Engineering Mechanics
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Module - 01
Statics
Lecture – 20
Friction – III

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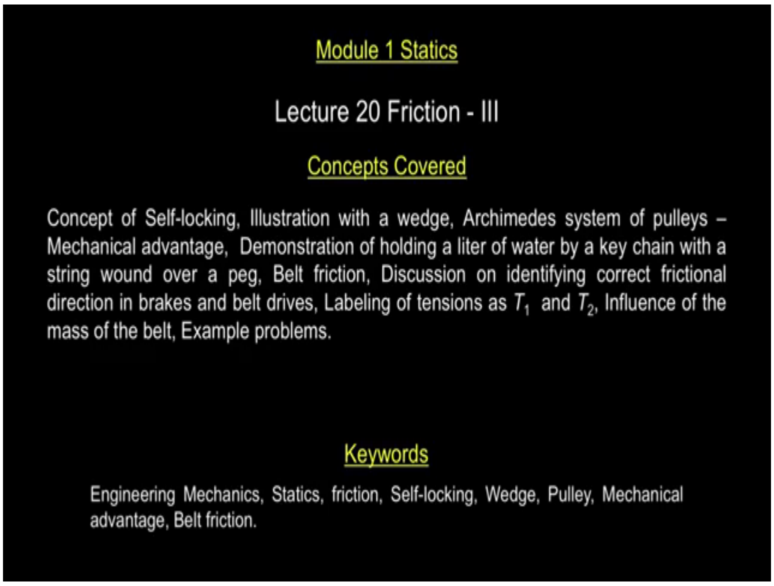

Let us continue our discussion on Friction.

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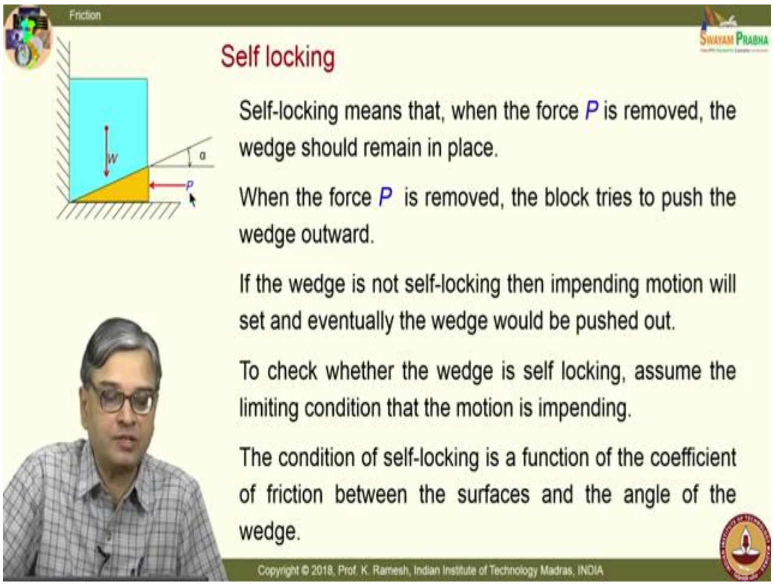

See in the last class, I introduced the concept of Self-locking and the idea is you have a weight lifted by a wedge, I had told you earlier, for illustration the wedge is shown very big, the actual angle is about 3 to 4^o. It is shown as 30^o, for the easy visualisation.

So, the idea is when I remove the force P, the block should remain at the place where it was lifted initially and when the force P is removed, normally the block will try to push this wedge out. And what you have is you have impending motion set and you bring in the condition of impending motion and investigate, whether the wedge what you have driven into the block is self-locking and self-locking is a desirable property you want to have it ok.

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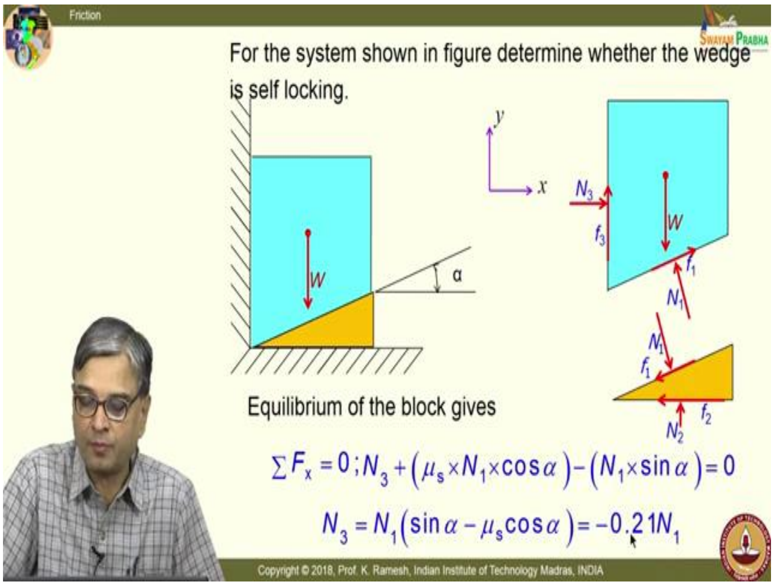
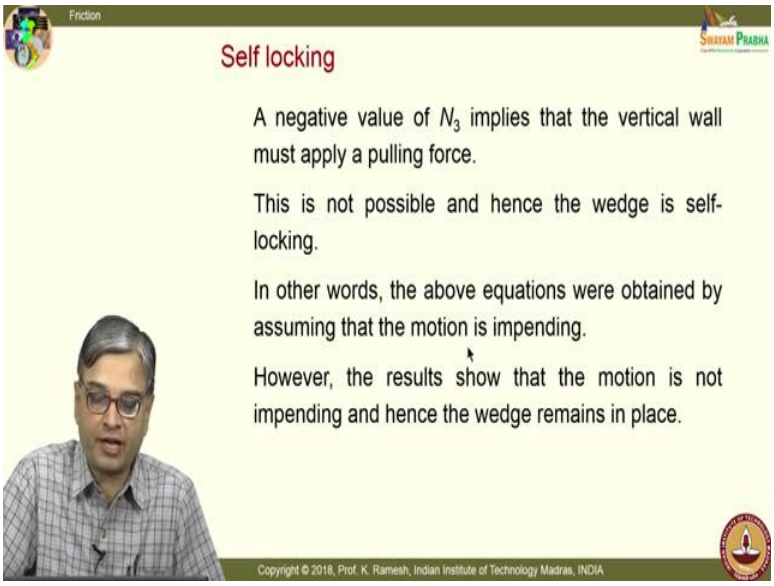


And if you look at this, I remove this force P and draw the block as well as the wedge; and the block comes down ok. So, you will have a friction will be in the opposite direction and the wedge also comes out so that is prevented by the frictional effects and you have to indicate the frictional forces correctly. The moment you indicate the frictional forces rest of it is straightforward. So, here we are investigating, when the load P is removed, what is the condition of the wedge as well as the block and the equilibrium of the block gives $\sum F_x = 0$.



It gives me this expression and when I simplify, I get an expression for the reaction from the wall N 3, that turns out to be negative, this is 0.2 1 N_1.

