

Engineering Mechanics
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Module – 01
Statics
Lecture – 15
Virtual Work- II

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Module 1 Statics

Lecture 15 Virtual Work- II

Concepts Covered

Methods of virtual work, Selection of independent variable, Mathematics involved in virtual work problem, Physics of virtual work problem, Sign convention for solving Problem in virtual work, Sample problems.

Keywords

Virtual work, Active force, Reactive force, Independent variable, Arbitrary displacement, Constraint displacement, Degrees of freedom, Couples, Displaced configuration.

Let us continue our discussion on Virtual Work.

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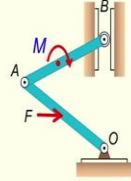
In the last class, we have looked at what is the method of virtual work I also said that you should look for an active force diagram was you learned that the active forces do not contribute to virtual work.

Method of Virtual Work

- This method is useful for determining the position of equilibrium of a system under known loads.
- Virtual work is calculated using virtual displacements and the respective active forces

$$\delta W = F\delta s + M\delta\theta$$

- The use of δ instead of d signifies that we are dealing with virtual quantities



The diagram shows a mechanical system with a pivot point O. A force F is applied at point A, and a moment M is applied at point B. The system is shown in a displaced configuration.

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And I also mention that the method is useful for determining the position of equilibrium of a system under known loads. And we have also noted that virtual work is calculated as $\delta W = F\delta s + M\delta\theta$ you should note that be

incremental component is given a symbol δ instead of d . I have also put for the work the symbol δ indicating that we are dealing with virtual quantities.

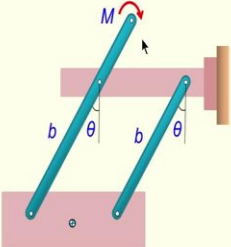
And I have just written a statement that I have two active forces, I have said that the force F has a displacement δs so, $F\delta s$. Here again I cautioned you the force in full value is acting and a virtual displacement is given as δs , the work done is force into displacement, it is not like force is gradually applied from 0 to F . So, these are all the distinctions that you should note while you are dealing with virtual work. So, this again emphasizes that use of δ instead of d signifies that we are dealing with virtual quantities.

You know we have also looked at in the last class what is the meaning of degree of freedom of a system. We have looked at a single degree freedom system and two-degree freedom system you will really understand a single degree freedom system when you look at a two degrees of freedom system because it requires two independent variables to define. And when I have a single degree of freedom system it requires only one independent variable to define the configuration of the system.

Virtual Work

SHYAM PRASAD

1. A block of mass " m " is suspended by two massless links as shown in the figure. Moment " M " applied at the end of one of the links maintains the configuration as shown. Neglecting friction determine the angular orientation θ .



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Now, the idea is when we want to solve the problem in method of virtual work; we should ensure whatever is the choice of our independent variable I should get one unique answer. You know books give the problem with one reference coordinate, it might label the value as θ or β or whatever the way

the problem is coined and we would see the nuances in that when we solve a problem you will understand the nuances.

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And let us take a very simple problem we had seen this in a vertical fashion in the last class it is a very similar one. So, you know what way the configuration of the system will change; it is easy for you to visualize and I expect you to visualize in the method of virtual work under the given application of forces how the displaced system will look like when you give a virtual displacement. And I want you to sketch it fine, and that is very important when you want to solve a problem in the method of virtual work.

So, the problem statement is I have a block of mass m and for convenience we have said that there are two links which are massless; these are all idealizations you do to simplify your mathematics. And here you are really learning a methodology; so, it is no harm in assuming that these two links as massless.

And a moment M is applied at the end of one of the links to maintain the configuration as shown and it is labeled in the diagram with reference to the vertical the link is oriented at angle θ . See normally when a problem is given you tend to use those quantities directly

for solving a problem; that is how we human mind will react fine.

Virtual Work

SWAYAM PRABHA

From the figure, h is given by, $h = b \cos \theta + c$

- The active forces involved in this case are,
 - ✓ Weight mg , acting through the centre of gravity
 - ✓ Couple M , applied to the end of the link

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We would solve the problem from the way it is posed to us. And I would like you to visualize when I apply a moment M , if I give a virtual displacement what way the system will get displaced; you put the

virtual displacement little exaggerated for you to sketch neatly.

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So, I have the system like this and for simplification we can simply represent the link by a line like this. And recognize that these are all pin joints that is how the system are shown to the functional in the previous class. We saw the system as deep whatever you

see as the horizontal bar was vertical and you had literally used the similar problem to see positions of equilibrium.

