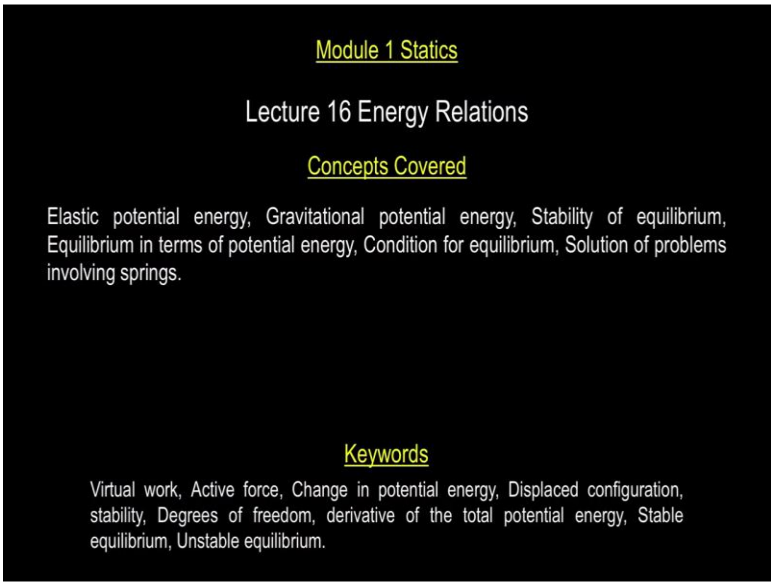
Engineering Mechanics
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module – 01
Statics
Lecture – 16
Energy Relations



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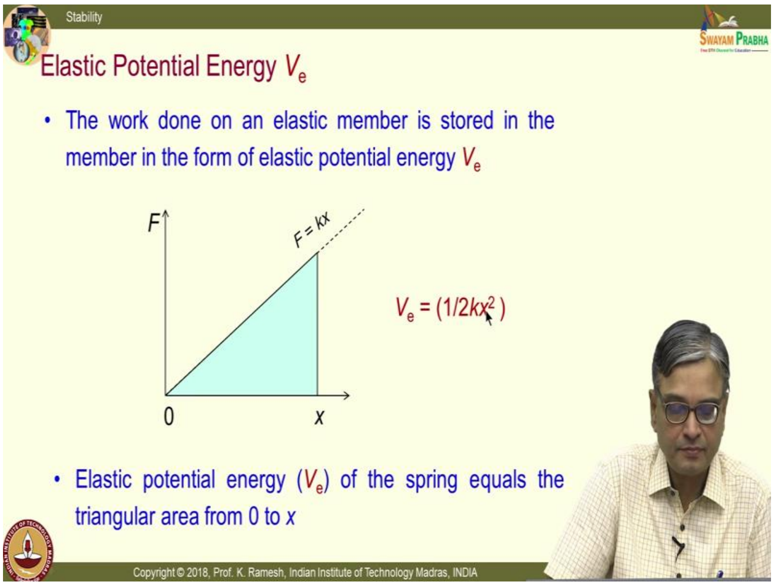
So, let us continue our discussion on Engineering Mechanics we will move on to the next chapter on stability. See in the force method, we have really investigated to keep the body in equilibrium what are the supporting reactions that you require. Then we investigated positions of equilibrium and in this chapter, we are going to discuss, what contributes to its stability how do you define stability and how do you verify for stability of a given system.



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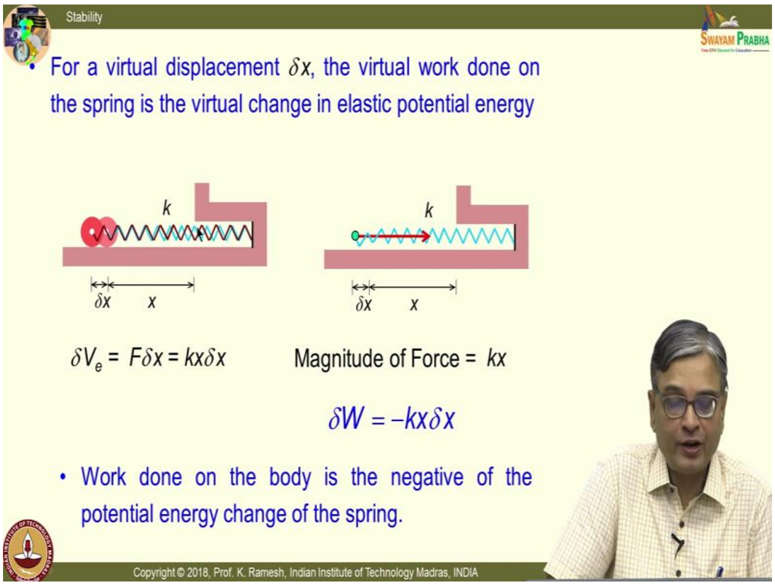
And you know springs are indispensable, you have many appliances that use springs for their regular functionality. Even though we say that we will confine our attention to rigid body mechanics, because springs are so, important we would like to analyse in some manner even in this course.

And one of the ways that you can recognise spring is it stores energy when it is pulled, and we know that the force is related to the displacement as $F = kx$. And whatever the force that you apply on the spring the work done gets stored in the form of elastic potential energy. That is nothing but, the triangle that you have and the value is $V_e = (1/2kx^2)$. In reality, even though you call and you have idealised the bodies as rigid when you go to the next level of course, where you relax this and say that they are deformable you recognise their behaviour similar to a spring, maybe it has a very high stiffness than a normal spring that you come across. So, whatever you analyse for this spring can be easily extrapolated to deformable solids, and I said that you would first start in the domain of small deformations then we will graduate to large deformation.



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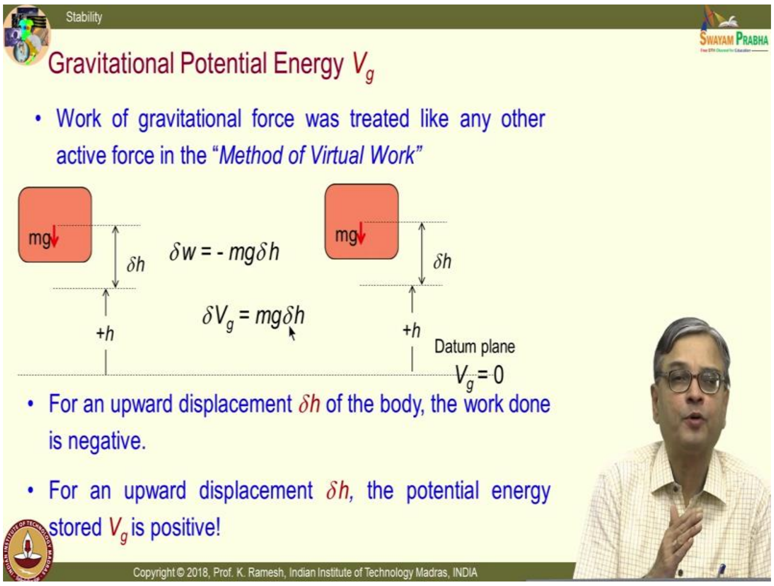
And let us look at the nuances when I say potential energy; I have one sign, when I have work done; I have another sign. We have to understand these sign conventions very clearly for us to apply the methodology, and I take a spring-loaded member I give a virtual displacement δx.

And whatever the virtual work that is done is also stored as a virtual change in elastic potential energy. So, we need to calculate what is the elastic potential energy, I have displaced it by a virtual displacement δx and we write the expression for the virtual change in elastic potential energy as $\delta V_e = F\delta x$ and we have seen for a spring force is proportional to the displacement. Now, how do I express this force?

Can I say this force is $k(x + \delta x)$. Can I afford to say it like this? We have to be very careful see we are dealing with virtual displacements. In virtual displacements, we give a virtual displacement we do not considered the force that it necessary to a virtual displacement. So, I cannot have the magnitude of force as $k(x + \delta x)$, I will have to have the force only as kx very subtle point. So, I would write the elastic potential energy virtual change of it as $kx\ \delta x$.

Now, let us look at what is the work done on the spring. I have given a displacement δx and I look at what is the restoring force, this is the restoring force in which way the force and the displacements are directed, the displacement is opposite to the direction of the force. When the force and displacement are opposite to each other what is the sign of the work done it has to be negative.

So, you have to understand the certain difference. So, when I say the elastic potential energy is $kx\ \delta x$, the virtual work done $\delta W = -\ kx\ \delta x$, this is what is written down this point. So, work done on the body is a negative of the potential energy change of the spring.



