Engineering Mechanics
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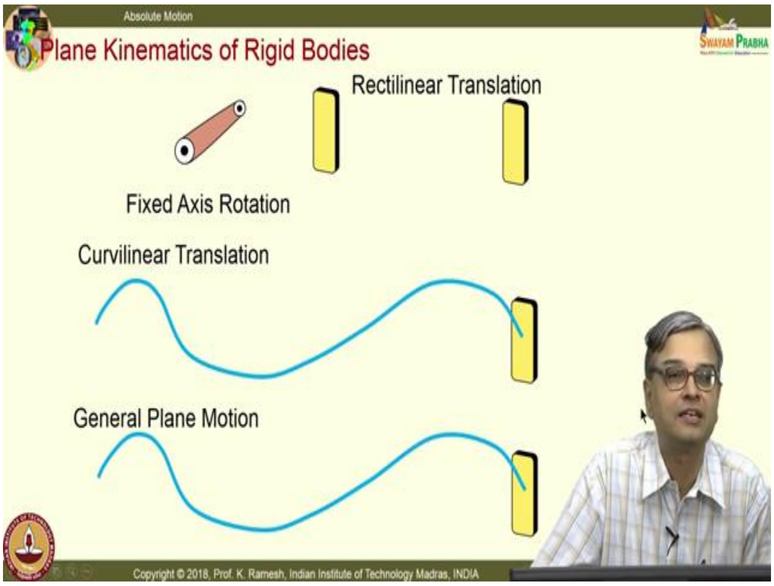
Module - 02
Dynamics
Lecture – 03
Absolute Motion

Let us move on to the analysis of rigid bodies using their Absolute Motion.



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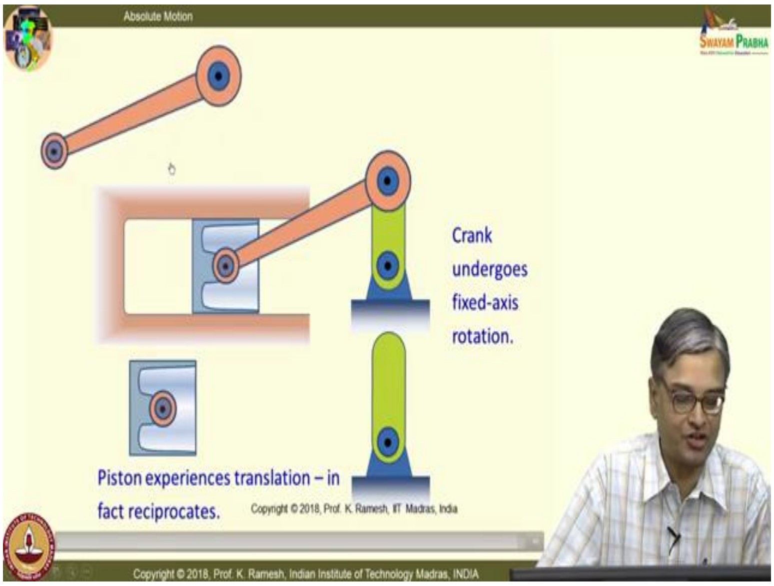
You know I move the duster that is what simulated here, this motion is very easy to comprehend and all of you would say at this is rectilinear translation; there is no difficulty at all in visualizing what is this. Then we have also seen fixed axis rotation, this is rotating anticlockwise and I can also rotate this clockwise; it could be clockwise or anticlockwise. Make a neat sketch of these diagrams and observe what is a kind of motion that I am giving to the duster now.

How will you classify this kind of motion? Look at this motion and also the motion that you have now. Are you able to perceive any difference between these two motions? For your benefit I will repeat the animations. So, what I have here is it is moving over a curve in one fashion, this is also moving over a similar curve in a slightly different fashion, do you observe the difference? And how can we label this kind of a motion; this is definitely a translation, but of a curvilinear in nature.

So, the difference is the body is not rotating when it traces a curve, this is slightly difficult for you to appreciate initially you know you all have your mini drafters when you want to move it, you have a parallel bar mechanism; you are able to have a curvilinear translation of this. If you have your scales fixed when you move it; you can have a curvilinear translation on your drawing board.



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And in the second case the object translated as well as rotated. So, you call that kind of a motion as general plane motion and we will have to understand how this has come about. Let us go back and visit our discussion on statics. You would also see that right now I have a very nice interesting animation here. This is IC engine and this is a very interesting problem that we would also solve later. It has all components of motion that we have discussed so far, you have a fixed axis rotation, you have a translation, you also have a general plane motion.

