**Engineering Mechanics**
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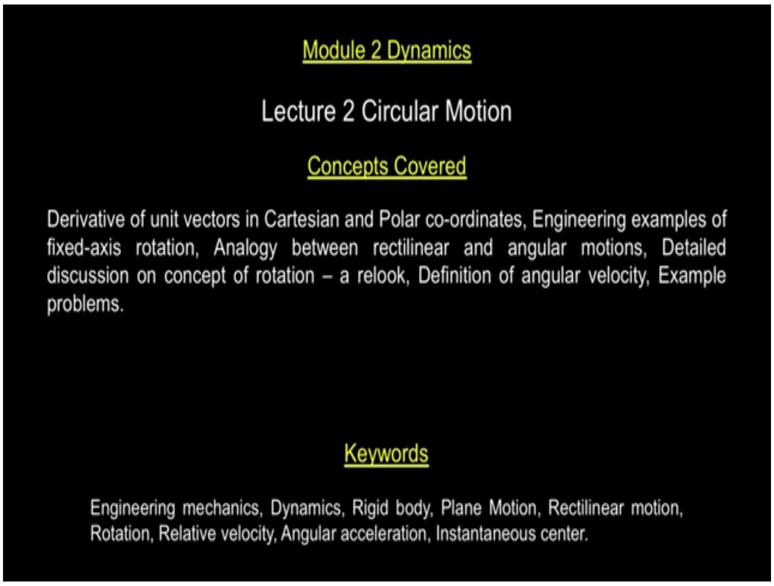
**Module – 02**
**Dynamics**
**Lecture – 02**
**Circular Motion**

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So, let us move on to Circular Motion.

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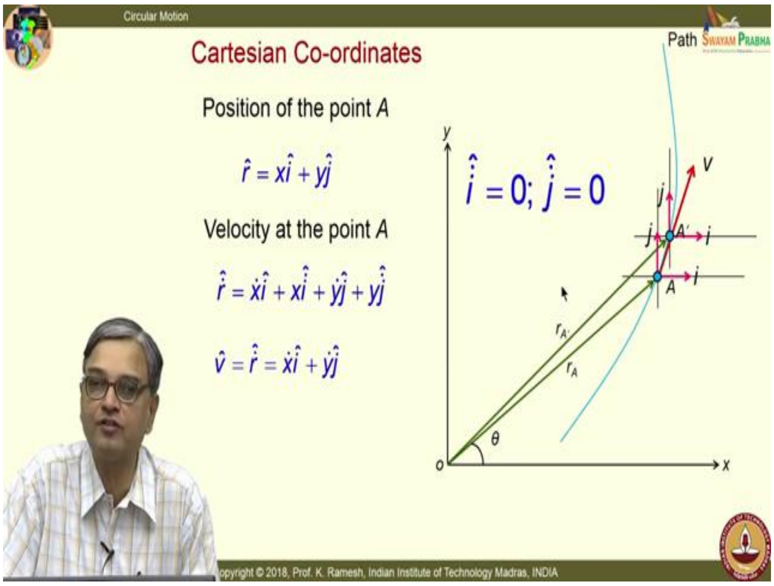
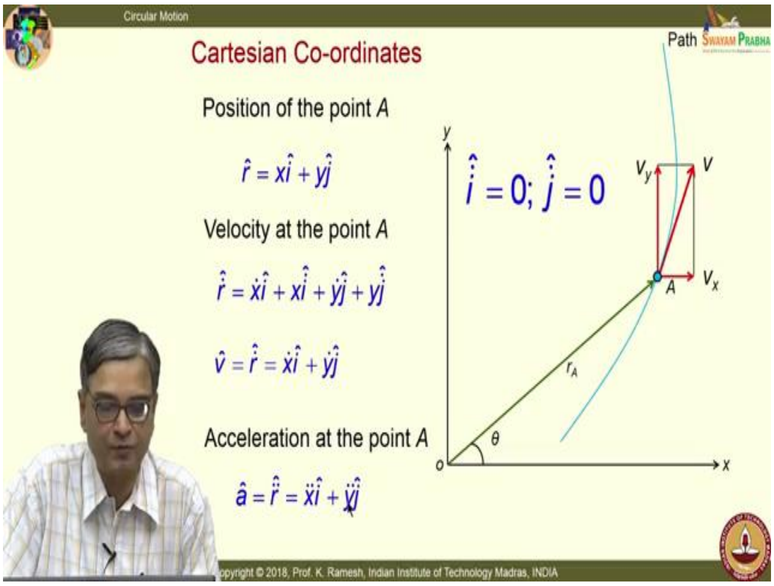
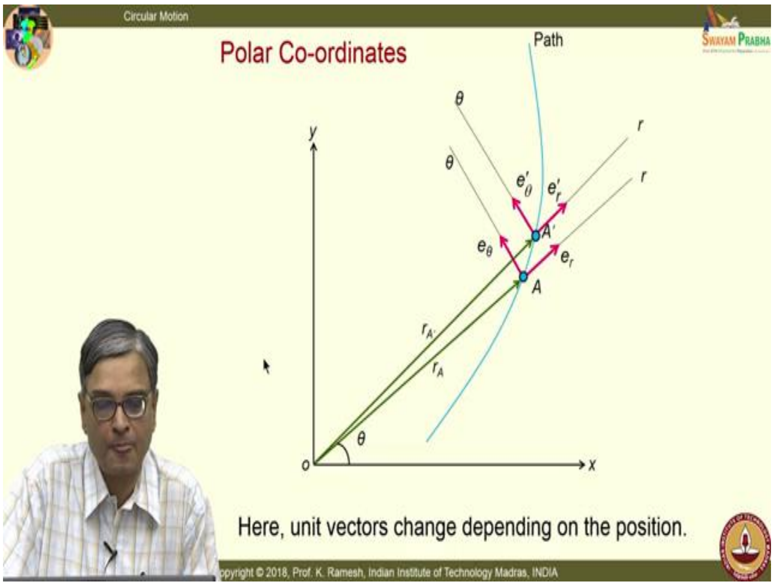
And let us take a particle moving along an arbitrary path. And let us look at quantities from a Cartesian coordinates system. I have the position vector for the particle and I can write the vector as $\hat{r} = x\hat{i} + y\hat{j}$. Now let me find out the velocity I will do the differentiation of this; when you do the differentiation, apply the product rule properly and write the all the quantities, do not leave any quantity untouched.




So, when I do that, I get this as $\dot{\hat{r}} = \dot{x}\hat{i} + x\dot{\hat{i}} + \dot{y}\hat{j} + y\dot{\hat{j}}$ and let us look at what happens when the particle has moved by a small distance to a position A'. I have the new position vector as $r_{A'}$ and let us look at what are the base vectors at point A and point A'. We are looking at the Cartesian coordinate system, when I do that at point A; I get this as *i* and *j* at point A' also I have this as simply *i* and *j*. Please make a neat sketch of all this even though you know these concepts earlier; clarity is required for further development of the subject if you have clarity then you do not need any preparation for your examinations.




So, by virtue of they being identical I can write $\dot{\hat{i}} = 0; \dot{\hat{j}} = 0$, this makes our life very simple in Cartesian coordinate system. So, I simply write the velocity as $\hat{v} = \dot{\hat{r}} = \dot{x}\hat{i} + \dot{y}\hat{j}$ and when I want to get the acceleration, the differentiation become simple because we already know $\dot{\hat{i}} = 0; \dot{\hat{j}} = 0$ that has made our life very simple.

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And I have the vector *v* can be looked at as components $v_x$ and $v_y$, then again acceleration as simply $\hat{a} = \ddot{\hat{r}} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$. So, this is fairly simple you have no difficulty at all and this is what we have learnt it in your earlier exposure also.

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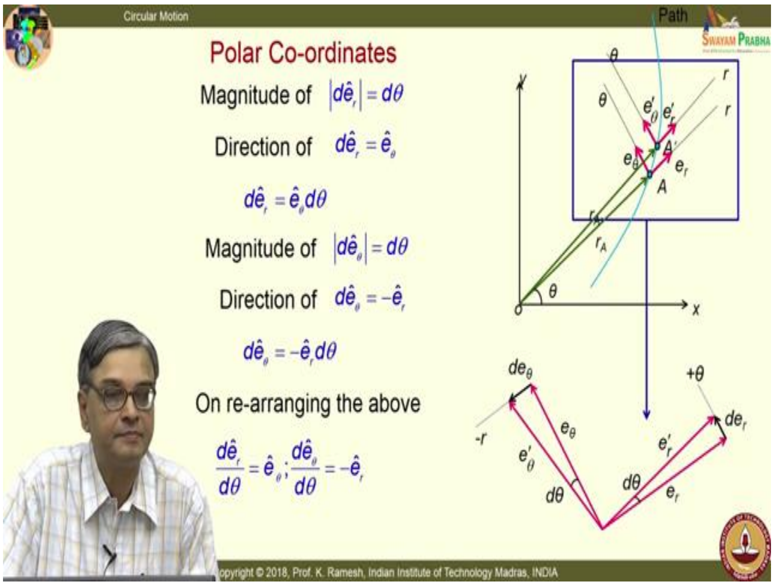
Now, let us move on to polar coordinates you know I make my animation slow; the reason is that you also make a neat sketch of what we discuss. I have an arbitrary path, I have the particle A, again I put the position vector, then I also draw the coordinate axis there I have *r* direction like this and *θ* direction like this and let me put the unit vectors on this direction as $e_r$ and $e_\theta$.

