

Engineering Mechanics
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Module - 2
Dynamics
Lecture - 1
Particle Dynamics

Module 2 Dynamics

Lecture 1 Particle Dynamics

Concepts Covered

When to idealise a problem on hand as a particle or a rigid body for analysis, Review of particle dynamics, Plane motion at constant acceleration, Projectile motion, Solving projectile motion when monkey is in free fall, Swirling of a stone, Terminal velocity, Relative motion, Apparent weight, Inertial frame of reference, Stopping distance of a car.

Keywords

Engineering mechanics, Dynamics, Co-ordinate systems, Unit vectors, Velocity, Acceleration, Projectile motion, Terminal velocity.

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So, we shall move onto the next module of the course on dynamics. I will start with Particle Dynamics it is mostly like a review because you know quite a bit of particle dynamics.

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And you know in this course I have always tried to give an importance to modeling to the extent possible.

Particle Dynamics

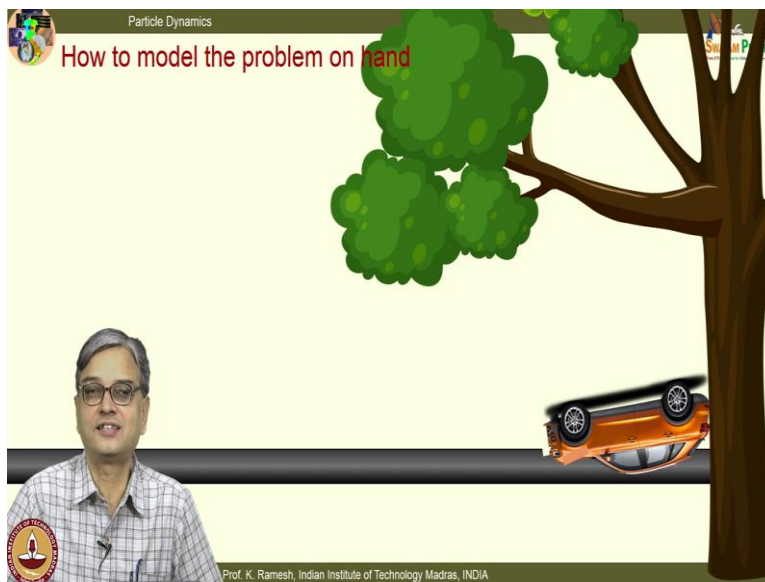
How to model the problem on hand

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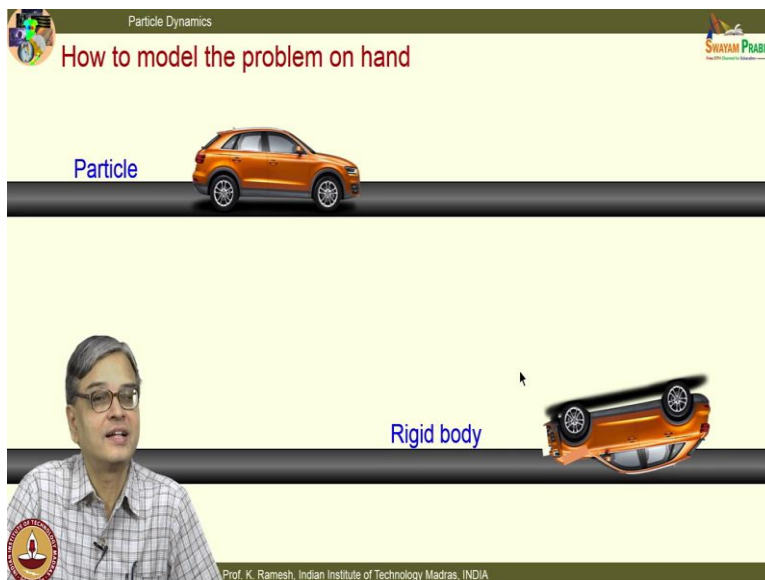
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I have a situation like this the car is traveling in a road, I have another situation. It is very unfortunate that this met with an accident and I would like you to observe what happened to the car ok.



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Now, the question is I have two situations how do I model it for the purpose of analysis the situation one, what is the minimum requirement to model the situation you can always complicate the problem, a complication bringing in is very; very simple engineers have to solve the problem with suitable idealizations. And;



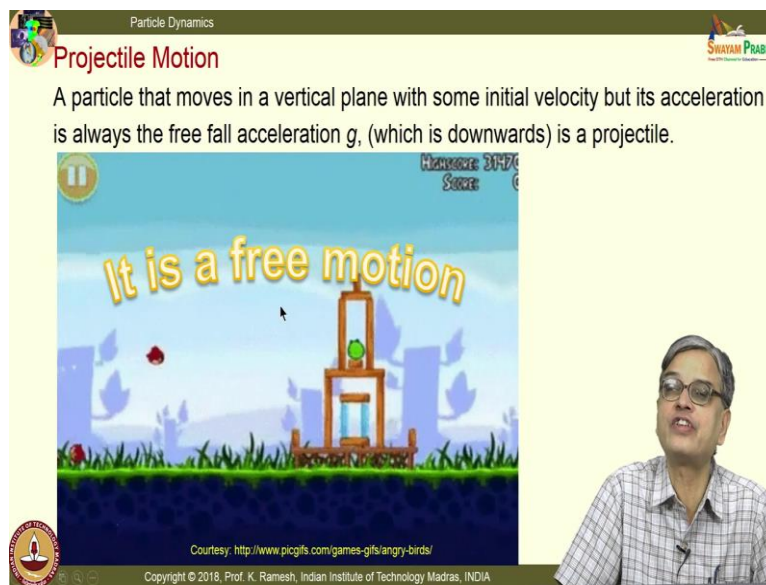
obviously, this is a huge body car you know 4 or 5 people can sit the situation was it was moving in a straight line; how can I model for the purpose of dynamic analysis? Can I model it as a particle? You can model it as a particle that is how we have solved the problems I can model

this as a particle.

