

**Engineering Mechanics**  
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**Module - 01**  
**Statics**  
**Lecture - 19**  
**Friction - II**

1) A block of size  $2a \times 2a$  and weight  $200\text{ N}$  is resting on a floor. Initially a force  $F_1$  is applied. Force  $F_2$  is applied at a height of  $1.5a$  from the floor. The force is gradually increased from zero to  $50\text{ N}$ ,  $90\text{ N}$  and  $100\text{ N}$ . Draw the FBD showing clearly the point of application of various forces and also state the condition of the block, if the coefficient of static friction between the block and the floor is  $0.3$  and the dynamic coefficient of friction is  $0.25$ .

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Let us continue our discussion on friction.

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And we will try to solve this problem. You have a block of  $2a \times 2a$  subjected to a force  $F_1$  and it is also subjected to a force  $F_2$ , whose magnitude is changed from zero to  $50$ ,

$90$  and  $100$  Newtons in steps. You are also given the coefficient of static friction between the block and the floor is  $0.3$  and dynamic coefficient of friction is  $0.25$ .

Equilibrium equations give

$$\sum F_x = 0; F_2 = f$$

$$\sum F_y = 0; N = 300\text{ N}$$

$$\sum M_c = 0;$$

$$-(N \times x) + (100 \times \frac{a}{2}) - (1.5 \times a \times F_2) = 0$$

These equations show that magnitude of  $N$  is independent of  $F_2$  but its location  $x$  is a function of  $F_2$ .

$$x = \frac{a}{300} \times (50 - 1.5 \times F_2)$$

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The idea is for different values of  $F_2$ , you have to get the position of the reaction on the block and also at what distance it will act.

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See this is the problem dealing with friction; so, you have to clearly

visualize, which way the frictional forces would develop. The problem statement is

simple and straightforward. I have force  $F_2$ , which is trying to push it to the right. So, I would naturally have the frictional force developed like this and I have indicated the frictional force simply as small  $f$ , you know this is a good practice, do not replace the frictional force as  $\mu$  times the normal reaction; never do that; although in very important problems this might be true. In general, one can coin a problem, where this as not reach the maximum frictional force.

Only when the motion is impending, you are allowed to write that. So, that is always a good practice to write this as  $f$  and the frictional force directions quite alright. I have this opposing the force that is the applied; so, this direction is all right and one of the common ways students solve the problem is, they put the normal reaction through the centre of gravity of the block. This is one of the common mistakes, please do not do that; the location of the normal force is dictated by the moment equilibrium.

And if you start the problem by putting it through the centroid, it clouds your thinking. So, it is always a good practice to put the normal reaction, at a distance slightly away, whose distance you have to determine as part of your mathematics. This is always the good practice to do that and, in all problems, dealing with friction, if you do not write the frictional force direction correctly, you are solving all together a different problem. So, spend a minute on observing, what way the frictional forces would develop? This is very very important do not rush through because if you rush through; then, you will be solving all together a different problem.

And we have the reference axis and we can write the equilibrium equations,  $\sum F_x = 0$ ,  $\sum F_y = 0$ , when I apply this  $\sum F_x = 0$ , I get the expression the force  $F_2 = f$  and I get the normal reaction. I have the weight of the block as 200 N and external force, which is 100 N; so, I get the normal reaction as 300 N and if I take moment about the point  $C$ , which is taken here. I can very well see that; this force will give a clockwise moment; this also will give you a clockwise moment and these two forces will give anticlockwise moment.

So, I get an expression  $-(N \times x) + (100 \times \frac{a}{2}) - (1.5 \times a \times F_2) = 0$ . This force since passes it through  $C$ ; it does not contribute to the moment all the forces are accommodated. On simplification, I get an expression for the distance  $x$ . So, in problems dealing with friction, do not put the

normal reaction passing through the centroid of the blocks; in certain problems it may be true, but in general do not put it passing through the centroid.

Determine the distance from the basis of your moment equilibrium; so, I get an expression

Case (1):  $x = \frac{a}{300} \times (50 - 1.5 \times F_2)$   $f = F_2$

$F_2 = 0 \text{ N}$   $x = 0.17a$

No Frictional forces are developed!

$F_1 = 100 \text{ N}$

$200 \text{ N}$

$N = 300 \text{ N}$

$x = 0.17a$

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expression

$$x = \frac{a}{300} \times (50 - 1.5 \times F_2)$$

and in the way, I have taken  $x$  is, I have implicitly taken this as a positive  $x$  and if I get negative  $x$ ; I will say that the distance will be put on the other side.

Now, let us investigate; when I have  $F_2$  is 0, what

happens?

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When I have the situation  $F_2 = 0$ , I do not have this force and you have to recognize, no

Observation of Various Cases

$x = \frac{a}{300} \times (50 - 1.5 \times F_2)$

$f = F_2$

$F_2 = 0 \text{ N}$	$x = 0.17a$
$F_2 = 50 \text{ N}$	$x = -0.08a$
$F_2 = 90 \text{ N}$	$x = -0.28a$
$F_2 = 100 \text{ N}$	$x = -0.33a$

$F_1 = 100 \text{ N}$

$200 \text{ N}$

$f = 75 \text{ N}$

$N = 300 \text{ N}$

$x = 0.33a$

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frictional forces are developed. Frictional forces develop only, if there is a tendency for relative motion; if there is no tendency for relative motion, no frictional forces are developed and you also have the expression, the frictional force equal to  $F_2$ .

Suppose I have and I also have, when I have this expression for  $x$ , I can find out what is the distance  $x$  that turns out to be  $0.17a$  and your normal reaction will act only, at this point. It will not pass through

the centroid and this would balance the moment equilibrium of the block; so, it is always determined from the moment equilibrium equation.

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Suppose I have  $F_2$  as 50 N, when I have this expression on  $x$ , that gives me  $x = -0.08a$  and from the way we have labeled  $x$ ; I put the  $x$  on the right side. The diagrams are not to scale; they are only illustrative. So, it is at a distance of  $0.08a$  and I have the frictional force is equal to the applied force. It has not reached the maximum value; you have to recognize that; it reaches a maximum value. Then, the block begins to slide, if the force is increased further.