So, make a neat sketch of it make a big sketch so that you have clarity and what we discussed. Then I move from position A to position A' separated by a distance *d*θ, I have moved to *A*'. So, I have this as $r_{A'}$ the distance as the position vector is denoted as $r_{A'}$ and here again, I can draw the *r* coordinate system and *θ* coordinate system when I have the polar coordinate system, let us look at how this coordinate change from position to position.

So, I draw *a* extension of line $r_{A'}$ , I get the r axis perpendicular to the I put *θ*. And the diagram is drawn so big for you to visualize unlike what we saw in the Cartesian coordinate system. If I put the unit vectors $e_{r'}$ and $e_{\theta'}$, they are not identical like what we had seen in the Cartesians coordinate system. It remains i and j when I moved from point to point, but here it changes from $e_r$ to $e_{r'}$; they are not parallel there is distinct difference and we will analyse these differences ok. Unit vectors change depending on the position; this is what you have to recognise.

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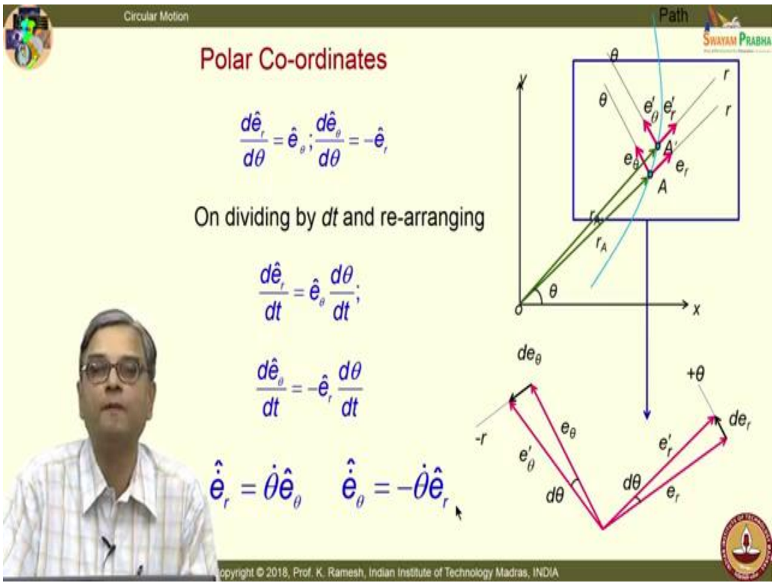
So, when I recognise this I would also go and put these with a much bigger magnification. So, I have $e_r$ it has changed to $e_{r'}$ and $e_\theta$ has changed to $e_{\theta'}$ and let me complete this vector diagram. So, this is the direction $\theta$ this is $+\theta$ and when I put a perpendicular to this, this becomes $-r$. So, that is very necessary for our discussion later and I would call this change as $de_r$. So, $e_r + de_r$ gives me $e_{r'}$, $e_\theta + de_\theta$ gives me $e_{\theta'}$.

Now, look at how do we designate the magnitude of $de_r$; mind you that these are unit vectors. So, when I say that these are unit vectors the magnitude simply turns out to be $de_\theta$, from the diagram it is very clear and we are discussing about $de_r$ and what is this direction it is along this direction. So, the unit vector is $e_\theta$.

Now, let us look at $de_\theta$ before that I have written it $d\hat{e}_r = \hat{e}_\theta d\theta$. Let us look at similarly the magnitude of $de_\theta$ that also turns out to be simply because these are unit vectors, I get this simply as $d\theta$ and what is this direction? It is clearly given in the figure. So, you can write the direction as $-e_{r\theta}$ is what is you have to be careful about; this $-e_r$ and I can write this as $d\hat{e}_\theta = -\hat{e}_r d\theta$. So, let me rearrange the terms above and I can write $\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta ; \frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r$.

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And let us move on and divide them by d t and rearrange it because we are interested as a function of time in dynamics. So, when I do that; I get $\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta ; \frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r$ $d\theta/dt$ which could be simply written as $\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta$ and $\dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r$; it is a very important relationship.

Because what you have to recognize is when we move on to polar coordinates which becomes convenient in certain class of problems. You have to leave it with polar coordinates, you cannot always leave it Cartesian coordinate, certain problems are easy to handle in a polar coordinate system, certain other problems are easy to analyze in n t coordinate or intrinsic coordinate system. So, when you use intrinsic coordinates or polar coordinates in both of these the unit vectors change, only in Cartesian coordinate system the unit vectors remain unaltered ok.

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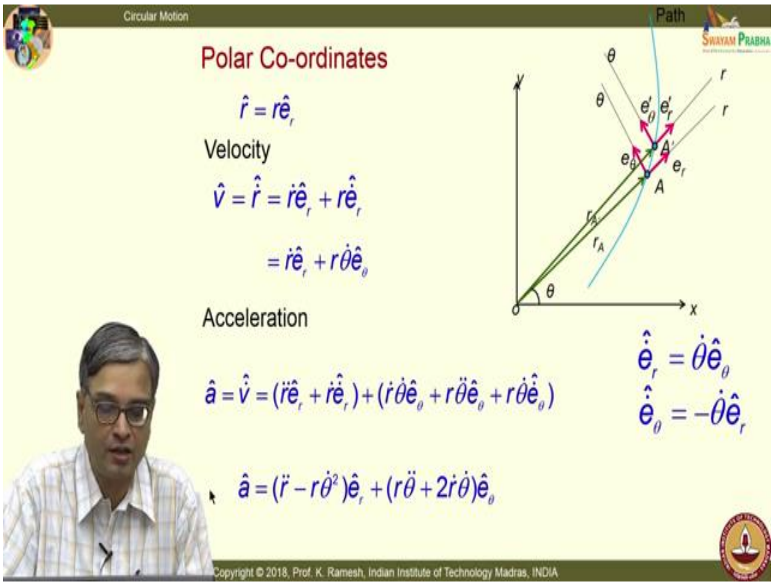
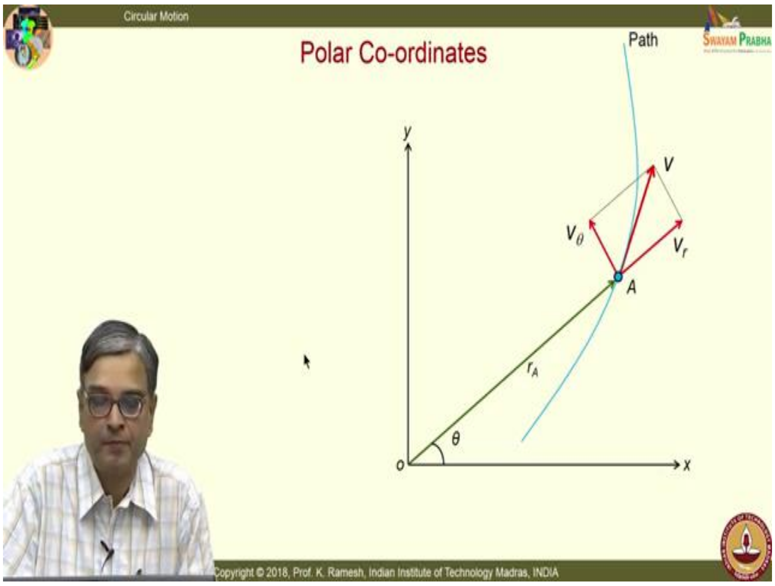
And I have done it for polar coordinates, I would like you to do it independently for the n t coordinates or the intrinsic coordinates. Now, look at how do we get the expression for velocity and acceleration. Now I can write $\hat{r} = r\hat{e}_r$ and apply the product rule properly and write the velocity as $\hat{v} = \dot{\hat{r}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$. And we know very well what is $\dot{\hat{e}}_r$; that is given as $\dot{\theta}\hat{e}_\theta$; so I get velocity as $\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$; I can also recognize what is the $r$ component and $\theta$ component and I extend the same for acceleration.




I would like you to differentiate and check your mathematics with my expression because you know when I make the slide, I can also make small mistakes in the expressions. Because these are usually cut and paste and, in the process, we miss out something, you apply the product rule, collect all the terms and put them in proper fashion and check it with my expression.

It is a very simple exercise. So, I have $\hat{a} = \dot{\hat{v}}$ and I differentiate this expression employing the product rule properly. So, I get this as $\hat{a} = \dot{\hat{v}} = (\ddot{r}\hat{e}_r + \dot{r}\dot{\hat{e}}_r) + (\dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta)$ we have not left out any term fine.

You might have used circle motion where the radius is constant, there some terms go to 0. What we are now looking at is a particle is moving along an arbitrary path, *r* is not constant it varies as a function of position. When I simplify this, I get the acceleration as




$\hat{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$, I am not telling the brackets and that is understandable ok. So, this is a most general expression that I get for acceleration and velocity if I employ polar coordinates to investigate the motion.