Again, the same car that has met with an accident I wanted you to see what happened in the accident; what is it that you have noticed? There is a particular thing happened to the car I brought it out very deliberately, the car was rotating, turning turtle is very common

if we apply brakes very heavily the car can turn turtle. So, what you see there it has a rotation, the moment the object has a rotation you can no longer analyze it as a particle you will have to bring in a minimum of idealizing that as a rigid body.

In an actual situation you know you will also have to bring in the deformability, you must say that this is elastic, you may want to find out a car moving on a speed breaker and we have funny speed breakers here. And you know if you have to find out the design



of suspension system and all other aspects of the car you may have to consider that as a deformable and make your life complicated and have complicated mathematics.

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And one of the simplest things that you come across in particle dynamics is projectile motion, a particle that moves in vertical plane with some initial velocity, but its acceleration is always the free-fall acceleration g is a projectile. You have learned it thoroughly, solve thousands of problems in your earlier exposure, nevertheless we will have a quick overview of it.

And in the case of statics I have said that when I apply a load try to visualize how the load could be applied, that brings in certain amount of engineering visualization. In dynamics, you must visualize when I say it has some initial velocity how the initial velocity has come about, think about it. That will make you what is the real problem situation or when you have a real problem situation you can use that information to model it appropriately, it will help you to understand the problem better.

And you have a fun fact here you have many games although I have not played this game, my student was so enthusiastic he had put this, in this and probably the game developers would have used projectile motion to simulate all this movement of the bird

because the bird here does not have its own energy you are only making it to move and you have to recognize that this is a free motion.

Particle Dynamics

Constant Acceleration

Plane motion (Different forms of Equations)

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$u_x, u_y, v_x, v_y \dots$$

These equations are valid only if the acceleration is constant!

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Let us understand this little more and before we get into it let us see the commonly used equations please write this down even though you know these equations. There are many symbolisms that are used, I can have the velocities represented as v

and u then I have the velocity $v = u + at$. I can also represent it as v and v_0 and I will also bring in the initial distance initial velocity and so on.

Particle Dynamics

Vertical Motion

The horizontal motion and vertical motion are independent of each other.

In both cases $u_y = 0$

$$y(t) = 0 \times t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

$$t = \sqrt{2h/g}$$

So the time taken to reach the ground is the same.

Balls dropped at the same time have same vertical motion and hence reach the ground at same time.

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So, I have a relation $v^2 = u^2 + 2as$ and the distance traveled is $s = ut + \frac{1}{2}at^2$. And in all this case you should note that the acceleration is constant, you have a variety of problems in engineering where you have acceleration is constant, fine. So, here large class of problems can

be handled and you can also have other symbolisms like u_x, u_y, v_x, v_y so many symbolisms people have adopted and you should remember that these equations are valid only if the acceleration is constant.

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Let us understand the vertical motion; I have a particle that is dropped from this height h . Now, I do it differently in the blue diagram, I will drop two balls one ball horizontally, one ball vertically; that means, I given horizontal velocity. Now, you had visualized from where this horizontal velocity can come, we will also solve some problem where I will tell you how this could be visualized fine.

What do you think it will happen when I drop the ball vertically down when I put it like this what happens? You look at the animation what is the striking observation both of them reached the ground down at the same time you can easily explain it from your projectile motion fine. But for a commoner it may be very difficult for them visualize, they will only say if I drop the ball straight it will reach faster, if I throw the ball horizontally it will take more time to reach that is how your normal thinking will happen, your so-called intuition cannot help. And if I look at the analysis you should recognize

Particle Dynamics

Projectile Motion

$u_y = 0$

H

y

x

Range (R)

The horizontal and vertical motions are independent.

$a_x(t) = 0$	$a_y(t) = -g$
$u_x(t) = u \cos \theta$	$u_y(t) = u \sin \theta - gt$
$x(t) = u \cos \theta \times t$	$y(t) = u \sin \theta \times t - \frac{1}{2}gt^2$

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the horizontal motion and vertical motion are independent of each other.

Though the horizontal motion of this ball is different, the vertical motion is identical to the red ball or the blue ball that is dropped straight. You exploit the equation that you have in both the

cases $u_y = 0$ and then I have $y(t) = 0 \times t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$ and get $t = \sqrt{2h/g}$. So, the time taken to reach the ground is the same, it is very important learning when you learn particle dynamics to start with.

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And let us move on to projectile motion and here again recognize the horizontal and vertical motions are independent. I use different symbols for different problems so that you will get accustomed to different symbolisms, people have used different symbols. So, I have a ball which is thrown at a velocity u oriented at an angle θ and this goes in a parabola even before you know the problem you understand all these quantities.