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We have looked at it for a spring, we will also investigate what happens to the body force due to gravitation. When we develop the method of virtual work of gravitational force was treated like any other active force. We have not treated it like an energy, if you treat it like an energy it is easier for you to visualise how do we bring in stability in a mathematical sense ok.

We have now replaced the elastic spring as elastic potential energy. We would also look at any change due to gravitational effects you know very well, that the force and the displacement are opposite in direction you will say the work done as $\delta w = -\ mg\delta h$. But I can also see when I have this as a datum where I say the potential energy due to gravitation is 0; I would recognise this displacement of δh as $\delta V_g = mg\delta h$ so there is definitely a sign change between the potential energy and the virtual work, they are opposite to each other in the sign.

So, this is summarised here for an upward displacement δh, the work done is negative, for an upward displacement the potential energy stored V_g is positive.

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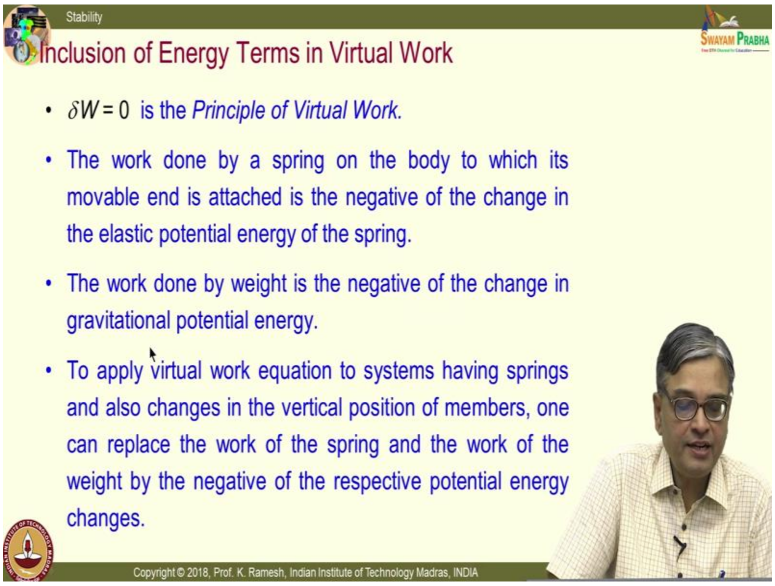

So, what is its influence then we look at in the principle of virtual work, we have studied that if virtual work done equal to 0 then the system is under equilibrium. I can now bring in the energy terms and rewrite the principle of virtual work. So, we have seen the work done by a spring is the negative of the change in the elastic potential energy of the spring. Similarly, work done by weight is the negative change in gravitational potential energy.

So, when I have a system that has springs as well as vertical position of members move. One can replace the work of the spring and the work of the weight by the negative of the respective potential changes you know in words it looks very complicated.

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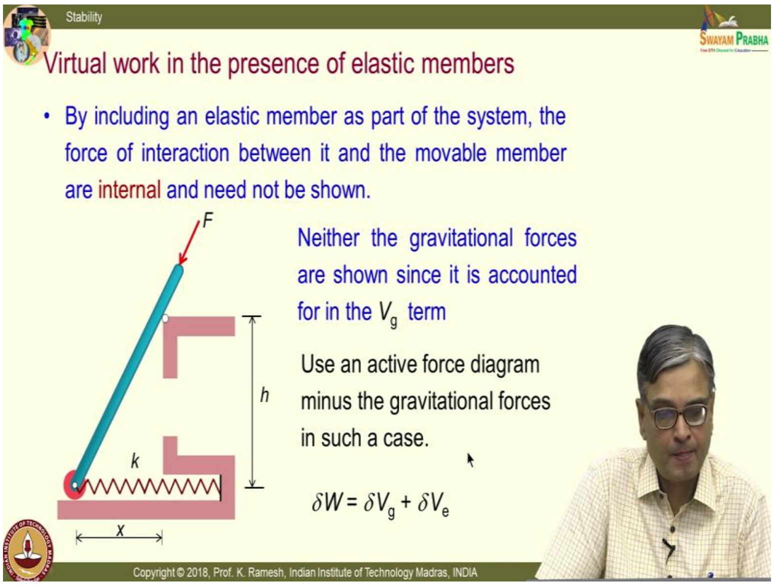
But if I put it as a mathematical equation it is as simple as this $\delta W = \delta V_g + \delta V_e$. This is nothing but, recasting your principle of virtual work equation involving change in potential energy. And when you say δW you qualify that these are the virtual work done by all external active forces other than spring forces and gravitational forces because they are separately included for spring and separately included for any vertical movement of the system. Now, the question is how do I solve problems involving springs, because springs gets compressed and they store elastic potential energy.



There are different ways of handling this particularly in Mariam, he treats this as a separate chapter, in other books they talk in continuation of the virtual work, without any major distinction. By including an elastic member as part of the system the force of interaction between it and the movable member are internal and need not be shown we have looked at in this chapter on virtual work.

We do not have to draw a free body diagram we need to have an active force diagram because we recognise the supports do not contribute to work done and internal forces the net work done is 0. So, we worry only about active forces, and we have included the role of an elastic member like a spring by energy term, I can treat that as an internal force.

And suppose, I have this link has some weight in many problems we have solved it without considering the weight of the member and in some problems, we have considered the weight of the links. So, when we use the energy approach you do not even show the gravitational forces, but we include that as a term involving Vg, because we have recast that virtual work equation as $\delta W = \delta V_g + \delta V_e$. So, we have to use an active force diagram minus the gravitational forces in such a case.

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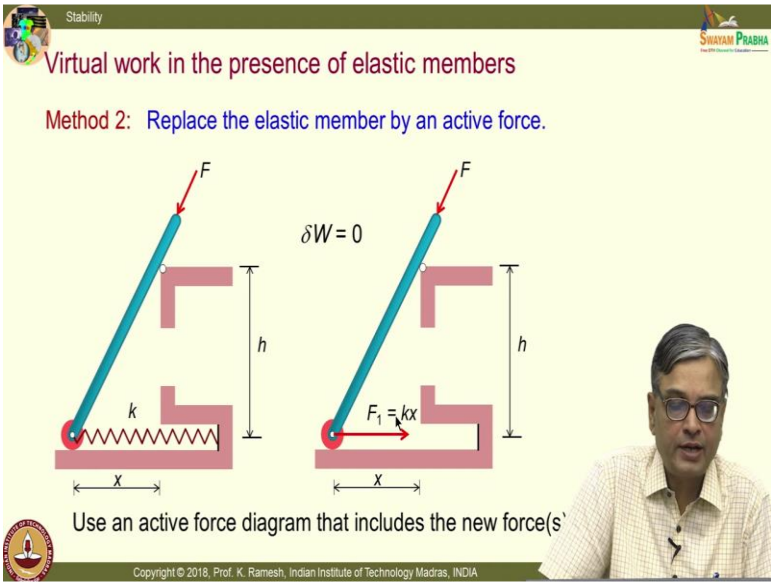

What is the way I can solve the problem, is there any other method? I can also replace the elastic number by an active force. See this is the trick that people employ when I have to find out the support reactions, there again replace the support by the respective forces which we have modelled earlier and then treat that as an unknown active force your virtual work equation will finally, give you what is the value of that reaction.

So, when you learn by using this method 2 you also know how to handle the spring as well as how to handle a reactive force. So, I would replace the presence of a spring by another force F_1 which is given as $F_1=kx$. So, you use an active force diagram that includes the new forces and then solve the problem. So, I have $\delta W = 0$ it is the basic equation.

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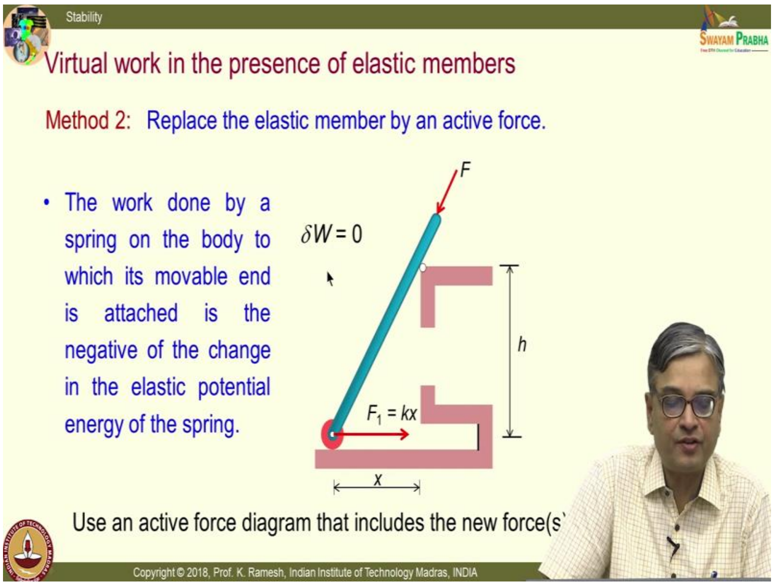

And you could have right hand side replaced by energy that is what is listed here or treat this spring as an active force in this direction and write basically this equation.