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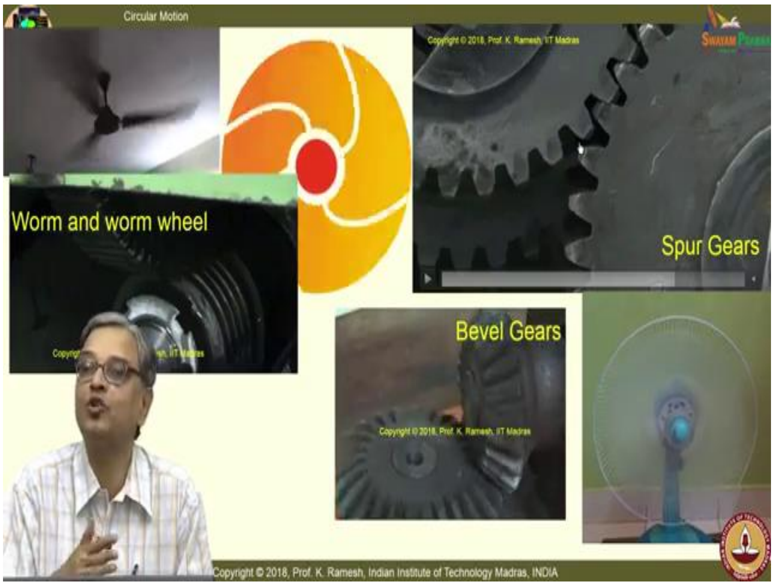


And this is finally summarized as I have velocity; it is now represented as a component in the r direction and a component in the $\theta$ direction, I can visualize this as $V_r$ and $V_\theta$.

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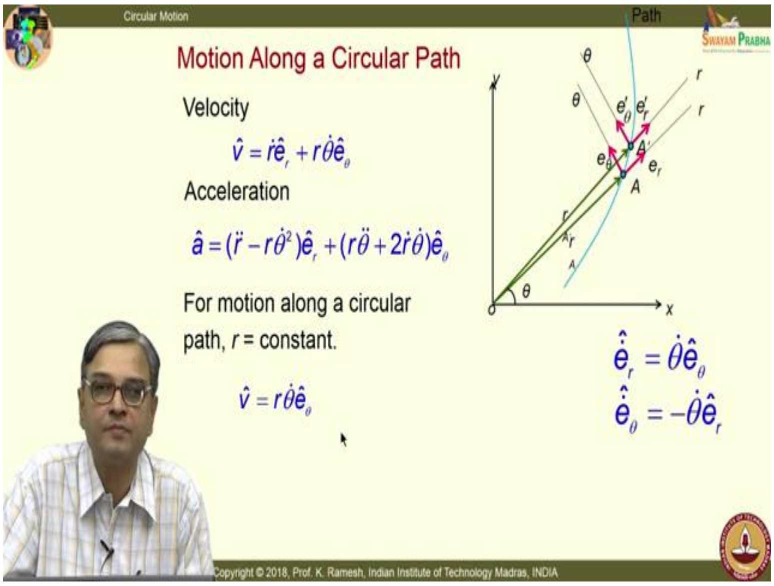
I know it is very important to see in our day to day life we have many applications where we come across rotation from a simple hanging fan like this or impeller blade and you have a gear drive here. See, I made it a point that this is a course on engineer mechanics when there is an opportunity to bring in engineering aspects, I would like to show that. I have different types of gears, I have one set of gears here, I have another set of gears here, I have a third set of gears here ok.

If you look at these are known as spur gears and these are known as bevel gears and this is called as warm and warm wheel. And look at in all of these cases how the axes are aligned? And you can look at here, the gears are rotating like this and you can closely look at what is the kind of profile that you have on the teeth that profile is known as involute profile, you learn in later courses and you need that for smooth functioning of the gear. And in the cases of bevel gear note that I have an axis like this, I have another axis horizontal they intersect at a point. So, you are in a position to translate rotation from one direction to another direction using this.

On the hand, you have a warm and warm wheel where you required a very high reduction in the speed this is used, you will find that this is a warm this is called a warm and this is the warm wheel the warm wheel will have a very slow rotation. And also look at the axis I have an axis of worm like this and I have an axis of a worm wheel like this. And what is common in all these cases? You have a fixed axis around which these objects rotate.




And I have also deliberately shown a table fan in this fashion table fan has a very interesting motion we will see that later. It also has an axis and because in many applications, you have an axis above which things rotate we also cloud our thinking and we always perceive rotation with respect to fixed axis this is a mental blog that we have, but we have to get out of it fine.

And suppose we would like to analyze something in a circle motion like this what is the kind of mathematics that we have to do? I have the general expression velocity and acceleration that we have derived just now.

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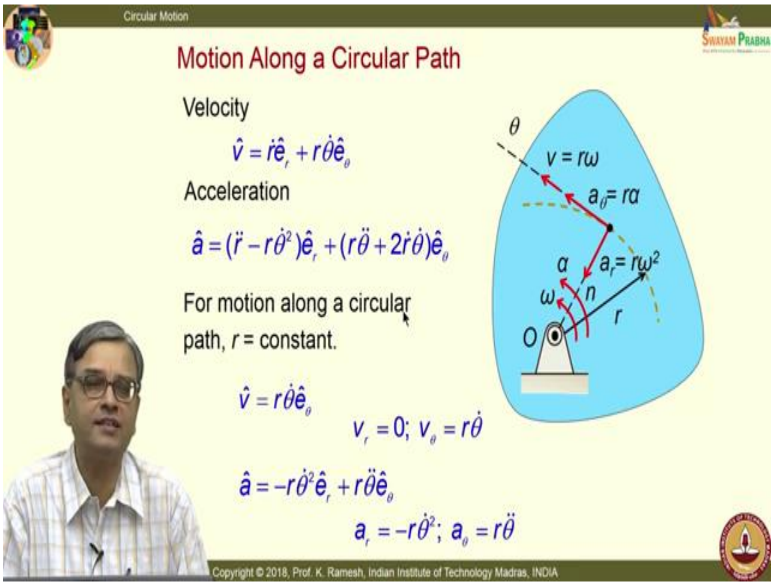
If I have a motion along the circular path first definition is r remains constant; it is not like this is varying in a generic path I have r changing, when I say *r* remain constant and I have r dot and r double dot goes 0 and a circle motion is simplified simply as $\hat{v} = r\dot{\theta}\hat{e}_\theta$.

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And I have this I have an object I have shown this as a rigid body, I have a reason for it I have axis above which the body rotates and I have taken a representative point and pictured its motion like this; this is at a distance fixed distance *r* from the point of rotation. And when I look at this expression, I get $v_r = 0;\ v_\theta = r\dot{\theta}$.

And I also represent v as tangential to the path as r$\omega$ and I have acceleration this reduces to $\hat{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta$. And I can also visualize it in the diagram a *r* as pointing towards this centre of rotation $a_r = -r\dot{\theta}^2;\ a_\theta = r\ddot{\theta}$. This you know very well you know many times you have looked at these expressions and you should not undermine this expression; you should know how to use these expressions properly.

Even though I say that we have looked at axis of rotation, even when we solve later our problems, we will somehow look at this as circle motion about some point which will make our calculations very simple. Even though there is no deliberate circular motion we would have instantaneous centre of rotation, all these concepts will develop. So, we would fall back to these expressions because we are accustomed to this and use this intelligently for solving different problem situations.

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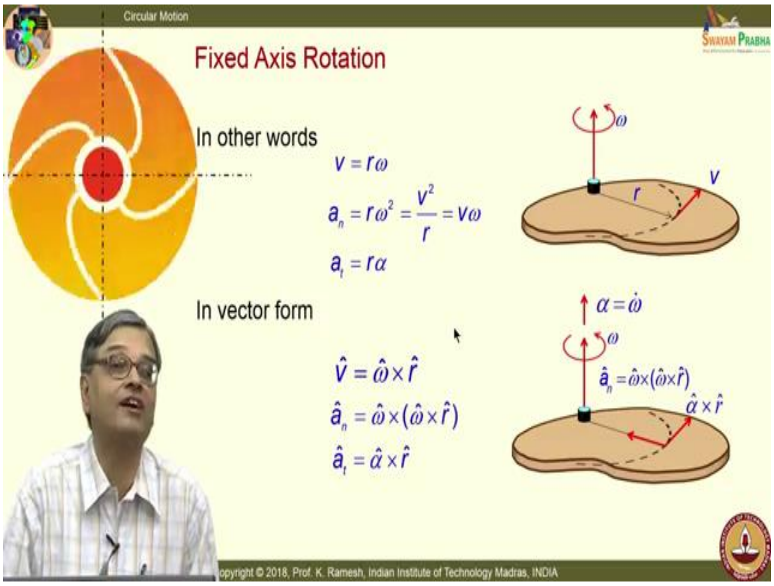


And I can also represent this in other fashion, I can also recognize this as *nt* coordinates and I have just shown that this is rotating in clockwise or antic lock wise both are possible.

Let us compare the expressions when I write in expression for *v*, we always say it as $v = r\omega$ , how you have learnt it has anybody told you as $\omega r$? Usually, we will call it as $r\omega$ but when I write it as a vectorial form what is that I should do? I have to do very careful I have to call it as $\omega \times r$ do not make a mistake because you always call it as $r\omega$, in vectorial form it does not become r $\times \omega$ it is $\omega \times r$ ok.

See these are all the usual mistakes students do in a hurry and I have a normal reaction, normal acceleration and that is given as $a_n = r\omega^2$ and that is nothing, but $a_n = r\omega^2 = \frac{v^2}{r} = v\omega$ and if I write it in vectorial form, I would write it as $\hat{a}_n = \hat{\omega} \times (\hat{\omega} \times \hat{r})$ and this is what I have shown it here; I have a acceleration towards this point.

This is the normal component and I can also write the tangential component; usually you call it as $r\alpha$ that is what you say $r\alpha$, but when I write it in vectorial form; I should say it as $\hat{a}_t = \hat{\alpha} \times \hat{r}$ . See we would solve problems graphically so that you get better visualization where we will deal with components there you can easily use $r\omega$ or $r\alpha$, you would also solve the problem by vectorial notation you have to handle those vectors carefully, $\omega \times r$ and $\alpha \times r$ you should never make a mistake there.