And this crank this is known as a crank; this crank has fixed axis rotation and your piston has a reciprocating motion, essentially on a line. On the other hand, when I have this connecting rod; this connecting rod what way can you classify the motion? The connecting rod has a very interesting type of motion.

Because on end this is moving in this slot, the other end is rotated by the crank and this in general has a general plane motion, it has both translation as well as rotation. I have this translation when I come to this connecting rod; this has a translation as well as rotation.

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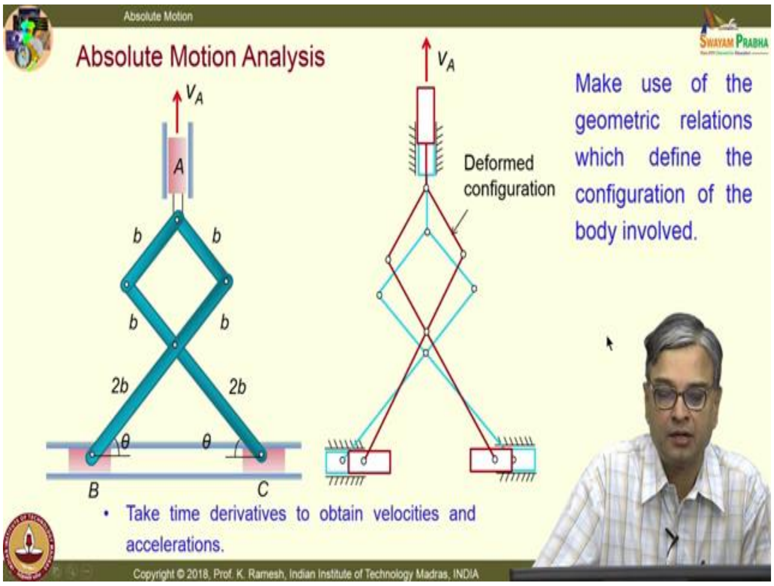
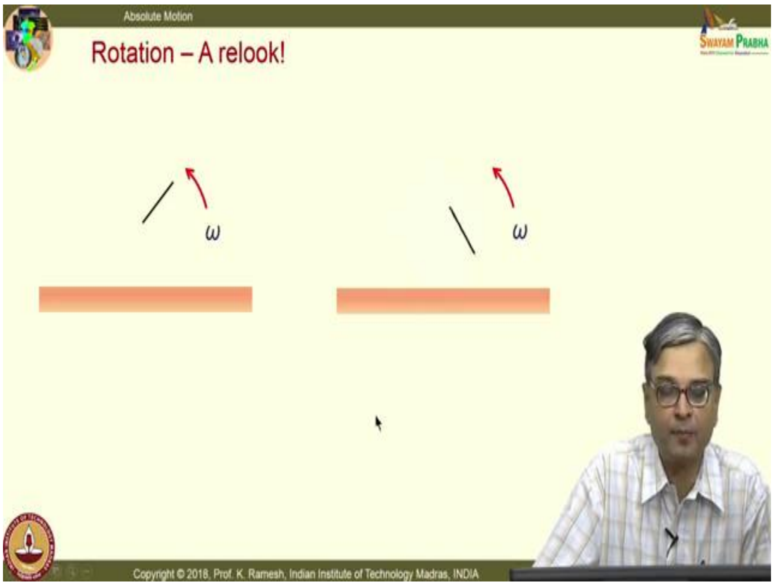


And in absolute motion analysis what you do is; make use of the geometric relations which define the configuration of the system involved. You know a class of problems you can attempt to solve by using absolute motion, but it may not be convenient to solve all problems, so you need to develop newer methodologies; that is why we have translating axis, rotating axis and so on, so that you are able to perceive the relative motion much more easily than an absolute motion.



In the case of absolute motion analysis you have a mechanism like this; I am having this with a velocity pulling it out and you have a groove and a slot and these body can move in this fashion in horizontal fashion and since you have already done virtual work where we have looked at pin jointed members like this, it is easy for you to visualize what would happen when I move this cylinder. Then you get into the geometry and take the time derivative to obtain velocities and acceleration. A class of problems can be easily amenable for solution when I look at the absolute motion.