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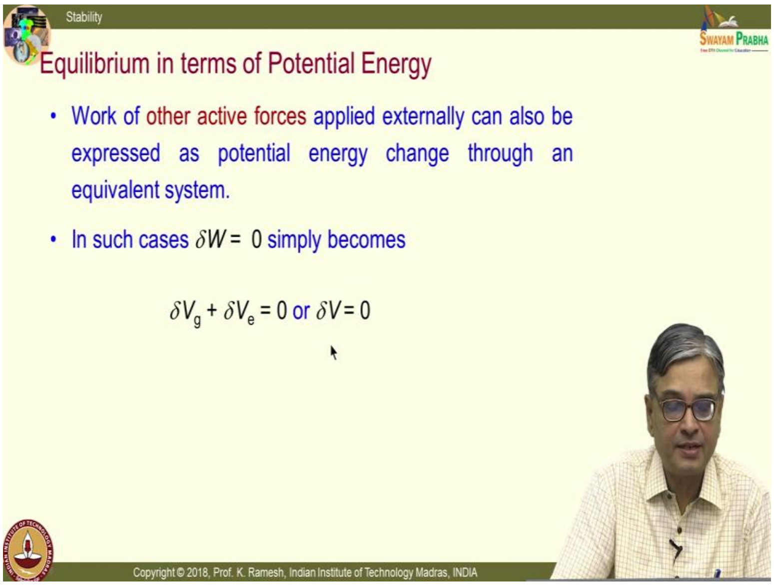

Now, we have seen spring can be replaced by an equivalent form of potential energy. Suppose, I considered every member forming the system is also deformable I can also find out the work done by other forces equivalent to an energy change. If I do that $\delta W = 0$ simply becomes, $\delta V_g + \delta V_e = 0$. In this we have accommodated the behaviour of all other systems which we have earlier treated as active force and calculated the work done you could also replace it as a change in elastic potential energy in that case the equation simply becomes δV=0.

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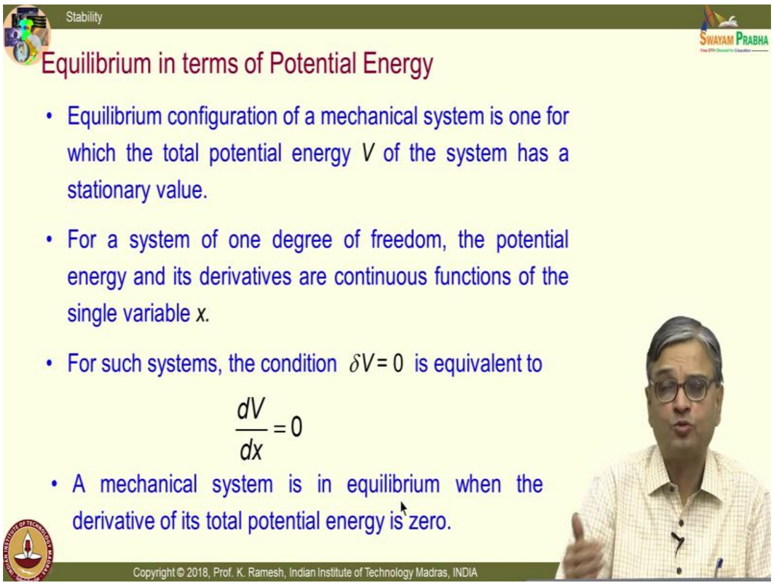
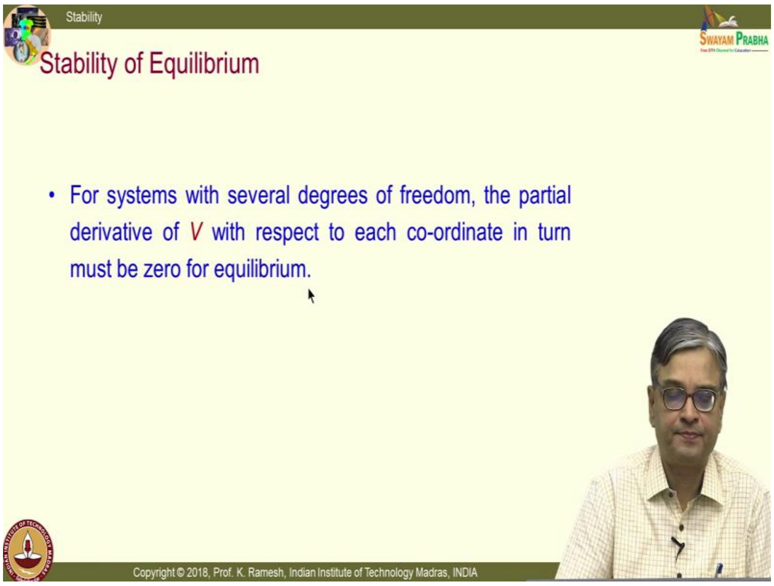


Another form of writing the virtual work equation in terms of potential energy and this makes it elegant for us to discuss about stability. So, equilibrium configuration of a mechanical system is one for which the total potential energy of the system has a stationary value, and for a system of one degree of freedom the potential energy and its derivatives are continuous functions of the single variable x we have discussed what is the meaning of degree of freedom.

And we said you have how many independent variables that are required to specify the configuration of the system for such systems the condition δV =0 is equivalent to simply

$dV/dx = 0$. So, a mechanical system is in equilibrium when the derivative of its potential energy is 0. So, to arrive at this statement we started with the virtual work then we included the action of a spring then the gravitational potential energy.

Then all the energy contribution because of all other active forces also replaced as equivalent to an elastic potential energy then we are able to write and state that a mechanical system is in equilibrium when the derivative of its potential energy is 0. This is for a single degree freedom of system.



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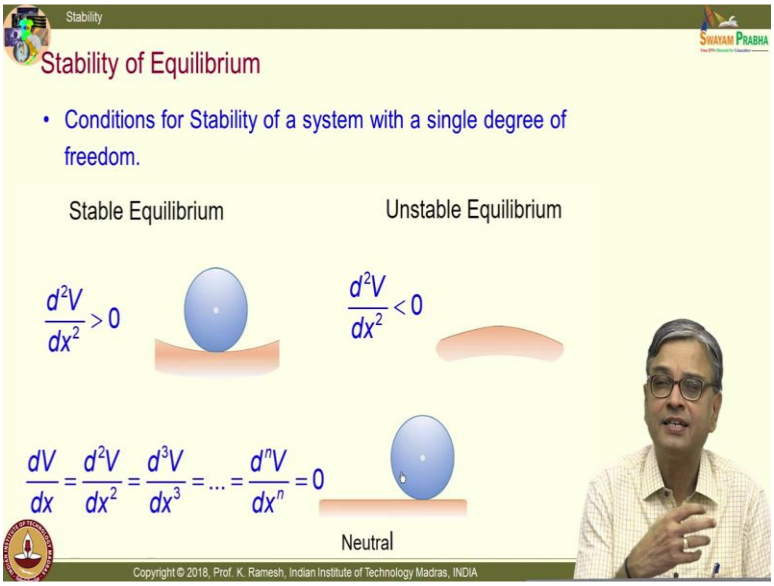
Suppose I have multi degree of freedom system instead of a total derivative like what we have seen the partial derivative of V with respect to each co ordinate in turn must be 0 for equilibrium. That is where the difference comes when you move from a single degree freedom system to multiple degree freedom system.



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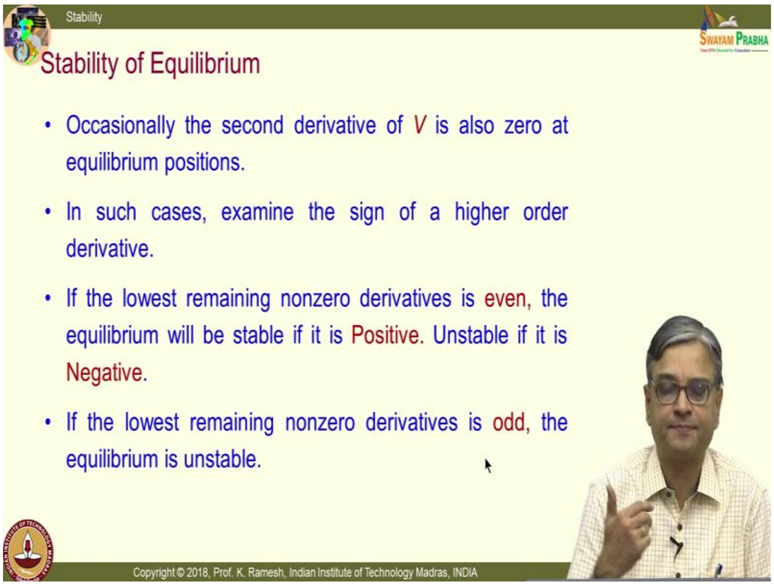
And you know this you must have learnt it in your earlier courses. So, we have a nice example here. So, what is a meaning of stable equilibrium unstable equilibrium and what is the neutral equilibrium, we also looked at the associated mathematical condition.

