

Engineering Mechanics
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Module – 01
Statics
Lecture – 17
Review Before Quiz I

So, in this class let us review the subject that we have learned so far and you know in the first chapter we discussed the force systems. We have looked at a concurrent force which is also seen in trusses and we discussed in practice all forces are actually distributed. When I have a pin and a hole contact; we replace it by a concentrated force and if you really look at what happens, you have a distribution rigid body is an idealization, concentrated force is also an idealization goes hand in hand with rigid body idealization.

Then we have also looked at body force you have from rotating platform like this and another important concept is we have to quantify what is the rotating effect of the force;

Transmissibility of Force

- Principle of transmissibility states that the external effects of the force are independent of the point of application of the force along its line of action.

Pulling or pushing is producing the same effect!

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we understood; what is the meaning of a bending moment and so on and so forth.

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And one of the important concepts that we understood in the process was transmissibility of force. When we are

looking at the external effects, it is independent of the point of application of the force along its line of action and if I make the board smart with this force; it will move forward. Suppose, I move the force to the back and then make the board smart it will also move forward. So, the external effect does not change and what you understand is pulling or pushing is produces the same effect on the body under consideration.

Review Before Quiz -1

Transmissibility of Force

The diagram illustrates the transmissibility of force. It shows three horizontal bars representing a body. The top bar is labeled 'Rigid Body' and has two red arrows pointing outwards from its center, representing tension. The middle bar is labeled 'If the body is Deformable' and has two red arrows pointing outwards from its ends, representing tension that causes elongation. The bottom bar is labeled 'If the body is Deformable' and has two red arrows pointing inwards from its ends, representing compression that causes shortening.

- Even though the internal effect is different, the shape of a rigid body will not change, by definition.
- Force is a Sliding Vector
- Slides along the line of action.

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Suppose I look at what happens when I apply two forces like this, this is applying a tension a rigid body by definition will not have any change. If the body is deformable it would get elongated, if the body has a compressive force it would get

compressed. And what we learnt was even though the internal effect is different the shape of a rigid body will not change by definition; it makes our life fairly simple at the initial stages of the course.

We have also learned force is a sliding vector, slides along the line of action. And particularly this is useful when you have to reduce a given force system into its resultant or when I have multiple forces acting on it. If I have to find out the resultant you use the principle of transmissibility to simplify the force system and find out the resultant comfortably.

Review Before Quiz -1

Resolution of a Force into a Force and a Couple

The diagram shows three stages of force resolution on a body. In the first stage, a force F acts at point P_1 . In the second stage, the force F is moved to point P_2 , and a couple M is introduced to maintain equilibrium. The distance between P_1 and P_2 is labeled d . In the third stage, the force F acts at point P_2 and the couple M is shown as a curved arrow.

This is a step that finds repeated applications in the study of mechanics.

Reversing the above procedure, it is also possible to combine a force and a couple to an equivalent force acting at a different point

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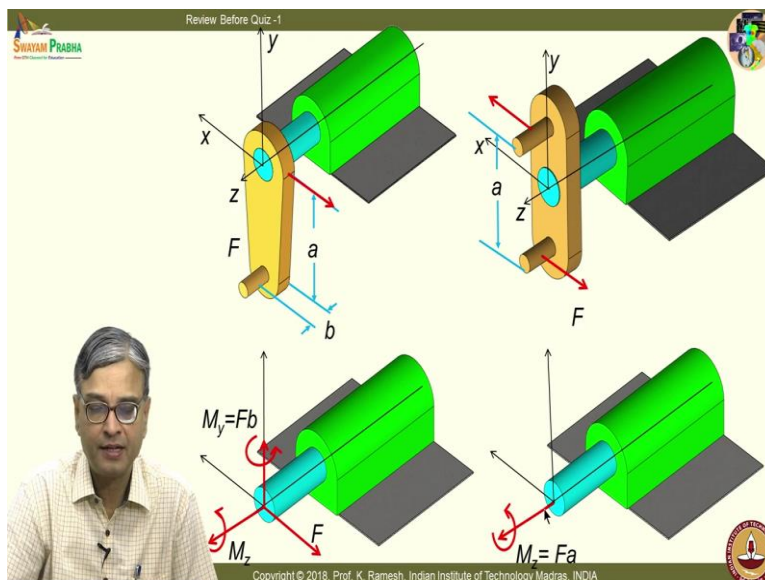
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Then the important concept that we learnt was what is the meaning of resolution of a force into a force and a couple, I take point P_1 and I have a force F acting on it I can freely move this along its line of action.

Suppose, I want to find out what is the effect of the force acting at point P_1 to point P_2 , I cannot simply move the force to point P_2 .

Suppose, I want to maintain the external effect as same; I can add another force and subtract here, I have put this force here and I have subtracted this force here this is similar to what is the situation here. Since, we have learnt what is the meaning of a couple force system; we could visualize the force here and this force here would form a couple and you have the perpendicular distance as d . So, I can visualize sliding a force from this point P_1 to P_2 ; it actually amounts to in addition to a force you also have a moment and we found that this is a free vector.

I can slide the force along its line of action; it is fairly simple, but if I have to move the force from point P_1 to P_2 , if I have to maintain the external effect, I have to have a force as well as a couple. And this is what we said finds repeated applications in the study of mechanics and I have also advice that you should master this technique and you can also



reverse the above procedure. If I have a couple and a force, I could find the force acting at some arbitrary point which we will have to find out the location.

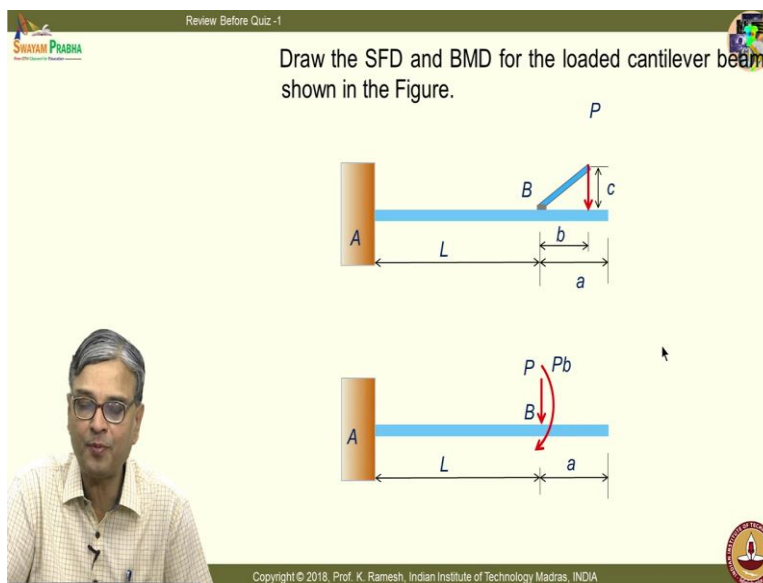
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And we understood the subtleties between the

moment and a couple by taking this example, I have a force acting on this handle. Now I want to find out what is the effect of this force on this shaft, the free body diagram is shown here.

Let us go in stages I have the force acting at this point at a distance b away from the plane on which I want to find out its effect. I would move this force horizontally like this, we know I cannot move this force horizontally unless I have a couple fine. So, this would result a couple which is given here which will be F into b ; then I would move this force from this level to the level of the x axis.

So, I have this force when it is moved what I will have is I would have a force acting on it as well as a twisting moment. My ultimate interest is to rotate the shaft, but if I apply the force with one pin like this it would introduce the force as well as an additional moment Fb . On the other hand, if I put an equal and opposite force system it would only produce a twisting moment on the shaft. So, this brings out a subtle difference between rotating the shaft with one handle, rotating the shaft with two pins attached to the handle fine.