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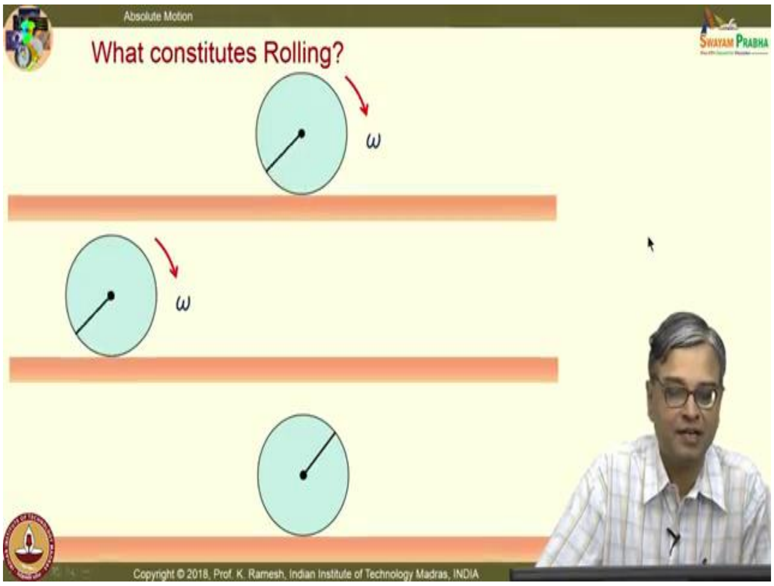

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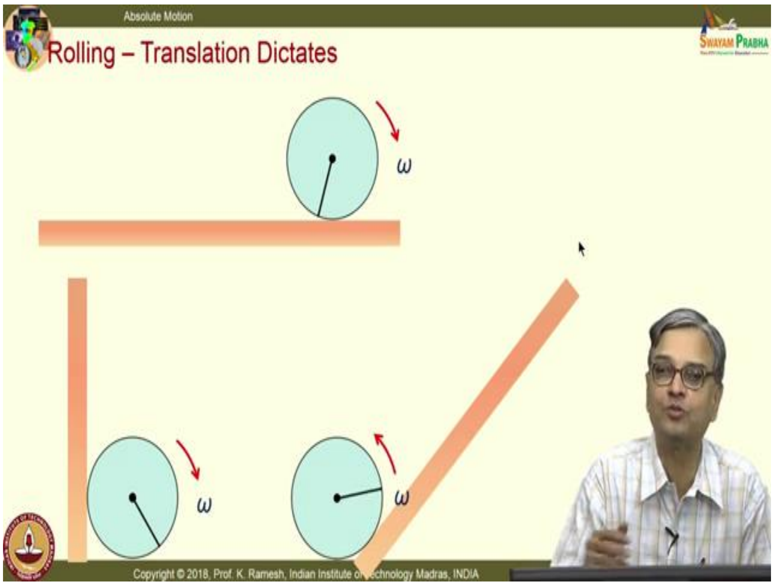
And we have looked at what was rotation in the previous class; this just a recapitalization. So, you will have to realize that it is enough I put a line on a body and monitor its rotation and able to find out what is the angular velocity and all of you have some idea of what is rolling. And I would like you to go back and see what constitute rolling; see I have three discs I have put. Can you guess why I have put three discs for me to explain?

If you have noted my initial slide that shows the background you would have got the clue. Is it same as rotating about its axis, is it same as fixed axis rotation? Or it has some connection with fixed axis rotation, you understand the question? You see a wheel rolls everybody sees, you come to the class using a bicycle; the bicycle wheel rolls and then come that why you are able to come to the class.

It is also rotating about its axis but it also has another component of motion and that is what is beautifully illustrated in this animation. I have a fixed axis rotation and I have a translation, the combinations of these two gives me rolling, sees again. What I show here is this disc rolls in this direction. Why it rolls? You will have to now bring in an understanding in statics you have read that when I have a rigid body, in general under the action of different forces it can have a force and a couple at an arbitrary point and in dynamics the arbitrary point we might take the mass center.

So, it can have a force which will translate, which will have another component, which will tend to rotate it, that beautifully happens in the case of rolling. So, it is a combination of fixed axis rotation and a translation; when I give these two this translates and rolls as long as I given the translation here. Suppose I change the direction of a translation, I am rotating the disc in the same manner, but I translated in this direction; it rolls in this way; make a neat sketch of it. So, you have looked at their rolling and you know the interrelationship and then you stop thinking and solve problem after problem; that helps you to get grades.