So, I have what is called a range of this projectile, what is a maximum height it can reach and what is the condition when it reaches the maximum height and we can also derive what is the equation of this curve, it is all very simple we are only recapitulating what you have learned already. So, let me write for horizontal motion separately and vertical motion separately. There is no acceleration in the horizontal motion I have acceleration

in the vertical motion that is $-g$ I have given the reference axis this way.

Particle Dynamics

Projectile Trajectory

Substituting $t = x/(u \cos \theta)$ in $y(t)$

$$y(t) = u \sin \theta \times \left(\frac{x}{u \cos \theta} \right) - \frac{1}{2} g \left(\frac{x}{u \cos \theta} \right)^2$$

$$y(t) = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

Equation of a Parabola $y = Ax + Bx^2$

T is time taken to reach ground.

$$u_y \left(\frac{T}{2} \right) = 0$$

$$T = \frac{2u \sin \theta}{g}$$

$$R = x(T) = \frac{u^2 \sin 2\theta}{g}$$

Diagram showing a projectile launched at angle θ , reaching a maximum height H , and landing at a range R . A coordinate system with x and y axes is also shown.

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And I can find out the x component of velocity as $u_x(t) = u \cos \theta$ and y component of velocity is $u_y(t) = u \sin \theta - gt$ and x is given as $x(t) = u \cos \theta \times t$ and y is given as

$$y(t) = u \sin \theta \times t - \frac{1}{2} gt^2, \text{ all of}$$

this you know you have to treat these motions independently. And then you can also go about and getting the expression for the range, for the height, for the time taken etc.

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So, if I substitute $t = x/(u \cos \theta)$ in $y(t)$ I am in a position to get an expression of this curve,

when I simplify this I get the expression for y as $y(t) = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$ and this is nothing, but a second degree curve. So, I have $y = Ax + Bx^2$ this is nothing, but an equation of parabola that is the reason why you put the curve the projectile motion as a parabola. And what are the other understanding that we have?

Let us say capital T is the time taken to reach the ground; that means, from here to this it takes time T and mind you in all these analyses we have not considered the effect of air around it, there is no friction acting on the projectile fine. And it will reach the peak

point at $T/2$, so I have an expression

$u_y(t) = u \sin \theta - gt$ from this I can easily get an expression for what is the

$$T = \frac{2u \sin \theta}{g}$$

time taken

Then I can also find out from the expression for $x(t) = u \cos \theta \times t$. I get the

range as $R = x(T) = \frac{u^2 \sin 2\theta}{g}$. Now I have to find out what is the height H ? So, if you consider the x motion and y motion independently you can derive all these quantities comfortably, there is no great difficulty in doing that.

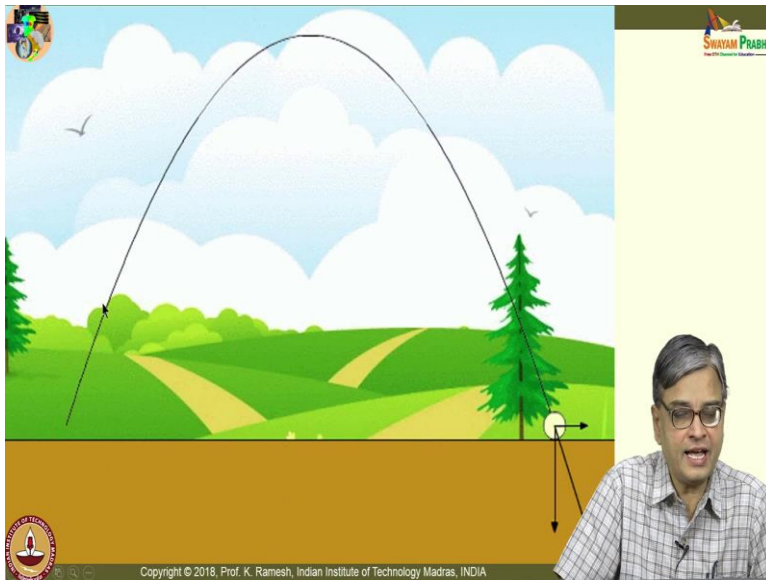
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To summarize I have $u_y(0) = u \sin \theta$, $u_y(T/2) = 0$, $a_y(t) = -g$ and I have this expression, so from this I am in a position to calculate the height maximum height possible is

$$H = y\left(\frac{T}{2}\right) = \frac{u^2 \sin^2 \theta}{2g}$$

and we have already got the expression for T and also the expression

for R . What is the range, what is the time taken, what is the maximum height reached? And this is one of the first things you do when you learn particle dynamics and this has many applications in day to day life right from cricket onwards you have many

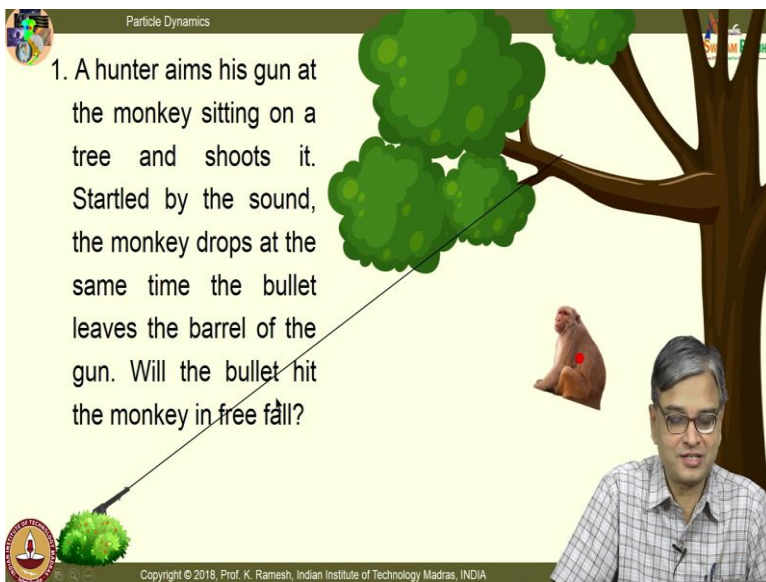


applications that is the reason why you have learnt all of this.