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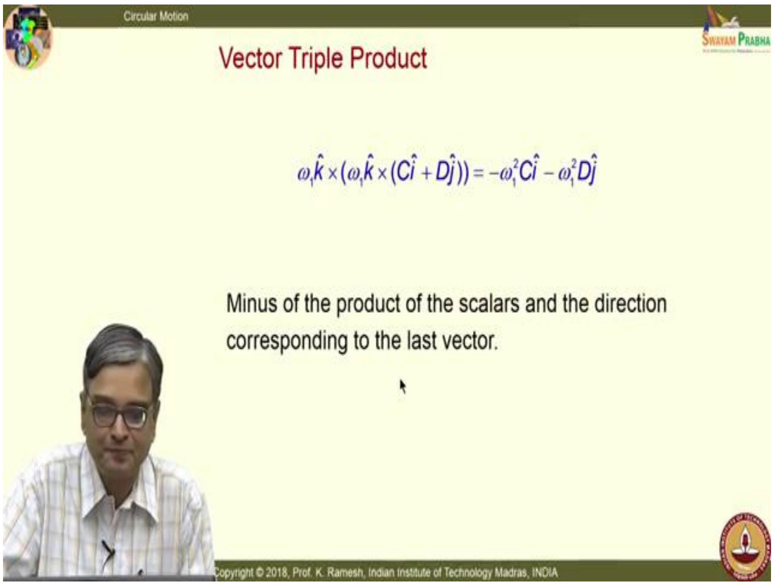
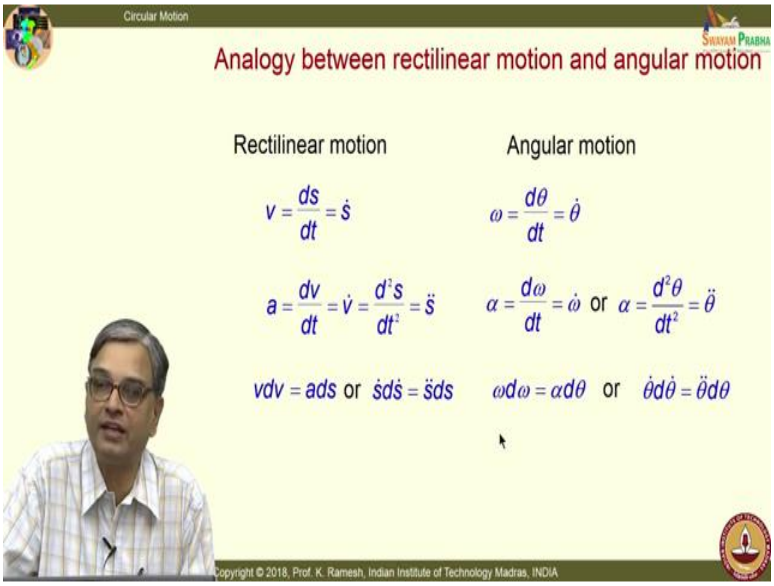
And you know you also have a vector triple product when you want to do a vector algebra; it is also desirable that how do you do this quickly. There is no point in spending time looking at *i j k* and then do the long hand calculation for a simple expression. If you look at, I would get the answer as $\omega_1\hat{k}\times(\omega_1\hat{k}\times(C\hat{i}+D\hat{j}))=-\omega_1^2C\hat{i}-\omega_1^2D\hat{j}$. The interpretation is what I get in the triple product is minus of the product of the scalars; please write down it is a very important property which you might have not noticed it, it will make your algebra faster.




So, it is nothing, but the minus of the product of the scalars and the directions corresponding to the last vector; in a vector triple product invariably, you will get only the answer like this. So, if you know the trick you can do the mathematics fast and you will not come back and say question paper is simple, but it is very long you need to apply all these tricks to solve your problem quickly ok.

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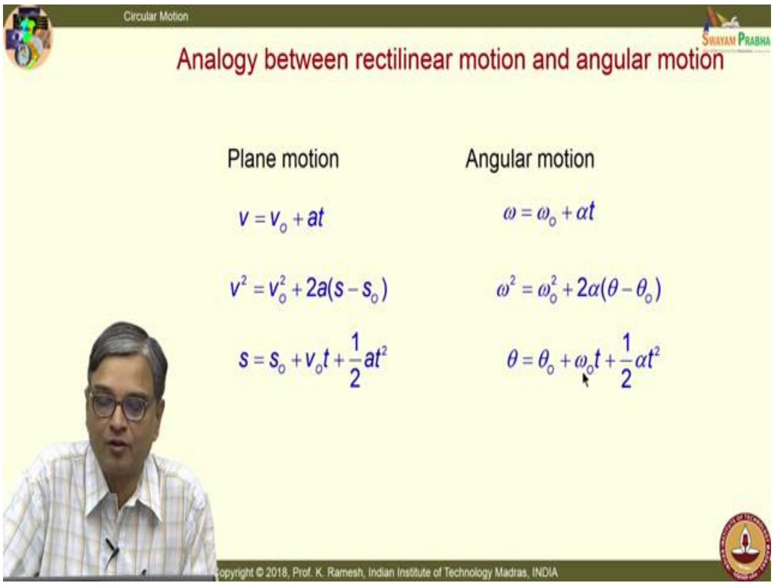
And before we move on to further discussion; let us also look at the analogy between rectilinear motion and angular motion. The repetition of what you all learnt earlier no harm in having these equations again in your notes; $v = \frac{ds}{dt} = \dot{s}$ and the parlance in angle




motion is $\omega = \frac{d\theta}{dt} = \dot{\theta}$ and acceleration is $a = \frac{dv}{dt} = \dot{v} = \frac{d^2s}{dt^2} = \ddot{s}$.

Here I have $\alpha = \frac{d\omega}{dt} = \dot{\omega}$ or $\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$ and you also have another interrelationship $vdv = ads$ or $\dot{s}d\dot{s} = \ddot{s}ds$. In angular motion it is $\omega d\omega = \alpha d\theta$ or $\dot{\theta}d\dot{\theta} = \ddot{\theta}d\theta$.

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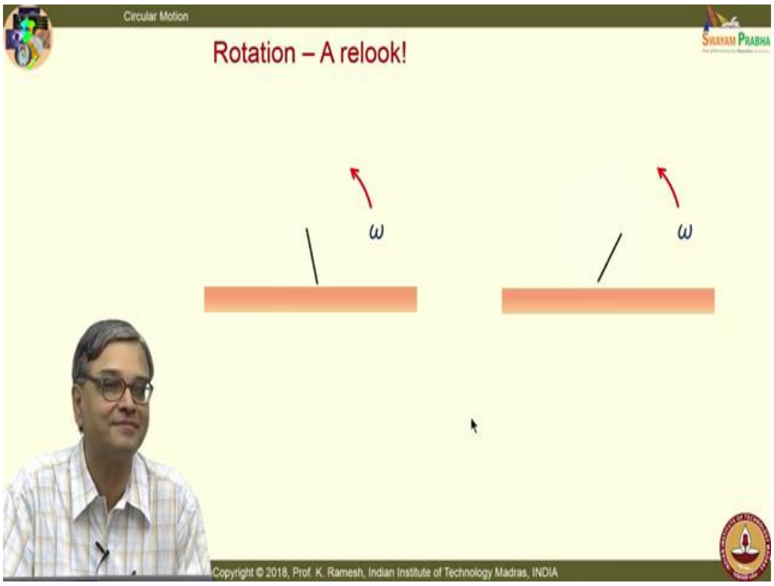


And we will also look at the other expressions and mind you that these are derived for constant acceleration; a has to be constant $v = v_0 + at$, $\omega = \omega_0 + \alpha t$, $v^2 = v_0^2 + 2a(s - s_0)$, $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ and $s = s_0 + v_0 t + \frac{1}{2}at^2$, $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$.

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And you know I am not going to bring in these equations again, for completeness these equations are necessary, fine. Now, look at what is happening to this line. Observe this and then visualize these are anything that you can recognise out of this I have just shown a line. What does it look like? Line is moving in space fine in a plane it is moving, it is rotation ok.

So, I have given this as I have already shown that this is rotating like this and I am showing another line like this. What is happening to this line; what is happening to this line? Let us compare these two, this line is rotating like this line is moving like this; are they one and the same or different? They are different; I am happy, I want you to say that I want to provoke you to say that I have done my job ok. Let us look at how these lines have come about fine, because you are accustomed to this, this looks weird, this looks different fine.

Let me tell you how I made this animation, I would say before deriving this; this is rotating with angular velocity $\omega$ this is also revolving with angular velocity $\omega$, they are not different ok.