But that does not help you to understand and appreciate what physically happens in rolling and how do you model when we have complicated array of things where you have to accommodate rolling from the fundamental principle. You have to recognize rolling is a combination of fixed axis rotation plus a translation; it has a rotation and a translation.



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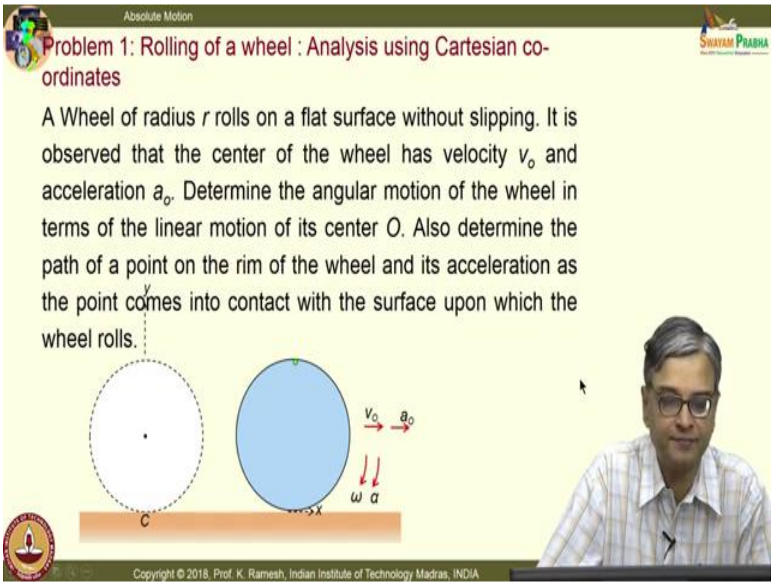
And the next slide shows, I have the similar rotation, I have a horizontal translation it rolls horizontally, I have a vertical translation it rolls vertically, I have an inclined translation, it rolls in an inclined fashion. It is a very nice example of general plane motion, very common example that you are accustomed to.



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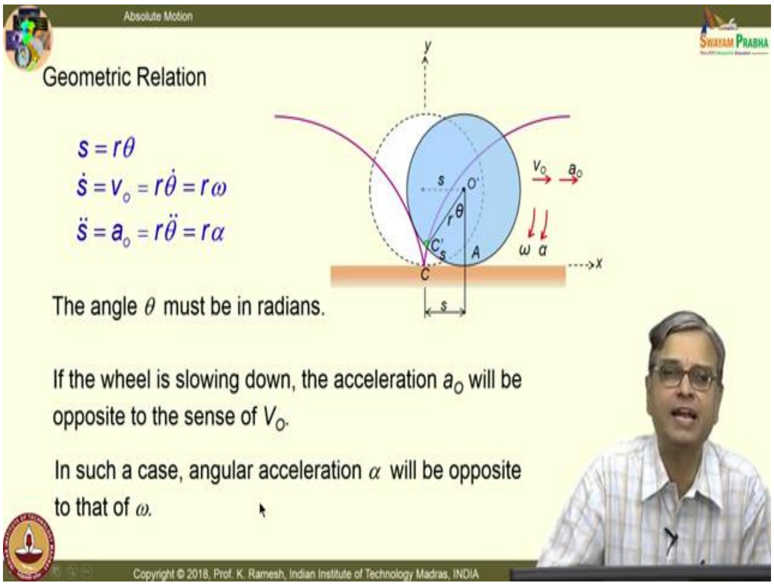
And let us get the mathematics for this, so I have a wheel of radius *r* rolls on a flat surface without slipping that is a very important statement, if you have slipping none of these kinematic relations are valid. It is observed that the centre of the wheel has a velocity v naught and acceleration a naught. Determine the angular motion of the wheel in terms of the linear motion of its center *O*.

Also determine the path of a point on the rim of the wheel and its acceleration as the point comes into contact with the surface upon which the wheel rolls. And you have nice set of animations; please make a neat big sketch of this in your notes. Here it just rolls from the next slide onwards you have much more details, so we move on to the next slide.



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And you have the disc rolls by a small amount please observe this; this animation will be repeated. First make an observation, then you have enough opportunity to draw, very carefully drawn so this is roll on the surface and the point *C* which was originally it was contacting it was moved to a point *C* prime. And it has horizontally translated by a distance *s* and we have already said it rolls without slipping fine.