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And this is a very nice animation it summarizes visually what happens in a projectile, see I start with a velocity and look at here the vertical component goes to 0 only horizontal component remains and another one for the entire motion, the horizontal component remains same.

Visually you understand see, visual understanding and appreciation will stay in your system much longer than just mathematical expressions and when it comes and hits the ground it is not coming and sitting it is coming and hitting with a velocity, it is not coming and simply hitting the coming and



landing on this with 0 velocity ok. So, this gives a nice appreciation of what happens to a projectile.

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And now we will solve some interesting problems, these are all fictitious problems to bring in some life you attach it to some kind of idealistic situation and you know the monkeys in IIT are very, very intelligent fine. So, if there is a gunshot this fellow will go and hide that is a natural instinct when there is some danger you would like to find out the least possible distance where you can go and hide.

But, in the problem, it states startled by the sound the monkey drops at the same time the bullet leaves the barrel of the gun, so it is fictitious fine; that means it has a free fall. The question is if the monkey has a free fall will the bullet hit the monkey. That is all we

Particle Dynamics

For monkey:

$$s = vt + \frac{1}{2}gt^2 \quad s = \frac{1}{2}gt^2$$

$$y(t) = h - s$$

$$y(t) = h - \frac{1}{2}gt^2$$

For bullet:

$$x = u \cos \theta \times t$$

$$\tan \theta = \frac{h}{x} = \frac{h}{u \cos \theta \times t}$$

$$h = u \sin \theta \times t$$

$$y(t) = u \sin \theta \times t - \frac{1}{2}gt^2$$

$$y(t) = h - \frac{1}{2}gt^2$$

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have to investigate fine. So, the question is if it falls will the bullet come and hit the monkey? A very interesting problem and you can analyze it comfortably.

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First thing what you learn is when you aim the monkey and put your gunshot it is not going to go travel straight, look these people who are not trained in mechanics will not be able to visualize, they will only think that it will go straight because of acceleration due to gravity it has a projectile motion fine. Now, we have fictitious situation. The moment it heard it sound it starts falling down freely. So, this blue this green falls down straight and this red ball follows the projectile motion.

So, let us look at what happens to the monkey, I have a coordinate system like this for

the monkey and I write the motion $s = vt + \frac{1}{2}gt^2$ and it starts from $v = 0$, so I get $s = \frac{1}{2}gt^2$ and I can also write this s with respect to the ground that is the height y which is nothing,

but $y(t) = h - s$. So, I have an expression at any point in time t , the height of the monkey

from the ground is $y(t) = h - \frac{1}{2}gt^2$.

Now, what we will have to see is what is the height of the gunshot which follows the projectile motion from the ground if these two heights match then there will be a hit on the monkey it is a fictitious problem, but very interesting to learn concepts related to

projectile motion. So, for the bullet, I can write the motion as $y(t) = u \sin \theta \times t - \frac{1}{2}gt^2$ mainly because it gets released from the gun with an initial velocity. Whereas, the monkey drops down with 0 initial velocity.

And what are the other things that I know $x = u \cos \theta \times t$ and from the geometry I can get

$$\tan \theta = \frac{h}{x} = \frac{h}{u \cos \theta \times t}$$

. So, when I go and replace this here, I get this as, I get from this simplification I can get $h = u \sin \theta \times t$. I will replace this here I find the height of the bullet

from the ground is $y(t) = h - \frac{1}{2}gt^2$, at any point in time no matter what is the distance between the gun and the monkey whichever distance from the strength of our mathematics provided there is a free fall, provided that the monkey has good hearing and

there is no friction, there could be a possible hit that is what your mathematics says. So, it is a very interesting, but fictitious problem.

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And now you have another interesting problem which relates your circular motion and projectile

motion, this you might have played in your school days. A boy whirls a stone in

Particle Dynamics

2. A boy whirls a stone in a horizontal circle of radius 1.5 m and at a height of 2 m above ground. The string breaks and the stone flies off horizontally and strikes the ground after travelling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone in circular motion?

2 m

10 m

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horizontal circle of radius 1.5 meter and at a height of 2 meters above the ground. The string breaks and the stone flies off horizontally and strikes the ground after traveling a horizontal distance of 10 meters, somebody has done the measurement and given it to you.

So, this gives you when I say something moves with an initial velocity you get an idea I have the swirling of the stone like this and suddenly the string snaps and it comes with an initial velocity, when I have a gun, you have a gun powder which provides the, which bursts and provides the initial velocity to the bullet and here the initial velocity comes from circular motion.