So, what is natural to go about you know very well that I am externally applying a moment M so that I have maintained the position of the block at this level h . So, it is natural to expect that I would put a virtual displacement for M fine; as if I have slightly increased the value of the moment, that is how you can visualize how I introduce a virtual displacement I said. Whatever the force that is required to introduce a virtual displacement, we do not know that consider it for calculations.

So, can I sketch when this is applied how the link will rotate; it will rotate and try to lift up the mass M you should recognize that; if you recognize that then you can make a neat sketch of the displaced quantity. I would put the displaced diagram first then we will look at it do you get this idea? You have to get this idea first in all problems dealing with virtual work; I would insist first step you should do is for a given virtual displacement what is that displaced configuration of the system consistent with the constraints of the system.

When I rotate it, I have this brown line and I would naturally expect the way the members are connected the mass would get lifted up. You should sketch this very neatly so that I am in a position to recognize what way the δh changes. I should see very clearly from my sketch that the mass mg has moved up whereas, the link has moved in the direction of the moment M .

And from the sketch of the links and the geometry given the length of the link is given as b and this height is what we want to find out this is nothing, but $h = b \cos \theta + c$. So, the idea is we would like to find out if there is an incremental change in h what way θ would change because they are interdependent. That are constraints in the system the way it is constructed I can arbitrarily have one value of δh and have another arbitrary value of $\delta \theta$ they are interconnected.

And let us look at the problem from mathematical point of view. So, you also identify what are the active forces involved in this case. I have the mass m , mg is the force and I have the external applied moment M . I do not have to worry about the reaction forces here and also internal forces in the system all that is not required like what we have done

for the force method where you had written down the free body diagram and systematically calculated the forces. So, whatever the diagram that I have written down

Virtual Work

- Now, the virtual work done by mg is,

$$+mg\delta h = mg\delta(b\cos\theta + c)$$

$$= mg(-b\sin\theta\delta\theta)$$

$$= -mgb\sin\theta\delta\theta$$
- Negative sign shows that the work is negative for a positive value of $\delta\theta$
- θ is measured positive in the clockwise direction (The external moment will produce positive work that way) and for $\delta\theta$, it is also taken in the same sense.

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here is known directly as an active force diagram, and for the given virtual displacement I should recognize the displacement of configuration very clearly.

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Now, let us look at what is the work done by the mass

mg mass m . So, I will put this as $mg\delta h$ and I have the expression for h which is given as $h = b\cos\theta + c$. I would like to know what is the incremental variation of $(b\cos\theta + c)$.

When I put δ for all practical purposes, I treat it as small b ; I would do the differential fine. While we will do virtual work, we will consciously use only that as δ to indicate that we are dealing with virtual quantities. And when I do this what way this expression will turn out to be when I differentiate $\cos\theta$ what happens? I also have another important aspect that I also get a minus sign. So, I get $-b\sin\theta\delta\theta$ and c is the constant and it goes to 0.

So, I get this work done by the mass m which is the actual mg as the force into δh ; I get this value as $-mgb\sin\theta\delta\theta$. So, the negative sign shows that the work is negative for a positive value of θ . So, this is what the explanation from mathematical point of view and books have lot of confusion in this chapter fine. From one edition to another edition the author tries to emphasize different aspects and people look at the virtual displacement individually whether it is positive or negative; here also we are discussing that.

When I have a positive value of $\delta\theta$, I have a negative value for this; they are oppositely related is what it shows. And we have always learned that counter clockwise positive and clockwise is negative; for convenience we change all that θ is measured positive in the

clockwise direction. And the explanation given is the external moment will produce

positive work that way and for $\delta\theta$ it is also taken in the sense ok.

Virtual Work

- The work done by the moment is $M \delta\theta$.
- Now, from the principle of virtual work,

$$\delta W = 0$$

$$M\delta\theta + mg\delta h = 0$$

- This gives,

$$M\delta\theta = mgb \sin \theta \delta\theta$$

$$\theta = \sin^{-1} \left(\frac{M}{mgb} \right)$$

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So, there are lot of confusions there; let us solve the confusions like this fine and then we will have a detailed discussion how should we solve the problem ok; what should be the emphasis.

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The work done by the moment is $M \delta\theta$, which is very obvious that is also positive because I have the corresponding displacement in the direction of the force. Now, from the principle of the virtual work, we are not dealing with friction. So, we would always use $\delta W = 0$; not only this as the focus is learning the methodology, we would confine ourselves to single degree freedom system.

So, you do not even have to verify whether the given problem means single degree freedom it is a single degree freedom problem. So, that you learn the basics of the method thoroughly then you can handle multiple degrees of freedom systems. So, I would just write whatever I have got $M\delta\theta + mg\delta h = 0$; we have already seen plus $mg\delta h$ from the mathematic we got this as minus. So, when I put it on the other side, I get the expression $M\delta\theta = mgb \sin \theta \delta\theta$.