When the second derivative $\frac{d^2V}{dx^2} > 0$ I have a stable equilibrium and when I have this is a stable equilibrium, when I have $\frac{d^2V}{dx^2} < 0$ I have a unstable equilibrium when I have dV/dx. Normally, people say $dV/dx = 0$, when you have $\frac{d^2V}{dx^2} = 0$ and $\frac{d^3V}{dx^3} = 0$ all these also leads to a neutral equilibrium condition.

So, this is the way you will verify when I write the potential energy find out its derivatives based on the second derivative you are in a position to say whether it is stable or unstable.



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Suppose, I have a situation the second derivative is 0, we also have situations occasionally the second derivative of V is also 0 is at equilibrium positions. In such a case examine the sign of a higher order derivative and if you do that if the lowest remaining non zero derivative is even the equilibrium will be stable if it is positive, unstable if it is negative.

On the other hand, if the lowest remaining non zero derivative is odd the equilibrium is unstable all this comes from study of mathematics. We use that as a basis for us to investigate the stability. So, we have a recipe, I have to get the potential energy form of the virtual work and then look at the derivatives and comment about stability.

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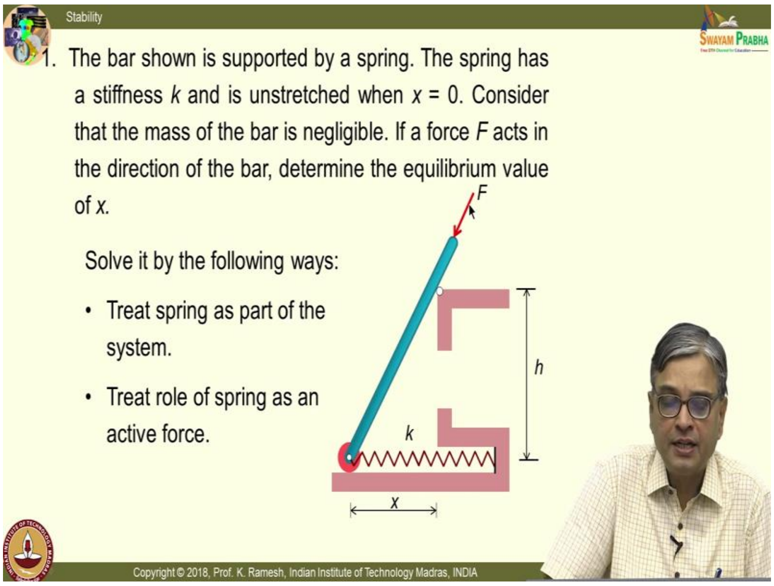
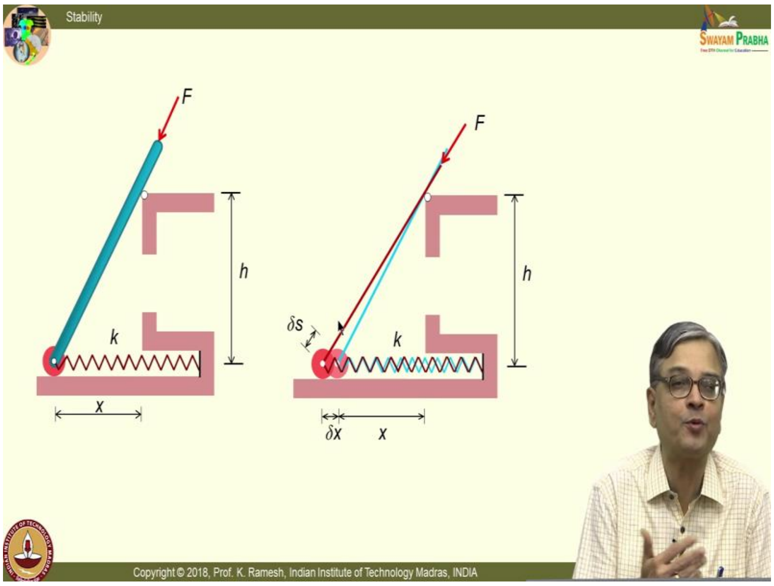
Let us solve one example problem, make a neat sketch of this problem first understand the physics of the problem what I have here is I have a member like this to make our



lives simple I have not considered the mass the problem says consider that the mass of the bar is negligible. So, we only understand the role of a spring and what you can do is, when you apply the force you can visualise the spring will get stretched horizontally when I leave the force it will come back to its original position and what is shown here as a circle you have to appreciate this is not a sharp corner.

But a very smooth corner so that this bar can move up and down smoothly. There is a very smooth interaction here, and what we would do is we would try to solve this problem in two different ways treat spring as part of the system then I bring in the role of a spring in terms of its potential energy, treat role of spring as an active force. Remove the spring replace it by an active force and solve the problem.



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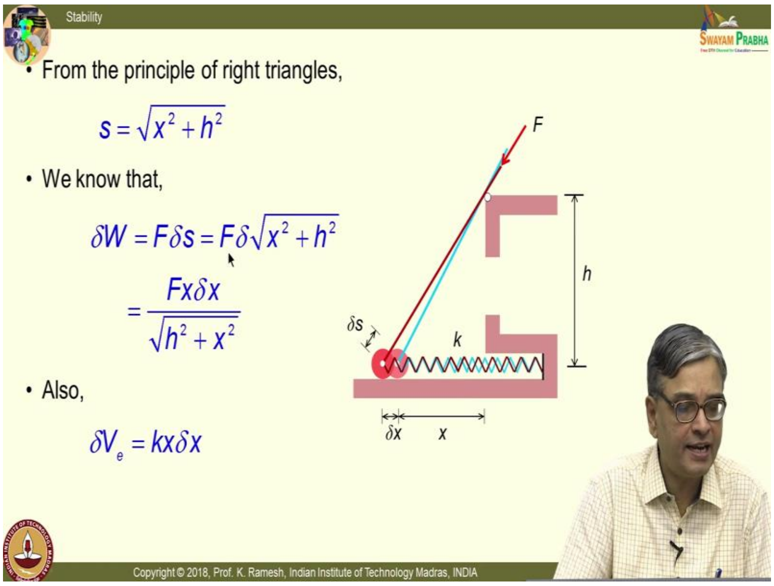
So, first aspect is we have to visualise when I apply the force what happens to the system and thing it is also stated in the problem when x is 0 spring is unstretched that is also given in the problem statement. Now, let me give a small virtual displacement to it. You have to recognise that this link also will move, see in all problems dealing with virtual work you have to bring back your knowledge of geometry.

Properties of triangles, properties of circles depending on the problem context you may have to use them appropriately, you have to physically visualise when I give a virtual displacement how the system gets displaced. And for example; if I have original length of this member is s, from this point to the roller here when I visualise a virtual displacement, I must also recognise this length changes by a small measure δs.



So, I have to bring that from the geometry of the system ok. So, the spring is stretched by a distance δx and the rod from this corner point to the roller has displaced by a distance δs.

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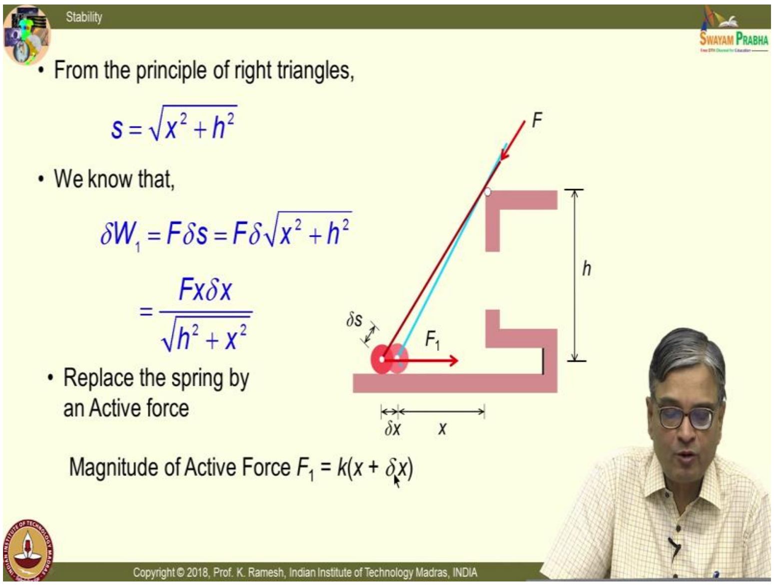
So, let us first solve it spring as part of the system. So, I would replace the spring by its energy and even before I do that, I get the relationship for the link *s* it is not labelled here. But, what I mean is s is the length from this corner to the roller comes straight from your geometry as $s=\sqrt{x^2+h^2}$ and then the work done by the force because when I say work done I am going to write only the active force the active force is only this. Spring I am going to bring in the form of elastic potential energy.