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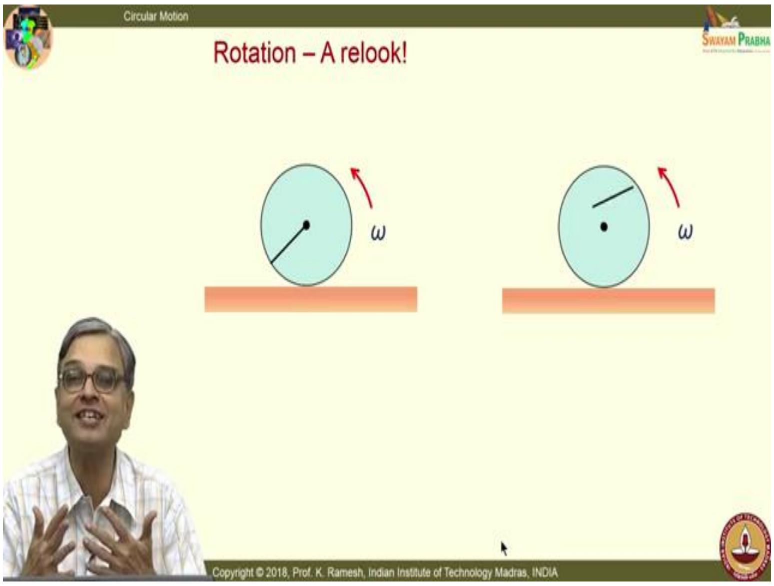
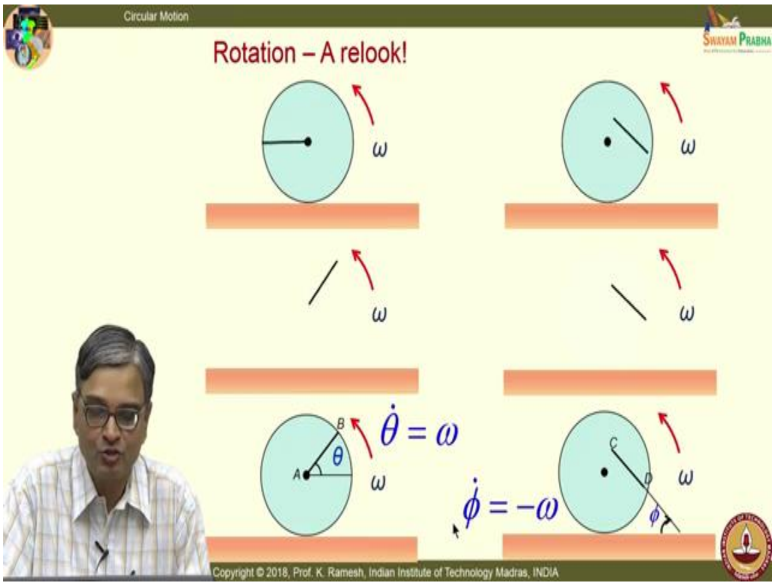



Let me put the cat out of the bag; I will see this is you have pictured it very clearly this is a rotation I have a object which is rotating about a axis and you are able to see this. And what I have done in the second case is; again I am rotating the same object, I have taken a different line that is all, we are not accustom to that; we have a mental block rotation means it should be like a spoke in a wheel, am I right. You have to get out of that if you get out of that feeling, then you can understand general playing motion of a rigid body.

And we have to understand what is the meaning of rotation; we are having a relook at it. When I say you have to have a relook at it, we will also have to evaluate this, quantify this, how do I quantify rotation? We all know you say angular velocity how do you define angular velocity?

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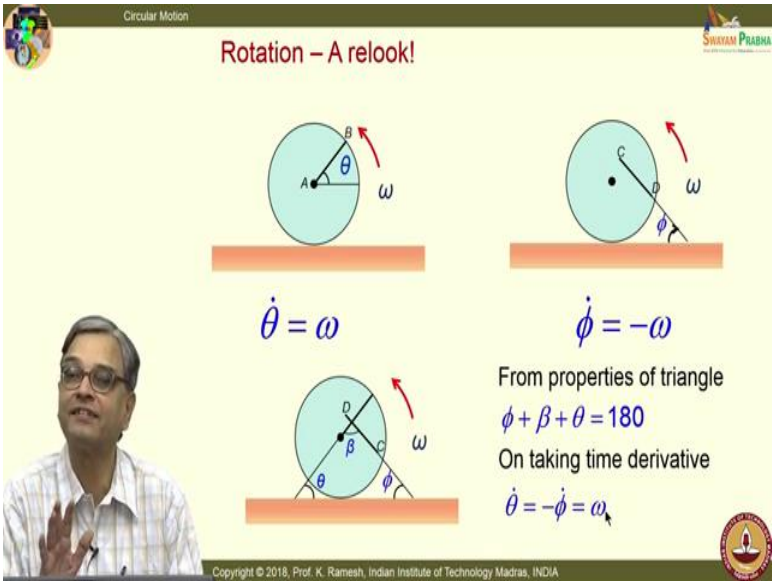
And the next slide shows to convince you; they are one and the same I am showing all of them in one plot. But you may think that you showed these diagrams differently, you show everything at all at the same time let me see this is rotating on this object. This is a line which is again rotating above this centre of rotation, but the line is different that is all the difference, but for appearance it looks as if it has a very complicated motion; it is because our eye is not tuned to it, suppose want to make a machine man as a scientist what is that I should do?




I should actually monitor what is its motion from a fixed reference. Here we know that this is $\theta$ and I can say $\dot{\theta}$ and then do it. If I have to monitor this, I should find out what is the angle with respect to a fixed reference and monitor as a function of time; what happens to that ok. And we would also derive, so this becomes $\dot{\theta} = \omega$ and here it would result

$\dot{\phi} = -\omega$ see here I am measuring it anti clock wise, here I am measuring it clock wise. So, that makes a difference; so, we will get on to this.

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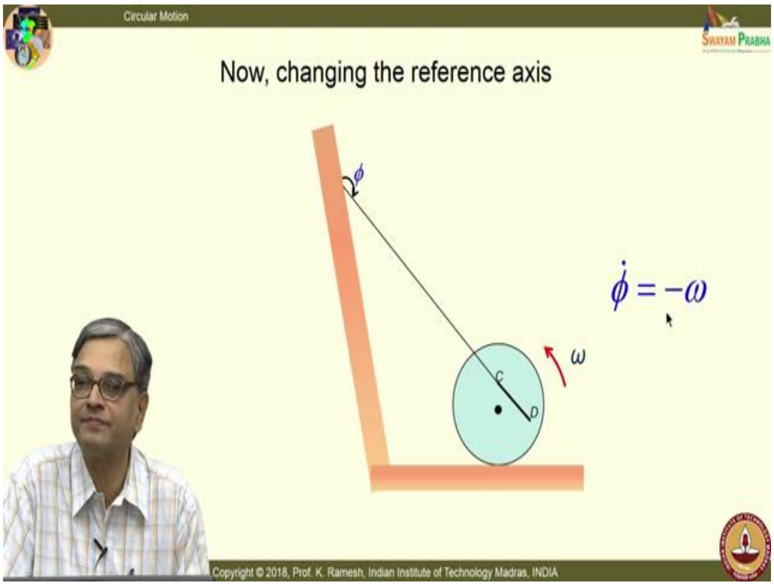
I have this shown again now look at this we know $\dot{\theta} = \omega$. So, I have this angle $\theta$, now on this diagram I put this line *CD* as it is like this here. And you know this is scribed on this rigid body whatever the angle in this case it appears to be degrees, it need not be 90 I have just put a generic angle beta that relative variation will always remain same, you agree with me? That does not change. Now, I put this angle $\phi$ and from the property of the triangle what is that I get? I get $\phi + \beta + \theta = 180$.




So, I take the time derivative I get this as $\dot{\theta} = -\dot{\phi} = \omega$; is the idea clear? You know your rotation very well, but you have understood certain aspect of it if something differently shown to you; you are shaky. Now, you will never be shaky we will again revisit it ok, I will again explicit it from a different perspective.

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Is there any sanctity that I should measure it only from horizontal? No, the requirement is I should have a fixed reference in space, if I measure it with phi, it will also go to the same angular velocity.

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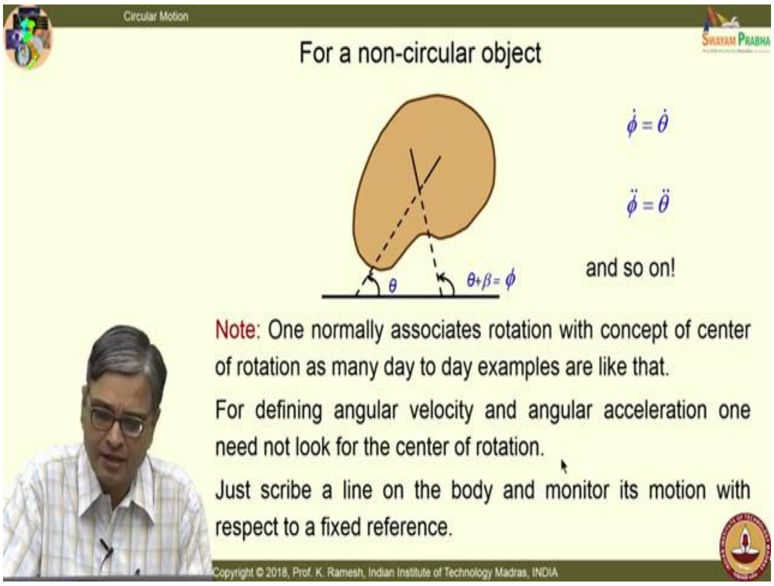
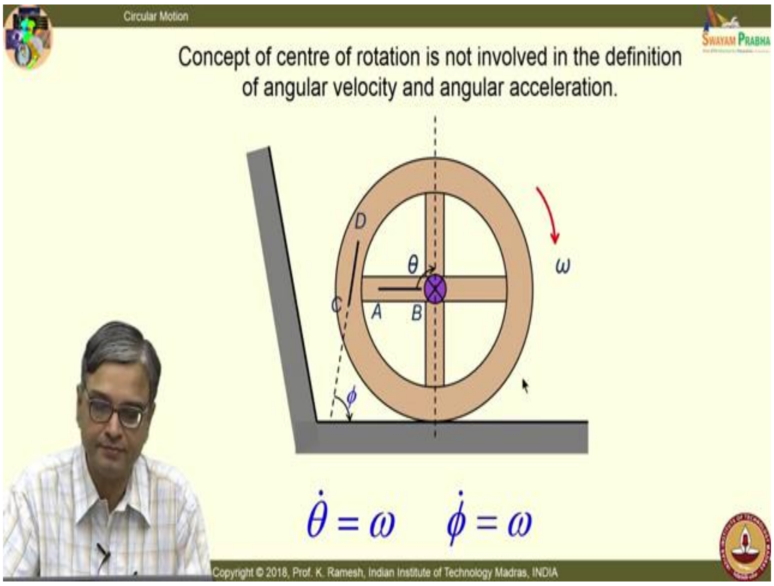
Let us take a noncircular object I again have scribed two lines, now you know very well how to find out what is the definition of $\omega$, scribe the line and monitor what happens to that line fine. And this is having a rotation like this and we want to find out what is the rotation of this object ok. So, I put a line to that the idea here is one normally one normally associates rotation with concept of centre of rotation as many day to day examples are like that.