So, I cancel out $\delta\theta$; so, I get an expression for θ as $\theta = \sin^{-1} \left(\frac{M}{mgb} \right)$. So, I am in a position to relate the configuration represented by the independent variable θ to the applied moment. So, when the applied moment changes, the configuration will also change meaning that θ will change θ is the function of M ; it is a very interesting result. Because I am in a position to find out the configuration because we have such mechanisms used in day to

day requirement. Like we saw the painters were able to climb up safely and then paint the building that is what we saw. And I also mention when you are loading a cargo in the airplane; they have just one it will quickly lift up very fast its very fast. So, you save time.

Solving Problems in Virtual Work

Two approaches to assign sign conventions for solving problems in virtual work:

- Mathematics
- Physics of the problem

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Now, let us look at the problem from two perspective one is I can solve the problems of mathematics, keep your eyes close depend only on your sign convention so on and so forth or look at the physics of the problem.

Obviously, you have been listening to me all along I have always been insisting use mathematics, never forget the physics; physics should be the basis for which you should solve the problem use mathematics to assist you.

Approach 1 – Mathematics

Method 1: Using θ

Method 2: Using β

From the figures, h is given by,

$$h = b \cos \theta + c$$

$$\Rightarrow \delta h = -b \sin \theta \delta \theta$$

$$h = b \sin \beta + c$$

$$\Rightarrow \delta h = b \cos \beta \delta \beta$$

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So, let us look at what is the importance of the statement.

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Now, the problem was originally coined with θ as reference, a natural step is you would use θ and solve the problem and it so turned out that when I do

the differentiation it turned out to be negative it was coming so elegantly, there is no confusion in getting the $\delta W = 0$.

Suppose I have given you the problem statement as with independent variable as β because what we have learnt when I have a single degree of freedom system, I need one independent variable to define the configuration. I can choose any angle to define the configuration, I can refer it with respect to the horizontal, I can refer with respect to the vertical; it should not make a difference.

So, let us first collect the data the way we solve the problem earlier, we would just collect the data and look at what is the kind of answer that we are getting. I can also write the expression for h , I can also draw the displaced configuration for displaced configuration I am taking the reference as M , I am actually increasing M so that I can visualize which way the link will rotate fine I am not making any difference in these two.

Let us get the expression for h ; in this case we have already seen that $h = b \cos \theta + c$ and in

this case, it is $h = b \sin \beta + c$.

And when I do the differentiation what happens? I get this as a negative quantity, I get this as a positive quantity.

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And if you have just followed the mathematics, you would have just said

$W_M = M \cdot \delta \theta$ and this is $M \cdot \delta \beta$.

Approach 1 - Mathematics

Method 1: Using θ

$W_M = M \cdot \delta \theta$

$W_{mg} = mg \cdot \delta h$
 $= mg \cdot (-b \sin \theta \delta \theta)$

Method 2: Using β

$W_M = M \cdot \delta \beta$

$W_{mg} = mg \cdot \delta h$
 $= mg \cdot (b \cos \beta \delta \beta)$

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And you would have just written $W_{mg} = mg \cdot \delta h$, I would get in one case a negative quantity I in another case I will get a positive quantity this is all you would have done.

You know once you start the mathematics you do not apply your thinking on the nuances when use mathematics you should also look at the nuances. Like we have looked at what is the definition of θ whether θ is positive, θ is negative, whether $\delta \theta$ is positive, $\delta \theta$ is all that we are not doing it and, in all likelihood, you will not even think in the examination fine.

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Approach 1 – Mathematics

Method 1: Using θ

Method 2: Using β

From the principle of virtual work,

$$\delta W = 0$$

$$M\delta\theta + mg\delta h = 0$$

$$\delta W = 0$$

$$M\delta\beta + mg\delta h = 0$$

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So, when I do this when I write the virtual work again the same thing.

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I would get this answer like this; I would get

$$\theta = \sin^{-1}\left(\frac{M}{mgb}\right) \quad \text{and} \quad \beta \text{ as}$$

$$\beta = \cos^{-1}\left(\frac{-M}{mgb}\right)$$

. You know at examination hall you will

always get all sorts of sudden intuition; you know $\cos\theta$ and $\cos(-\theta)$ are one and the same ok.

You may also jump to the very convenient conclusion \cos inverse is also same.