So, I write δW as $\delta W = F\delta s$ and you can visualise that force and displacement on the same directions so this is positive. So, I get this as $F\delta\sqrt{x^2+h^2}$ and when I put the differentiation, I get this finally, as $\frac{Fx\delta x}{\sqrt{h^2+x^2}}$.

Now, I find that I have moved it by a distance δx, whatever the movement has increased the elastic potential energy of the spring that is given as $kx\ \delta x$. You know I would not solve the problem right here; this is a set of relations I get when I treat spring as part of the system bring in the role of spring by replacing it auctioned by an elastic potential energy. Then I will say $\delta W = \delta V_e$ and then solve for the value. What I will do now is I will replace this spring by an active force, I do not have a spring but I have replaced it by a force F_1.



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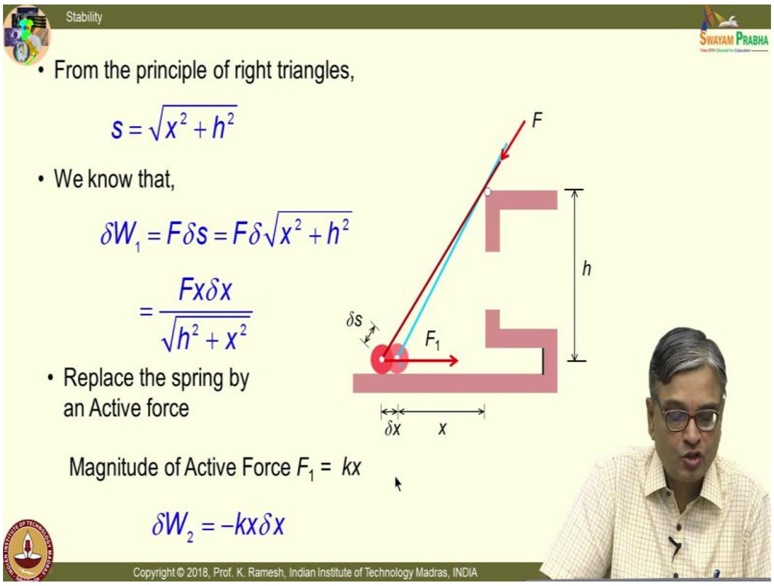
So, I have on this system two active forces force F and F 1 are acting. So, when I write $\delta W = 0$, I must write the contribution of work done by this force as well as the work done by this force, we borrow the same information from the earlier slide. So, this is known $\delta W_1 = F\delta s$, when I write for this the force and the displacements are in opposite direction. Here again, I raise the question can I write it as $F_1 = k(x + \delta x)$ just to caution you we are not worried about the virtual force required to cause the virtual displacement δx.



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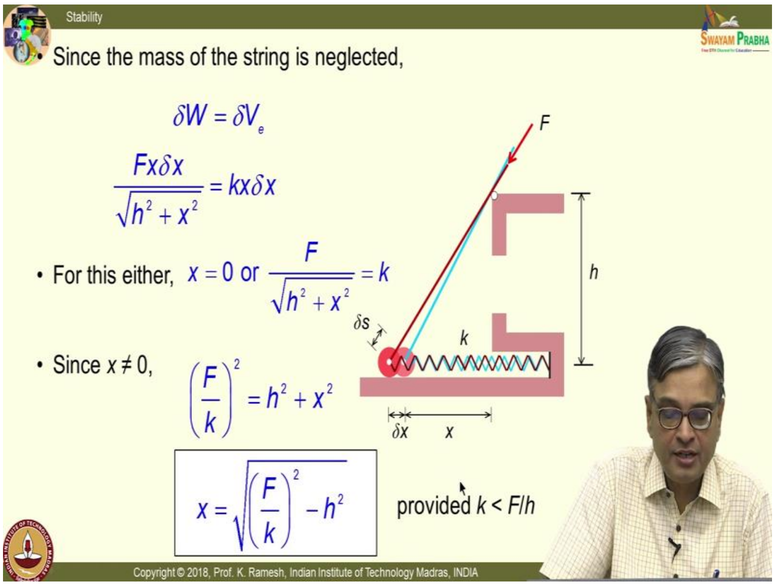
So, the force still remains as $F_1 = kx$ force and displacement are in opposite direction. So, I get this as $\delta W_2 = -kx\delta x$. So, you find the ultimate equation is same here when I put δW = 0. I am going to get $\frac{Fx\delta x}{\sqrt{h^2+x^2}}$ = $kx\,\delta x$.



I get the same answer whether I treat spring as part of the system or replace the spring by an active force, there are one and the same.

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So, I get this $\delta W = \delta V_e$ when I treat spring as a part of the system. So, I get this expression, I have already mentioned then even if I use the virtual work method replacing the spring by an active force final expression would be identical to this there is no change. So, I can easily solve for relationship between x and F. So, $x = 0 \text{ or } \frac{F}{\sqrt{h^2+x^2}} = k$ when you solve for this, I get the expression for x as $x = \sqrt{\left(\frac{F}{k}\right)^2 - h^2}$ and you also have a condition provided $k < F/h$.

See in many of these instances you may have to design what should be the spring I should use for this system we would also solve a problem like this, you know we have looked at a linear spring we would also take a torsional spring which is available in your cycle carrier, when you lift the carrier, when you leave your hand that automatically comes down any angular displacement you give it comes back to its normal position there is a restoring force that brings it back. So, when I say spring it can be a linear spring or it can be a torsional spring like we have a linear displacement and angular displacement. We would also see a problem a little while later.

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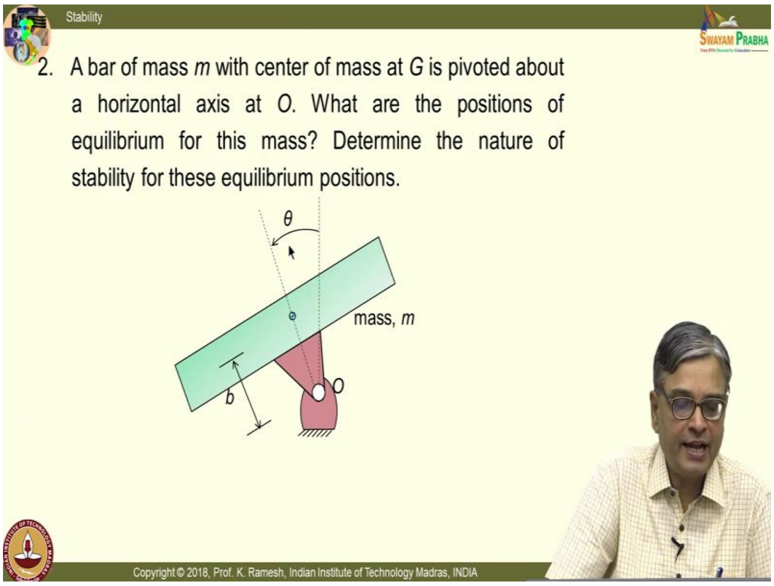

Now, we would take another very simple problem, we already know even before solving the problem whet is the stability of this system. This is just to verify, what are the basic mathematical equations we have got or convincing for us to carry on. So, we have taken a very simple problem make a neat sketch of it and in the problem statement itself it is shown at a generic position which is oriented at an angle θ and I have this mass pivoted at the joint O; that means, it is a pin joint the mass can move freely over it. The question is, what are the positions of equilibrium for this mass?

Not only that of these equilibrium positions you have to identify the position which is stable even before solving the problem you can visualize which is the stable position for θ; $\theta = 0^o$ could be one answer $\theta = 180^o$ could be another answer. And it is also easy for us to see when the mass is at the lower end it is very stable if the mass is vertically up even if a small wind comes and hits it from left or right it will display its position. All problems dealing with stability you have to visualise a perturbation small perturbation.