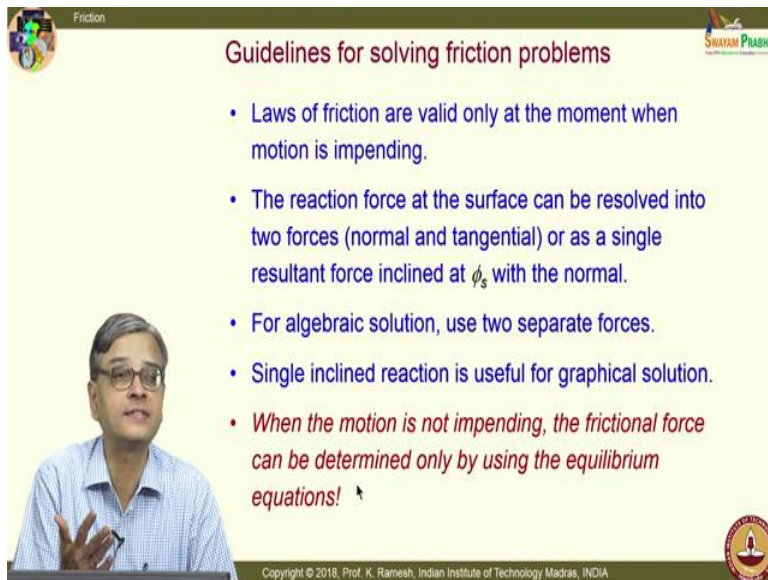
Now, let us look at the case,  $F_2 = 90$  N; so, when I put it in this expression, I get the value of  $x = -0.28a$  and in the diagram, it is put on the right side and I have this as 90 N. What does the problem statement as given? It has given the coefficient of static friction as 0.3 and I have the normal reaction as 300. So, the maximum frictional force, it can reach is only 90 N. So, this is the situation where, the block is in a state of impending motion.

If the forces are slightly increased; then, the block will begin to slide and you make a neat sketch of all this and as I told you earlier, if you select key problems in any chapter, you understand all concepts related to that very systematically. The problem statement is definitely very simple, but illustrates important aspects, what you have to follow when you have a problem dealing with friction?

We have also been asked to find out, what happens, when  $F_2$  is 100 N? and we find that  $x$  is  $0.33a$  from this expression and you also find that, the reaction developed because you are given a kinetic coefficient of friction that is given as 0.25. It can develop a maximum force of only 75 N in this case; that is dictated by the dynamic coefficient of friction; even though my applied force is greater than this, you would not have frictional forces to balance it.

Suppose I make the screen as smart, what would happen? The block would begin to slide to your right; it will no longer remain in equilibrium. So, this problem clearly brings out frictional force is developed only when, there is a tendency for relative motion, initially

depending on whatever the force is applied, the frictional force balances it and reaches the maximum and the maximum frictional force at the point of impending motion is  $\mu N$  not otherwise; unless you satisfy yourself that, the motion is impending. You cannot blindly put the value as  $\mu N$ ; if you do that, then you are solving all together a different problem ok.



**Guidelines for solving friction problems**

- Laws of friction are valid only at the moment when motion is impending.
- The reaction force at the surface can be resolved into two forces (normal and tangential) or as a single resultant force inclined at  $\phi_s$  with the normal.
- For algebraic solution, use two separate forces.
- Single inclined reaction is useful for graphical solution.
- *When the motion is not impending, the frictional force can be determined only by using the equilibrium equations!*

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So, what are the guidelines for solving friction problems? As I mentioned you earlier, laws of friction are valid only at the moment, when motion is impending, that needs to be investigated. The reaction force at the surface can be resolved into two forces

normal and tangential or as a single resultant force inclined at the friction angle  $\phi_s$  with the normal and whenever, we use the algebraic solution handling them as normal and tangential forces is easier to simplify.

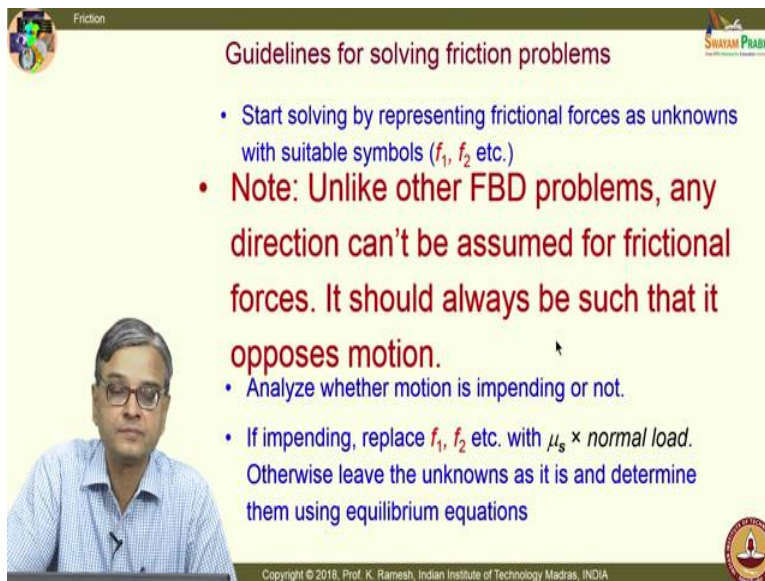
On the other hand, if you use a graphical solution. See, in fact, in early part of development of engineering; they were not having calculated; they were having only slide rules and they were depending more and more on graphical solution, graphical solution is very important from visualization point of view and engineers have use graphical solution earlier.

And if you use graphical solution, a single inclined reaction is useful and it is cautioned, when the motion is not impending, the frictional force can be determined only by using the equilibrium equations. You cannot replace it as  $\mu N$ , unless the motion is impending.

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So, the advice is you start solving by representing frictional force as unknowns. See, in the previous problem we had only one surface where, there was frictional interaction. If I



**Guidelines for solving friction problems**

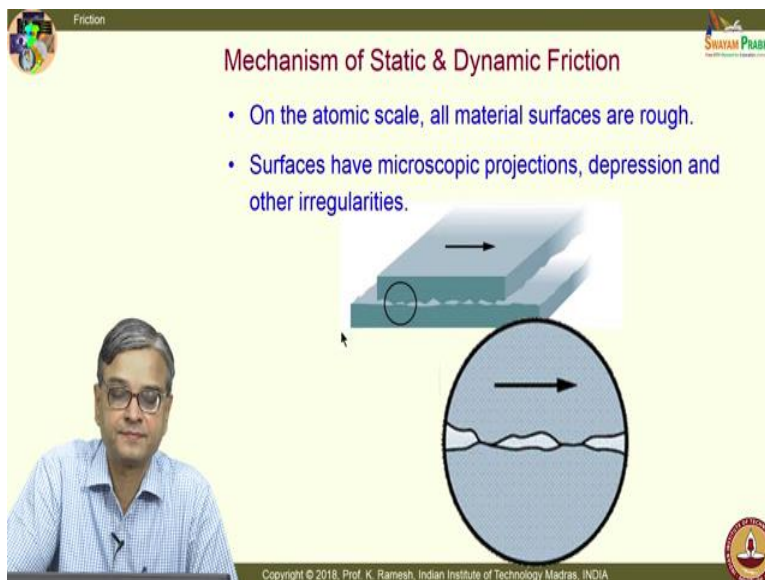
- Start solving by representing frictional forces as unknowns with suitable symbols ( $f_1, f_2$  etc.)
- **Note: Unlike other FBD problems, any direction can't be assumed for frictional forces. It should always be such that it opposes motion.**
- Analyze whether motion is impending or not.
- If impending, replace  $f_1, f_2$  etc. with  $\mu_s \times \text{normal load}$ . Otherwise leave the unknowns as it is and determine them using equilibrium equations

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have multiple surfaces, it is always desirable that, you indicate the frictional forces  $f_1, f_2$  etc. Do not jump to  $\mu_1 N_1$  or  $\mu_2 N_2$ ; do not do that. You investigate; you make a statement specifically in your problem analysis, whether the motion is impending. This is a good

practice to handle problems dealing with friction.