I do not need to define rotation based on a centre of rotation; it is enough that I mark a line on the rigid body and monitor its motion from a fixed reference, then I can find out what is the angular velocity either this line or the other line this is one and the same ok. I have measured both of them anti clock wise; when I measured both of them as anti clock wise I would get $\dot{\phi} = \dot{\theta}$ and $\ddot{\phi} = \ddot{\theta}$.

So, it is a very subtle concept it is better that we revisit this. For defining angular velocity and angular acceleration one need not look for the centre of rotation, just scribe a line on the body and monitor its motion with respect to a fixed reference. You know books do not pay so much attention on explaining this; it is a very subtle concept that you need to know.



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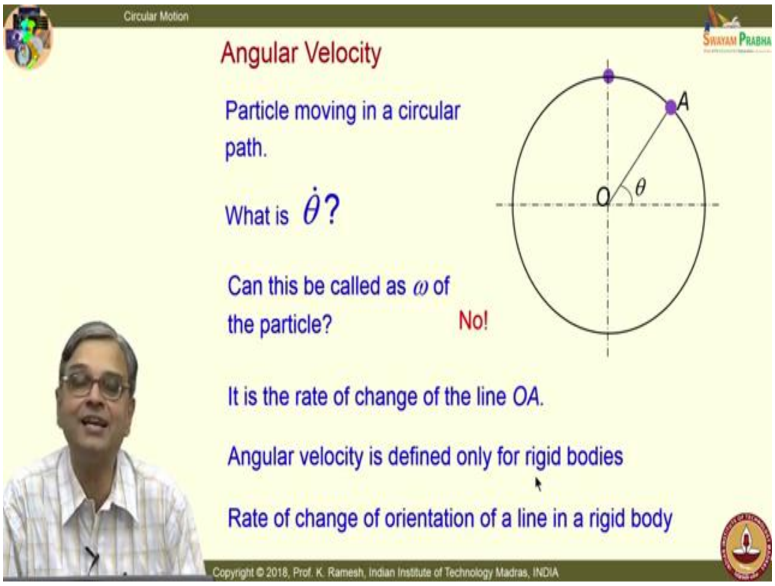
And I also have another example you know I have taken a wheel; it is not a rolling it is just rotating above this axis, I just shown one reference here, another reference here.

Normally, if you are asked to find out what is the angular velocity; involuntarily you will take a line on the spoke because, you always associate rotation with centre of rotation and put a radial line, this is how you do it. And this I have measured it as clock wise angle $\theta$, I can also have another line which is put on it is this portion of the wheel, another line CD monitor it from a fixed reference. In the same sense if I measure it clock wise; this also I measure it clock wise and monitor what it is this would also be simply $\omega$.

So, the fundamental learning what you learn here is if I want to investigate whether the body is rotating or not; scribe a line on the rigid body, monitor it from a fixed reference, whatever the answer you get you would be able to specify that as $\omega$ or $\alpha$ as the case may be fine.

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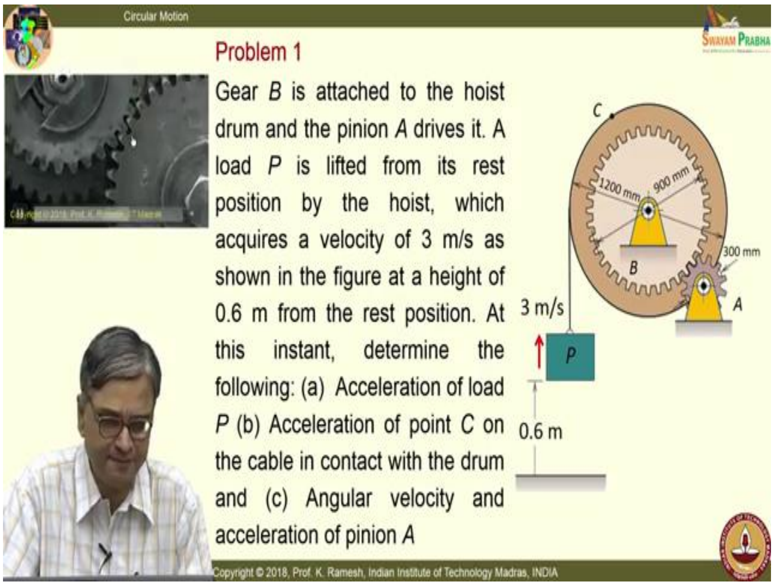
And before we complete this let me go back to a particle moving in a circular path; I have a particle moving in the circular path and this is what is happening to the particle. And then if I want to define the motion of the particle at an instant of time, I would have taken the centre, I would have labelled it as *OA* and $\theta$.

And tell me what is $\dot{\theta}$ can I call this as a $\omega$ of the particle? What would you do? I have $\dot{\theta}$; it is rotating in a circular path can I call this as $\omega$ of the particle? Tempting, it is tempting that why not I call it I cannot call it for a particle; particle by definition is a spec fine.

We have looked at I have a big car which is moving in a straight line without any hit against a tree, I said I can analyze it as a particle. Have you played with a magic ball when you are kids you have rubber ball and then you spin it? It will go forward and backward and come to a halt; it is like a boomerang. You throw it comes back to you, but you have to spin it, if you do not spin it properly it will not do.

How will you analyze that ball from dynamics point of view? It is very small can you call it as a particle? You cannot take it as a particle; the moment it has a spinning, you have to treat it as a rigid body. So, please understand when I say $\dot{\theta}$ in the contrast of particle analysis; you have to recognize $\dot{\theta}$ as a rate of change of line *OA*, angular velocity is defined only for rigid bodies. Again, a very subtle concept, we all know about it, but we do not handle it properly it is a very subtle concept.

And if I have to find out the angular velocity, scribe the line and find out its rate of change of orientation from a fixed reference. So, you have a recipe I am going to use this recipe in tomorrow's class and find out what is rotation and understand what is general plane motion fine.

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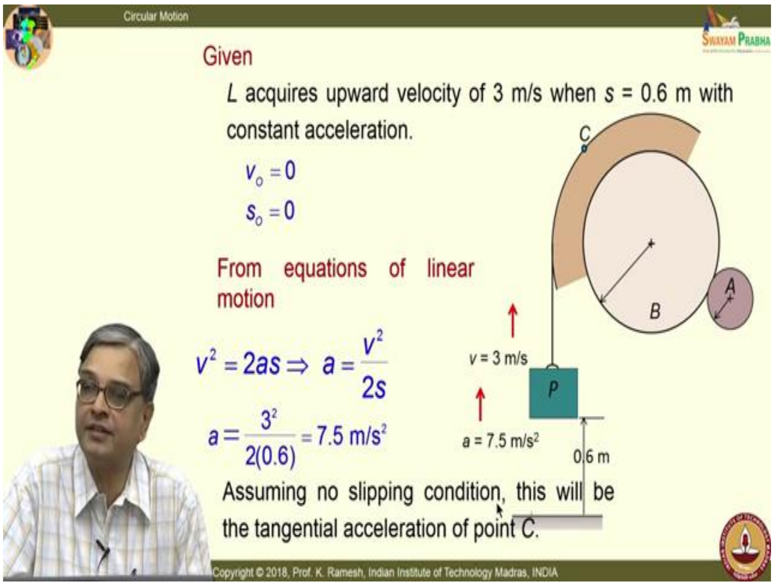
Let us move on to solving simple problem which combines linear motion as well as rotary motion and I would like you to look at the gears here. What you see in the first place? What is the rotation of this; this is rotating anti clock wise, what is the rotation of the other gear? It is clock wise understand this inter relationships, if you have a gear in mesh; if one is rotating anti clock wise, other will be rotating in clock wise.

And it moves very smoothly. So, I have teeth engaged on this; this is not the replica of this I could get some gears moving, I thought you have not exposed to gear it is better that something of a physical reality, then you can correlate what happens to these gears; I can simply replace these gear with a mean radius of the contact at the point ok.

I will not put these teeth when I look at the solving of the problem. And take down the statement here, what it is gear *B* is attached to the hoist drum and the pinion A drives it this is the hoist drum. The gear *B* is integral with the drum *C* and I have a smaller gear you usually call it as a pinion that is the language used in engineering.