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The negative value implies that the vertical wall must apply a pulling force, which is not possible. So, that is how you establish, that the wedge is self locking and what contributes to self locking?

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If you look at, the angle of the wedge and the friction contributes to self locking and this is very well used in many engineering applications, what you see here is a tensile testing machine. This is screw driven and the material what you put between the jaws is pulled by it.

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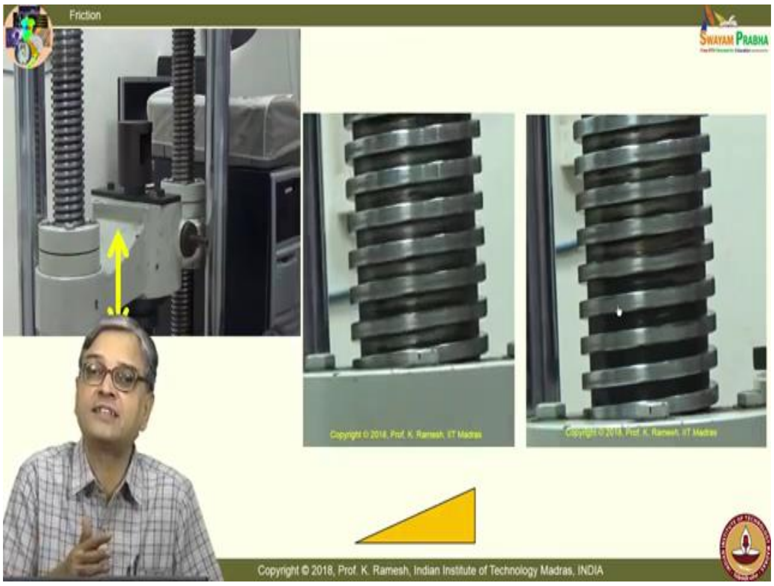
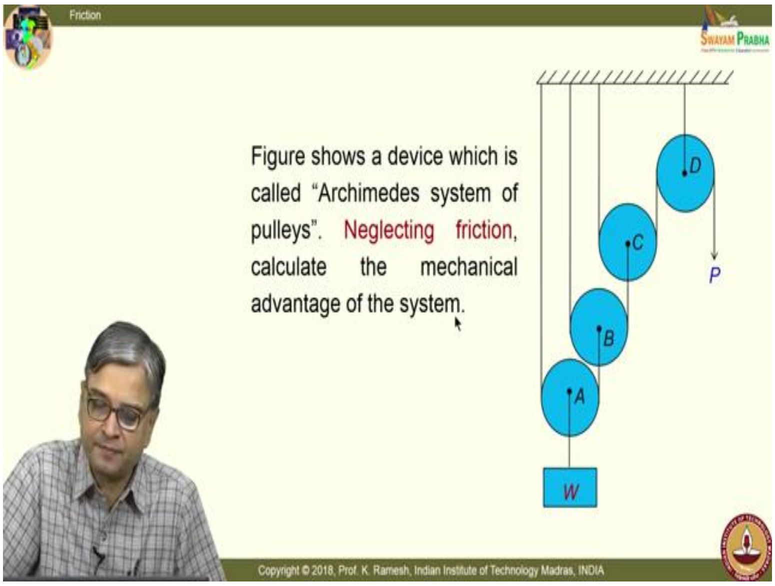


And it is rotated on one side it comes up and then rotation is in the opposite direction it comes down and what you need to notice is this is a square thread. Many of the power screws, if you go and look at a lathe, where you have a you have a tool post, that slides that is also activated by a lead screw and most of this power screws have square threads and for the purpose of analysis, this can be simply visualised like a wedge, wrapped around the cylinder. It is very easy to analyze and all of these appliances require self locking.

It is a desirable property not only that, in all these cases motion is impending. So, this is a reason when you learn friction but, when you actually apply to mechanical engineering, you find many of the applications you have self, I mean you have impending motion, people tend to replace friction by mu times N blindly. You can also coin problems where in some portions of the system motion is not impending in some portions motion is impending; you should never forget that.



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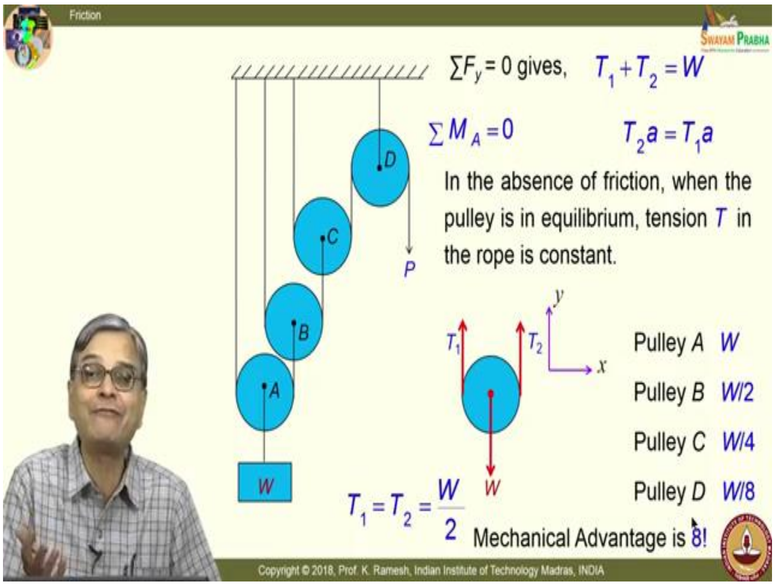

And this is also very interesting use of frictionless pulleys and this is known as Archimedes system of pulleys and important aspect is neglecting friction, calculate the mechanical advantage of the system. See if you look at engineering development, you know you need to achieve things with what material availability, with our minimal effort, how can I lift a very large weight that is where the Archimedes has given this contrivance.

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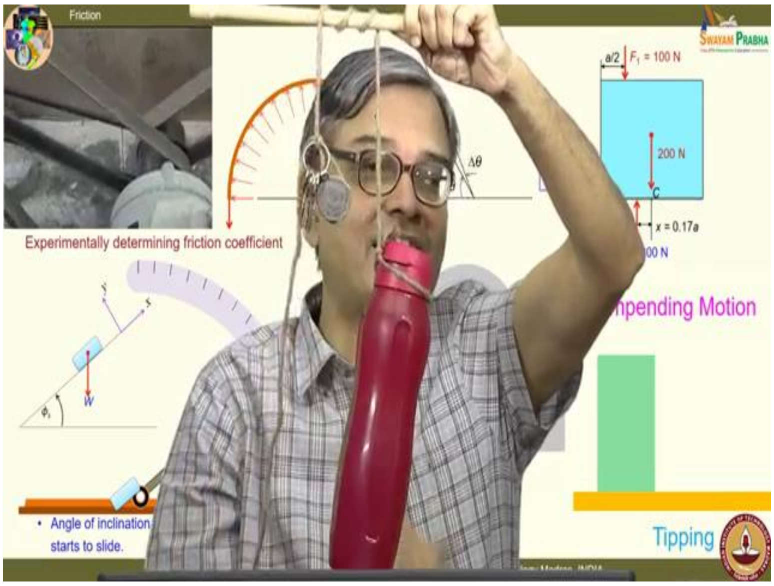
And we will just go on investigate, how do we handle this. So, I have a pulley separated weight is acting on this.

