angle, I do not need anything other than one independent variable to decide the configuration of the system ok.

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Suppose I make this as a five links look at what happens I have a five links now ok; I can rotate this independently she can also rotate it independently. So, I need one independent variable to define this, I need another independent variable to define this it is because it is a replication of two links coming in different fashion. And you know degrees of freedom is a very important concept that you will be coming across any engineering education. And only when we discuss the principle of virtual work this concept is introduced. So, it is better that you understand what is the meaning of degrees of freedom.

So, in this class we have looked at what is the principle of virtual work we initially discuss what is work, then we also looked at what are the forces that do not contribute to work, we develop the concept of active forces and we also looks at the reaction forces and internal forces do not contribute to work and we also learnt the symbolism that is used in virtual work I would use delta to denote their virtual quantities and we have also seen them in some demonstrations on what is the meaning of position of equilibrium and what are the degrees of freedom of a given system.

And degrees of freedom of a given system is a very important in engineering, it is usually introduced when you learn principle of virtual work. Although we have learnt in general what are the degrees of freedom of a system, we would confine our attention to solving problems involving single degree freedom systems. Thank you.