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So, we have a necessary foundation and one of the common doubts the students had was you had shown a beam and there is a welded arm like this I have a force P acting on it. And when we discuss in the class, we replace the

force which is acting at a distance on an extended arm as a force and a couple acting at point B .

This comes out naturally if you look at how do you apply the principles that you have learnt. But the doubt was you said the force is following a principle of transmissibility why not I simply move the force and find out the bending moment acting on the B . Why not I do that, it is because you are not understood the concept very clearly and I would counter it by another example.

See in this case when I move this force there the member is still there and then you are able to have the force hitting on the member. The member as such does not feel the force like this, it is feeling only at the point of weld that is where the confusion is.

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Suppose I have another problem where I attach a bar like this and I weld it here, I apply the force here. Now let me use your argument of transmissibility, I have this is the line of

Review Before Quiz-1

SHYAM PRABHA

Draw the SFD and BMD for the loaded cantilever beam shown in the Figure.

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action and then the force is moved along the line of action, it is not hit; touching the body of the beam anywhere like in the previous example.

When I have a force system like this you would naturally feel, I cannot move the force like this and then try to find out

what is the effect on the body. You would only see that this is the arm attached to this body and you would naturally reduce this as a couple acting like this. You have to look at; yes, principle of transmissibility tells pushing or pulling has no difference.

But you cannot apply it randomly when I have a beam, when I have an extended arm where I put the load; the beam senses that force only at the point where it is attached. So, you have to apply the principle properly when you want to understand what is the force system that is happening on the beam, is the idea clear? This is one of the doubts that students have raised why not I simply transmit the force?

See once, I put here this will be a force which can go in the line of action, that is fine simple for the resolution or simplification you can do that. But the beam as such does not feel this force, the way that you want to translate and then look at it; it feels this force as a moment acting at this point the same argument is valid for the previous example also.

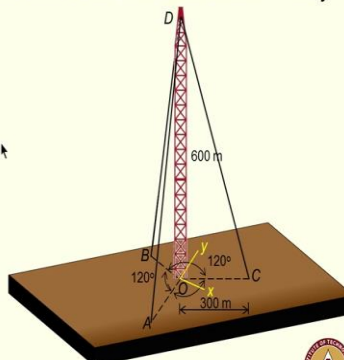
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Now, we move on to solving a simple problem in three-dimensions, my idea is I have deliberately focused on two-dimensions. I said at this level of the course you have to visualize what happens and visualization is far simpler when you are handling two-dimensional problems.

Review Before Quiz -1

SHAYAM PRABHA

Three guy wires are used in the support system for a television transmission tower that is 600 m tall. Wires A and B are tightened to a tension of 60 kN, whereas wire C has only 30 kN of tension. What is the moment of the wire forces about the base of the tower? The y axis is collinear with AO.



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I would like you to visualize whether the bending moment is acting in the clockwise direction or anti-clockwise direction that training has to come; which can come when you solve two-dimensional problem it is easy to scale up to three-dimensions. It

is not a big deal at all, fine; and this is also a problem given in your assignment sheet. So, you have a transmitting tower and you have this supported by three guy wires and the

problem also specifies what should be the selection of the coordinate system fine.

Review Before Quiz -1

SHAYAM PRABHA

First, determine the unit vectors along the cable directions A, B and C

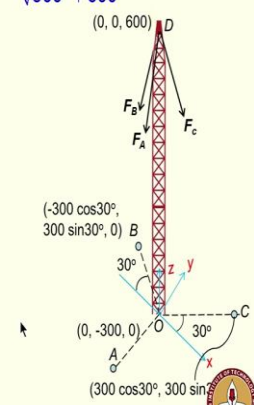
$$\hat{f}_A = \frac{(\text{coordinate of A}) - (\text{coordinate of D})}{\text{Length of AD}} = \frac{-300\hat{j} - 600\hat{k}}{\sqrt{300^2 + 600^2}}$$

$$= -0.4472\hat{j} - 0.8944\hat{k}$$

$$\hat{f}_B = \frac{(-300\cos 30^\circ\hat{i} + 300\sin 30^\circ\hat{j} - 600\hat{k})}{\sqrt{300^2 + 600^2}}$$

$$= -0.3873\hat{i} + 0.2236\hat{j} - 0.8944\hat{k}$$

$$\hat{f}_C = \frac{(300\cos 30^\circ\hat{i} + 300\sin 30^\circ\hat{j} - 600\hat{k})}{\sqrt{300^2 + 600^2}}$$

$$= 0.3873\hat{i} + 0.2236\hat{j} - 0.8944\hat{k}$$


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And when I have to do a problem like this, I have to first identify the coordinates of the key points. I have point A and then I have point B, I have

point C that comes from your geometry, then you have to have the coordinate of point D.

Then I have these guy wires are cut and they would essentially transmit tension, we also label them as f_A , f_B and f_C and the first and foremost step is, I need to find out the unit

vector along these forces fine. And for me to write the unit vector its easier if I write the coordinates first and I take if I have to find out the unit vector from D to A . So, final minus initial divided by the distance that gives me the unit vector like this.

Review Before Quiz -1

Tensile Force acting on guy wire A,
 $F_A = 60(-0.4472\hat{j} - 0.8944\hat{k})\text{kN}$
 $= (-26.832\hat{j} - 53.664\hat{k})\text{kN}$

Tensile Force acting on guy wire B,
 $F_B = 60(-0.3873\hat{i} + 0.2236\hat{j} - 0.8944\hat{k})\text{kN}$
 $= (-23.238\hat{i} + 13.416\hat{j} - 53.664\hat{k})\text{kN}$

Tensile Force acting on guy wire C,
 $F_C = 30(0.3873\hat{i} + 0.2236\hat{j} - 0.8944\hat{k})\text{kN}$
 $= (11.619\hat{i} + 6.708\hat{j} - 26.832\hat{k})\text{kN}$

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And similarly, I will have to do for the f_B and f_C . So, there is no big deal I mean you have to do little more mathematics. So, that is the reason why I said that let us not focus on three-dimensional problems, concepts can be better understood when you solve two-dimensional problem and you will also be forced

to visualize, that visualization is very important for engineering fine. So, I get the unit vectors along these directions.

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Review Before Quiz -1

Total guy wire force $= F_A + F_B + F_C$
 $= (-11.619\hat{i} - 6.708\hat{j} - 134.16\hat{k})\text{kN}$

Moment of the guy wire force about the base of the tower O
 $M_O = r \times (F_A + F_B + F_C)$
 where $r = 600\hat{k}$
 $= 600\hat{k} \times (-11.619\hat{i} - 6.708\hat{j} - 134.16\hat{k})$
 $= (4024\hat{i} - 6971\hat{j})\text{kN-m}$

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And then you have a simple mathematics I have to find out what is the tensile force acting on the guy wire, I get this value because it is given in the problem what is a tension in A and I have a unit vector. So, similarly I can find out what is the force

acting on guy wire B, I can also find out the force acting on guy wire C; it is all fairly straightforward you have a little bit of arithmetic involved other than that the procedure is very simple.

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Then I find the net force this is $F_A + F_B + F_C$; I get this quantity like this then I have the moment I simply put it as $r \times F$. You have already learnt $r \times F$ and it is very difficult to visualize in a problem like this and if your r cross F will automatically give you the necessary result that you are looking for. So, I finally, get this as $4024 i - 6971 j$ kN-m.

So, scaling F from two-dimension to three-dimension is fairly simple you have all the fundamental that you have that are required to solve such problems. The only requirement is you have to apply them systematically, you have to be patient in

Review Before Quiz-1

Simplified Use of Equilibrium Equations

- At least two forces are required to keep a body in equilibrium

Collinear and equal and opposite

- A member acted upon by 2 forces: *two force member*.
- How the forces need to be aligned for the member to be in equilibrium?

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identifying the coordinates and find out the unit vectors properly and then do the arithmetic.