So, this is again caution, I have told you many times. Unlike other free body diagrams problems, any direction cannot be assumed for frictional forces. It should always be such that it opposes motion. It is very important. Then, you are solving all together a different



**Mechanism of Static & Dynamic Friction**

- On the atomic scale, all material surfaces are rough.
- Surfaces have microscopic projections, depression and other irregularities.

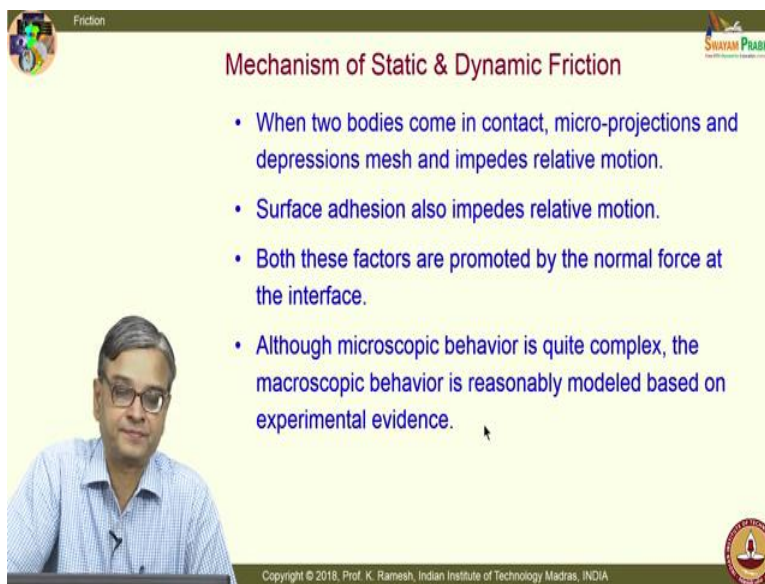
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problem. All your effort in doing the simplification will go to a waste, if you rush through in putting the frictional forces and it is again advice analyze, whether motion is impending or not. If the motion is impending, replace  $f_1, f_2$ , whichever is appropriate. All maybe appropriate or some maybe

appropriate. In such a case, replace them with  $\mu$  times the normal load; otherwise leave the unknowns as it is and determine them using equilibrium equations. So, that is a good practice for you to follow.

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And we will also spend a short while, just to understand the mechanism of static and dynamic friction. See, on the atomic scale, all material surfaces are rough; which is shown in a magnified fashion here. So, they have microscopic projections, depressions and other irregularities. This is just a magnified version of what you see here; so, I have projections and depressions.



Friction

### Mechanism of Static & Dynamic Friction

- When two bodies come in contact, micro-projections and depressions mesh and impedes relative motion.
- Surface adhesion also impedes relative motion.
- Both these factors are promoted by the normal force at the interface.
- Although microscopic behavior is quite complex, the macroscopic behavior is reasonably modeled based on experimental evidence.

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When two bodies come in contact, micro-projections and depressions mesh and impedes relative motion. Surface adhesion also impedes relative motion. Both these factors are promoted by the normal force at the interface. So, this is where the difficulties come, if you extrapolate it and then, visualize it without doing an experiment; you can come to wrong conclusions.



Friction

### Mechanism of Static & Dynamic Friction

- Pushing force less than maximum force of static friction leads mainly to elastic deformation of the micro-projections and the contact points where the forces of intermolecular cohesion are exerted.
  - The resulting elastic force is the force of static friction.
- Sliding friction is due to plastic deformation
- Frictional force reduces as motion begins because of the reduced meshing and adhesion of the surfaces.
- Frictional forces result in a loss of energy (such systems become non-conservative), which is dissipated in the form of heat.

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The microscopic behavior is very complex and people have efficiently modeled it by careful experimentation; that is what is summarized here. Although microscopic behavior is quite complex, the macroscopic behavior

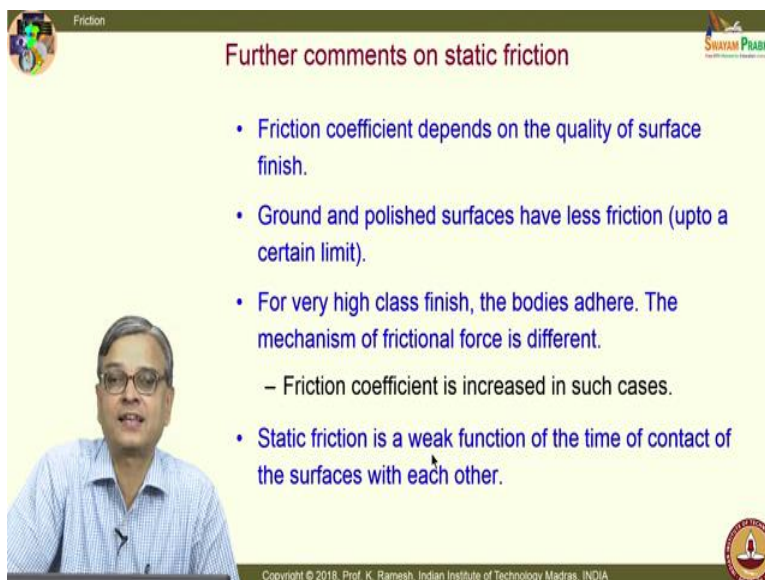
is reasonably modeled based on experimental evidence.

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What happens when I have the pushing force less than the maximum force of static friction? It leads mainly to elastic deformation of the micro projections and the contact points, where the forces of intermolecular coefficient are exerted; that means it is reversible. I have only elastic deformation; when the force is released, it comes back to its original position.

On the other hand, and this is also summarized that the elastic resulting elastic force is the force of static friction and sliding friction is due to plastic deformation. Frictional force reduces as motion begins because of the reduced meshing and adhesion of the surfaces.