Approach 1 – Mathematics

Method 1: Using θ

Method 2: Using β

$$M\delta\theta = mgb \sin\theta \delta\theta$$

$$\theta = \sin^{-1}\left(\frac{M}{mgb}\right)$$

$$M\delta\beta = -mgb \cos\beta \delta\beta$$

$$\beta = \cos^{-1}\left(\frac{-M}{mgb}\right)$$

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So, be the whatever the relation I have obtained; so, β is erroneous because I have a relationship between θ and β . So, what we have got by blindly applying the mathematics; if I take another independent variable you end up with a wrong

answer.

Let me revisit the same problem and bring in a beautiful way of interpreting the expressions you get, which will help you to solve a problem consistently no matter which

Approach 1 – Mathematics

- The relation obtained using β is erroneous since one knows that $\theta = 90 - \beta$!
- Hence, the sign convention from mathematics alone is not reliable if some other angle is used to define the system.

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independent variable you select and you interpret finally, the sign of the work. We have seen work is a scalar and if I have the corresponding displacement in the direction of the force then I have the positive work. If the corresponding displacement is opposite to the force then I have

negative work. This is a very strong sort of an idea that would help you to solve the problem, no matter which independent variable you select that is a best way to do it fine.

Approach 2 – Physics of the problem

Method 1: Using θ

$$h = b \cos \theta + c$$

$$\Rightarrow |\delta h| = |-b \sin \theta \delta \theta|$$

Method 2: Using β

$$h = b \sin \beta + c$$

$$|\delta h| = |b \cos \beta \delta \beta|$$

Only the modulus of the quantities are considered without their sign

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So, if I have to do that what I have to look at is when I get this h expression when I get δh ; do not worry about the sign, you just look at the

modulus. Let us assign the sign of these quantities when we write the virtual work statement, investigate whether I am doing a positive work or a negative work.

And what is important here? Your neat sketch of the displaced configuration is extremely important for this; if you do not do that then you connect physically visualize. Here I can

physically visualize when I write the $\delta W = 0$, I can physically visualize that mg is moving up. So, mg is doing what kind of work it is doing negative work.

On the other hand M and $\delta\beta$ are all in the same direction, so it is doing a positive work there is no way I will make a mistake whether I use θ as the independent variable to start way or whether I take θ as my independent variable to define the configuration is the idea clear. See some books do say this, but they do not emphasize it fine; we have all along seen in the case stresses, we had a detailed discussion on sign convention, how to interpret the force acting on the trust member as tension of compression.

And also, we had a detailed discussion on what is the sign convention used in plotting shear force diagram and bending moment diagram, I said why sign convention is important. And how you can solve problems on different books when they have use different sign convention you can still solve the problem without any difficulty.

On the similar note, when you come to the method of virtual work my recommendation is draw the displaced configuration as neatly as possible, take 2 minutes in doing that

Approach 2 – Physics of the problem

Method 1: Using θ

$W_M = M \cdot \delta\theta$ same sense

$W_{mg} = -mg \cdot \delta h$ Opposite sense

$= -mg \cdot (b \sin \theta \delta\theta)$

Method 2: Using β

$W_M = M \cdot \delta\beta$

$W_{mg} = -mg \cdot \delta h$

$= -mg \cdot (b \cos \beta \delta\beta)$

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then in one step you will write the expressions. Because the mathematics involved as very simple the configurations you are going to come across are again going to be simple. The problem where you can make a mistake is when you are writing $\delta W = 0$, which one is positive virtual was which one is

negative virtual work you can visualize it when I have a displaced configuration like this.

My recommendation is; please adopt this approach; we would see what is the way the expressions are going to come.

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Please write down I am not finding any one of you are writing the notes, it will look obvious if you go and want to try it from a book; you will thoroughly get confused. Because each book is different and it is very difficult to figure out what is the sign convention that they have used and they do not spell it out very clearly. Many of it is left your imagination. So, you will understand one problem with same problem is asked you might max the paper in some other problem is given you will not even know how to approach it; it is very important.

So, I have this the forced and the corresponding displacement on the same sense. So, I write the work done as positive and mind you I think I should have put δW_m and δW_{mg} anyway even then it is not put its quite ok; we are dealing in the chapter of method of virtual work. And here I have indicated the sign as negative because the force and the corresponding displacement are in opposite directions.

Virtual Work

SHAYAM PRABHA

Approach 2 – Physics of the problem

Method 1: Using θ

Method 2: Using β

$M\delta\theta = mgbsin\theta\delta\theta$

$\theta = \sin^{-1}\left(\frac{M}{mgb}\right)$

$M\delta\beta = mgbcos\beta\delta\beta$

$\beta = \cos^{-1}\left(\frac{M}{mgb}\right)$

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Then the next step to straight forward; I have this work done by the force mg is $-mg.(b\sin\theta\delta\theta)$.