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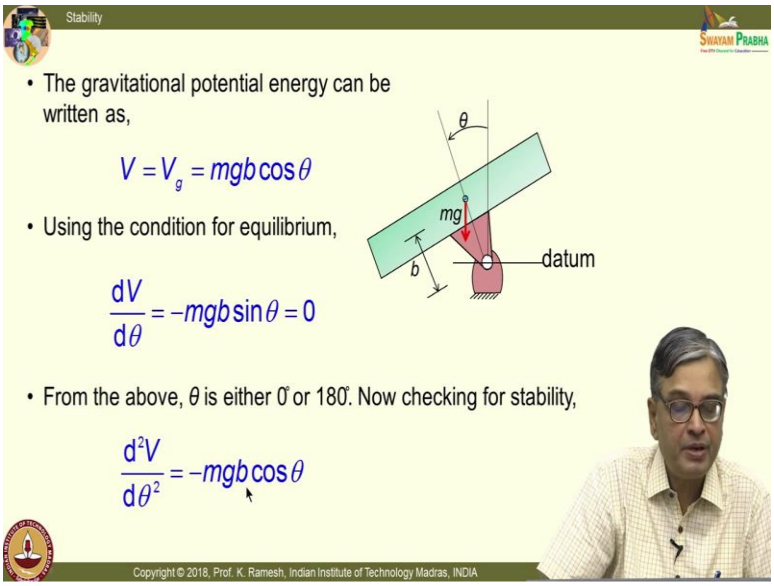
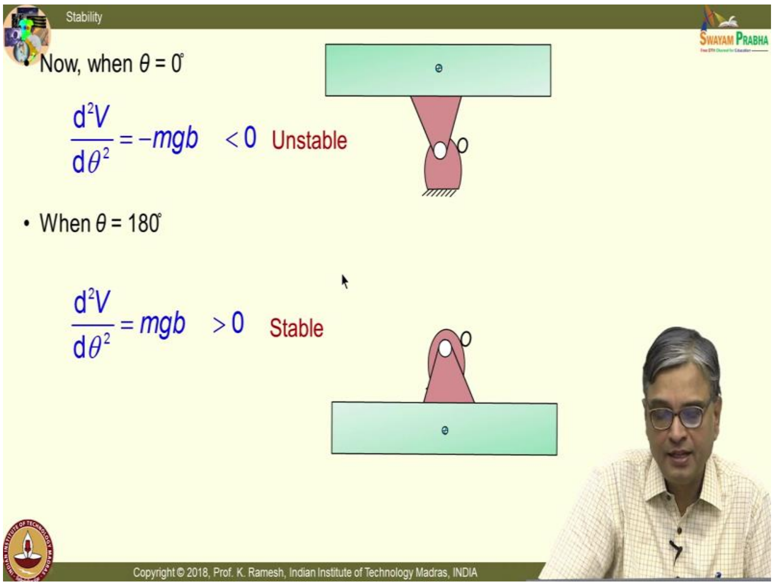


So, here it is completely dealing with gravitational effects. So, I need to recognise a datum in all problems dealing with gravitational potential energy, I must have a reference datum from which I make the measurements.

So, I have identified the datum here and I can simply write this, it is so simple and easy that this is nothing, but $V_g = mgb\cos\theta$ because the mg is at a height of $b\cos\theta$ from the datum. So, I have an expression for V, now I have to take the derivatives I get $\frac{dV}{d\theta} = -mgb\sin\theta = 0$.

So, that gives me two solution for θ and $\sin\theta$ is 0; $\sin\theta$ is 0 when θ is 0 or 180^o this is to check the equilibrium and I need to get the higher derivative to check for stability. So, when I do this, I get this as $\frac{d^2V}{d\theta^2} = -mgb\cos\theta$. Now, let us investigate what happens to this when θ is 0^o or 180^o.



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So, you have cos 0 is 1. So, I find $\frac{d^2V}{d\theta^2} = -mgb < 0$ is negative and this is unstable. So, if I sketch the configuration this is the configuration when $\theta = 0$ because θ is referred with respect to the vertical and you can see I can balance it at this position even if there is a small disturbance this will get dislodged. So, our mathematics is also agreeing to that intuitive appreciation of the system. So, our mathematics is right, and when I have θ is 180^o I have cos (180) becomes -1. I get this as positive and this is a stable configuration.

So, when I plot the same configuration of the system at $\theta = 180^o$ this is at the bottom and you can very well see that its quite stable, it is a naturally most stable configuration for the given system fine. So, this problem is fairly simple and straight forward.

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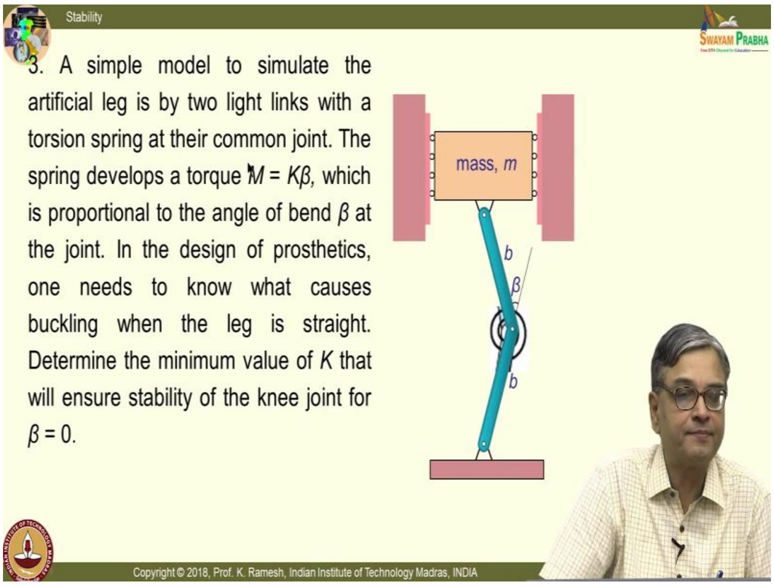
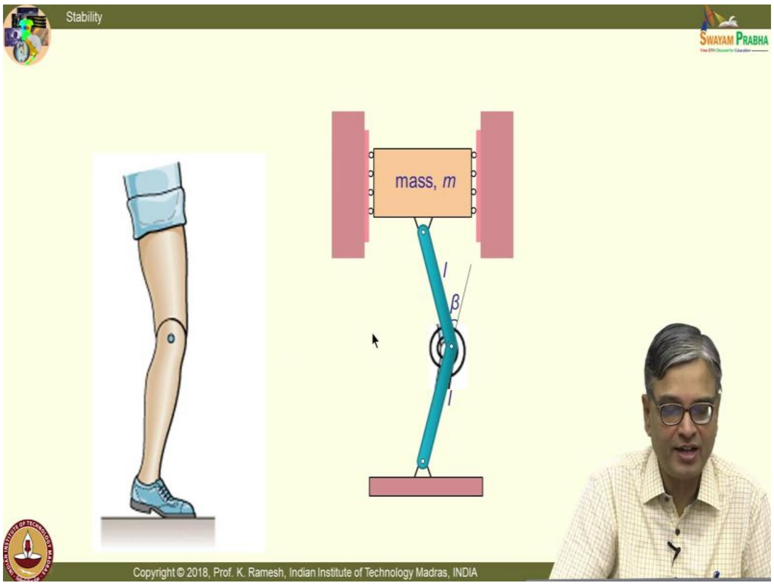


Then we move on to another interesting problem, you make a neat sketch of this, see these days prosthetics design is becoming very prominent mainly because of our own making because you know we have lot of avenues for people to get involved in accidents and they lose their limbs and you need to have some kind of prosthetics support and this is a very interesting problems.

It is also dealing with practical stuff and you have to understand what is the stability see it is a common experience when you stand on your legs, if somebody comes and put a small force at the knee joint from behind you cannot remain stable you would buckle. Now, the question here is you have idealized the persons weight as mass m and then you have also modelled his leg as having two links like this I have a link of length *b* and then another link of length b which is not true because your leg is not like that it is a very cool and simple approximation. So, you solve a problem and then have a joy that you have solved something pragmatic and practical ok.

And replace your knee functionality by a torsional spring. So, this will develop the restoring force depending on the angular position of the leg and you have given in the problem statement the spring develops a torque $M = kb$. I am sorry it is k into β and this β is measured from this arm. And you measure what is the orientation of this see this is the problem is basically a symmetric problem that is how we have modelled it and that is why it says a simple model to simulate the artificial leg it is a very simple and crude model gives a very nice problem in this course to solve.

The idea is what should be the spring stiffness for a given mass so that we would be in a position to ensure stability for the knee joint when $\beta = 0$, when the leg is straight what should be the minimum value of the spring stiffness see that is a design problem. If you put an inappropriate spring the person with this prosthetic leg can never stand straight, he will always collapse. You need to have a minimum value you can have a value larger than that from a design perspective you should know what should be the minimum value ok.