And obviously, when I have a frictional force, you have dissipation of energy and that is felt as heat; in many appliances you find heat is generated and this is also one of the reasons, when the frictional forces are not that significant. We use conservation of energy as a very important tool for analysis to make our life simple; even though some energy is lost in friction, if the frictional forces are considerably less; it is not considered for simplicity in analysis.



Friction

SHUKAM PRADHA

### Further comments on static friction

- Friction coefficient depends on the quality of surface finish.
- Ground and polished surfaces have less friction (upto a certain limit).
- For very high class finish, the bodies adhere. The mechanism of frictional force is different.
  - Friction coefficient is increased in such cases.
- Static friction is a weak function of the time of contact of the surfaces with each other.

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And you should also recognize friction coefficient depends on the quality of surface finish. Ground and polished surfaces have less friction up to a certain limit; it is not that as I have honing operation done on the blocks, which will live a very smooth surface finish or very high class of finish the bodies adhere. There the phenomena totally change; it is not engagement of micro projections. You have adhesion becomes predominant. Particularly, when you go to a work shop, they will have blocks to find out the height



and they are very highly polished and when you put them together; it is very difficult to separate.

So, in general when I have a ground and polished surface, you will have less friction; but only up to a limit. The mechanism of surface behavior changes, when I have a very high-class surface finish and I have adhesion and in such a case friction coefficient is increased rather than a decrease because of smooth surface.

And another question is what happens if I have a normal force acting on the object? Thus the length of time has an influence on the frictional behavior and what is observed from experiment says static friction is a weak function of the time of contact; it is a function of time, but it is only weak function; so, in all practical purposes i can neglect the time of

contact of the surfaces for analysis.

**Rolling Resistance (Friction?)**

- Rolling resistance is the force resisting the motion, when a body rolls along the surface of another body.
- Deformation at the point of contact introduces a resistance to rolling.
- This resistance is an entirely different phenomenon from that of dry friction.
- In analogy with sliding friction, rolling resistance is often expressed as a coefficient times the normal force.
- There is no slipping or impending slipping in the interpretation of rolling resistance.

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And before we move on to solving problems dealing with dry or Coloumb friction, we will also have a peep into, what is rolling resistance? It is always labeled as rolling the systems and you can also

call it as rolling frictions debatable. There is only an analogy fine?

Rolling resistance is the force resisting the motion, when a body rolls along the surface of another body. It is again a very complex interaction and deformation at the point of contact introduces a resistance to rolling. There this resistance is an entirely different phenomenon from that of dry friction, dry friction is because of tangential forces developed; but rolling resistance why do you study is the way you write the expression for frictional forces, you use a similar analogy even for rolling; other than that the phenomena are quite different like I mentioned to you earlier. In analogy with sliding

friction rolling resistance is often expressed as a coefficient times a normal force, this is where it shares the commonality.

There is no slipping or impending slipping in the interpretation of rolling resistance. Do not extra plate beyond the point; rolling resistance is also expressed as sum coefficient of rolling resistance multiplied by the normal force; that is the only meeting point. It is actually dictated by deformation at the point of contact and you have many parameters

that contribute to rolling resistance.

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**Rolling Resistance (Friction?)**

- Factors that contribute to rolling resistance are the (amount of) deformation of the wheels, the deformation of the roadbed surface, wheel diameter, speed, load on wheel, surface adhesion, sliding, and relative micro-sliding between the surfaces of contact.
- A rubber tire will have higher rolling resistance on a paved road than a steel railroad wheel on a steel rail.
- Rolling friction has the least value.
- This has led to the development of ball and roller bearings.

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If you look at, what is the amount of deformation of the wheels; the deformation of the roadbed surface, the wheel diameter, speed, load on wheel, surface adhesion,

sliding and relative micro-sliding between the surfaces of contact.

So, it is a very complex mechanism by itself. People are still doing research on this and if you look at a rubber tire will have a higher rolling resistance on a paved road than a steel railroad wheel on a steel rail. All your rails, what you have as the train, you have a steel wheel rolling on a steel rail. On the other hand, you have automobiles going on the roads, which has a very soft rubber tire; so, it will have a higher rolling resistance and one of the significances of rolling is, rolling friction has a least value. See, one of the aspects, which we have seen is, how do you support a bearing? We have seen earlier that people put oil and then separate the metal to metal contact.

After rolling people also have device, what are known as ball bearings and you can see the resistance is very very small; it has a very nice movement and you have different types of ball bearings, you have this as spherical balls and you have rollers. You can see this as rollers ok. The inner race is taken out, this is; there is a inner race and there is an outer race. So, this will take more radial load than a simple spherical ball and you also

have tapered ball bearings. You look at the kind of complexity; engineering has developed into. Once they have recognized, the rolling has the least friction; they have devised various gadgets and in this what you find is, it can take both axial and normal load, the tapered ball bearings are there.

And you also have in important applications; the end points are completely sealed. This is ball bearing; you do not see the inner casing here. This is also required; you have many submersible applications, where you do not want any fluid get into it and affect its performance. So, you have this hermetically sealed. So, that is what is summarized here, rolling friction the motivation is. This has led to the development of ball and roller

**Sliding or Tipping?**

- Determine the exact point of application of the normal force (not necessarily passing through centroid) by moment equilibrium.

If the point of application of normal force  $N$  (or total reaction  $R$ ) lies within the base, sliding occurs.

- If it lies outside the base, block will tip before sliding.
- Limiting condition is that it is just at the end of base.

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bearings, which are normally, used in many engineering applications routinely.

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Ok, let us come back to our dry or Coulomb friction, when you are dealing with problems in friction, you must also recognize; it is

not that sliding will always happen. See, for example, if your mother wants you to push the Almirah in the house; intuitively you know, where to hold your hand on it and then push it. If you differ, the height, at some height it will not slide; but it will only tip. You do not do as calculation there and find out. You just have a physical feeling and then do it. If I have to do that mathematically, what is the way I will go and analyze it?