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And here again I would get the final answer is like this and when I converted into θ it is one and the same. So, one and the same I

have not got a different answer by changing the independent variable. Let me also give you one more variation see in the both the methods where I have looked at whether I should use θ or β ; I have given virtual displacement only to M to start with, then I figured out what way it will happen to mg .

I can also do it from a different perspective, I can give a virtual displacement to mg do you get the idea? When I give a virtual displacement mg , mass will come down the mass

will do a positive work whereas, M will do a? Negative work a lot of clarity in this I would appreciate you try that out as a home work. See when you want to learned a subject take a simple problem look at it from various perspectives you get the concepts

clear, it is not that you need to solve hundreds of problems to do it. A single problem is sufficient to get the nuances of the concepts and these are very subtle.

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And I have also taken another problem which is very similar to what we have seen yesterday.

2. A block of mass " m " is suspended by two massless links as shown in the figure. A force of " P " applied at the end of one of the links maintains the configuration as shown. Neglecting friction determine the angular orientation θ .

The diagram shows a block of mass m on a horizontal surface. Two links of length b are attached to the top of the block and extend upwards to a horizontal bar. The left link is at an angle θ to the horizontal and has a force P applied at its free end. The right link is also at an angle θ to the horizontal. The distance between the attachment points on the bar is $2b$.

Instead of applying a moment, I have applied a force you can also lift it by applying a force. And here I have deliberately taken the θ as this; I have not taken θ with respect to vertical I have taken with respect to horizontal. Just to illustrate the point normally whatever is it is given in the problem; you start with and if you directly apply the mathematics without thinking which is in all likelihood you may end up when you are in

a hurry to solve the problem.

The diagram shows the system in two states: an initial state and a displaced state. In the initial state, the links are at angle θ and the height of the block is h . In the displaced state, the links are at angle β and the height is $h + \delta h$. A coordinate system with x and y axes is shown.

- Height h , is given by, $h = b \cos \beta + c$
- The active forces involved in this case are,
 - ✓ Weight mg , acting through the centre of gravity
 - ✓ Force P , applied at the end of the link

So, do not take the approach of blind mathematics, draw the displaced sketch, investigate while writing $\delta W = 0$, whether this contributing positive work or negative work; at that

time, you apply the sign appropriately then you will never make a mistake.

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So, with that perspective this problem will become very simple; I think this also I have solved it in two different ways; let me see. And to save space what I have done is I have cut this because this is too long; it is not drawn proportionately in the sketch; it should have been longer than this. So, indicated that I have just made a cut you indicate that this is longer than what is actually shown.

So, initially find out; so, in all problems you first label the active forces or recognize the active forces, you have the active forces are P and mg . So, first step is identified the active forces because this practice would help suppose I want to find out the reaction what way I would do is, I would remove the support put the reaction there and treat it as an active force that is a trick. When I have to find out reactive forces using principle of virtual work, I cannot do it the way what I have done it in the case of force method.

Here I remove the support put the unknown force as an active force and allow it to move the way dictated by the constraint of the system it is the trick that you will have to do. So, I have the replaced configuration, so when I put force like this, I get this displaced the mass moves up. So, your sketch should be sufficiently neat enough to show the relative movements, if you do not draw a neat sketch you can add even when you want to assign sign later while writing the expression $\delta W = 0$. Sketch is very important if you do not do the proper sketch then you can mess up the whole thing.

So, I have the value θ and I should recognize this as θ decreasing; see what happens is one of the conventions people uses, they look at from the coordinate point of view. If the distance increases that virtual displacement is positive, if the angle increase the virtual displacement is positive, if the angle decreases virtual displacement is negative. All that can lead to confusion because if you individually assign a sign for displacement, individually assign a sign for the forces you are ultimately looking at the product and if you do not handle these independent sign conventions properly you can mess it up.