In general, I can replace the tension in the cable as T_1 and T_2 because, I have a situation where there is no friction, we can also investigate what should be T_1 and T_2, that will turn out to be $T_1 = T_2$. And from the equilibrium of the pulley I have T_1 + T2 = W and this is what I said because, I do not have friction, I get an expression, the tension remains constant only the pulley changes the direction, that is all it does. And if I look at what is the value of T_1 and T_2 for the pulley A, that turns out to be W/2 and this is a load, that needs to be supported by the pulley B.



So, pulley A supports W, pulley B supports W/2, pulley C supports only W / 4 and pulley D supports only W by 8. So, you have a mechanical advantage of 8. So, with a small pulling force, you can lift a very large weight, fine.

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Let me ask, see I am going to have a keychain and I also have a water bottle fine; this is the 1 litre water bottle. So, it should be 1 kilogram fine. Can I support this water bottle with this key bunch, we weighed it yesterday it is about 80 grams fine, shall we try that; ok.

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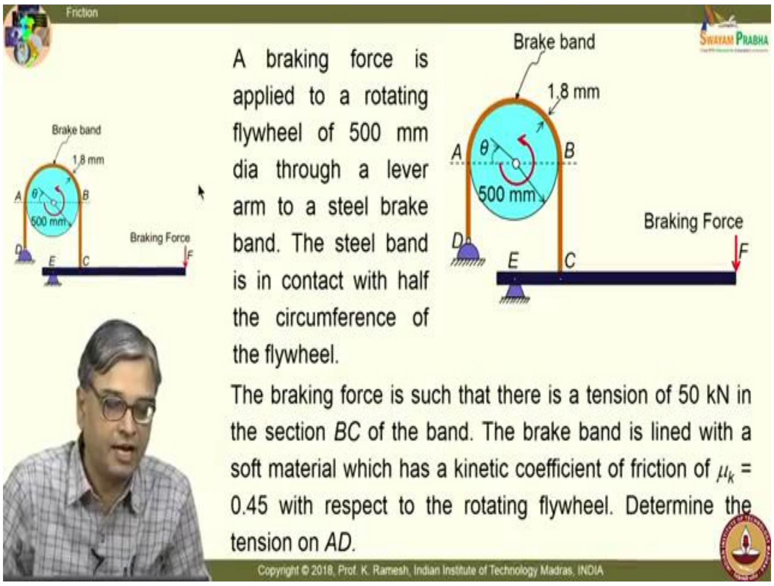

So, I will do is I will first tie this and what I have here is I have a peg, which has friction fine, what I do here is, I put it like this is not safe 1 round is not going to be safe, it is not safe, I put one more one round, even this is not safe, I put another round and you see here, I have only 80 grams, supporting 1 kilogram of weight. And this is having a friction, friction aids, it looks very surprising fine, we will go on investigate the mechanics of it.

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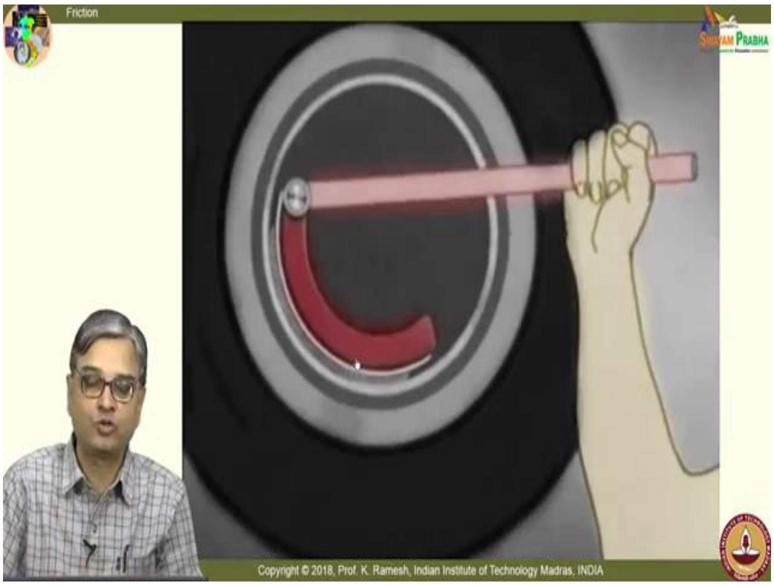

You know to develop the mechanics we will take a real-life system, where I have a flywheel, this is prevented from rotation by the use of a break band and thanks to Archimedes. We have a nice lever arm system. So, I need to apply a smaller force to apply a very large force in the segment B C and the main difference between what we saw as Archimedes pulley and this is you have friction between the drum, flywheel and the belt and the value of kinetic friction is given as 0.45 and you have certain geometric description, the diameter is about 500 mm, the thickness of the band is 1.8 mm and this is a steel break band. And initially the flywheel was rotating in the anticlockwise direction all that is very important.

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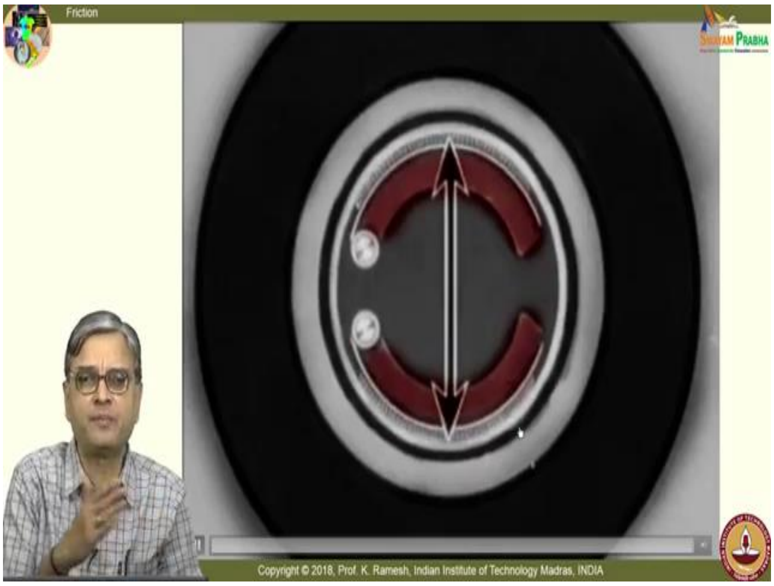

Now, let us go and draw the free body diagram, before that you know I also have another interesting animation. Suppose I have a braking system which has a very small length of the shoe, the braking action can burn this off intense heat is generated. So, it will not have a life suppose, I increase the length little longer, here again it is slightly improved but, this also can burn off because of friction. And you have a sufficiently long shoe for you to affect the braking and if you want to have a braking system to have a life, you should not only have just one shoe like this.