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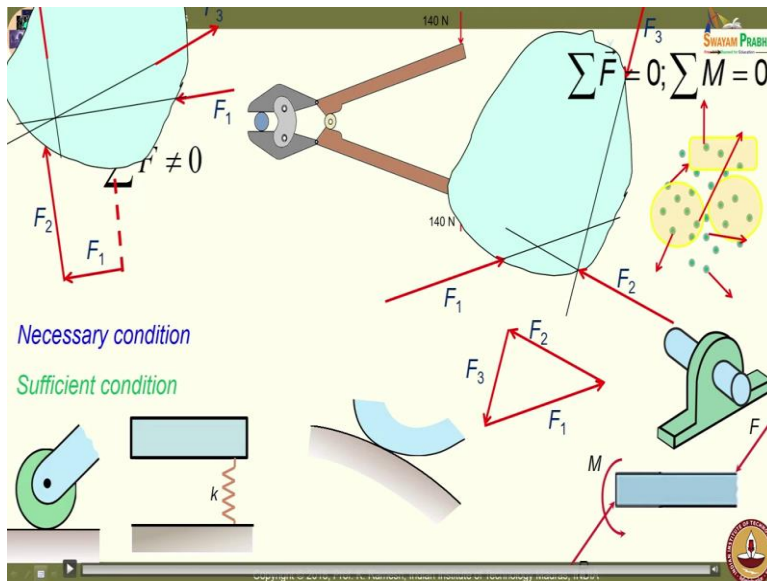
Then we have also looked at in many systems you could identify equilibrium very easily if you recognize that I have a two

force member or a three force member and the question raised here is if you have to keep a body in equilibrium you need at least two forces and how these forces should be? They should be collinear and equal and opposite.

You would use this property when you recognize in an assembly of rigid bodies if there is a two-force member you use this information effectively to reduce the complexity of the problem. We have done the problem of a crimping tool initially it was looking as if it is too difficult problem to solve. One we split it up as links and identified two force members the problem was very simple, two force and three force member assembly is what you come across in many problems, it is a very important clue.

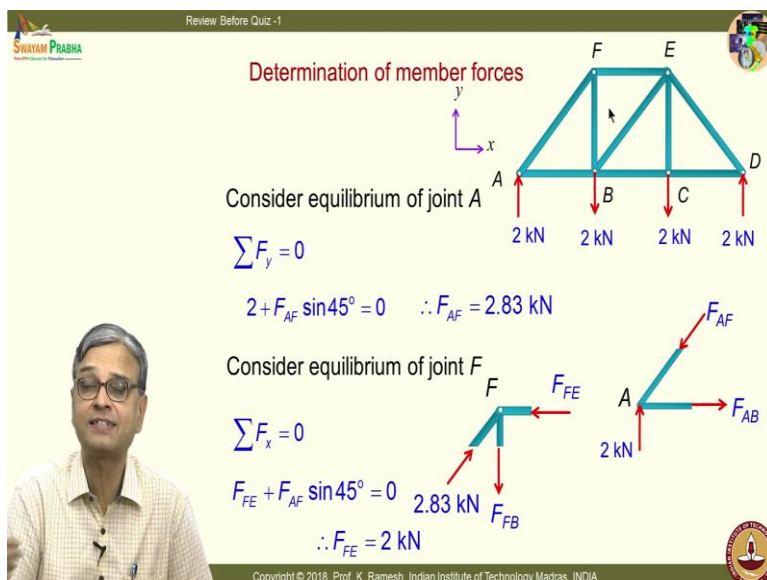
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And we have also looked at what way you will do when you have the three-force member. I have 1 situation where the force is not balanced so, it will have a translation.



In the other case the force polygon is closed, but it has a net moment and the way that you can visualize is the third force has to pass through the point where the other two forces meet.

And we have also discussed if I have to have a necessary and sufficient condition for equilibrium I have to look at for every conceivable subsystem. And we have also looked at different types of supports, I have a roller support, I have this bearing so on and so forth.



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Then the question is yes, when we had discussed the necessary and sufficient condition in that chapter, I took up three example problems. I did solve for the system as a whole and for some sub assemblies, we verified whether they F

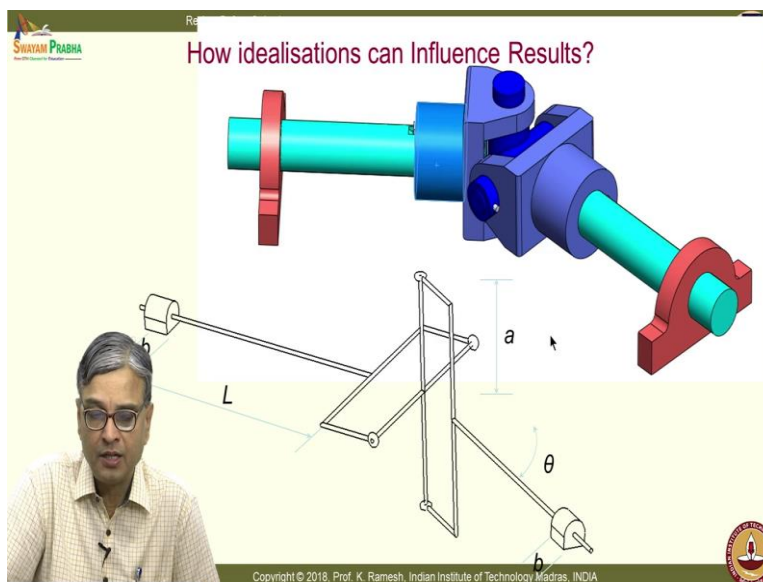
$= 0$, $M = 0$ satisfied. Have is done that for the all other problems you have discussed later, if you go back and look at when we looked at the truss problem by the method of joints.

In fact, we did that, we have taken joint A ; isolated the joint and we have also put the forces to start with as away from the joint, then I had used the arithmetic and found out what is the actual force reverse the direction, then I move to the joint F ; I moved to the joint F and I put these forces properly and my arithmetic gives me what should be the correct direction of F_{FE} and so on.

So, what we did was the isolated the sub-assembly, isolated this sub-assembly at least in the method of joints even without your knowledge you had verify the equilibrium condition for every conceivable subsystem. In the method of sections, when you take a section you should analyze the left free body as well as the right free body which we are

not done, where does error that precipitates when I do not follow this.

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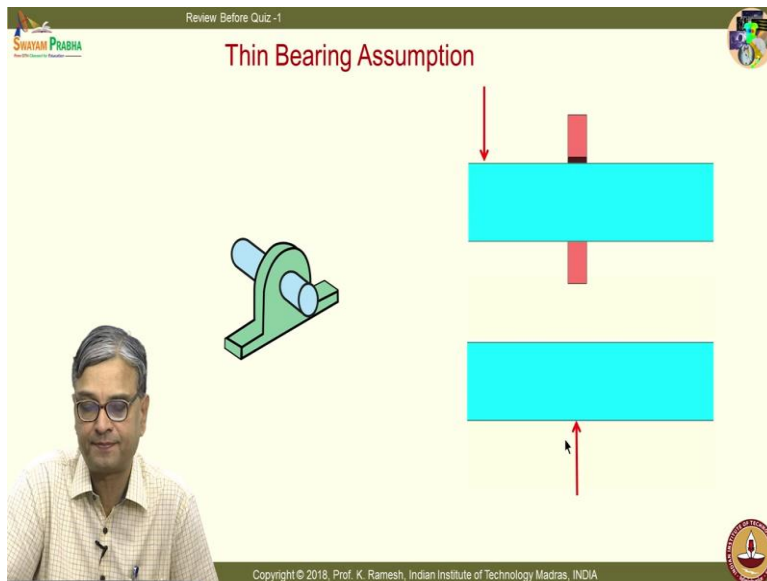
There is one nice example I said in the case of in the Crandall and Dahl in the first chapter he has given a nice problem I wanted you to do the reading assignment and this is the

universal joint if you look at a truck. At the bottom of the truck you will have this from the engine you will have the rear the wheels are driven through this universal joint.