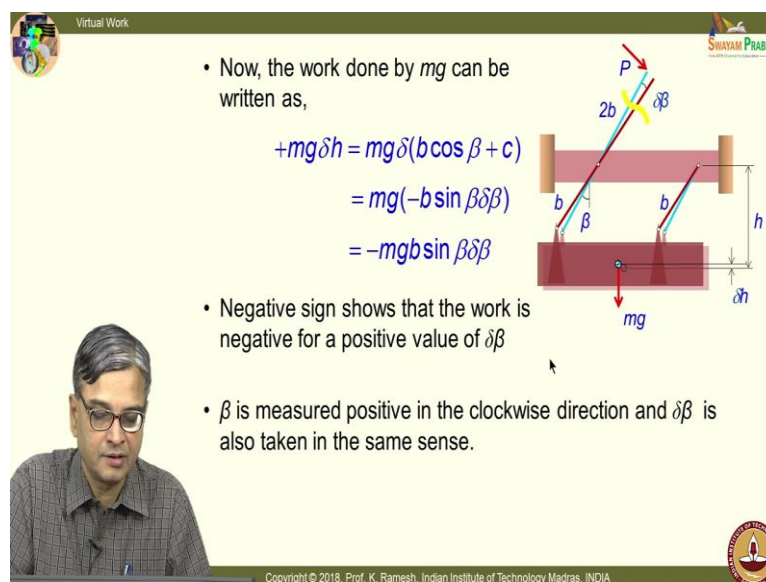
On the other hand, you look at the product from the perspective whether force and the corresponding displacement are on the same direction. For example, we are accustomed to calling an anti clockwise moment was positive, clockwise moment as negative. On the other hand, if a clockwise moment adds and the corresponding displacement is clockwise

the net product is still positive which you can easily visualize. So, do not go in for a sign convention independently for a force, independently for a virtual displacement look at the product that is a message I am trying to convey.

If I have β then β is increasing θ is decreasing, but β is increasing for this configuration.

So, it is desirable that my work is $h = b \cos \beta + c$. So, the idea here is one way of looking at the problem is look at the problem and choose an independent variable, so that your mathematics is consistent or if I have to use this if I want to have θ to increase I can also visualize that M comes down and θ increases and you have a negative work at P from mathematics also you can solve.

But there are more pit falls while applying at you can make a mistake. On the other hand, whichever the force you are taken for the virtual displacement as a reference draw the displaced configuration from the geometry identify the internal relationship do not put the sign for the incremental quantities, investigate whether the work done is positive or negative and write the virtual work equation. That is full proof you will never make a



Virtual Work

- Now, the work done by mg can be written as,

$$\begin{aligned}
 +mg\delta h &= mg\delta(b\cos\beta + c) \\
 &= mg(-b\sin\beta\delta\beta) \\
 &= -mg b \sin\beta\delta\beta
 \end{aligned}$$

- Negative sign shows that the work is negative for a positive value of $\delta\beta$
- β is measured positive in the clockwise direction and $\delta\beta$ is also taken in the same sense.

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mistake no matter which independent variable you select whether you handle one active force as a reference or the other active force as a reference because the problem can be solved in multiple different ways. So, here what I have by indicated this I will strict to the mathematics instead of

writing my interrelationship in terms of θ I would write in terms of β .

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And rest of the problem is straightforward I have the same procedure and I have this and this turns out to be like this and I get the expression.

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Virtual Work

- The work done by the moment is $P \times 2b \times \delta\beta$.
- Now, from the principle of virtual work,

$$\delta W = 0$$

$$P(2b)(\delta\beta) + mg\delta h = 0$$

- This gives,

$$P(2b)(\delta\beta) = mgb \sin \beta \delta\beta$$

$$\beta = \sin^{-1}\left(\frac{2P}{mg}\right)$$

$$\beta = 90 - \theta$$

$$\theta = \cos^{-1}\left(\frac{2P}{mg}\right)$$

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And this is very clear that this just positive work, so I have this like this and this gives me the final expression

$$\beta = \sin^{-1}\left(\frac{2P}{mg}\right),$$

when I put $\beta = 90 - \theta$, I get this as $\theta = \cos^{-1}\left(\frac{2P}{mg}\right)$. So, what I have attempted to show

here is if the problem is coin with a particular independent variable and you are going to give a virtual displacement in one of the forces is convenient to you; you may change the independent variable and solve the problem from a mathematical perspective.

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Virtual Work

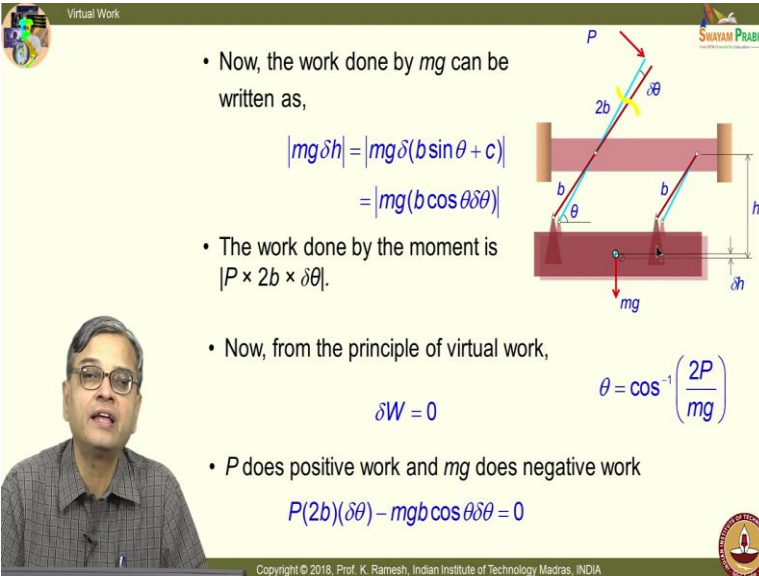
- Height h , is given by, $h = b \sin \theta + c$
- The active forces involved in this case are,
 - ✓ Weight mg , acting through the centre of gravity
 - ✓ Force P , applied at the end of the link