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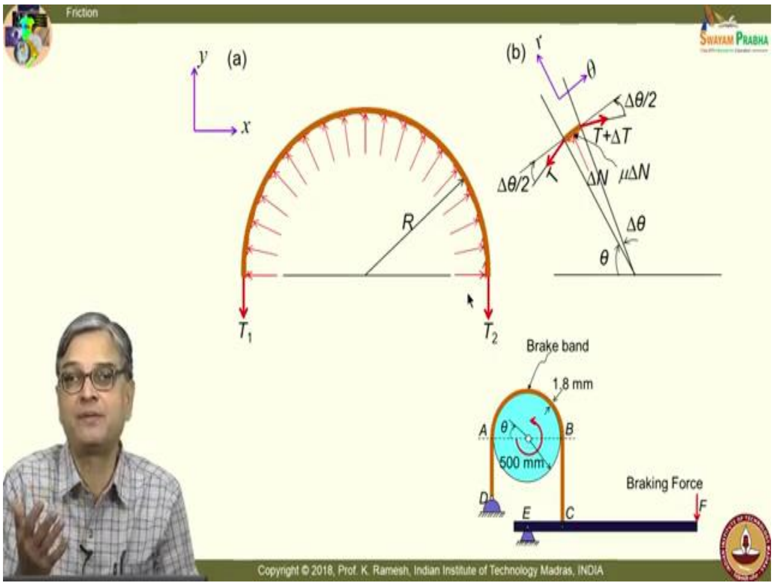

We will mean to have 2 shoes likes this; this is how many of the braking systems are available commercially now. So, the angle of lap is very important, you cannot have a very small angle of lap. So, you have a concept from a practical example.

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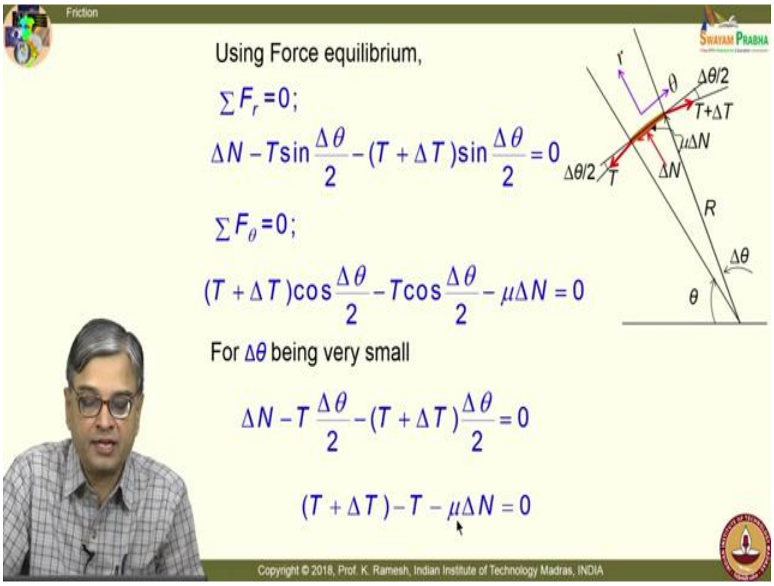
And we will look at, what happens in the braking. Here the braking is more efficient because, I have applied a smaller force because, of the advantage of the lever system and as before I will put the tension as T_1 and T_2, we do not know what are these values and if there is only normal reaction, if there is no friction I will have only normal reaction and now we have to correctly put the frictional force on the band. Please understand that, the band is preventing the flywheel from rotating, the flywheel is rotating anticlockwise.

So, what is the direction of friction acting on the brake band? The drum is applying so it is anticlockwise fine. So, I will have a fictional interaction all around the circumference of the belt and if I have to analyse, I need to take a typical section out of it and it is convenient to analyse it in the polar coordinate system. So, I will take a section at angle θ , I have taken it from clockwise and I take a small segment $\Delta\theta$, I separate it out and I put the force interaction on this and I should recognise, that the tension does not remain constant along the length of the brake band.



Because, I have friction the situation is different. So, if I say T at one section, I will say $T + \Delta T$ in the other section. So, I said that it is convenient to analyse it in a polar co ordinate system. So, I have the r and θ axis shown like this and I have the normal force is depicted as ΔN and the motion is impending because, you know you want to stop the brake band and it is in the limiting condition. So, I can replace the friction interaction conveniently as μN. Now, I will put the tension, this segment is taken as $\Delta\theta$.

So, if I put the normal on these faces, there will be oriented angle $\Delta\theta / 2$ and if I take the tension as T on this side of the belt, when I move by an angle $\Delta\theta$, the tension on this side would be say $T + \Delta T$. So, now, we are equipped with all the forces depicted on the belt and write the equilibrium equation along the r direction and θ direction that is straight forward. So, the key point here is, you have to recognise tension does not remain same along the length of the belt because, I have friction. And secondly, you will have to identify what is the correct direction of friction on the belt, these are very important.

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So, I use the equation of equilibrium, when I say $\Sigma F_r = 0$, that gives me $\Delta N - T\sin\frac{\Delta\theta}{2} - (T+\Delta T)\sin\frac{\Delta\theta}{2} = 0$, please verify whether this expression is correct and when I put sigma $\Sigma F_\theta = 0$, I get $(T+\Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu\Delta N = 0$.

And you know very well that, $\Delta\theta$ is very small, I can replace sin $\Delta\theta$ / 2 as $\Delta\theta$ /2 and cos $\Delta\theta$ / 2 as 1 when I do this, I get an expression like this, $\Delta N - T\frac{\Delta\theta}{2} - (T+\Delta T)\frac{\Delta\theta}{2} = 0$ and this expression reduces to $(T+\Delta T) - T - \mu\Delta N = 0$. Please verify these expressions including the sign, there could be some typographical errors; please correct them if they are.

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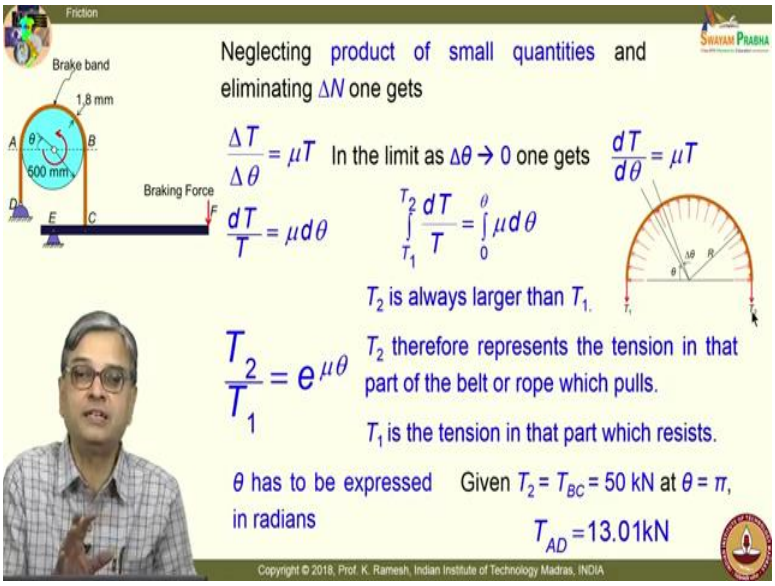

So, what I get this finally, neglecting product of small quantities and eliminating ΔN one gets, $\frac{\Delta T}{\Delta\theta} = \mu T$. So, in the limit as $\Delta\theta$ goes to 0, I get $\frac{dT}{d\theta} = \mu T$, this can be written alternately like $\frac{dT}{T} = \mu d\theta$. And if you look at the way that we have labelled θ and $\Delta\theta$ and the other quantities, I can integrate it over the complete length of the belt, which is wrapped over the drum, $\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^\theta \mu d\theta$.