It is very prominently seen in trucks you have a look at it you learn engineering only when you look at systems around you. So, you need to transmit torque from one axis to another axis. A simplified model was taken up in Crandall and Dahl and they have analyzed it.

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And what they did was they have taken the bearings to start with as thin bearings when you have the bearing as thin suppose, I have the load this is also slightly thick I should

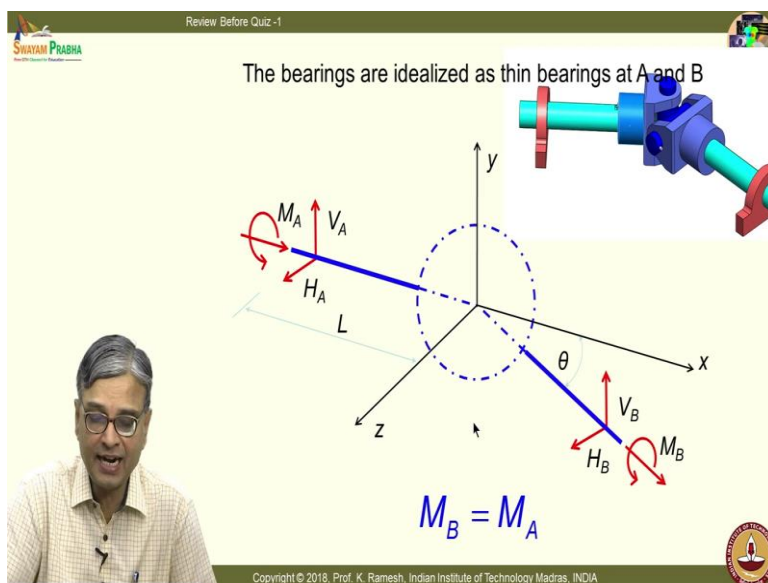


have had just half of it, and when I apply a force it will just come down and I could replace the role of a bearing simply as a single concentrated force.

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So, if you do this kind of an assumption; the problem is initially solved

you have reactions at the bearing a single reaction in this plane, single reaction in the

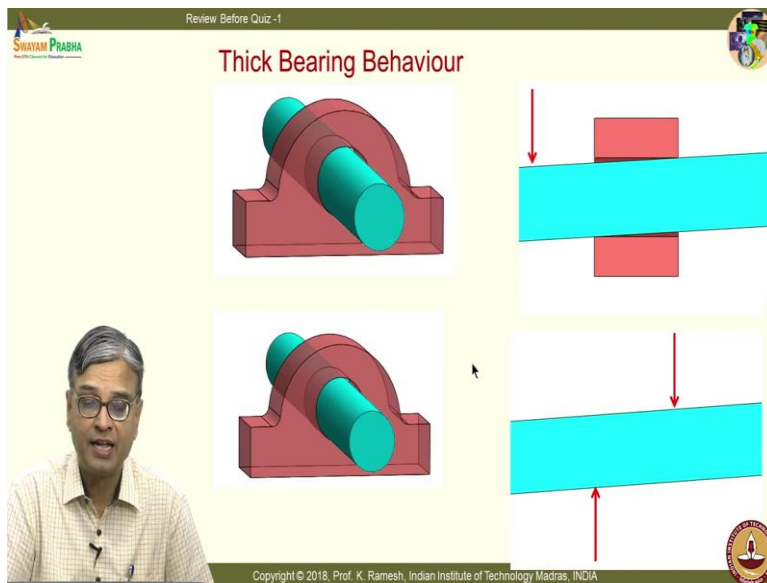


other plane. And they have analyzed the system as a whole and got an expression relating the torque M_A and M_B and the result was, they are equal torque. Then he goes about and dismembers these sub assemblies and he finds equilibrium condition is not satisfied in a sub

assembly. Then he raises a question what has gone wrong in the process? Because when I say the system is under the equilibrium, it should satisfy for the system as a whole as well as all the sub-assemblies.

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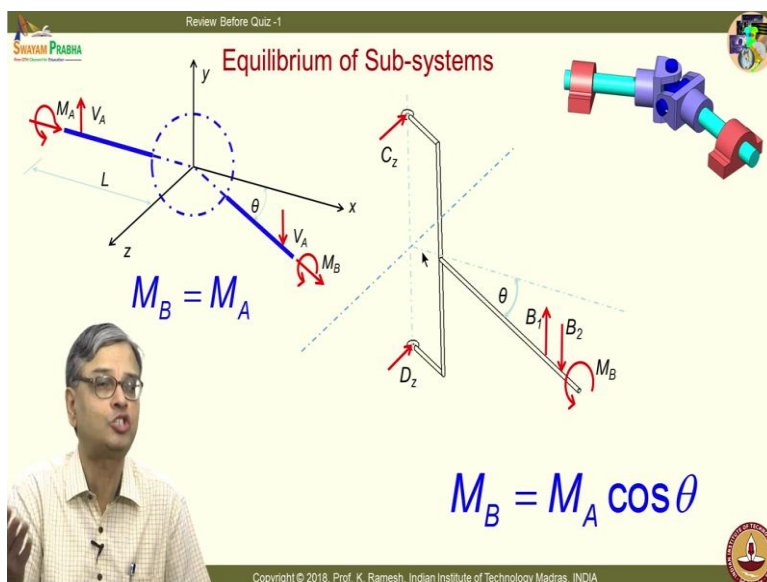
When he does that exercise, he brings in an important factor we have considered the bearings as thin. If it is a thick bearing when I have a force; it will have a situation like



this, it will be inclined and it will make contact at two points; this happens on the vertical plane this happens on the horizontal plane when I show it for two different planes you can visualize it for any other plane.

So, replacing the bearing reaction as single vertical force is not going to solve. And when I have a two force it is also going to transmit a moment. So, the mathematics has helped in improving the idealization.

So, replacing the bearing reaction as single vertical



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So, this is where the mathematics helps and when they do this analysis; the bearing is no longer can be idealized as a thin bearing, it should be idealized only as a thick bearing. And when you translate it in the free body diagram, I would have two

forces acting on the; at the place of the bearing. And when you do this subsystem as well as the system as a whole remain in equilibrium and you get an altogether different answer the torque here is $M_A \cos \theta$ for this particular configuration, for another consideration the expression would change.

So, the focus here is as long as your idealizations are clean and neat and that reflects the physics of the problem, when you find out the condition satisfied for the system as a whole subsystems also will satisfy because the problem dealing with the truss or being or so simple, you idealize the supports these idealizations are reasonably good. Even though we have not specifically insisted on analyzing the sub-assemblies; the answers were correct.

Review Before Quiz-1

Verification of Idealisations

- Compare the predictions with the behavior in the actual system by an experiment
- If satisfactory results are not achieved, reconsider the steps of analysis
- A frequent difficulty is a failure to select an appropriate system or systems to define actions on it by its surroundings
- It may be necessary to alter the assumptions regarding the characteristics of the system
- This can lead to different idealised model of the system

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So, the message here is compare the predictions with a behavior in the actual system by an experiment. It is not that you say that I will idealize like this and solve the problem, what for you are idealizing it? You are trying to understand what is the actual problem that you have how well you have understood and how to verify whether your modeling is correct ultimate goal is it has to satisfy an experimental result fine.