Particle Dynamics

1.5 m

2 m

10 m

As the horizontal and vertical motions are independent, it can be solved easily.

The ball falls from a height of 2 m.

$$h = ut + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2 \times h}{g}}$$

Time taken to reach ground $t = \sqrt{\frac{2 \times 2}{g}} = 0.64 \text{ s}$

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You know in many of the problems that we have solved I have consciously taken effort to bring in the physical reality to the extent possible to aid your visualization fine. How do I go and solve the problem? I have a circular motion initially and I have

a projectile motion to follow. And you know if you look at a problem in dynamics, there are many; many ways of solving it, there is no one unique method to solve the problem.

The method which is comfortable to you should adopt and then solve and we will solve it from first principles from the simplest way of handling it and I can look at the horizontal and vertical motion as independent and investigate what is the height the ball has to fall

on the ground that is $h = ut + \frac{1}{2}gt^2$ and I get $t = \sqrt{\frac{2 \times h}{g}}$.

So, the time taken to reach the ground when I substitute the quantities this turns out to be 0.64 seconds. So, I have looked at the vertical motion initially and I have calculated the time. And let me look at the horizontal motion next, see before I go into this I would like

to caution the reason why I said from where the initial velocity comes, when you want to coin a problem you simply say there is an initial velocity, but you should also visualize in which of the applications you can get that initial velocity.

See one of the commonest things that I find is you know young mothers when they bring their children to the school in IIT they come at a terrific speed and the children are not clinging onto the vehicle or the mother firmly when they come. Imagine if there is some obstruction and the boy or the girl travelling in the scooter falls off, they do not realize it can be a cause for danger ok. If you are travelling at slow speed of 20 km/hr when you dislodge yourself you have 20 km/hr.

When you are firmly fixed to the body nothing happens to you that is the reason why they want you to wear seatbelt in a car or when you are sitting in a scooter you should put the child properly see elders will know how to manage, but children will not know how to cling onto it and if you really observe the scooters children are not kept in a safe position. And if you go abroad there are regulations on what should be the child seat, how they should be secured, they will be never allowed to sit in the front seat of a car, a car is much safer than a two-wheeler fine.

So, as educated students when you have the role of parenting please apply your mechanics understanding and handle them safely, it is very unsafe what you see on the common ground is very unsafe and you should also realize there are thumb rules which says if you are travelling at a speed of 20 km and have a fall, it is equivalent to falling from two-story building.

You realize when you go to a storey building you will not jump off, you know that it is very dangerous, on the other hand, you think 20 km is not a very great speed and 40 kilometers is like falling from fourth floor. So, do not make your life very uncomfortable, you learn mechanics bring in that understanding in your day to day dealings fine it is not for just learning and getting the grades it should percolate into your lifestyle where it is needed.

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So, consider the horizontal motion I get $vt = 10$ and $v = \frac{10}{t} = 15.63 \text{ m/s}$; and now I have this as a circular motion, I can easily find out what is the centripetal acceleration

$$a = \frac{v^2}{r} = 163 \text{ m/s}^2$$

Particle Dynamics

Considering horizontal motion

$$vt = 10$$

$$v = \frac{10}{t} = 15.63 \text{ m/s}$$

Centripetal acceleration

$$a = \frac{v^2}{r} = 163 \text{ m/s}^2$$

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. So, it is very simple, but it is also very interesting. So, always visualize when I say there is an initial velocity; what is the source of that initial velocity? That will make you a better engineer because you start visualizing and then relate it to physical situations

fine.

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Particle Dynamics

Another Method

$$v = \sqrt{\frac{g}{4}} \cdot 10 = 15.63 \text{ m/s}$$

From projectile path

$$y(t) = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

Substituting $\theta = 0$ and $u = v$

$$-2 = 0 - \frac{g \cdot 10^2}{2v^2}$$

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I can also solve this by another method with just one equation provided I know the equation which I would not recommend because you do not have to remember anything, but here I should know the projectile path, if I know the projectile path and I

substitute the value I get the velocity straight forward I get the same answer, I cannot get any different answer. There are multiple ways that you can solve a given problem. Choose a method that is comfortable to you, I would always prefer a method where I start from the first principle where I do not have to remember anything fine. So, that is my recommendation to you.

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Particle Dynamics

Influence of Air Resistance

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So, far we have never looked at the air resistance; what happens when there is air resistance? I am not going to solve it mathematically, but I would give you an idea what would happen to the range; obviously, the range will get shortened and that needs to be accommodated

when you are designing a missile you want to hit your enemy. You should calculate all of this an intercontinental ballistic missile you need to even bring in relativity theory and then calculate.

Particle Dynamics

Terminal Velocity

SHYAM PRABHA

When an object starts falling, its velocity is zero to start with.

It gains velocity as it falls due to gravitation.

Due to air resistance, drag force develops and it increases due to increase in velocity.

mg

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If they travel for 500 km one thing if they travel for 20,000 km the story is different, you cannot apply the same mechanics for short distance and long distances. So, there is all the more important for you to learn mechanics.

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And let us look at another situation what happens we have seen when I drop a ball because of acceleration due to gravity it acquires speed. Now, I do not drop the ball at short distance I go to a very tall building and drop the ball. What happens? So, you have the concept of terminal velocity, you should thank God; God has created all of this it is

not that anything you drop will acquire infinite velocity, ok, the velocity cannot reach beyond a particular point.