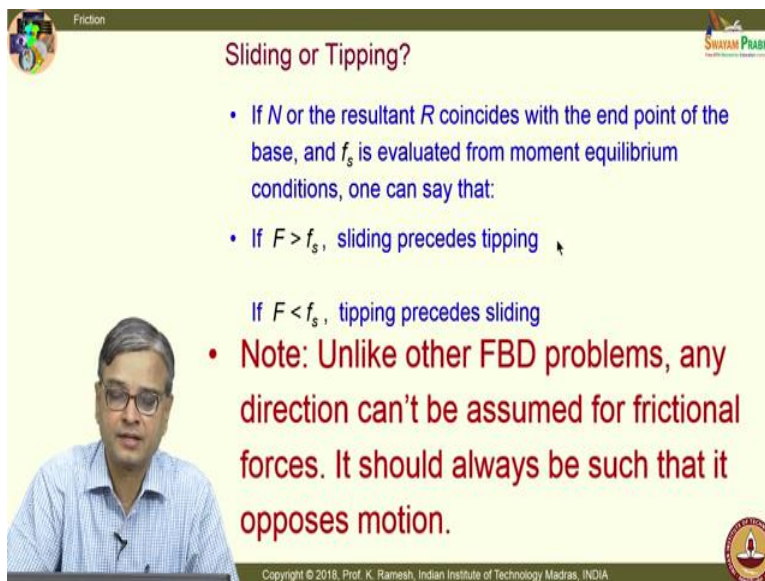
So, we have to find out the point of application of the normal force by moment equilibrium and obviously, when I apply the load like this, what will happen? If the point of application of normal force  $N$  or the total reaction lies within the base, sliding occurs. The force happens here; there is a small spelling mistake here, please correct it. So, you, we have already seen, how to find out the location of the reaction force and if this is so; then, you will have only sliding to happen.

On the other hand, if I apply the force at a different location and if I investigate, what is the point of action of the reaction? If it lies outside the base, block will tip before sliding. What is the limiting case? Limiting case is the edge of the object; so, here it becomes the corner. So, a limiting condition is that, it is just at the end of the base. So, if I have to investigate whether the object under the action of the force, whether it will slide or tip. One way of doing it is, you put the reaction force here and then, find out what should be the force? Or what should be the weight? Problems can be coined on multiple ways.

So, if you want to mathematically investigate, whether sliding will precede or tipping will precede; you will have to find out, what is the location of the reaction force? If the reaction force locates within the objects, sliding will occur; if it lies outside the object, tipping will occur. If I have to investigate tipping, the limiting condition is that will happen at one of the edges, either this edge or this edge depending on, which way you

apply the force ok.

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**Sliding or Tipping?**

- If  $N$  or the resultant  $R$  coincides with the end point of the base, and  $f_s$  is evaluated from moment equilibrium conditions, one can say that:
  - If  $F > f_s$ , sliding precedes tipping
  - If  $F < f_s$ , tipping precedes sliding
- **Note: Unlike other FBD problems, any direction can't be assumed for frictional forces. It should always be such that it opposes motion.**

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So, if I summarize this mathematically; so you see that it has tipped, if  $N$  or the resultant force  $R$  coincides with the end point of the base and  $f_s$  is evaluated from moment equilibrium conditions, one

can say that by investigating the value of  $f_s$  and compare it with the actual force; if the applied force is greater than this force  $f_s$ , sliding precedes tipping.

If the applied force is less than  $f_s$ , tipping precedes slide. When you solve a problem, you get to know, how do you distinguish between the two? So, this is again emphasized in problems dealing with friction. Find out the direction of the frictional force by reflecting the possible relative motion? Do not rush at this stage; that is very important.

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Let us, understand this from a very simple problem; I have two blocks, block *A* and

2. The mass of block *B* shown in the figure is 125 kg. The value of  $\mu_s = 0.40$ , between *B* and the horizontal floor over which it is placed, while  $\mu_s = 0.45$ , between the blocks. What is the mass of block *A* if the system is to remain in equilibrium when a force  $P = 200$  N is applied on the block *B*, as shown in the figure.

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block *B*; the geometrical dimensions are given. This is given as 1.2 m, height is 0.3 m and you have triangular block *A*. This is about 0.5 m height and you are given the frictional coefficients between the value of  $\mu_s$  is 0.40 between block *B* and the horizontal floor; the frictional

coefficient is only 0.4.

But between block *B* and block *A*, the frictional coefficient is given as 0.45 and the whole system is pushed by a force 200 N, which is acting at a height of 0.15 m and the question asked is, what should be the weight of the block *A*; so that the system remains in equilibrium under the action of the external forces 200 N and a 600 N applied at the tip of the block *A*.

It is a very fictitious problem to understand various aspects of visualizing and taking a strategy to solve this problem. First of all, we will have to recognize, what are all the possible motions that can happen under the action of the forces; that is what you have to investigate.

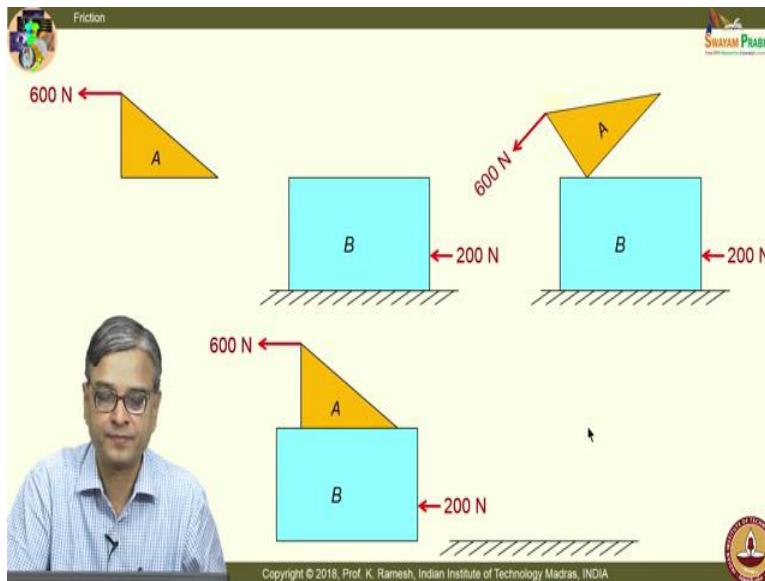
Then, find out for each case, what should be the minimum weight from among these solutions which is the minimum weight that is required for equilibrium? Ultimately, what you want is; when I apply the force 600 N and 200 N, the two blocks should remain in that place; it should not move. That is what they are really looking at.

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What all things can happen? I can think of three different situations. I can have one situation, where in; when I apply the force 600 N; only the block A slides on block B;

this is one possibility. The other possibility is when I apply a force 600 N, the block A tips; this is another possibility. The third possibility is both these blocks slide together fine and you know, what you will have to recognize is; for understanding I say that this slides like this; that mean just prior to that the



motion was impending.

Just to indicate when the motion was impending; when I have to consider this, I have to

consider the block A was impending. This is what I have to do to verify whether this will happen or not fine. Similarly, when I have this; both blocks A and B would be under impending motion. Then, I can replace the frictional force as  $\mu$  times a normal force and let us solve the problem systematically.

Case (a): Impending motion of A to the left.