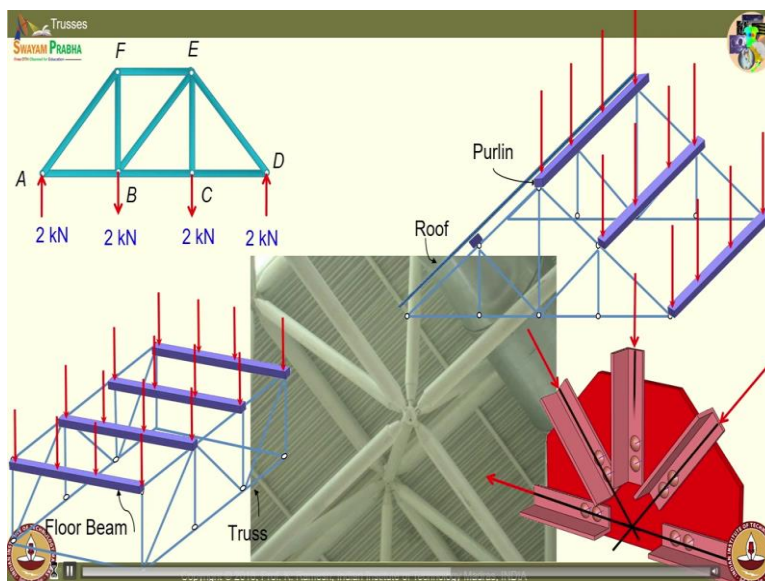
If satisfactory results are not achieved, reconsider the steps of analysis. A frequent difficulty is a failure to select an appropriate system or systems to define actions on it by its surroundings. So, there could be a problem in what you identify as a system this is one type of problem that you may encounter.

It may be necessary to alter the assumptions regarding the characteristics of the system. This can lead to different idealized model of the system; this is what we had seen in the nice example of universal joint. See it is also difficult to coin simple problems to illustrate concepts, you should at least appreciate the team that they coin this problem and then figured out that mathematics ultimately helps. See we have all along said and if you are not dealing with problems involving friction, I can assume the reactions in any direction unknown forces in any direction, my mathematics is will tell me what is the correct direction.

And here you have an example if I verify the necessary and sufficient conditions it also helps you to verify whether the idealizations you have chosen to solve the problem are indeed appropriate fine. And you have to know I said whether some of you have had an experience of river rafting, you see the ship is going in the sea what is the kind of forces that happen on this? Just have a look at it. It is very it is caught in a storm and so, another most difficult aspect in any engineering is how to identify the forces acting on your system.

Do not think that whatever you are studied in this course is great enough for you to feel proud of you have to keep in mind that we are taking baby steps. We are learning idealized systems and idealized force interaction which somebody has given you and whole of engineering lies in how to identify the forces that act on your system for you to analyze. Look at the wave it goes through the complete body of the ship and it should be about 20 to 30 feet high and your system has to survive it cannot collapse. So, have

humility that is very important.



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Then we moved on to discussing about trusses and we emphasized that no member is continuous at a joint, you have a gusset plate whether it is a two-dimensional problem or a three-dimensional truss. I

have idealized for a two-dimensional case as a pin joint, for a three-dimensional case it is idealized as a ball and socket joint here.

And we have also seen that loads are applied only at the joints that is a very important aspect of it. And we also learned two methods one is a method of joints; I take out a section like this and then I analyze this and I can also take out a section like this and then you call it as a method of sections. In the case of method of joints, it is a concurrent force

system so, I can write only two independent equations. In the case of method of sections,

I can have non-concurrent force system and I can do three equations.

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And we have also discussed how to associate the member force as tension or compression. And we discussed at length when I have a pin at a highly magnified

Association of Tension or Compression

Using principle of transmissibility move the force to the other side

Force away from the pin is indicating tension in the member.

Force towards the pin is indicating compression in the member.

Assume that the member is in Tension

Actual force interaction is below the pin.

Convention is to indicate the force at the pin on the same side of the member.

Same is true if the member is cut and force is assumed to be outwards.

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perspective, we will have a clearance and if you assume that the member is under tension the force interaction has to happen here and here. So, that this member is under tension.

But if you analyze the pin you would naturally put the force like this and I said

engineering is one profession where you use convention. If you use this kind of an approach then I should put this force interaction only on this direction. Indicate the force at the pin on the same side of the member then you can interpret if I have a force away from the pin

If the unknown member forces except one have the same line of action, the force in the remaining member can be easily determined by appropriately choosing the co-ordinate axis while analysing the joint.

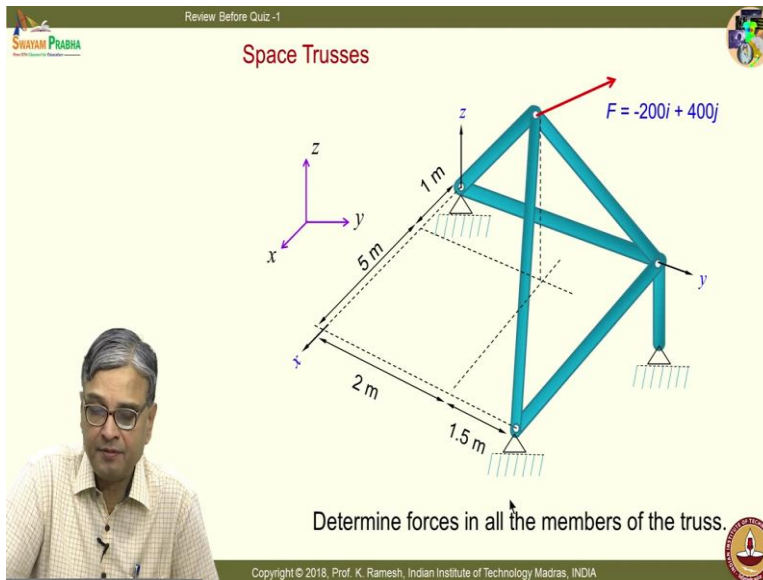
Force F_{AD} can be found by using

$$\sum F_y = 0$$

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you call it as a tension. And if it is towards the pin you call it as compression ok; force away from the pin is indicating tension, force towards the member is indicating compression. All that circus is much simpler; if you follow our discussion of isolating a joint along with the member.

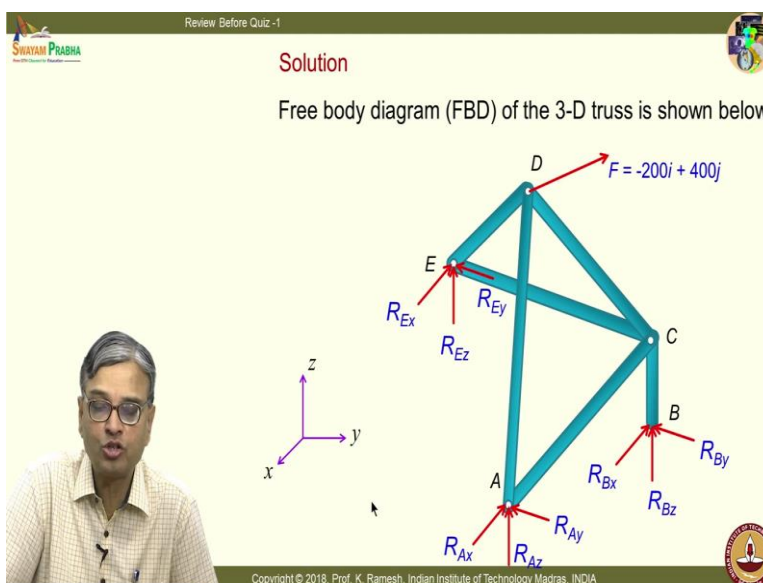
It is easy to visualize when you put the force away from the member, it is easy to perceive that it is tension and I have also discussed our mathematics is such when I get



negative it automatically becomes a compressive force, you have taken advantage of the choice of unknown force direction to start with.