This you can easily do the integration and get the expression as $\frac{T_2}{T_1} = e^{\mu\theta}$, it is a very famous expression. What are the contributing things? When I have this tension, the tensions are totally different and they are exponentially related, T_2 is very high compared to T_1 and what contributes to this, do you find the size of the drum coming into the picture? The 500 mm diameter is not come in error, radius is not coming into the picture at all, what is coming into the picture is, what is the angle of lap, how much it is on the drum? And another important aspect is it is also written here.

When you write θ, even if the problem statement gives the angle of lap in degrees, please converted into radians, these are all the common mistakes peoples do. They will know that the expression $\frac{T_2}{T_1} = e^{\mu\theta}$, you should not substitute θ in degrees, you should substitute only in radians, only then you can evaluate these quantities correctly and in the problem you are given the force in the segment BC, in our derivation we have taken that as T_2.

So, $T_2 = T_{BC} = 50$ kN and θ is very obvious from the problem statement, that this is one half of the drum it is lapping. So, $\theta = \pi$ radians and you are also given the value of kinetic friction. So, you can easily find out what is the value of tension T_1 in this case, any of the tensions you will be able to find out and we want to find out what is the tension in segment theory and that turns out to be 13 kN ok.

Let us reflect upon this expression, see in our labelling what we have done is, we have cleverly put T_1 on this side and T_2 on this side, knowing fully well how I want to get the result. In our expression, when I write $\frac{T_2}{T_1} = e^{\mu\theta}$, the expression also tells you, T_2 is always greater than T_1. T_2 represents the tension in that part of the belt or rope, which pulls. See in all friction problems, you will have to identify the direction of friction from the physics of the problem, you have to visualise, you have to visualise it correctly and if you visualise it wrongly, then you are solving all together a different problem.

You should understand how T_1 and T_2 are when you draw the frictional direction, one way of looking at is, I have T_2 here and the friction is just opposite to that. I have tension T_1 and I have the friction direction and tension T_1 or aligned in the same way. So, what I have here is, when I use this expression, you must consciously label T_2 and T_1 correctly.

T_2 should be the higher value of tension, T_1 should be the lower value and it should be mark correctly on your free body diagram, looking into the direction of frictional force.

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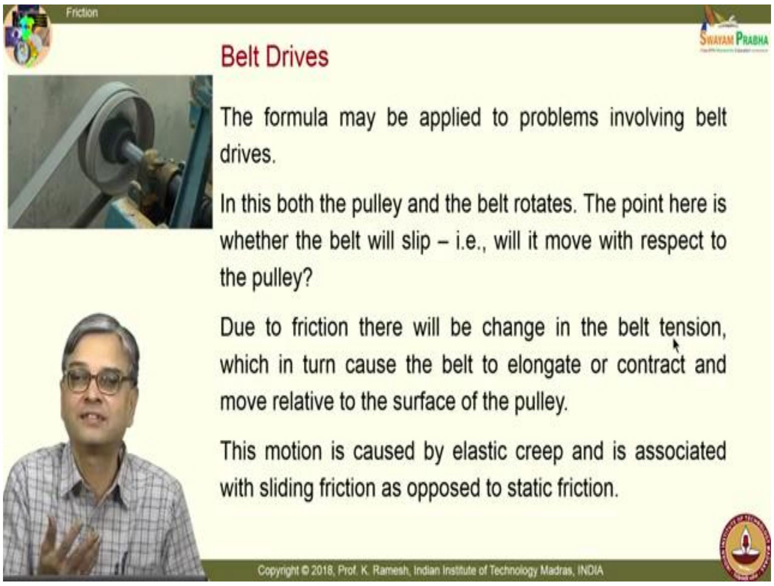

And let us look another application of friction is your belt drive in your common 'atta chakki' and what I would like you to point out here is.

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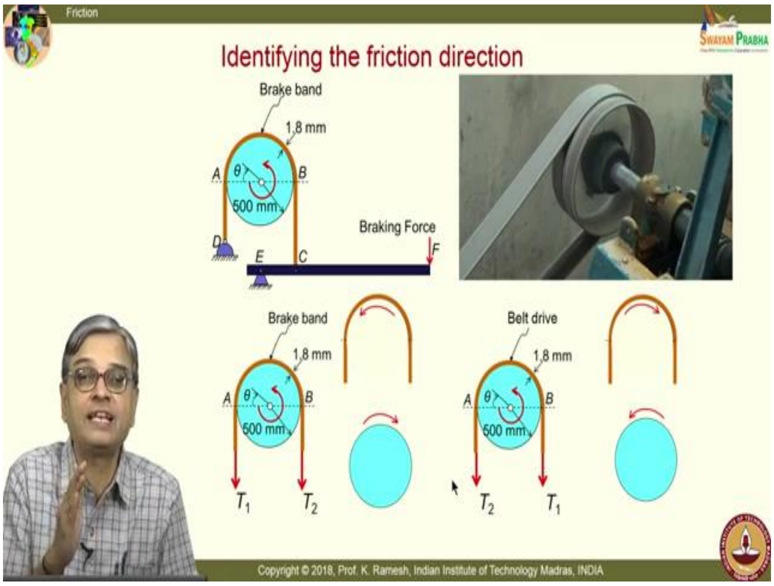
I have a driver pulley here and this is a driven pulley, that is done by a belt drive and you will have to distinguish between a driver and driven pulleys.



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Whatever the expression that we have got, can be applied to problems involving belt drives. Here both the pulley and the belt rotate. The point here is, whether the belt will slip, that is will it move with respect to the pulley. Due to friction, there will be change in the belt tension. So, you recognise that the belt is deformable, see even though we confine our attention to rigid body mechanics, we have relaxed it in brought in springs and then we have analyzed them.

Similarly, in this application, whatever the belt that you have, that can deform. Due to friction there will be change in the belt tension, which in turn cause the belt to elongate or contract and move relative to the surface of the pulley. This motion is caused by elastic creep and is associated with sliding friction as opposed to static friction.