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And the next slide shows your leg and usually we will have this shorter and this is slightly longer ok. So, this is a very idealised condition that is what you have to look at.

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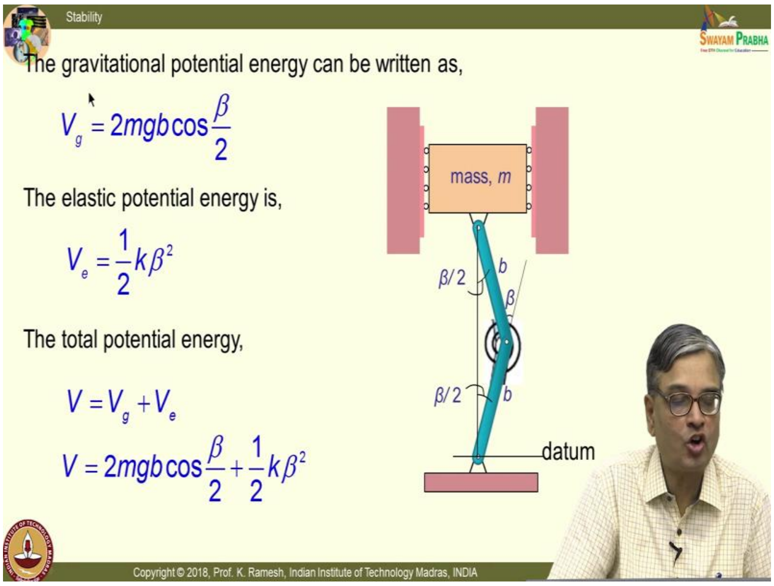
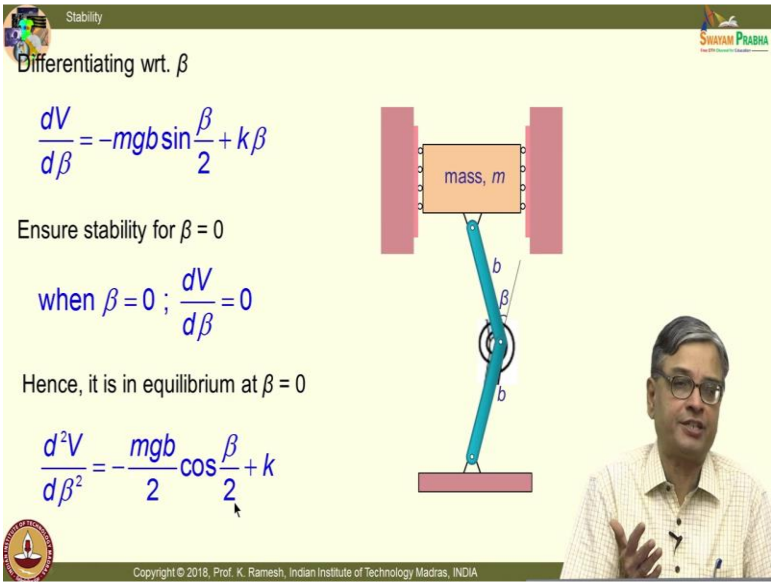
And so you have a datum here and I have the mass lumped here, the entire weight of the person is idealised as like this and we have to bring in your property of the triangles and find out what is this angle that is also again very simple, but I am only altering you please go back and refresh your understanding of geometry. We will not pay attention on that in discussing that in the course, but that background is needed for solving problems.



So, I have these angles as from geometry it is $\beta/2$, I have these angles at $\beta/2$ from the property of a triangle you get this then I can easily write what is V_g; V_g is nothing, but $2mgb\cos\frac{\beta}{2}$ I have $b\cos\frac{\beta}{2}$ here another $b\cos\frac{\beta}{2}$ here. So, twice of that contributes to the potential energy and if I take this as the datum.

So, in all your problems dealing with energy where you have to calculate the gravitational potential energy precisely indicate the datum position. If the datum position is different all your calculations will be appropriately modified and you have the spring the stiffness is given as k.

See usually stiffness is given by the small k in the problem statement it is put as big k, but in the problem solving it is taken as small k I think you can understand it based on the context. So, I get the elastic potential energy as $\frac{1}{2}k\beta^2$ here only gravitational potential energy and elastic potential energy is there, we have considered this as mass less links. So, the total potential energy is simply $V = V_g + V_e$. So, I get this as $V = 2mgb\cos\frac{\beta}{2} + \frac{1}{2}k\beta^2$.



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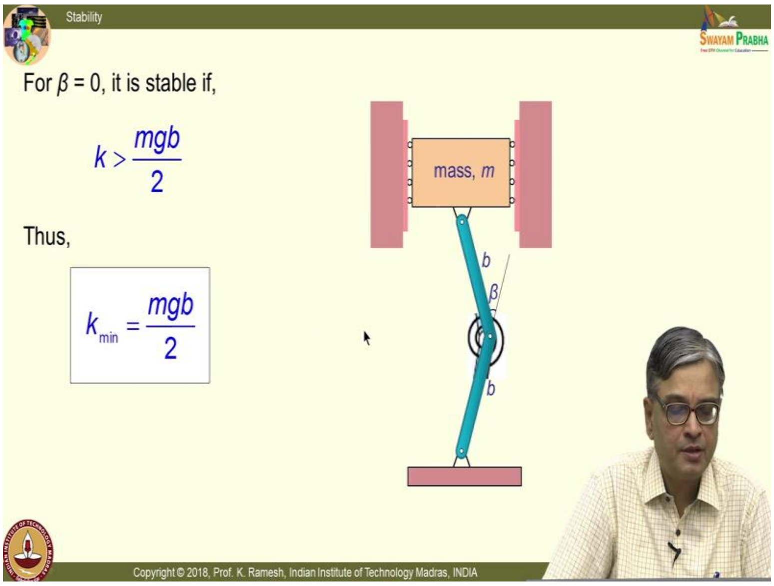
So, now you differentiate this, differentiating with respect to β $\frac{dV}{d\beta} = -mgb\sin\frac{\beta}{2} + k\beta$. I ensure stability for $\beta = 0$ this goes to 0 there is no problem.

So, at $\beta = 0$ it is in equilibrium, to look for stability I have to get the second derivative $\frac{d^2V}{d\beta^2}$ from this if I differentiate, I get this as $\frac{d^2V}{d\beta^2} = -\frac{mgb}{2}\cos\frac{\beta}{2} + k$. So, this gives me a minimum condition for it to be positive it has to be 0 then only it becomes positive ok. So, I am in a position to get a condition to ensure what should be the torsional stiffness of the spring that I should put it if I have to develop a prosthetics to replace your knee joint; very simplified problem.



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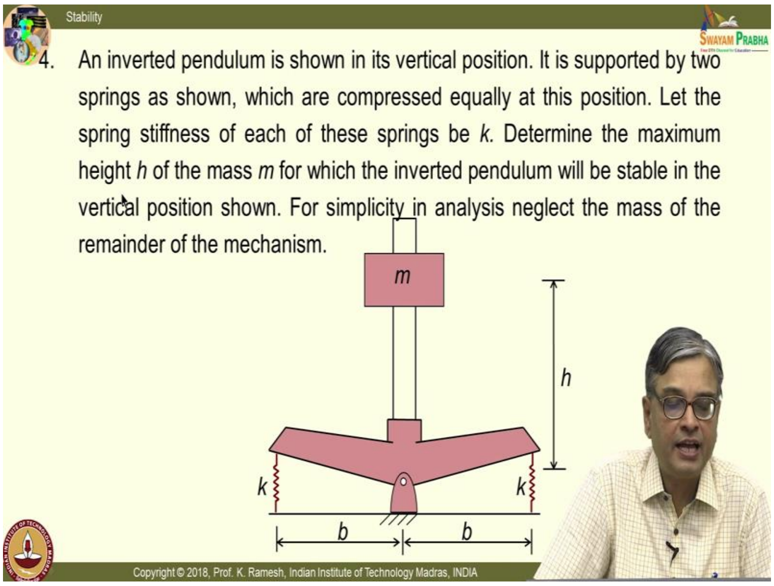
We have given colour to that problem by bringing it similarity to a leg one can improve the model in several fold and for $\beta = 0$ it is stable if $k > \frac{mgb}{2}$ if it is equal to you have $d^2V / dx^2 = V\,\theta^2 = 0$ we would like to have this as positive for stability. So, I have to get this stiffness greater than $\frac{mgb}{2}$ very interesting problem and then we move on to another problem. So, I have this $k_{min} = \frac{mgb}{2}$, we will also move on to another problem.