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And you know in the case of method of sections we had taken a section with four unknowns, but we could still solve it, I can also do the same trick if I have method of joints. Suppose I have unknowns F_{AC} , F_{AD} and F_{AB} ; in this case, F_{AC} and F_{AB} are along the same line of action you choose a coordinate system along this line of action. Even though I have three unknowns at the joint I can find out the third unknown, if my interest is only on F_{AD} .



See the basic requirement is if I use method of joints; find out the joint which has only two unknowns. If I use method of sections cut a section which has three unknowns, we have seen exceptional cases in the case of method of sections. Method of joint also I can do the trick by identifying a

suitable coordinate system.

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So, it is better that you learn many methods that you have and I said that we have focused mainly on two-dimensional trusses. Suppose I go for a space truss like this what

Review Before Quiz -1

Checking for Determinacy

- Number of joints, $j = 5$
- Number of members, $m = 6$
- Reactions, $r = 9$

Since
 For a 3D space truss problem
 $m + r = 3j$

It is a statically determinate problem.

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is the difference it is going to be? It is all supported by ball and socket joint and you will have to go back to your vector algebra and solve the problem.

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Without vector algebra it is difficult to handle the problem like this and you have three reactions at each

one of these supports and they are handled in a similar fashion.

(Refer Slide Time: 31:00)

Review Before Quiz -1

D is at a height of 2 m. Origin at E.

Unit vectors of truss members are

$$\hat{f}_{EC} = 0\hat{i} + 1\hat{j} + 0\hat{k}$$

$$\hat{f}_{ED} = \frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$$

$$\hat{f}_{AC} = -1\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\hat{f}_{AD} = \frac{-2}{\sqrt{5}}\hat{i} - \frac{0.6}{\sqrt{5}}\hat{j} + \frac{0.8}{\sqrt{5}}\hat{k}$$

$$\hat{f}_{CD} = \frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}$$

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And we can also look at whether the problem is statically determinate or not. The main difference is instead of $m+r = 2j$, I would replace it by $3j$ because at each support when I have a ball and socket joint, I can write three independent equations that is the only

difference rest of it becomes vector algebra for you to do it and you lose out on visualizing what happens physically.

And here again I can use a; this is a statically determinate problem because I have number of joints as 5, number of members as 6 and reaction as 9. So, I can solve this problem with $m + r = 3j$.

Review Before Quiz -1

SHAYAN PRABHA

Considering joint D is in equilibrium

$$\vec{F} + F_{DA}\hat{f}_{DA} + F_{DE}\hat{f}_{DE} + F_{DC}\hat{f}_{DC} = 0$$

Equating \hat{i}, \hat{j} and \hat{k} terms

Member	Magnitude of Force(kN)	Nature
DE	343	T
DC	397	C
DA	186	T

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(Refer Slide Time: 31:51)

And when I do this, I have to find out the unit vectors after I find out the unit vectors isolate a joint; isolate a joint like this

(Refer Slide Time: 32:03)

And then again you do the forces like this and since I have vector as i, j, k. I can solve for three unknowns so, equating the terms i, j and k here

again you can find out the member forces associate whether it is tension or compression so on and so forth. So, scaling up from two-dimension and to three-dimension is not at all difficult that is what I am trying to say, it is only little bit involvement in mathematics. But while

Beams

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learning the subject; learn it thoroughly for two-dimensional system, you can easily graduate to three-dimensional system.

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I have all these forces where the directions also modified and so on. Then we moved on to discussing beams, we have understood any member that supports transverse load we

Review Before Quiz-1

SHAYAM PRABHA

Sign convention in Meriam

The diagram shows a beam of length $a+b$ with a coordinate system (x, y) starting from the left end. A point x is marked at distance a from the left end. The beam is divided into two parts: (1) the left part of length x , and (2) the right part of length $a-x$. External forces include A_y at the left end, B_y at the right end, and a point load P at distance a from the left end. Internal forces at the cut at x are shown as M (bending moment) and V (shear force).

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call it as a beam and we discussed that I should have the loading diagram, shear force and bending moment diagram one below the other.

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And in this chapter also we have discussed about sign convention. It is a discipline because depending on the problem complexity you may want analyze the left side of the beam or the right side of the beam and I should follow one single convention so that whatever the result I get from analyzing the left portion or the right portion, it is unique in magnitude as well as sign for me to plot shear force and bending on diagram; magnitude may be same, but the sign may become different. So, it is more of a discipline

Review Before Quiz-1

SHAYAM PRABHA

Sign Convention

The diagram illustrates the sign convention for internal forces. For a **Positive Surface** (cut on the left), a positive moment M is shown as counter-clockwise and positive shear V is shown as downwards. For a **Negative Surface** (cut on the right), a positive moment M is shown as clockwise and positive shear V is shown as upwards.

- On a positive face positive moment is positive and negative shear is positive
- On a negative face negative moment is positive and positive shear is positive!

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ok.

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And what is the sign convention? We have to identify this as a positive surface and then we have to identify this as a negative surface. Since we have adopted Merriam, we will use that sign convention on a positive face positive moment is positive and negative shear is positive; this is what you have in the case of Merriam, I take the moment anti clockwise and shear force downwards.

And what is the sign convention? We have to identify this as a positive surface and then we have to identify this as a negative surface. Since we have adopted Merriam, we will use that sign convention on

This is to draw the isolated section. Once I draw the isolated section, all my signs I have to refer it with respect to the reference axis ok. You should treat this force as negative when you do the force balance, you should not miss that. On a negative face negative

moment is positive and positive shear is positive.

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For $0 < x < a$

$$\sum F_y = 0 \quad V = 0 \text{ N}$$

$$\sum M = 0 \quad M = 0 \text{ Nm}$$

For $a < x < a + L$

$$\sum F_y = 0 \quad V = P \text{ N}$$

$$\sum M = 0 \quad M = P(a - x - b) \text{ Nm}$$

And we have also looked at the concept of you know what I have said is you take a section and then do it, I expect you to do this in your examination. So, find out suitable planning of

your answer script. So, that you do all this calculation separately and reserve a space for you to draw the bending moment and shear force diagram one below the other; we have these expressions you have seen it earlier.

(Refer Slide Time: 35:24)

Principle of Superposition

SFD

$0 < x < a \quad V = 0 \text{ N}$

$a < x < a + L \quad V = P \text{ N}$

BMD

$M = P(a - x - b) \text{ Nm}$

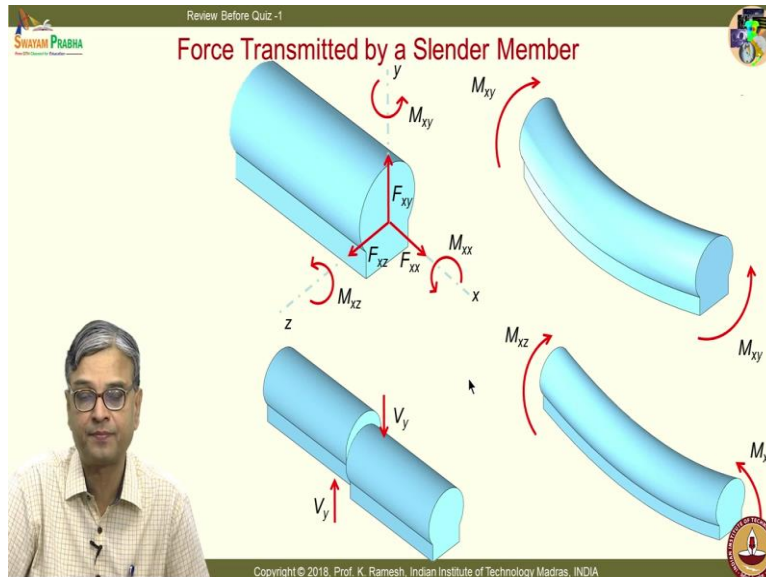
$M = 0 \text{ Nm}$

$-P(b + L)$

I have drawn the bending moment and shear force diagram one below the other, I have these expressions. And in this context, I also discuss all systems in practice are in some way non-linear, we

idealize that as linear system why do we do it? We do this primarily because I can exploit the principle called principle of superposition.