When an object starts falling it starts with a 0 velocity, it gains velocity as it falls due to

Terminal Velocity

When an object starts falling, its velocity is zero to start with.

It gains velocity as it falls due to gravitation.

Due to air resistance, drag force develops and it increases due to increase in velocity.

At one point of time the drag force equals the gravitational force and the body reaches equilibrium.

$F_d = \frac{1}{2} \rho v^2 A C_d$

$F_d = \text{Drag force}$
 $C_d = \text{Drag coefficient}$
 $v = \text{Velocity}$
 $A = \text{Projected area}$

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gravitation, but you should also look at air resistance. See in engineering we bring in features when they are significant when you have learnt the projectile motion there was air around you; you are not done that experiment in vacuum. When it is not significant you do not

bring in into your modeling, when it is significant you bring it into your modeling and then understand what happens.

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Terminal Velocity

$\sum F_y = 0$

$F_d = \frac{1}{2} \rho v^2 A C_d = mg$

Since the body is at equilibrium, it falls at constant velocity and hence the drag force remains same and equal to the gravitational force.

$v_t = \sqrt{\frac{2mg}{\rho A C_d}}$

$v_t = \text{Velocity of body at this point}$

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Drag resistance develops and you have a drag force F_d and that is given as $F_d = \frac{1}{2} \rho v^2 A C_d$ where C_d is the drag coefficient depending on which fluid you are moving in and A is the projected area, v is the velocity and this keeps increasing. So, it reaches a situation that it balances

the weight of the object, at one point of time the drag force equals the gravitational force and the body reaches equilibrium.

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That is what you see here $F_d = \frac{1}{2} \rho v^2 AC_d = mg$ and I get this final expression and I can also find out what is the limiting velocity, it will move at a constant velocity downwards and

this is $v_t = \sqrt{\frac{2mg}{\rho AC_d}}$.

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So, this is the velocity of body at this point and you have called this as terminal velocity, people also have calculated for different situation what is the terminal velocity possible

Object	Terminal velocity (m/s)	95% DISTANCE* (m)
Shot (as in shot put)	145	2500
Skydiver	60	430
Baseball	42	210
Tennis Ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop	7	6
Parachutist	5	3

* This is the distance through which the body must fall from rest to reach 95% of its terminal speed

Courtesy: Fundamentals of Physics, Halliday, D. Resnick, R. Walker, J., John Wiley & Sons (P) Ltd
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and if you take a shot put it can have terminal velocity of 145 meters per second it takes 2500 meters for it to achieve this, 95 % of the velocity for it to reach. And for a skydiver he will have the highest velocity of 60 meters per second you need to be trained to be a skydiver, everybody cannot do skydiving.

Because now you have all these sports have come in abroad you have skydiving also possible, people go for skydiving and come back and if your health is not alright then do not try out all of this.

There is a difference between skydiving and a parachutist; parachutist has a velocity of only 5 m/s, whereas a skydiver he will have a terminal velocity of 60 m/s and your body should be strong enough to withstand, it is constitutional strong does not gain age. You can have our honorable past president doctor Abdul Kalam travelling in the jet airplane and comeback safely at the age of close to 75 or 76, even as young boy can have a problem ok, so do not try out skydiving without knowing this.

And another interesting thing is the raindrop, see all of this are falling at some known height raindrop from the clouds it comes, imagine it acquires velocity and then you will have injections you will not go out in the rain you cannot have a nice film song in rain fine ok. Thankfully it has a terminal velocity of only 7 m/s and it acquires this with a very short distance of 6 meters ok.

Particle Dynamics

SHANTANU PRABHA

Relative Motion

The plane moves at an acceleration a in the x -direction.

Velocity of plane = $v_p = u + at$

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See, in dynamics what you do is in problems where you can see absolute motion you try to solve based on absolute motion that is what we have seen earlier, but a variety of practical problems you can handle if you understand the concept of relative

motion fine and you know you had recent floods in Kerala, so our air force was dropping food to all of these people. The question here is for an observer on the earth he would see the packet following a parabolic path because it follows the projectile motion. What would be the motion perceived by the person sitting in the aircraft that is what we have to find out ok.

Let us understand the concept of relative motion. So, I have given the axis like this the particle is labeled as A and this moving object is labeled as B and let us analyze the motion and it is given in the problem the plane moves at an acceleration you are given the acceleration a in the x direction it is not moving with constant velocity. It is picking up speed because it has to go from one place to another place fine fast enough. So, the velocity of plane is $v_p = u + at$.

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And velocity of body in x direction is because at this point when it dropped it has a same

Particle Dynamics

Velocity of body in x-direction = $u_x = u$

Velocity of body in x-direction w.r.t plane = $u_{x/p} = u - v_p$
 $= u - (u + at) = -at$

Position of body in x-direction w.r.t plane = $x = - (1/2)at^2$

Velocity of body in y-direction = $u_y = gt$

Velocity of body in y-direction w.r.t plane = $u_y = gt$

Position of body in y-direction w.r.t plane = $y = (1/2)gt^2$

$y + \frac{g}{a}x = 0$
 Equation of a **Straight Line**

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velocity like the plane. So, this is where I said from where does the initial velocity come if it is dislodged, from the moving object it has a same velocity as that ok.