$\sum F_x = 0; f_A = 600 \text{ N}$

$f_{A \text{ max}} = 0.45 \times N_A$

This gives  $N_A = 1333.33 \text{ N}$

$\sum F_y = 0; N_A = m_A g$

Hence,  $m_A = 135.92 \text{ kg}$

$\sum M_O = 0;$

$d \times N_A + 0.5 \times 600 - 0.3 \times m_A g = 0$

One gets,  $d = 75 \text{ mm}$

So sliding is possible.

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Now, I consider the case impending motion of  $A$  to the left. Though, I say impending motion my animation will emphasize, if a force is lightly increased relative motion like this is possible. So, do not link impending motion to sliding; do not do that. We are talking of a situation just prior to this; to illustrate that, to communicate that I am putting it like this; understand it in the context.

So, I have the free body drawn here; I have to put the forces and I have the weight acting like this. The question is, what is the value of  $m_A$  we should find; this is the question and I would have in general frictional force developed and this direction of friction opposes the force of pulling ok; so, that is fine and as we have discussed already, I would not put this normal reaction passing through the centroid. I would put it up at an arbitrary position. Let that location be at a distance of  $d$ . Once you have drawn the free body, rest of it is very simple; there is no great mathematics involved, but take a minute to draw the free body correctly. Do not rush there; that is my advice.

So, now I can write  $\sum F_x = 0; f_A = 600\text{N}$  and  $f_{A\max}$  that is dictated by the maximum frictional force  $f_{A\max} = 0.45 \times N_A$  and this gives the normal reaction as 1333.33 N and I get  $\sum F_y = 0; N_A = m_A g$ . So, I can calculate the value of  $m_A$ , this turns out to be 135.92 kilogram.

We should not stop here; we should also find out, what is the exact location of the reaction force? Because that would tell me whether it will slide or tip. So, let me write the moment equilibrium about point  $o$ , when I write moment equilibrium about point  $o$ , I get this as  $d \times N_A + 0.5 \times 600 - 0.3 \times m_A g = 0$ .

So, when I substitute the values of  $m_A$  and  $N_A$ ; in this I get the value of  $d = 75$  mm, which is within the block; so, sliding is possible. The block is not going to tip, if the block is of weight 135.92 kg. It is not going into tip; it is going to only slide.

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Let us now, assume block A is assumed to tip about the corners. Let us investigate this

situation; in this case  $d = 0$ .

So, I will have the normal reaction acting at the corner; this is what, we had seen mathematically if I have to investigate, whether sliding is possible or tipping is possible, I will have to take the reaction force at one of the edges, depending on the problem context. In this problem

Case (b): Block A is assumed to tip about the corner O.

In this case,  $d = 0$ ,

From moment equilibrium, one has

$$\sum M_O = 0$$

$$0.5 \times 600 - 0.3 \times m_A g = 0$$

$$m_A = \frac{1000}{9.81} = 101.94 \text{ kg}$$

This gives,  $N_A = m_A g = 1000 \text{ N}$ .

Maximum  $f_A$  possible is 450 N

Thus,  $F > f_s$ .  
Hence, sliding will precede tipping.

this is the appropriate edge.

From moment equilibrium  $\sum M_O = 0$  one has the value like this  $0.5 \times 600 - 0.3 \times m_A g = 0$ . This gives me a value  $m_A$  as 101.94 kg. Now, we have to go back and investigate; this gives  $N_A = m_A g = 1000 \text{ Newtons}$ . Maximum  $f_A$  possible from frictional forces is 450 N

because the coefficient of friction is given as 0.45.

$d \times N_A + 0.5 \times 600 - 0.3 \times m_A g = 0$

For,  $N_A < 1000 \text{ N}$  ( $m_A < 101.94 \text{ Kg}$ ), the block will tip as  $d$  will go out of the block for equilibrium.

And what I find is, the force what you have applied is greater than  $f_s$ ; even though we have investigated for possible tipping; what we find is, when I have the mass as 101.94 kg, sliding will proceed tipping; it will still

slide only ok; only if the mass is less than nine 101.94 kg, which we can see in the next slide.

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I have the expression; you have to look at the expression for this. We have determined  $m_A$  keeping  $d = 0$ , when I have this  $m_A$  value is less than that; then, the sign changes ok,  $d$  will go outside of it. So, for  $N_A < 1000$  N, indicating  $m_A < 101.94$  kilogram, the block will tip as  $d$  will go out of the block for equilibrium that comes from the moment equilibrium.

Now, let us look at the third possibility; both the block slide.

Case (c): Block A and B together having impending motion to the left.

$$\sum F_x = 0; f_B - 200 - 600 = 0;$$

$$f_B = 800 \text{ N}$$

$$f_B = 0.40 \times N_B$$

$$N_B = \frac{800}{0.40} = 2000 \text{ N}$$

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Now, let us look at the third possibility; both the block slide; block A and B together have an impending motion to the left. So, this is what I have said that, you have to impose this condition and find out the requisite answers. I show that it is

sliding; it is not sliding, just to put a recognition that when I increase the force slightly, it would start sliding. So, it is at the wedge of impending motion.

So, I have this and I take them together and I am investigating what happens on this surface. So, I put this at an arbitrary position at a distance  $s$  from the edge Q; I have the reaction as  $N_B$  and frictional force as  $f_B$ . When I apply the condition  $F_x = 0$ , I get  $f_B - 200 - 600 = 0$ ; this gives me  $f_B = 800$  N and  $f_B = 0.4 N_B$ ; so, I get  $N_B = 2000$  N.

And I can also go and find out, what should be the value of  $m_A$  for this. Fairly simple algebra; and if I put  $F_y = 0$ ;  $N_B - 125 - m_A g = 0$ .

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This gives me  $m_A=78.87$  kilograms and we will also see whether its slides or tips; so, I have to take moment about  $Q$ , shifting  $N_B$  equal to this point. So,  $M_Q = 0$  is what you are

$\sum F_y = 0; N_B - 125g - m_A g = 0$   
 $m_A = -125 + \frac{2000}{9.81} = 78.87 \text{ kg}$   
 $\sum M_Q = 0$   
 $s \times N_B + 0.15 \times 200 + 0.8 \times 600 - 0.45 \times m_A g - 0.6 \times 125g = 0$   
 $s = 0.287 \text{ m}$   
 $m_A < 78.87 \text{ kg}$   
 System as a whole will slide.

doing it. So, I have this mathematics and I do get  $s$  as 0.287 meters; so, this is within the block. So, then the mass of block A is 78.87 kilogram. We have confirmed at it will only slide ok; it will not tip.

Now, let us go back and analyze, what should be the minimum value for the

block A So that it remains in equilibrium.

$m_A < 101.94 \text{ kg}$   
 Based on block A to tip. But sliding precedes.  
 $m_A < 135.92 \text{ kg}$   
 Block A will slide  
 Thus minimum value For equilibrium  $m_A = 135.92 \text{ kg}$ .  
 $m_A < 78.87 \text{ kg}$   
 Based on system as a whole to slide - but Block A will tip!