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Instead my recommendation is look at the same problem because I have the luxury, I can put the same slide again and then start the discussion.

So, I have this displaced, I retain the same independent variable indicated in the problem, I get an expression for h , find out what is the incremental change, do not assign a sign here look at only the quantity.

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Virtual Work

- Now, the work done by mg can be written as,

$$|mg\delta h| = |mg\delta(b\sin\theta + c)|$$

$$= |mg(b\cos\theta\delta\theta)|$$
- The work done by the moment is $|P \times 2b \times \delta\theta|$.
- Now, from the principle of virtual work,

$$\delta W = 0$$

$$\theta = \cos^{-1}\left(\frac{2P}{mg}\right)$$
- P does positive work and mg does negative work

$$P(2b)(\delta\theta) - mgb\cos\theta\delta\theta = 0$$

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And also, for the work done by the I think it should be the force it is not the moment because it is cut and paste in preparing all these slides this is the force here ok, so $|P \times 2b \times \delta\theta|$. Now, while writing this expression

recognize that the force and the corresponding displacement on the same direction that it is positive. The force and the corresponding displacement δh in this case are in opposite sense. So, you get the expression like this I directly get

$$\theta = \cos^{-1}\left(\frac{2P}{mg}\right).$$

So, my recommendation is; get the incremental quantities, do not assign the sign, investigate for each of the active forces look at the virtual displacement and find out whether they are in the same direction or opposite direction. If the corresponding displacement is the same direction as the force, then you take that virtual work contribution as positive, if they are opposite it is negative then you will find method of virtual work is very elegant and simple any problem that is given to you; you can comfortably solve it there is no difficulty at all.

So, that is the approach I would recommend you finally, investigate from the neat sketch, if you draw the sketch wrongly then this method also can give surprising distance, for me to identify that P does positive work or mg does negative work I must have clear sketch drawn.

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Let us take a one more problem it is very interesting this is what I said in some of the

3. A force P is applied at the end of the massless links of length b each, connected as shown. Determine the magnitude of the couple M required to maintain the equilibrium of the mechanism by the method of virtual work. All the joints can be assumed to be frictionless pin joints for analysis.

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toys that they have this, they will not have just one unit like this several units and suddenly they will put like this it will come and pop and hit at you that toys use to enjoy I could not get one I was seen it in the trains that they come and sell.

So, here a force P is applied at the end of the mass less links of length b each and they are all pin jointed without friction. And when the force P is applied to pull it what is the moment required to maintain the equilibrium of the mechanism you have to get it by the method of virtual work. And you know you should also look at what are the constraints illustrated in the problem.

Obviously, when I pull this, it has to have a freedom to come down, so I have a slot here

- The active forces/couples involved in this case are,
 - ✓ Moment M , acting at the end of the link
 - ✓ Force P , applied as shown

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which allows that end to move up or down depending on which direction I apply the force P , if I pull it will come down, if I reverse the direction this will go up. So, you have the facility provided in your mechanism there is the slot which allows this pin to

move up or down. So, that is a constraint; that constraint is very clearly specified. And the first step in virtual work is identify the active forces; active forces are P and M .

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And I have the angle θ referred with respect to the horizontal and make a line sketch of this problem and what I am going to do is? See it is easier for me to visualize I am applying the force P ; I am trying to pull it out. So, I would visualize what is the displaced configuration when I slightly increase the force P it is easy for me to do. On the other hand, if somebody wants to increase M and then draw the displaced configuration that will also have to give the same value for this problem is the idea clear.

See I am giving a virtual displacement for a given configuration to establish the interrelationship. I am not supposed to get one result if I have a virtual displacement, I give it to force P or I give a virtual displacement to force M . If I give it for M recognize what way P will get displaced? If I give it for P recognize what way M will get displaced.

And when I look with P it is natural, I expect virtual displacement in the direction of P is the idea clear. On the other hand, if I go and look at M as a reference its natural for me to visualize in the direction of M , this is what people will do easy for you to do that. So, if I take P as a reference P will do a positive virtual work, if I take M as a reference M will do a positive virtual work. Either way I should get only one answer I should not get different answers, so, this is one aspect that you have to keep in mind.