So, velocity of body in x direction is $u_x = u$ and velocity of body in x direction with respect to

plane $u_{x/p} = u - v_p$ and substitute this relevant expressions I get this as $-at$ and position of body in x direction with respect to plane because I want to find out what happens to this with respect to plane, the x position is given as $x = - (1/2)at^2$ and I can also find out

Particle Dynamics

Relative Motion

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velocity of body in y direction $u_y = gt$ and velocity of body with respect to vertical plane is same there is no change because it is moving in the same height.

Position of body in y direction with respect to plane is $y = (1/2)gt^2$, now I

combine these two expressions I get an equation as $y + \frac{g}{a}x = 0$. So, this is nothing, but a straight line.

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So, if a person sitting in the air plane he would see the object falling as a straight line this is what he would feel. So, this is just to give you an idea on relative motion.

Particle Dynamics

Apparent Weight

Spring scale shows only the force that it exerts on the man.

$$\Sigma F_y = ma_y$$

$$R - mg = ma$$

$$R = m(g + a)$$

Man in the lift

Apparent weight shown by the scale is higher!

Comment what will the spring scale show when the lift is coming down and decelerating.

Accel a

Spring scales

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Then we move onto how to apply Newton's law, you will also have to appreciate inertial frame of reference and lifts are very; very common these days and here you have a few stories and if you go to New York you have 75 stories, 100

stories. So, you really travel for a very, very long distance and they have very efficient lifts which will cover 60 floors in 10 seconds or 15 seconds and so on fine.

So, there you will definitely feel that there is an accelerations, steady motion and deceleration all that you will have an opportunity to feel for it, in a short distance of two floors or four floors you may not feel that. Now, what I do is I have a spring scale in the lift and the man is standing on it and let us analyze what happens to this air as again we just model this a particle and I have this weight acting downwards. And I have the reaction from the floor; the floor is here the spring scale the idea is we can also note down what is the reading of the spring scale, is the diagram complete?

This is that you have to make a difference between statics and problems dealing with dynamics. The particle is having an acceleration a you have to indicate that very clearly only then the diagram is complete fine and I am going to write the Newton's law standing on ground I am not going to stand inside the lift and then write it, here I can say

$F = ma$ fine. I can write the motion is in the y direction, so I can put $\Sigma F_y = ma_y$ and from this I get $R - mg = ma$ and what is the surprising answer you are getting I get

$R = m(g + a)$. What would the spring scale show as the reading on it? It will only show the reaction force.

So, you will find the weight of the man is more than what he is actually is because the spring scale shows only the force that it exerts on the man. So, the apparent weight shown by the scale is higher. So, do not get annoyed when you are on weight reduction because that is a common thing that is happening, governments are putting levies on people who are overweight and you are on a regime of dieting and then you find that you are overweight in a fun do not be annoyed, you can also be underweight when you are travelling in a lift ok. So, now, that you comment and think about it what will spring

scale show when the lift is coming down and decelerated ok.

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Apparent Weight
 The lift has reached a steady speed of V .
 What the spring scale will show?

$$\Sigma F_y = ma_y$$

$$R - mg = 0$$

$$R = mg$$

Apparent weight is same as the actual weight!

Man in the lift

Let us move onto another situation, it has come to the this level, then it is moving at a constant velocity imagine that I am going for 100 floors in rock feller tower and let us write the

free body diagram I take this as a particle I have reaction R I have mg . Now I recognize that this is moving at a constant velocity, there is no acceleration we have learnt from Galileo's principle of relativity moving at a constant velocity or stationary one and the same. So, in this case I will have $\Sigma F_y = ma_y$ and I have $R - mg = 0$ and I get the reaction equal to the actual weight.

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Now, let me think of third situation it is coming down accelerating, let us see what happens, I put the free body diagram. I have this. The diagram is not complete is should also put in the acceleration until then this diagram is not complete, we are dealing

Apparent Weight

Spring scale shows only the force that it exerts on the man.

$$\Sigma F_y = ma_y$$

$$R - mg = -ma$$

$$R = m(g - a)$$

Apparent weight shown by the scale is lower!

Comment what will the spring scale show when the lift is going up and decelerating.

Man in the lift

Accel a

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problems in dynamics. Then simply apply these expressions $\Sigma F_y = ma_y$; $R - mg = -ma$ because the acceleration is in the opposite direction of the coordinate axis, so I get this as $R = m(g - a)$.

So, if I or on a weight reduction regiment do not be happy that your weight is low when you are coming down in the lift, you have to look at the actual weight and then decide about it. Apparent weight shown by the scale is lower and now comment what will the spring scale show when the lift is going up and

decelerating.

Apparent Weight

What happens when the cable snaps?

$$\Sigma F_y = ma_y$$

$$R - mg = -mg$$

$$R = m(g - g)$$

Apparent weight shown by the scale is zero!

Man in the lift

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Let me move onto another unfortunate situation that the lift is snapped, so it is falling at what acceleration and you know now all this space agencies, they want to generate more resources, they want to send people and feel 0 gravity, so millionaires can fund some of their space programs. Early only astronauts were doing it, you get something similar to this in this when I put the equation

and you know now all this space agencies, they want to generate more resources, they want to send people

$\Sigma F_y = ma_y$ and when I put the free fall acceleration, I get an answer the reaction is 0 ok. So, you feel weight less apparent weight shown by the scale is 0.