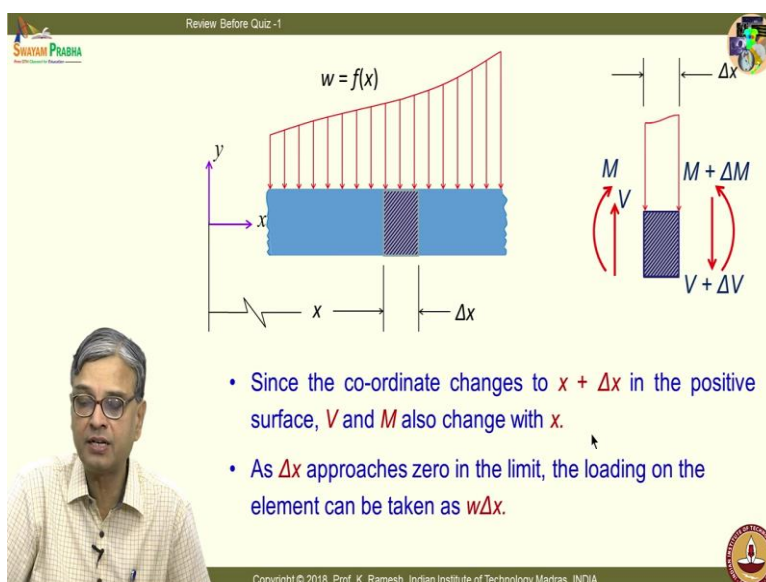
I can rethink that this problem is consisting of two different loads; load P and load P_b ; I can get the shear force and bending on diagram for the load P . I can get the shear force and bending on diagram for the moment, I can simply add this I would get this answer. This problem is very simple I just brought it for illustration and this is a very powerful principle that will be used in your later courses.



(Refer Slide Time: 36:24)

Not only this when I have a slender member it can in principle transmit three forces and three moments and we have seen bending moment in the plane, xy we have looked at the bending moment as M_{xz} . Suppose,

you have a member which is transmitting force in one direction also transmitting force in a direction perpendicular to that. You can apply it in specific position draw the bending



- Since the co-ordinate changes to $x + \Delta x$ in the positive surface, V and M also change with x .
- As Δx approaches zero in the limit, the loading on the element can be taken as $w\Delta x$.

moment diagram for this loading draw the bending moment for the other loading.

So, principle of superposition can be exploited for this also even though we have not solved any problem you can also handle such situations.

(Refer Slide Time: 37:04)

Then we looked at when I have a variable load acting on the beam, we develop the interrelationships. We realized that we have made a deliberate cut at a section of Δx , it

has a length of Δx so, I have to recognize that there will be variation shear as well as variation bending moment.

Review Before Quiz-1

Distributed load on the element has been replaced by a resultant force $w(x)\Delta x$ that acts through the centroid.

$$\sum F_y = 0 \quad \Delta V = -w(x)\Delta x$$

Dividing by Δx and taking the limit $\Delta x \rightarrow 0$

$$\frac{dV}{dx} = -w(x)$$

Summing moments about the right side of the element gives

$$\sum M = 0 \quad \Delta M = V\Delta x - \frac{w(\Delta x)^2}{2}$$

Dividing by Δx and taking the limit $\Delta x \rightarrow 0$

$$\frac{dM}{dx} = V$$

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And when I use this, we have got the final interrelationships that relates, what is dV/dx and what is dM/dx and we have also discussed how you can exploit these expressions to correctly draw your shear force and bending on diagram ok.

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And I have asked all of you to solve this problem; it is no big deal to make your problem

Review Before Quiz-1

Draw the SFD and BMD for the simply supported beam with the loads shown in Fig

Reaction forces on the beam

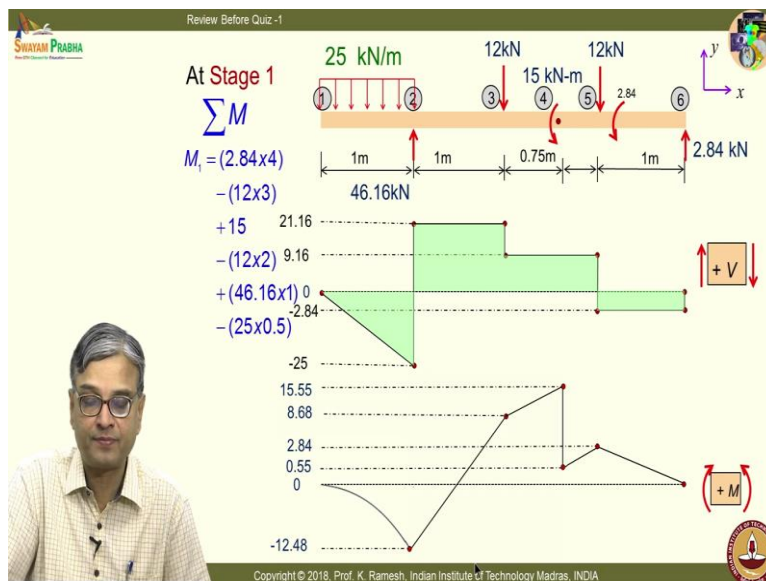
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involved in calculation by putting several forces. We have solved series of simple problems; it is no difficulty at all to handle a problem like this and I think I have also given you the reactions and the supports so that there is no difficulty in comparison of your answers.

For me to find out the reaction I can replace this distributed force as a concentrated force and all of you have determined the shear force and bending moment diagram by taking appropriate sections; draw the free body.

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Now, I am going to discuss a method to verify it, use this for verification for all your problems in quizzes and exams you have to take a section, show the free body write $F = 0$, $M = 0$ find out the unknown forces. But once I have obtained all these how do I verify it?



See for me to plot the shear force I move from left to right with the Merriam convention when I start, I have this W_{xx} is 0 so, I have this as 0. I start at 0 and this is uniformly distributed so, from section 1 to section 2; it has to be a linearly varying shear force, and I find out what is

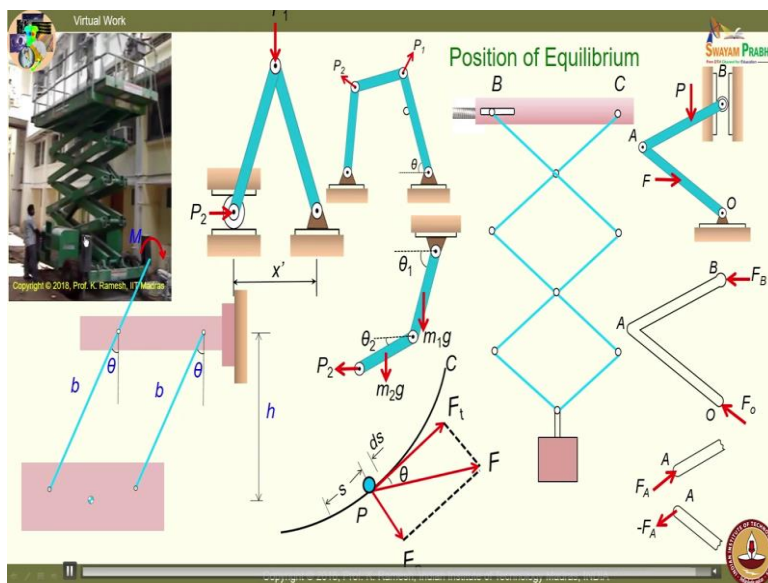
the shear force at section 2, I mark this point and I would join these two by a straight line.