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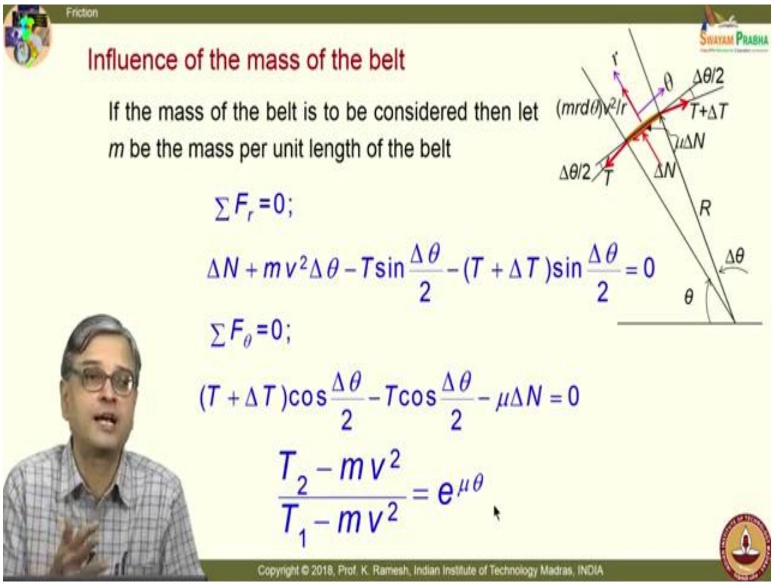
Suppose, I have to solve a brake problem or the pulley problem, how do I identify the friction direction, let us revisit this, there is a subtle difference. You will have to recognise that subtle difference and then handle it, fine. Now, let us take the case of a brake, the flywheel is rotating in the anticlockwise direction and I separate the belt and the drum, here you should recognise, that drum is providing the frictional force on the belt, which we have seen just now.

So, it is acting on this direction and naturally by Newton's third law, I will have on the drum the action is opposite to that. And how do I label which is T_1 and T_2? I have already said, T_2 and friction should be opposite to each other. So, I will label this as T_2 and this as T_1. Now, let me go to the belt drive and I am looking at a driven pulley, this pulley is driven by the belt. So, the friction is brought in by the belt to drive the pulley, it is a driven pulley. So, in this case the drum is rotated in anticlockwise direction by the belt.



So, what is applying friction on what? The belt is applying a friction in the anticlockwise direction to the drum, when I go to the belt, the belt will have a friction in this direction. How do I label which is T_1 and T_2? This will be T_2 and this will be T_1. Final interrelationship between T_1 and T_2 is same, $\frac{T_2}{T_1} = e^{\mu\theta}$.

So, you need to identify am I analyzing a brake, am I analyzing a driven pulley or driving pulley, you think about how you will analyze a driving pulley, take that as an exercise ok. So, in all problems dealing with belt friction, you have to be very careful in identifying the friction direction, as well as labelling T_1 and T_2. You cannot apply the formula blindly, you have to physically assess the problem situation, label them correctly and then only use the expression.

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And suppose I want to improve my analysis of the belt drive; I would like to bring in the mass of the belt. How do I bring it, then I will have to put the centrifugal force because, of it is rotation it is a body force and you have this standard expression; I have taken m as mass per unit length.

So, mass of this segment becomes, I have $m\,r\,d\theta$ so, mv^2 / r is your centrifugal force. So, that you are putting it here, your final expressions change slightly, in addition to whatever the expression we had, I will have an additional term $mv^2\,\Delta\theta$ and this turns out to be the final expression is, $\frac{T_2 - mv^2}{T_1 - mv^2} = e^{\mu\theta}$. So, you can accommodate the body force due to the belt.

If it is significant, if your operating speeds are very high then it becomes significant, then you will have to improve your predictions on the tension by incorporating the body force appropriately.

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And look at this example, see you have to recognise, it is very difficult to lift yourself, if you do not have assistance from friction.

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So, he goes up stays in place by friction and this is a fiction knot, there is also a special knot here and while coming down he just releases that and he is able to come down at a controlled speed easily. Imagine I have a frictionless pulley, you put your rope and then you hold yourself, you can probably hold yourself but, you cannot make yourself to move up or move down because, you have to exert a very large force for you to move up equal to your weight.

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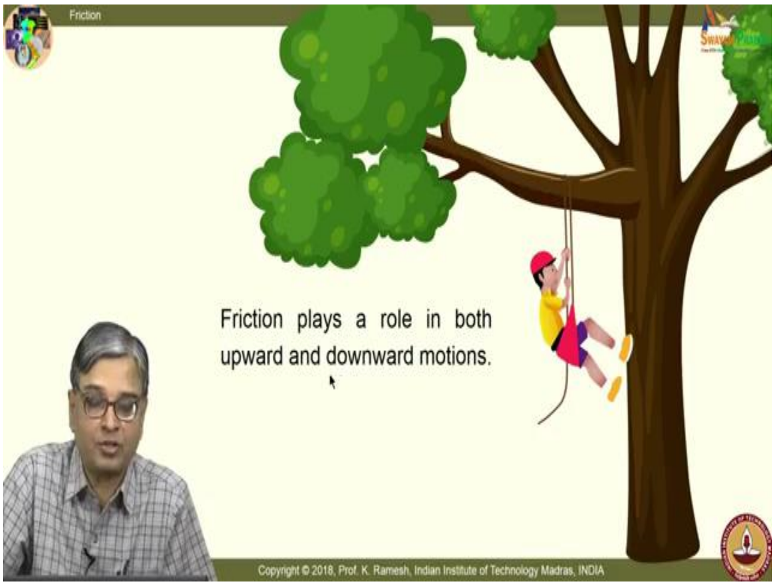

On the other hand, friction is very well used, it plays a very important role both upward and downward motions.

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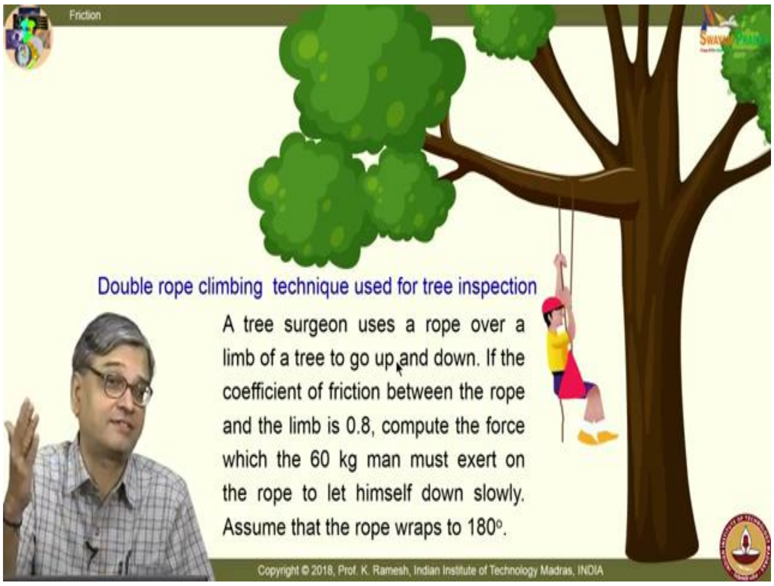
And let us analyse the problem given here, you know in western countries they call them as tree surgeon, they go inspects and you have very tall trees, in all cold climates even if you go to Shimla, you will find very tall trees and you cannot climb on to the tree by conventional methods, you need to have gadgets available for you to do that and in this case a tree surgeon uses a rope over a limb of a tree to go up and down. If the coefficient of friction between the rope and the limb is 0.8, compute the force which the 60 kg man must exert on the rope to let himself down slowly.