Pinion means if I have a 2-gear system smallest gear you call it as a pinion which acquires a velocity of 3 metres per second, I have a block which is resting here at a height of 0.6 metre from the rest position. At this instant determine the acceleration of load P, acceleration of point C on the cable in contact with the drum, angular velocity and angular acceleration of pinion A; all these quantities you have to find out.

So, it is nice problem combining linear motion and circular motion; I am not quite sure whether you have solved such problems earlier it is very simple for you to solve ok.

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Just to refresh how to solve the equations and I have replaced the sketch of these gears as simple circles. And the essential geometries listed I have $v_o = 0$ and $s_o = 0$, from equations of linear motion I can write this as $v^2 = 2as$ and $a = \dfrac{v^2}{2s}$. I get $a = \dfrac{3^2}{2(0.6)} = 7.5\ m/s^2$ please check my arithmetic ok.

That is a reason why I told you that you should bring your calculators, then and there you check and verify if there is any typographical error please bring it to my attention. And assuming no slipping condition this will be the tangential acceleration of point *C* do you recognize this? This is a rope which is winding on the drum and this has a particular acceleration, this is the same as tangential acceleration of point *C* ok.

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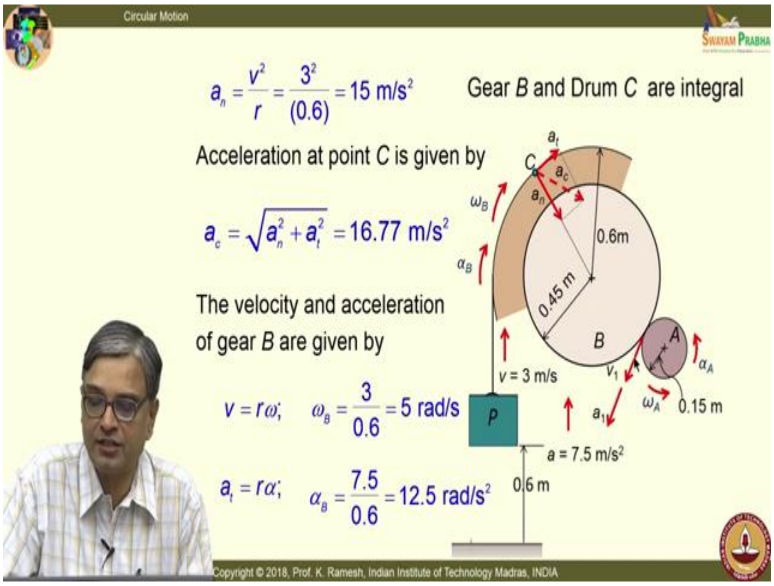
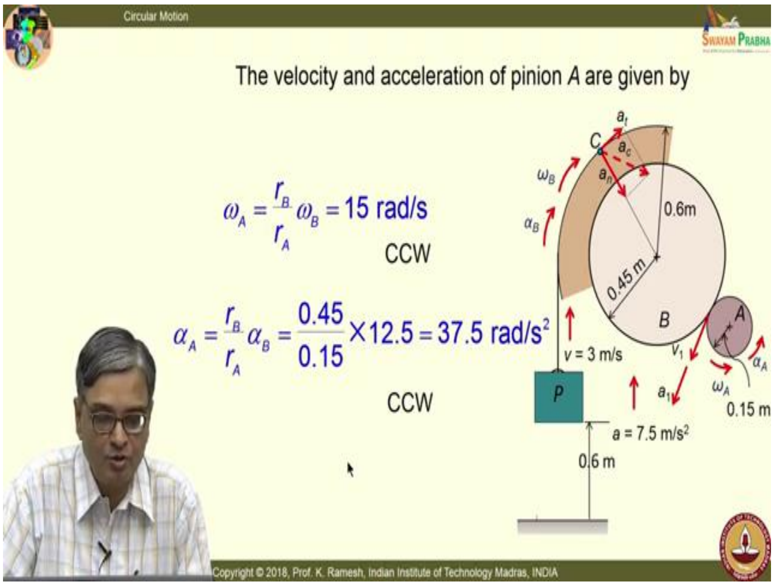
And what is the normal acceleration? I have the dimensions marked and I have this velocity and I have the expression $a_n = \frac{v^2}{r} = \frac{3^2}{(0.6)} = 15 \text{ m/s}^2$. And let me put the angular accelerations here normal accelerations here and I put the tangential acceleration this is same as linear acceleration of this block *B*. So, I get the net acceleration $a_c$.




So, I have a net acceleration of point *C* as 16.77 metre per second square; from which what is that I can find out? I can find out the velocity and acceleration of gear, gear *B* are given by $v = r\omega$ and B and C are integral. So, I get this as 5 rad/s, from this acceleration value I can also find out the angular acceleration of gear *B* I have $a_t = r\alpha$; and is nothing, but 7.5 m/s$^2$.

So, I get $\alpha_B = \frac{7.5}{0.6} = 12.5 \text{ rad/s}^2$ and what is common in between these 2 gears the pinion and the gear you have a common point here at the common point both of them should have same linear velocity. So, I have gear *B* as $\omega_B$ and $\alpha_B$ shown like this and I have for gear A; $\omega_A$ like this now you know very well this is anti clock wise this is clock wise that is how a gear pair operates.

And I have $\alpha$ a like this and I have this velocity $v_1$ if I measure it from this gear or this pinion it should be identical, I use that for me to find out the quantities of interest. So, it can have velocity $v_1$ and acceleration a 1. So, that gives me the requisite expressions to get the answers.

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So, I get $\omega_A = \frac{r_B}{r_A}\omega_B =$ 15 rad/s, this is counter clockwise motion and I have $\alpha_A = \frac{r_B}{r_A}\alpha_B = \frac{0.45}{0.15} \times 12.5 = 37.5 \text{ rad/s}^2$. Please check the numerical calculation and alert me if there are any typographical errors. Let us move on to another simple problem.

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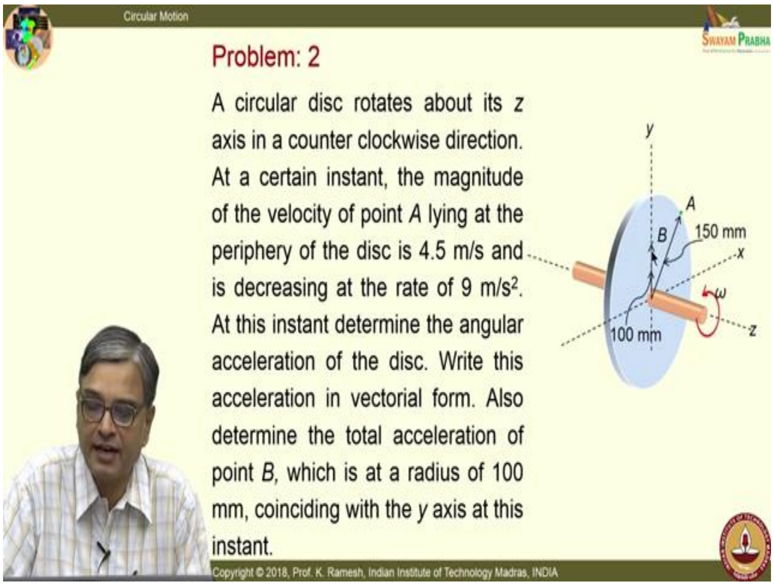
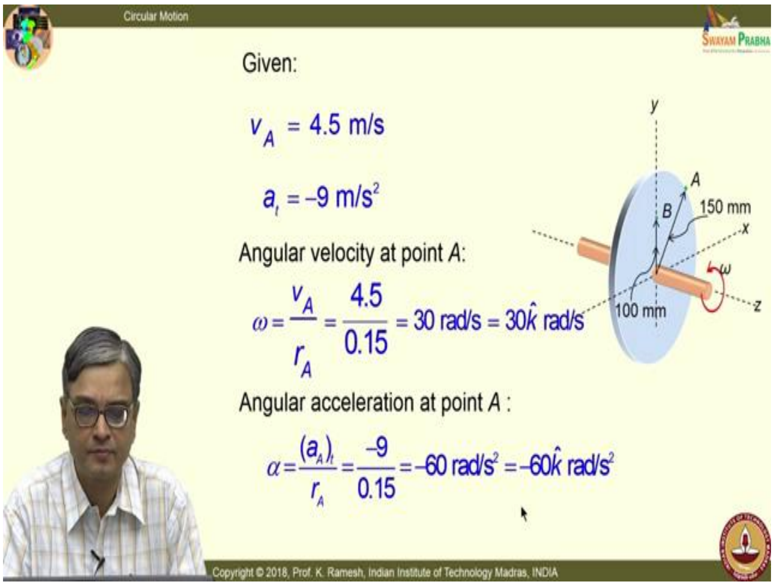



I have circular disc rotates above the z axis, I have the point A on the periphery marked and I have a point B which is marked at this point in time along the y axis at a distance of 100 millimetres and the velocity of point *A* lying at the periphery of the disc is 4.5 metres per second.

And it is decreasing at the rate of 9 m/s$^2$. At this instant determine the angular acceleration of the disc and I would also like you to write it in vectorial form. Also determine the total acceleration of point *B* which I have already said on the y axis at a distance of 100 perimeter. Again, a very simple problem the idea is to get familiarized with handling those equations it is fairly straight forward.