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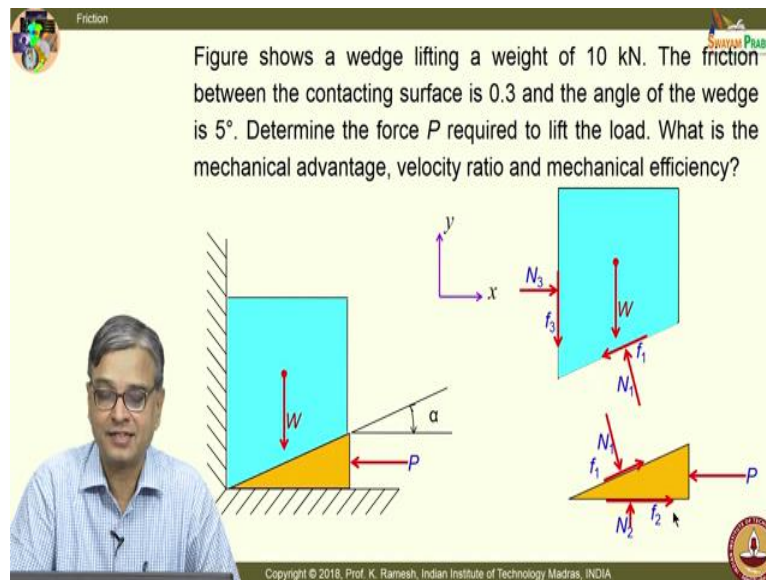
We have got one answer for when block A slides, I got this as 135.92 kilograms. I have got another answer as 101.94 kilograms, below that it will tip. If it is 78.87 kilogram the slide, but we have seen from this answer that the block will tip. So,

the minimum value for the system to remain in equilibrium turns out to be 135.92 kilograms; while you have to investigate all aspects of possible motion; it is a very nice problem; which illustrates the basic principle whether you investigate for sliding or tipping, whether the block tips or slides all that you are able to investigate.

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Then, we move on to another very interesting problem which is very useful from practical application point of view; again, very simple. I have a huge block  $W$ ; it is a weight of 10 kN.



The friction between the contacting surface is 0.3 and the angle of the wedge is  $5^\circ$ . See, for illustration purpose the angle is shown almost close to  $30^\circ$ ; if it is actually  $5^\circ$ , it will almost like parallel lines. To aid

your visualization in problem solving, it is deliberately done. So, do not take that this angle as  $30^\circ$ . It is given only as  $5^\circ$  in the problem statement.

And the question is, determine the force  $P$  required to lift the load and you would have seen if you have gone to any civil engineering construction, which they do it in a conventional sense; where they put bamboos to support the centering. They will have small shims they will go on hit at the bottom; so, that they will level the top fine and this is also used in an extended fashion, when I have this wedge; this wedge is rolled over a cylinder; you get what is known as a square threaded screw, that is used in many mechanical engineering machines and you want one important property call self locking. We will also see, what is the meaning of self locking? From that perspective only we have taken this problem.

And you will have to draw the free body and, in all problems, dealing with friction; you have to indicate the frictional direction correctly. Here, I am driving the wedge into the block  $W$ . So, what way should I indicate the frictional force? Which is the way should I indicate the frictional force? Because I am driving the wedge into it, you are deliberately applying a force in a direction; you have to recognize that, that is very important. So, I have the weight of the block acting downwards and this is the direction of frictional

force. You should recognize that. That is the key point here ok, because I am driving a wedge into it.

The moment you recognize this; then, it is simple. Since, I am driving the wedge, what way can I idealize the force  $f_1$ ? I am introducing a relative motion or I can definitely assume that the motion is impending. These are all key important aspects, when you handle a friction problem; you have to investigate and then only write. Under the action of this, what will happen to the block?

What do you think the block will do in relation to the wall here? The block will tend to move upward; so, what way the friction will be? Friction will oppose this; this is very important. You should visualize physically, what could happen. You must visualize imagine that you are wedge and find out what will happen to you, imagine you are a block, what would happen to you based on that you identify the frictional direction.

Then, the problem is very simple. Solving  $f_x = 0$ ,  $f_y = 0$ , you can do much better than me. This is very important here and once I have assumed this force directions; automatically direction of force on the edge is fixed, at least in this surface and this surface what way the friction would be? Because I am driving this like this, the friction will try to oppose this ok. I have this force, which is applied. This force is opposing this, that we have

Friction

$f_1 = \mu_s \times N_1$ ;  $f_2 = \mu_s \times N_2$ ;  $f_3 = \mu_s \times N_3$ ;

Equilibrium of the wedge gives

$$\sum F_x = 0;$$

$$-P + (\mu_s \times N_2) + (\mu_s \times N_1 \times \cos \alpha) + (N_1 \times \sin \alpha) = 0$$

$$\sum F_y = 0;$$

$$N_2 - (N_1 \times \cos \alpha) + (\mu_s \times N_1 \times \sin \alpha) = 0$$

Equilibrium of the block gives

$$N_3 - (\mu_s \times N_1 \times \cos \alpha) - (N_1 \times \sin \alpha) = 0$$

$$(\mu_s \times N_3) - W - (\mu_s \times N_1 \times \sin \alpha) + (N_1 \times \cos \alpha) = 0$$

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already seen and this force will also will oppose.

And what you can look at here is in all the surfaces one, two and three, the motion is impending ok. No place you have to determine the frictional force by solving the free body diagram.

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The way the problem is posed to you that clearly says that, this is the case and this is what we are going to write in the next slide that I have  $f_1 = \mu_s \times N_1$ ;  $f_2 = \mu_s \times N_2$ ;  $f_3 = \mu_s \times N_3$  because in all the cases the frictional coefficient is given.

So, it is not difficulty for you to find out this and equilibrium of the wedge gives  $F_x = 0$ ; then, you can easily do. I get some long expressions ok; which you can easily write down and solve. Let me pay attention on the physics behind it ok. So, what I am going to have is I am going to have these many equations and I have to solve for these unknowns and it

is in principle possible for you to solve this and I get the final answer as like this.