Suppose, I put the angle with respect to the vertical then also I should get one unique answer because it is the single degree freedom system no matter what independent variable I select, no matter which active force I give a virtual displacement to start with because they are interrelated for me to start with I had visualize. So, here I would visualize when P is given virtual displacement how does the links get displaced. I am pulling it out and you should visualize that this has to come down and θ would decrease. You should recognize all that your sketch should be sufficiently big enough to recognize this aspect; this aspect is very important. If you get this then virtual work is the simplest methodology for you to solve a variety of problem.

So, make a neat sketch and recognize that when I give a positive displacement δx it also does positive virtual work, and you can see very clearly here the virtual displacement $\delta\theta$ is opposite to the moment M . Now, we have to establish what is $\delta\theta$ and δx from the given

configuration of the system. So, do not assign sign there that is a message I am trying to give, when you differentiate some cases you may get positive value, some case you may get negative value; If the problem is posed in such a manner that negative value is acceptable and then you get a right answer, you may get satisfy that you have solve the problem correctly, but you are not have got the concepts clearly.

On the other hand, if you investigate the sign of the virtual work there is very small likely hood for you to make a mistake. So, the recommendation is; investigate the product, do not investigate independently the sign convention for the force and

Virtual Work

- Choosing a coordinate system with origin as shown and let $\delta\theta$ be the virtual rotation to the link

$$x = 3b \cos \theta$$

$$|\delta x| = |-3b \sin \theta \delta \theta|$$

- Hence, by virtual work method,

$$\delta W = 0$$

$$M \delta \theta + P \delta x = 0$$

$$-M \delta \theta + P(3b \sin \theta \delta \theta) = 0$$

$$M = 3Pb \sin \theta$$

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independently the sign convention for virtual displacement.

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So, I get this interrelationship between θ and x as $x = 3b \cos \theta$. This you should be comfortable in doing a by looking at the inspection of

the geometry, there you have to visualize very straight forward and you can find out what is it, this expression you can easily write.

Now, I differentiated I do not recognize this sign because I am looking only at the modulus. I write this expression as $\delta W = 0$, then I can write the expression like this and I get the final step I have recognize that the moment is contributing to negative virtual work and the force P is contributing to positive virtual work. So, then I get an expression in one shot, I knock of this $\delta\theta$ and I get an interrelation between the two, I can also determine θ from this $M = 3Pb \sin \theta$. So, it is very interesting, so in this chapter you are in a position to find out the position of equilibrium.

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Since you are solved this you know can also make an attempt as a home work to solve an

Virtual Work

A screw with lead L , is rotated by a torque M , which decreases the distance BC and in turn lowers the mass m . Find the relation between the angle of rotation and θ .

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extended version of this and this is also giving you some exposure to certain other aspects of engineering. What I have here is? I have a screw here and when I have a screw it will have a pitch, it will have a lead these are all the terminologies I would like you to Google it and refer what is a pitch of a screw

what is a lead of a screw, and in your workshop practice you would have done lathe cutting, you would have a leads screw rotating whereas, this tool post is linearly moving ok.

So, you can also go and try it out take a bolt and nut hold the nut and rotate the screw you will find the nut will have a linear motion fine, if you hold the nut. Something like that is used here, so when I rotate this nut the point B can move forward or backward depending on the sign of the moment you must recognize that. So, you should recognize go to the definition of what is the lead of a screw I want you to do some homework on that. So, that you also learned little bit of engineering terminologies.

The only thing new here how to relate the rotation to the linear movement, rest of all instead of one link which we saw earlier I have two links, I will have change in expression, the problem is straight forward. The only aspect which you do not have a background now is what way to interpret rotation of a screw to linear movement of point B you figure that out fine, take that as an exercise for you to do it should also understand what is the lead of a screw.

So, in this class we have learned how to solve the problem in the method of virtual work. I have given a very important recommendation. Whenever you encounter a problem in virtual work, first identify the active forces, then visualize the displaced configuration

and make a neat sketch out of it. From the displaced configuration identify what is the sign of the virtual work by each of the active forces. So, look at the contribution if the force and the corresponding displacement on the same direction, then the contribution of the active force for a virtual work is positive.

If the force and the displacement are in opposite direction then its contribution is negative, this would help you to solve the problem without any difficulty. But the important requirement is you need to have a neat sketch of the displaced configuration and you should also have visualization of the physics of the system. The physical system visualization is very important and, in a sense, it will also help you appreciate engineering better. Physical visualizations are very important in engineering, so use virtual work as the vehicle for you to learn that.

Thank you.