And; obviously, a tree branch is of an odd shape to make the problem simple, assume that the rope wraps to 180°. I have deliberately given this angle as 180°, just to alert you, even the problem statement gives it degrees convert it into radians and then use it. Do not make a mistake; do not rush to solve the problem.



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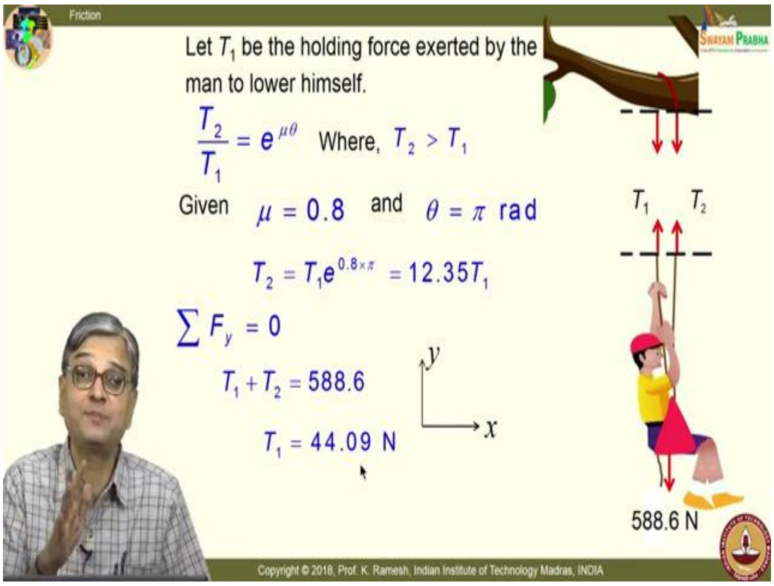
And what is the challenge here? You need to identify, what is T_1 and T_2 and the situation is that the person is climbing down, he is just coming down smoothly. So, he is just having a force to hold it, he comes down automatically by his self-weight. So, which is the pulling force and which is the holding force? The pulling force is weight ok; that is happening on this part of the rope and this is where he holds it. So, he holds this portion. So, this becomes T_1 and this becomes T_2, the moment you identify what is T_1 and T_2 the problem is solved.



There is nothing else in the problem, whole of this problem requires correct visualization of the frictional interaction and correct labelling of the tension T_1 or T_2, rest of it is very simple. Once I have identified this, I get $\frac{T_2}{T_1} = e^{\mu\theta}$ and $T_2 > T_1$ and you are given friction coefficient as 0.8 and θ is now written down in radians and I apply this I get $T_2 = 12.35\ T_1$. And if I use $\sum F_y = 0$, I get $T_1 + T_2 = 588.6$ (weight of the man). So, for him to come down, he needs to just hold a very small force of 44 N.

Precisely the similar thing happens when I was supporting the water bottle of 1 kilogram with a key chain of 80 grams ok. So, it is possible, it is possible because of friction and I am not quite sure, when your anyone is from coastal areas, where you are accustomed to going in a boat coming back to the shore or if you have an occasion to visit Andaman and Nicobar islands, you go from one island to another island by a boat ride.

And once it comes to a shore, the person will throw the rope and the person in the shore will take it and put it on the peg several rounds and that is how the boat is halted and it stays in place and you can get on out of it and friction is very well used in this applications. So, friction is very useful, it is exploited in many engineering applications. So, friction helps so it is a very interesting problem.

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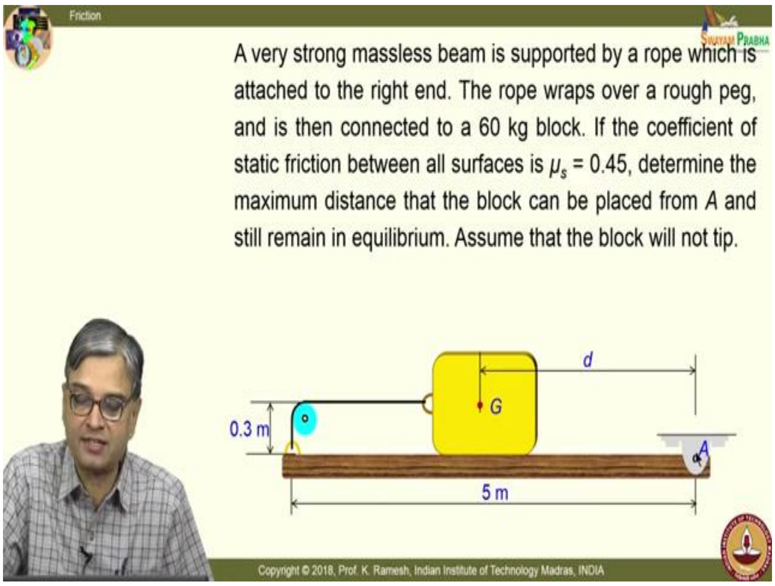
I will just make this problem statement and you think about it ok, you know this is a very nice problem, which encompasses both a sliding friction, as well as a rope friction, the

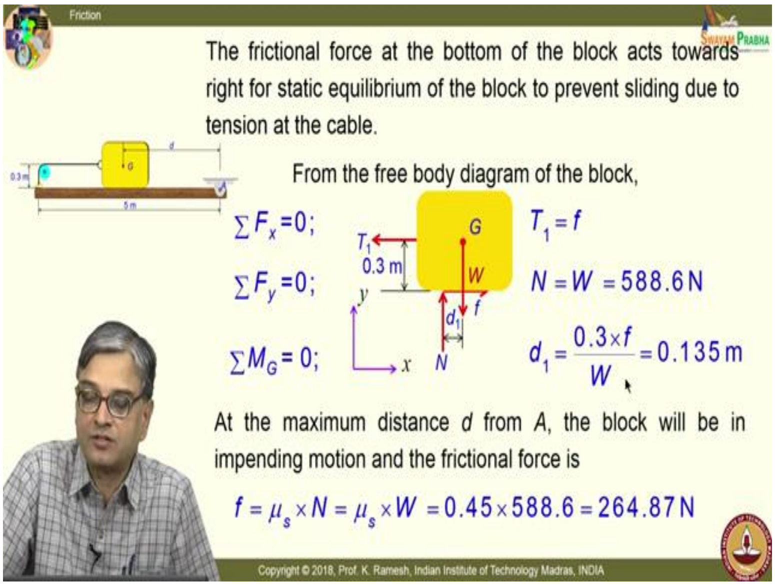


problem statement is like this you have a very strong massless beam is supported by a rope which is attached to the right end, the rope wraps over a rough peg.

You have to notice the language, all along we have said that it is frictionless peg, frictionless pulley. Now, we are bringing in friction and to make our life simple, the friction is taken as 0.45 at all surfaces and the question asked is determine the maximum distance that the block can be placed from a, I have labelled this as a and this is hinged and I have a hook here, I have a pulleys supported by some other means and the geometric dimensions are given.