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Particle Dynamics

Importance of Inertial Frame of Reference

Spring scale shows only the force that it exerts on the man. a

$\Sigma F_y = ma_y$ Newton's law applied from the Inertial reference.

$R - mg = ma$

Net force acting on the man is $(R - mg)$ for any observer.

Man in the lift

Can this be equated to the mass times acceleration of the lift as perceived by an observer in the lift?

For an observer in the lift acceleration is a

No! One will get absurd answers as the lift is accelerating and not an inertial frame of reference.

Spring scales

Accel

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Now, let us go back and analyze from a different perspective. Now, I have an observer inside the lift noting down what happens in the spring scale, it is going up and accelerating fine. We have already seen this diagram earlier, the same diagram and I will also have to put the

acceleration and $\Sigma F_y = ma_y$. I can apply only from inertial frame of reference, when I am standing on the ground it is the initial frame of reference it is not having an acceleration. I can write this expression $R - mg = ma$ and the spring scale shows only the force that it exerts on the man.

However, for an observer standing outside, for an observer standing inside the lift the expression for net force is again mathematically $(R - mg)$ we are not qualifying what is the value R what is the value of this one, this is same for the observer inside the lift also. Can the person inside the lift use this and write the Newton's law? Because this where people make a mistake see you will have to understand when to apply Newton's law, you can apply only in an inertial frame of reference. For an observer going in a lift with an acceleration does he perceive any acceleration? He will perceive no acceleration. He cannot write the Newton's law there.

One will get only absurd answers as the lift is accelerating and not an inertial frame of references very; very important for you to note ok. So, for an observer in the lift acceleration is 0, so apply the Newton's law very carefully. See this what we are going to

learn, how do we analyze relative motion from various perspectives for a rigid body,

then move onto kinetics where we apply the Newton's law properly fine.

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Particle Dynamics

Stopping Distance depends on friction

$v^2 = v_0^2 + 2a(x - x_0)$
 $0 = v_0^2 + 2ad$
 $d = -\frac{v_0^2}{2a}$

Minimum stopping distance
 $d_{\min} = \frac{v_0^2}{2\mu_s g}$

Maximum value of a is when $f = f_{\max}$
 $f_{\max} = \mu_s N = \mu_s mg$
 $\sum F = ma \Rightarrow -f_{\max} = ma_{\max} \Rightarrow -\mu_s mg = ma_{\max}$
 $a_{\max} = -\mu_s g$

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And we would also see one more interesting application because in India the roads have become very good now and also have very

powerful cars. And often we hear accidents happens by young drivers fine, you learn mechanics please apply your mechanics when you drive.

And you have to recognize when the car has to come to a stop, you need a finite distance though in advertisement they say that I have a good braking system it is going to be instant do not believe all of that. People are developing; developing definitely braking system you have ABS so many parameters people are finding it out so that it does not become a turtle all that they are doing it let them do all of it.

And if you look at this and I can analyze this as a particle. So, I get the distance required for it to stop is a function of the velocity and I look at this I have normal reaction, I have this weight of the car. And you have a frictional force generated because of your braking action between the tire and the road and this has an acceleration like this it is actually deceleration, but we do it like this, but mathematics will help me to get the correct sign. So, this happens when f is f_{\max} .

So, $f_{\max} = \mu_s N = \mu_s mg$ and I am in a position to get the expression which relates the distance as a function of the quantities finally. The minimum distance required is a function of the velocity as well as the coefficient of friction.

See now, we are having rains in Chennai you have to be very careful when you driving because of water the friction decreases fine and what you find is if the friction decreases by half the d_{\min} increases by two times.

Particle Dynamics

Stopping Distance depends on friction

$$d_{\min} = \frac{V_0^2}{2\mu_s g}$$

Generally μ_s between tyre and dry road comes around 0.8

V_0 (kmph)	d_{\min} (m)
40	7.87
50	12.29
60	17.7
70	24.08
80	31.46
90	39.82
100	49.16
110	59.82

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So, you have to understand there is a minimum stopping distance that is required. So, do not drive the vehicle on a rainy day carelessly. You also have a table which shows I do not think any driver in India maintains when they travel

at 40 kilometers per hour a distance of 7.87 meters. So, accidents are waiting to happen, in many of the industries they will say accidents do not happen they are caused ok, so this one way of bringing in safety.

So, you have to recognize, you have to maintain a respectful distance on the road in case of any eventuality you need to brake your car, you often hear in the Yamuna expressway you have a pile of cars, when somebody applies a break you have 100 cars come and hits which used to happen in the west occasionally. Now it is happening in India that is why you need to have all these cars to have crash worthiness. Earlier we were travelling at very slow speed, we were not travelling at very high speed.

So, once you are travelling at a high speed you have to bring in airbags for your protection, seatbelts for your protection and as engineers when you drive the car recognize that there is a minimum stopping distance which is the function of the velocity as well as the friction coefficient of friction and the friction drops when you have a water on the ground because of rain. So, you educate yourself and educate your friends and relatives and have a safe driving.

So, in this class we have looked at an overview of particle dynamics and we find a variety of situations can be modeled as a particle and we have also looked at what is

known as terminal velocity and we have also looked at how to apply the Newton's law when the lift is accelerating or decelerating. And finally, we have also looked at what is a minimum distance for a car to stop which is very; very essential particularly India nowadays because we have very good roads and very high powerful cars.

Thank you.