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Friction

We have four equations and four unknowns namely  $P$ ,  $N_1$ ,  $N_2$ , and  $N_3$ . Solving the simultaneous equations one gets,

$$P = 8.1\text{N}$$

$$N_1 = 12\text{ kN}; N_2 = 11.6\text{ kN}; N_3 = 4.68\text{ kN};$$

$$\text{Mechanical advantage} = \frac{W}{P} = \frac{10}{8.1} = 1.23$$

$$\text{Velocity ratio} = \frac{\text{distance moved by } P}{\text{distance moved by } W} = \cot \alpha = 11.43$$

$$\text{Mechanical efficiency} = \frac{1.23}{11.43} \times 100 = 10.8\%$$

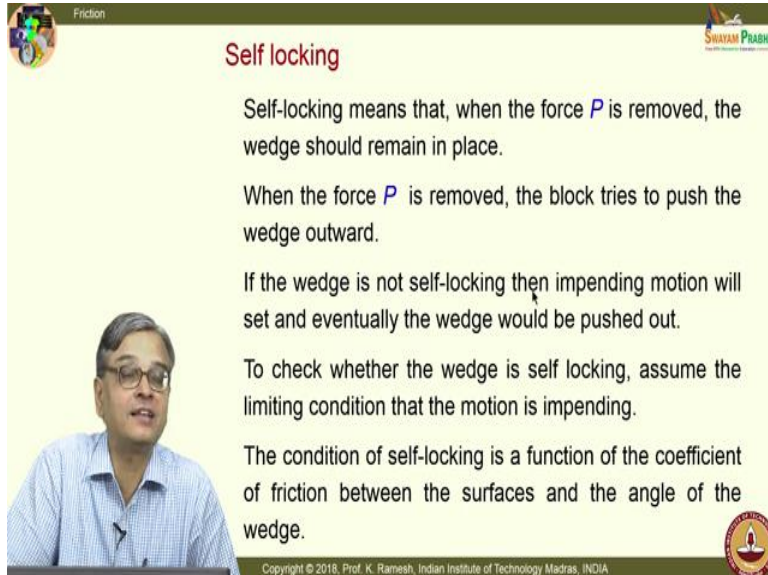
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I get this  $P = 8.1\text{ N}$ ; very small force and  $N_1 = 12$  kilo Newtons;  $N_2 = 11.6$  kN and  $N_3 = 4.68$  kN and you also learnt certain mechanical engineering

terms. You know, when you have this; they classified this as a machine ok. So, there are certain definitions, what is the mechanical advantage of a system? I apply force  $P$ ; I think you have to go back and verify the numbers; have I miss some this one we have to see whether it is 8.1 N or 8.1 k N, you just find out because here I have handled it as kilo Newtons, I just put as 10 by 8.1 kN; I get this as 1.23.

Otherwise, the mechanical advantage will be different; we have already seen the mechanical advantage concept, when we have looked at the crimping tool; that is again classified as a machine. My effort is something that gets magnified many times and you also have another definition; these are definitions ok. I have, what is known as velocity ratio? Distance moved by force  $P$  and distance moved by the block  $W$ , that is given as  $\cot \alpha$  that is 11.43 and I also have a terminology called efficiency, mechanical efficiency is given as mechanical advantage divided by velocity ratio into 100.

This turns out to be 10.8%; this indicate that this should have been 8.1 kN; so, please go and check up this. So, I have only 10 % efficiency; that is quite alright ok, but I am in a position to lift a weight comfortably.



**Self locking**

Self-locking means that, when the force  $P$  is removed, the wedge should remain in place.

When the force  $P$  is removed, the block tries to push the wedge outward.

If the wedge is not self-locking then impending motion will set and eventually the wedge would be pushed out.

To check whether the wedge is self locking, assume the limiting condition that the motion is impending.

The condition of self-locking is a function of the coefficient of friction between the surfaces and the angle of the wedge.

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Now, let us look at, what is self locking? Self-locking means that when the force  $P$  is removed, the wedge should remain in place; it is a desirable aspect; this is the way we want the system to behave. I do not

need to keep the force applying; I just hit the wedge and leave it at that. When the force  $P$  is removed, the block tries to push the wedge outward; if the wedge is not self-locking; then, impending motion will set and eventually the wedge would be pushed out; which is not desirable.

To check whether the wedge is self locking, assume the limiting condition that the motion is impending. The condition of self-locking is a function of the coefficient of friction between the surfaces and the angle of the wedge; so, it is a desirable property. When you have a mechanical power screw, you want that power screw to be self locking. It is very well used in mechanical engineering applications and the way to solve the problem is given; if the wedge is not self-locking, then impending motion will set and eventually the wedge would be pushed out.

So, to check whether the wedge is self-locking; assume that the motion is impending and re solve the problem.

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So, I have the same situation. Now, I do not have the force  $P$ ,  $P$  is 0. Now, I would have to put the frictional direction opposed ok. Now, this will try to push this out; so, look at the frictional direction, they are all different.

For the system shown in Figure determine whether the wedge is self locking.

Equilibrium of the block gives

$$\sum F_x = 0; N_3 + (\mu_s \times N_1 \times \cos \alpha) - (N_1 \times \sin \alpha) = 0$$

$$N_3 = N_1 (\sin \alpha - \mu_s \cos \alpha) = -0.21 N_1$$

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So, when I solve this problem in this fashion, I get the value  $N_3$ ; I get the value as minus  $0.21N_1$ . That means, I have assumed it like this; I am getting this opposite, which is not possible. So, that means, motion is not impending and the wedge

is self-locked; you need the self locking as a desirable property. Is the idea clear?

See, we solved the same problem with two different frictional directions and this is one of the reasons, when you have many of the mechanical engineering applications. People

### Self locking

A negative value of  $N_3$  implies that the vertical wall must apply a pulling force.

This is not possible and hence the wedge is self-locking.

In other words, the above equations were obtained by assuming that the motion is impending.

However, the results show that the motion is not impending and hence the wedge remains in place.

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simply write frictional force as  $\mu N$  because many of them operate at the condition of impending motion; only while you learn the frictional concepts in the initial stages, you coin problems where deliberately some surfaces as known impending motion, some surfaces

there is impending motion. So, this distinction you should be able to appreciate.



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So, I have this, a negative value of  $N_3$  implies that the vertical wall must apply a pulling force. This is not possible and hence the wedge is self-locking. So, we have established that the wedge was useful and the system was under self-locking.

It is a very desirable property; which is used in many mechanical engineering applications. So, in this class, we have solved a variety of problems dealing with friction. We have listed out a systematic procedure on, how do you approach to solve the problem dealing with friction? It is worth that you spend a minute in investigating the direction of that are a frictional force developed.

Then, investigate whether the motion is impending; then only replace the frictional force as  $\mu$  times the normal reaction and we have also discussed how to investigate mathematically whether the block will slide or tip. Finally, we have also learnt a very important concept dealing with self-locking; which is a very desirable property in many mechanical engineering applications.

In addition, we have also seen certain basic definitions that are used like mechanical advantage, velocity ratio and mechanical efficiency which is used in your higher studies.

Thank you.