And assume that the block will not tip, that is also a very obvious from the way it is being connected, I have the hook aligned with the centre of gravity of the block and I have this like this. Can you visualize the physics, can you visualize the physics?

I can keep this block anywhere here. So, the block will try to keep the beam horizontal, we want the bean to the horizontal. Now the question asked is, what should be the maximum distance of the block can be placed from A, can I keep it there, where the motion is impending, that is that is what is implied. If the block goes further and further the whole beam can fall down, if it is towards A, then it can remain horizontal. So, you have to investigate the frictional problem very systematically; very interesting.



You have sliding friction as well as belt friction accommodated in one problem, only if your concepts are very clear, you would be able to solve this problem.

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Let us attempt to solve this problem here. Now, let me draw the free body diagram of the block. So, I have the weight acting downwards and I have a tension on this string, we do not know what is it is value, it is acting at a height of 0.3 metres. And what is the direction of friction you expect between the block and the plank? It opposes ok, I am pulling it like this. So, friction will oppose and I will have the friction force like this, frictional force.

I have just put frictional force as f but, we know very well from the problem statement we are looking at a condition, when the motion is impending. So, I can also replace it by so on later but, we will have a look at it and when I put the normal force interaction, I deliberately put it at a distance d_1, I do not put it through the centroid because, you have to recognise the normal force is determined by the moment equilibrium. Now, let us write the equations of equilibrium one by one. So, from the free body diagram of the block, I get $\sum F_x = 0$, that gives me $T_1 = f$, $\sum F_y = 0$, that gives me the normal reaction equal to weight of the block, that is given as 588.6 N ($N = W = 588.6\,N$)

So, this is what the justification, at the maximum distance the motion will be impending. So, when the motion is impending, I can write $f = \mu_s \times N = \mu_s \times W = 0.45 \times 588.6 = 264.87\,N$. So, I know what is the tension T_1 acting on the string now, you have to recognise, which is T_1 and T_2, I have already written down as T_1 ok.

So, you have to recognise that, you have to recognise which is T_1 and T_2, when you look at the pulley and before that I also get one more expression so that I get what is the value of d, d_1 is nothing but, 0.3 f, this is anticlockwise and then this is clockwise and $N = W$. So, I get this value as 0.135 metres.

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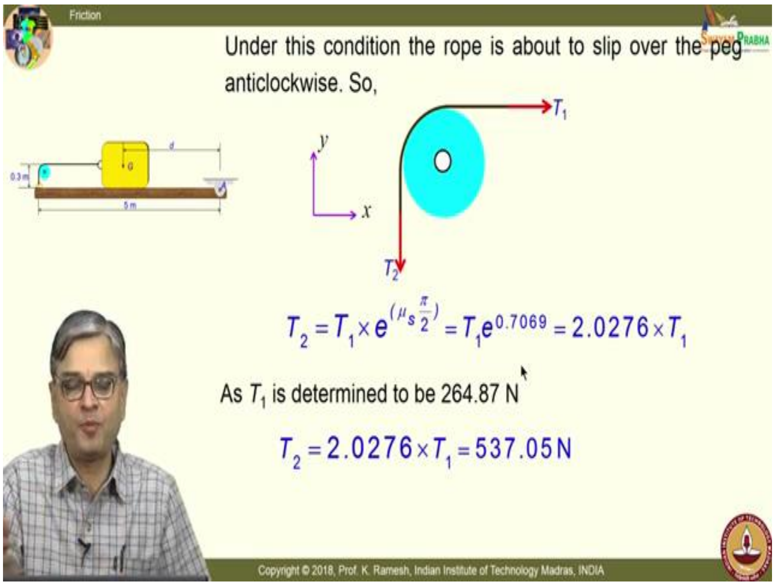

So, under these conditioned the rope is about to slip, this is also given as a problem statement. Assume that all interactions are impending. So, tell me which is T_1 and T_2, think about it. I have already told you that as T_1, I should not have told you that T_1, I should have put that as T and then ask this question anyway. So, I have there this labelled as this is holding and this is pulling force, moment I put this as T_1 and T_2, I can handle the pulley with friction and what is the angle of lap, we can take it as 90^o, that is $\pi/2$. So, I get T_2 as, 2.0276 T_1.

Since, we have already determined what is T_1, I can get what is T_2, T_2 is 537.05 N and now we go back and find out what is the minimum distance d.

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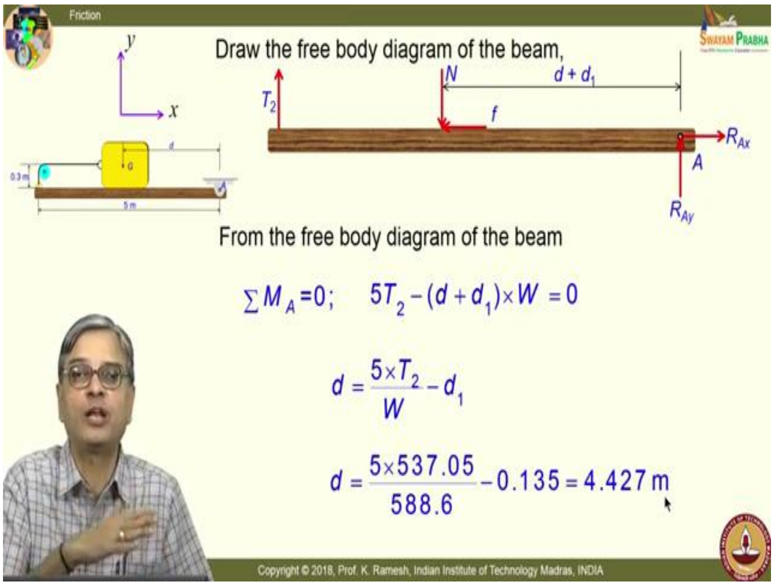

The problem is very interesting in the sense, it accommodates all that what you have learnt in static before we leave statics, I have the pin joint here, replace it by the 2 forces and I have the block, the interaction is a normal force and frictional force like this and you have T_2. So, from this free body diagram, I can estimate what is the value of $d + d_1$. So, that turns out to be, when I say the moment about A, I get this as $5T_2 - (d + d_1) \times W = 0$. So, I get d equal to this expression and finally, d turns out to be 4.427 meters.

It is a very interesting problem it brings in sliding friction as well as rope friction in one problem; it makes you to think carefully and also gives you raining on drawing the free body diagram of the plank so that you have a good revision of your statics. So, in this lecture, we have looked at what happens when the, when there is frictional interaction between the belt and the pulley or the drum, we have determined the expression as $\frac{T_2}{T_1} = e^{\mu\theta}$. We have noticed that size of the drum or the pulley does not come into the expression it is only the angle of lap that is coming into the expression.

And this angle has to be expressed in radians and you have to be very careful in looking at the physics of the problem and decide the frictional direction, as well as label what is T_2 and T_1. T_2 is always higher than T_1, if you do that the problem is completely taken care of and we have seen that this is quite useful in many engineering applications like belt drive and also for a person to climb on a tree and even for the demonstration, I have been able to show a simple keychain weighing about 80 grams to support a water bottle of 1 kilogram.

Thank you.