Then at section 2, I have a shear force that is the support I have; I can find out what happens at section 3 the value I can mark them like this, I should not join this to this point. I should recognize at this point I have a shear force, I would join a vertical line like this then, it remains constant till 3; fine. So, if you do like this I will come down when I have a force like this, I will come down that is how the calculations will be ok.

And then I will it will remain constant the moment has no role to play in the shear force then I will come down by 12 kN then I will go up by 2.84 kN. So, the plot will start from the left to right you can trace the forces. So, the key points are wherever you have external force, they serve as key points and you should also decide how many sections you need to take to solve this problem. And let us go and draw the bending moment diagram.

For bending moment diagram move from right to left, simply find out whether it is clockwise or anti clockwise moment and then keep plotting it, that is all you have to do. For the first one I have translated the force and indicated that I need to get a moment like this and I have got this value here and many students have asked me the question how do I handle a concentrated moment acting on the beam? Very similar to how you handle the force acting when you plotted this shear force.

So, when I go here, I should recognize that there is a step increase in the bending moment fine. I should recognize that there is a step increase at this point and if you



recognize that, then it is very simple to do. And wherever you have a load, you have a change in the moment diagram they are all key points when a shear crosses 0; you have extremum values and you can figure out. Because I have the slope is 0 here, I can figure out this or even otherwise I can draw a

straight line and find out what is the average, if my equation gives more or less, I can find out what should be the curvature of this ok.

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Then we moved on to position of equilibrium; there we said that we need to focus on active force diagram, we also discussed concept of degree of freedom. I have a single degree freedom system; I have a multiple degree freedom system and I have two links and in all these cases you need to plot the displaced position and you have a nice animation of a lift like this which can be deployed quickly.

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And let us solve one very interesting problem by force method as well as virtual work ok. And this kind of a truss you will come across normally in many industries. I have a handle like this and somebody pulls this down then it presses this object here and it is a

Review Before Quiz-1

SHAYAM PRABHA

4. A force of 200 N is applied on the knob as shown in figure. If $\theta = 75^\circ$, using the principle of virtual work and force method, determine the reaction at A.

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pin joint here, it is also a pin joint at D, it is also a pin joint at C.

Suppose I have to find out what is the reaction at A by the force method I have to dismember all of these members, then only I can find out the force. We would solve by both the methods and you will find

virtual work two, three steps you get the final answer. Whereas, a force method requires computation and also in the process you learn how to find out reaction by the method of virtual work, you have to replace it by an active force which we have learnt by looking at

stability, we have replaced a spring by the restoring force.

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Review Before Quiz-1

SHAYAM PRABHA

• Let R be the reaction force at A

$2 \times 120 \sin \theta$

Idealisation

Joints C, D and E are pin joints.
 "A" acts like a roller.
 Part B Moves smoothly over the column

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And when you are solving with a force method, I expect all of you to write you know all the problems idealizations. So, what are the idealizations you have joints C. D and E are pin

joints, A acts like a roller, part B moves smoothly over the column we are avoiding friction that makes our life simple. And you have the geometry which is shown as a line

sketch which is useful for method of virtual work and I can also visualize when I pull this; what would happen to the link.

So, in the case of method of virtual work I have to draw the active force diagram which is mentioned here, I have to find out the reaction here so, I just put the reaction by an active force and when I pull this will also move if I do not consider that this moves I will not be in a position to solve the problem.

Review Before Quiz-1

SWAYAM PRABHA

Force Method

• First the FBDs of each part is drawn

• GED is one member

$R_{Dy} \tan 15^\circ = R_{Dx}$

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So, first let us solve this as a by force method the problem looks complicated, it is not complicated at all if you recognize a few important things. So, you have the member DEG is one unit ok, it is a one physical

member and when I have a pin joint, I can replace the interaction at the pin joint as two forces we do not know the direction.

And I have assumed a direction like this on this, this is connected to the linked DC . So, I have to put the link DC and then put these forces opposite to this by Newton's third law. And you should recognize that link DZ is a two-force member, the moment you recognize this as a two-force member the force has to be along its line of action ok, that makes your life fairly simple.

So, once you determine the force R_D ; I can put this R_D here for me to write my interest is to find out in this what is the reaction here. So, if you look at here, I need to find out the vertical component of D ; force at D would be identical to this. So, I would replace this; these are all simple mathematical simplification I have R_{Dy} as $R_{Dy} \tan 15^\circ = R_{Dx}$ from the given problem geometry.

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Review Before Quiz -1

SHAYAM PRABHA

$\sum M_E = 0$

• Now finding the moment about E

$$-200(300) + R_{Dy} \times 120 \sin 15^\circ + R_{Dy} \tan 15^\circ \times 120 \cos 15^\circ = 0$$

$R_{Dx} = 259 \text{ N}$ $R_{Dy} = 966 \text{ N}$

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Then I go to the next step, I put only essential things here take moment about E, I get the value like this, this gives me $R_{DY} = 966 \text{ N}$, I get R_{DX} as 259 N. So, I have dismembered this; determine the forces here then I have to find out the force this force I can easily

find out because I have R_{DY} . Then I have to identify what happens the interaction of C with respect to this bottom block that is what I have listed it here.

Review Before Quiz -1

SHAYAM PRABHA

• In the bottom member

Reaction from pole

R_{Cy} , R_{Cx} , R

$$R_{Cy} = R_{Dy} = R$$

$$R = 966 \text{ N}$$

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So, you need to have reference axis, this is again a pin joint can be replaced by two forces and you will have a reaction from the pole that is put as the normal force. Because I have a smooth movement upward and downward

there is no friction, and this is a roller with a flat edge and this is also smooth so, I have a reaction R .

So, this clearly shows I have $R = R_{CY} = R_{DY}$. So, the question asked is what is the reaction force, we have got the reaction as 966 N; circuitous process, I have to dismember and do it. This is what is asked, but we will also see what are the forces acting on this member?

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Review Before Quiz-1

SHIVANI PRABHA

$R_{Ex} = 311 \text{ N}$

$R_{Ey} = -773 \text{ N}$

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Can you recognize what is this member, how many forces acting on this member? You should recognize that this is nothing but a three-force member. See many mechanical appliances you can see them as assembly of two force and three force members I have a force acting on this which is what the operator is pulling it down, then you have the interaction at D , you can easily say what should be interaction at C . I have these forces meet at a point and it automatically says the force acting at E has to follow this line of action.

Review Before Quiz-1

SHIVANI PRABHA

Method of Virtual Work

- From the principle of virtual work,

$$\delta W = 0$$

- Include reaction R as an active force
- This gives,

$$(200)(300\delta\theta) - R\delta(240\sin\theta + c) = 0$$

$$(200)(300\delta\theta) - R(240\cos\theta\delta\theta) = 0$$

- Hence,

$$R = \frac{250}{\cos\theta} = 966 \text{ N}$$

How to find the reaction?

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So, this also gives you the force R_{Ex} as 311 N and $R_{Ey} = -773$ Newton's. This is just to appreciate seemingly practical problem can be thought of as assembly of two force and three force members.

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Now we move on to method of virtual work and simply solve it in no time ok. You see the power of virtual work; I have $\delta W = 0$, I have a geometry and we have to identify the reaction.

So, make that as an active force and you have to look at what is the work component of this. Here the force and displacement on opposite direction so, I have negative, here force and displacement on the same direction so, I put this as positive because this is the

sign convention we discussed, do not individually assign a sign for the force or the displacement, look at the work component ok; look at the product and then assign the sign and this gives me in one step I get $R = 966 \text{ N}$.

So, this brings out the elegance of method of virtual work nevertheless, you will also have to know how to solve it by the force method, I thought it is a nice problem where I can review because it is a class on review; I could bring in how we have solved it by the force method, how we can solve the same problem by method of virtual work. I suppose we had a fairly good discussion on all the key concepts including the sign convention used in trusses, beams and virtual work.

Thank you.