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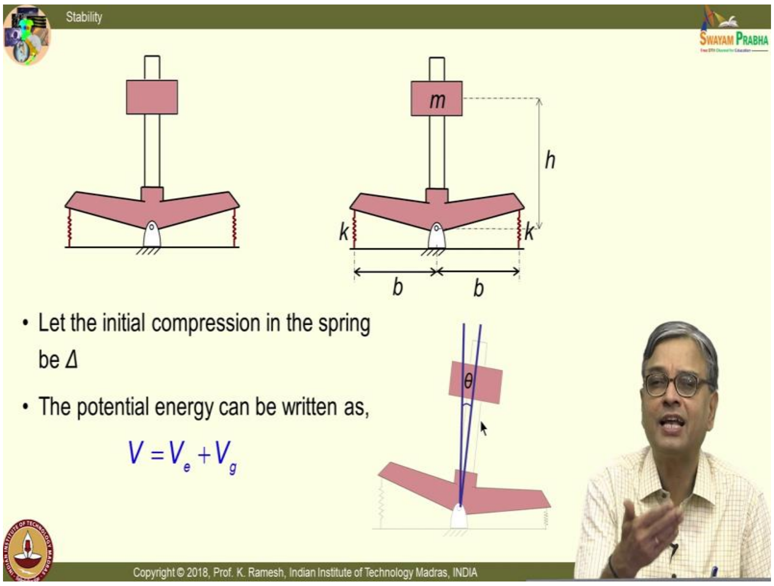
This is slightly involved with you have a mass you have two springs and this springs help it to maintain stability of this and what is stated in the problem is the vertical pendulum is supported by two springs as shown, which are compressed equally at this position they are not in the normal condition they are compressed because of the weight. Let the spring stiffness of each of these springs be *k*.

Determine the maximum height h of the mass m and you have determined the height from the pivotal position, from the pivotal position to the centre of mass it is h. Determine the maximum height h of the mass m for which the inverted pendulum will be stable in the vertical position shown, to make our life simple the problem itself states for simplicity in analysis neglect the mass of the remainder of this mechanism. We do not worry about all this limbs we will get a first approximation get a physics of the problem see in all problems dealing with stability we should start with some displaced position you should understand if there is a displacement from where the restoring force comes to make it bring it back to the original position.



You have to visualize that. In some problems the problem statement itself gives you the displaced position. So, it is easy for you to solve in this case you are asked to find out what happens at the vertical position.

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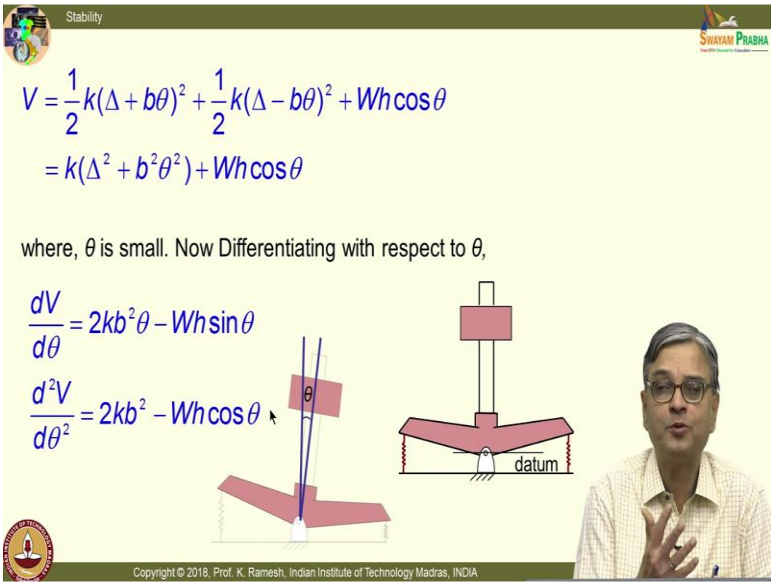
For me to write the mathematics I must first visualise what is the kind of change when there is a disturbance and that is shown in a nice animation here. So, if there is an angular motion one of the springs will compressed more. So, it will have a restoring force that is what is shown in the; I think we can go back and then see the situation our self. So, if it is displaced like this and you will have a restoring force because of the springs.

The springs help it to bring it back to the equilibrium position. So, let the initial compression in the spring be taken as Δ and here I have no other active force, we have a energy due to gravitation because of this mass and later on we will not write it as m into g we will simply replace it by the weight W, and I have two springs, instead of one spring I have two springs. So, I have to take care of both the springs when I write the elastic potential energy, I should not ignore one spring I should handle both the springs properly. So, I have the total potential energy is given as $V = V_e + V_g$ and I have given you a generic displacement position.



I have the vertical axis it is displaced by a small angle θ. For this configuration if you write the contribution of gravitational potential energy the contribution of elastic potential energy by the springs the problem is done. But then the rest is simple mathematical expressions you have to get $dV/d\theta$ and then $d^2V/d\theta^2$.

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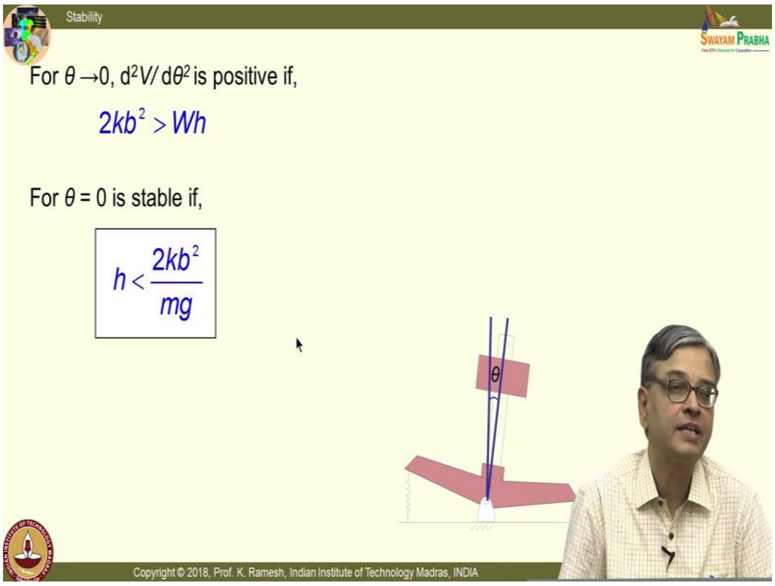
So, I have this let me write that basic equation which is written down very nicely. So, you have to visualise that this is displaced this spring is compressed more and this spring is slightly elongated.

So, when I have this, I have this compressed, when I have some θ, I move by an angle $\delta\theta$ I have this as $\frac{1}{2}k(\Delta + b\theta)^2$ and for the other spring it is $(\Delta - b\theta)^2$. In one spring it is a positive compression and another spring is stretched so the original compression is released originally, we have seen that the springs were compressed in this position. So, you need to have that otherwise if the springs are not compressed there will not be any restoring force fine and the contribution of potential energy from the weight is very clear.

From this datum this is simply $h \cos\theta$, this is easy to write probably you would write for one spring you must also right for the other spring carefully. That is where your sketch here one spring is compressed another string is stretched would help you to write for one spring it is on half of $k(\Delta + b\theta)^2$ for another spring it is $k(\Delta - b\theta)^2$ rest of it is simple.

I find out what is $dV/\ d\theta$ now differentiating with respect to θ I get $dV/\ d\theta$ as



$\frac{dV}{d\theta} = 2kb^2\theta - Wh\sin\theta$ and $\frac{d^2V}{d\theta^2} = 2kb^2 - Wh\cos\theta$. So, in all this problem dealing with virtual work you should be able to visualise the displaced configuration, that is the training. So, always put the displaced configuration then you would be able to write the mathematics clearly.

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For θ tending to 0 $d^2V/d\theta^2$ is positive if $2kb^2 > Wh$. So, that gives me h should be $h < \frac{2kb^2}{mg}$.

So, you run a position to establish what should be the height of the mass that can be kept in the vertical pendulum so that it functions like a pendulum h cannot exceed this value it should be less than $\frac{2kb^2}{mg}$. So, this chapter we have discussed about stability of equilibrium.

So, this completes statics in all its totality in the absence of friction. We initially looked at by force method what are the reactive forces that are necessary to keep the body in equilibrium in the method of virtual work; we investigated what are the different positions of equilibrium. In stability we are forced to recognise deformable solids, even the members, which were originally considered as rigid were treated more like a spring. So, we were able to rewrite the virtual work expression in terms of potential energy from that we could discuss about and find out the stability of the equilibrium of the system.

Thank you.