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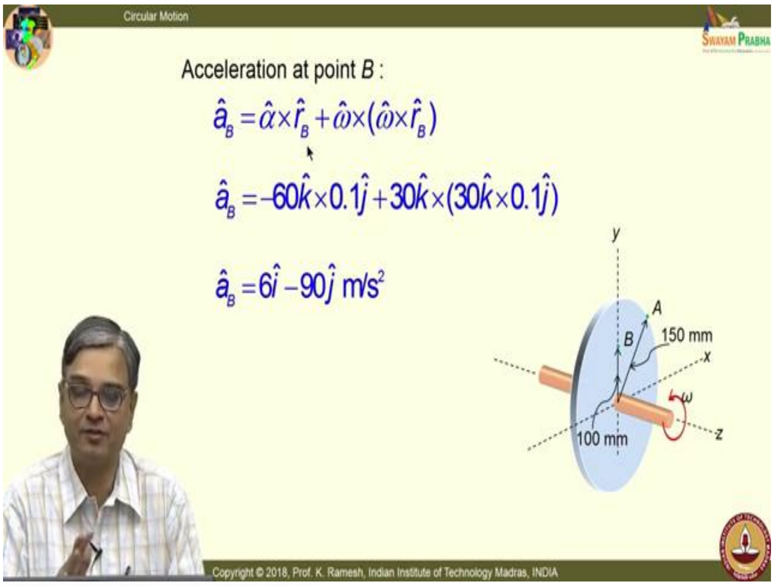
So, what is given is $v_A = 4.5$ m/s and you have to recognize what is given as the acceleration as $a_t = -9$ m/s$^2$. You have to recognise from the problem statement the value given is a t that is what you have to recognize ok. This is - 9 m/s$^2$, the mathematics is very simple angular velocity at point A is $\omega = \frac{v_A}{r_A} = \frac{4.5}{0.15} = 30 \text{ rad/s} = 30\hat{k} \text{ rad/s}$. So, in vectorial form it is 30 k and angular acceleration at point A is $\alpha_A$ is $\alpha = \frac{(a_A)_t}{r_A} = \frac{-9}{0.15} = -60 \text{ rad/s}^2 = -60\hat{k} \text{ rad/s}^2$ it is fairly a straight forward problem.

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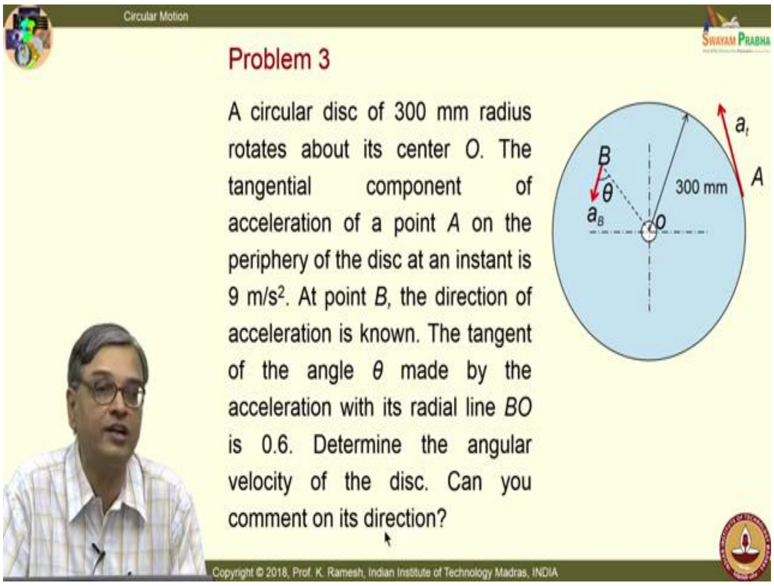
And we get the acceleration at point B. So, I have to write it as $\hat{a}_B = \hat{\alpha} \times \hat{r}_B + \hat{\omega} \times (\hat{\omega} \times \hat{r}_B)$ and you could substitute these expressions the values for these expressions and we have already seen how to do a triple product. So, when I simplify; I get this as $\hat{a}_B = 6\hat{i} - 90\hat{j} \text{ m/s}^2$.

The focus of this example is to interpret the problem statement and in vectorial analysis put it as $\alpha \times r$ and $\omega \times r$; do not put it as $r_\omega$ $r \times \omega$ or $r \times \alpha$ it is the other way you have to recognize that.

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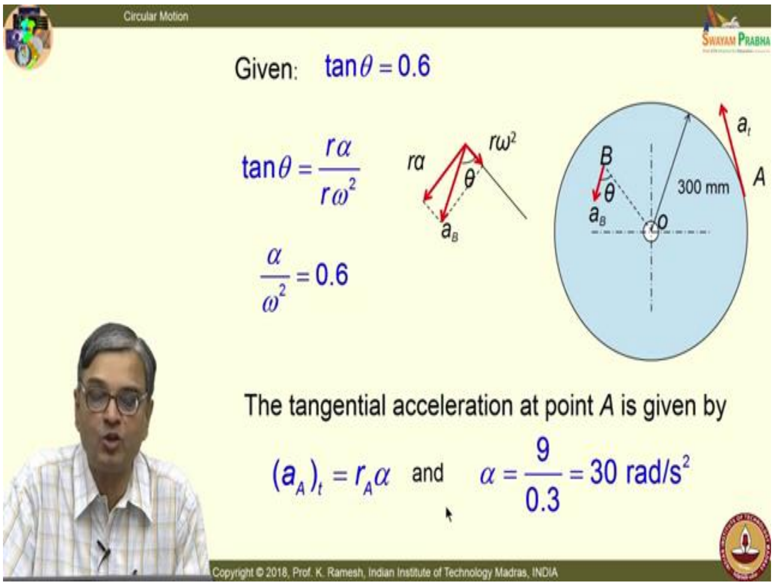
Let me move on to a 3rd problem it is again a very simple; what I have is I have disc of radius 300 millimetres. For a point on the periphery that the tangential acceleration is given for another point the orientation of the acceleration is given ok.

So, the tangential component of acceleration of a point A on the periphery of the disc at an instant is 9 m/s$^2$. By some measurement at point B only the direction of acceleration is known and what other quantity that is known is the tangent of the angle $\theta$ made by the acceleration with its radial line *BO* is 0.6. This tangent of $\theta$ tan$\theta$ is 0.6. Determine the angular velocity of the disc and the final part of the question is can you comment on its direction ok.

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So, tan$\theta$ is 0.6 the interpretation of the statement problem statement I have a B at this orientation and I have this as $r\omega^2$ and this is r$\alpha$. So, I get $\tan\theta = \frac{r\alpha}{r\omega^2}$ I get $\frac{\alpha}{\omega^2} = 0.6$. The tangential acceleration at point $A$ is given by $(a_A)_t = r_A\alpha$ that is tangential component and I get $\alpha = \frac{9}{0.3} = 30 \text{ rad/s}^2$. From this, I can go and find out what is the value of $\omega$.

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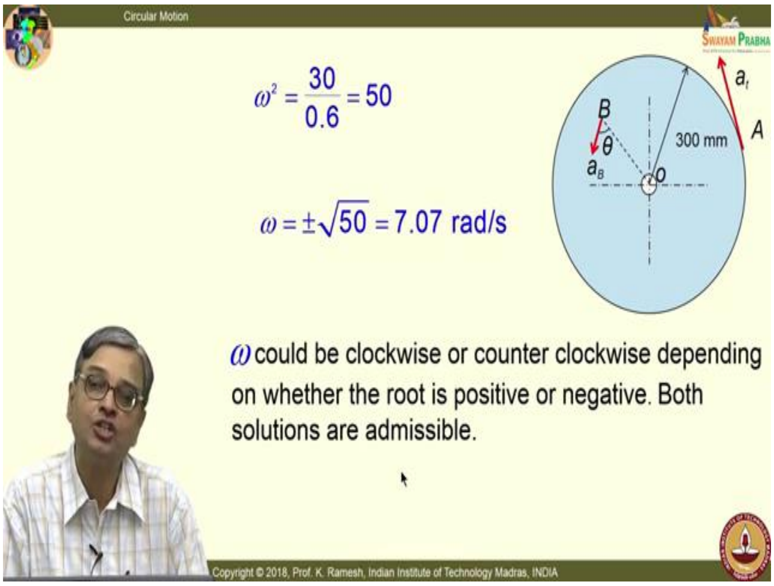


I get $\omega^2 = \frac{30}{0.6} = 50$ and I get this as $\omega = \pm\sqrt{50} = 7.07 \text{ rad/s}$. So, the idea is to say do not think if $\alpha$ is given in one direction, $\omega$ will be in the same direction; $\omega$ is an independent quantity, $\alpha$ is an independent quantity fine.

So, both the answers are possible; $\omega$ could be clockwise or counter clockwise depending on whether the root is positive or negative; both solutions are admissible, we do not have additional data to find out which is the direction for this problem. So, you will have to anticipate at both the solutions are possible. So, respect mathematics; do not say I like positive I will say $\omega$ is positive and then deal with it ok.

So, in this lecture we have looked at circular motion basically we looked at nuances of employing polar coordinates, we saw that the unit vectors change, we develop the interrelationships. Then we have also looked at what is the meaning of rotation? Finally, we said if you want to find out the rotation of a rigid body; simply scribe the line and monitor its motion from a fixed reference, you get the angular velocity and the angular acceleration of the body. Then we solved certain simple problems and the concept of rotation is subtle and I hope I have made that idea clear for you to appreciate general plane motion of a rigid body.

Thank you.