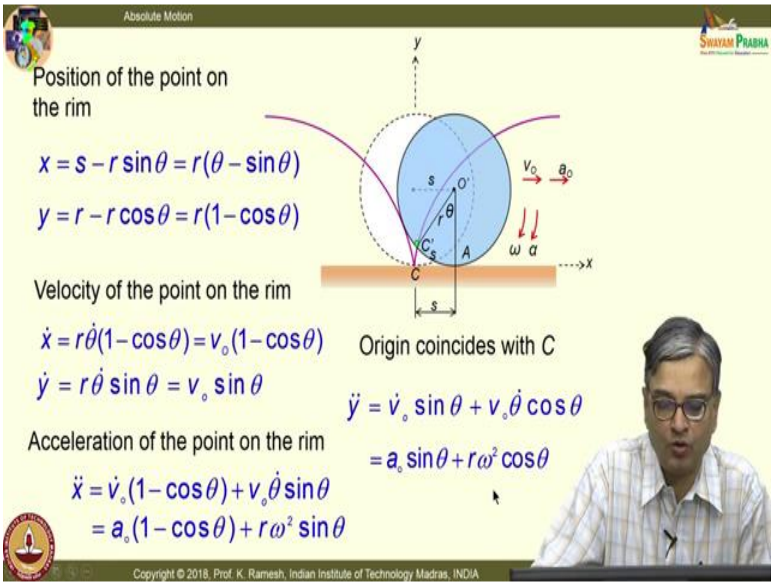
And what was C originally is now *A* here, so from A to C prime what should be the distance without slipping? That is same as the distance *s*, then you are in a position to write down the kinematical relationship; when the absolute motion is simple and very clear finding out velocity and acceleration are straight forward ok.

So, I have this $s = r\theta$ and how do I put θ? θ should be expressed in radians do not substitute it in terms of degrees. So, once I have this relationship getting $\dot{s} = v_o = r\dot{\theta} = r\omega$ It is a very important kinematical relationship; the moment you recognize in your system I have a wheel that rolls. How does it roll? That also you have to look at.

See you have thick cables that are used for electrical cable laying; you will have a spool and hub. There again you need to have a rolling and in some of those instances you will have the hub rolls on the cable. So, you have to look at which part of the system is rolling on what; that you have to recognize, then you straight away apply this kinematical relationship then the problem is taking care of and I have $\ddot{s} = a_o = r\ddot{\theta} = r\alpha$.

Very simple and straightforward; is a classical example that we start when we discuss absolute motion, as I mentioned earlier θ must be in radians. And in this case the acceleration is shown in this direction as it was increasing, suppose the wheel is slowing down; you will have a deceleration it will opposite to the sense of V_o.

I can have accelerating wheel as well as decelerating wheel both are possible. And this just summarizes in such a case angular acceleration alpha will be opposite to that of ω. The idea to emphasize here is once you know the direction of ω without a reflection on what kind of motion, do not jump to write the direction of acceleration identical to ω that is a message here. You have to investigate what is a motion, then appropriately put the acceleration direction.



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And here again to facilitate visualization I have the same animation repeated. So, make a neat sketch and put it or if you have drawn the sketch half way complete the sketch by looking at this and I can also write the quantities what I have this as *x*. I have *x* axis like this, *y* axis like this and origin coincides with *C* and *x* you can easily write it as $x = s - r\sin\theta = r(\theta - \sin\theta)$ I can also write $y = r - r\cos\theta = r(1-\cos\theta)$.

Once I have this, I can also get x dot, y dot, x double dot, y double dot all of it you can calculate, they are all very simple; please work it out yourself and check your arithmetic with the final answers I have on my slide. I have $\dot{x} = r\dot{\theta}(1-\cos\theta) = v_o(1-\cos\theta)$; $\dot{y} = r\dot{\theta}\sin\theta = v_o\sin\theta$.

Then I do the second differentiation I get this as

$$\ddot{x} = \dot{v}_o(1-\cos\theta) + v_o\dot{\theta}\sin\theta$$
$$= a_o(1-\cos\theta) + r\omega^2\sin\theta$$

So, these are all straightforward. In the absolute motion you should be in a position to get a configuration, how the relative motion, how the motion takes place and if you are able to depict that; differentiate and get the velocity, differentiate and get the acceleration.

I can also get

$$\ddot{y} = \dot{v}_o\sin\theta + v_o\dot{\theta}\cos\theta$$
$$= a_o\sin\theta + r\omega^2\cos\theta$$

.

We will also understand little more about the motion of a wheel when it rolls.

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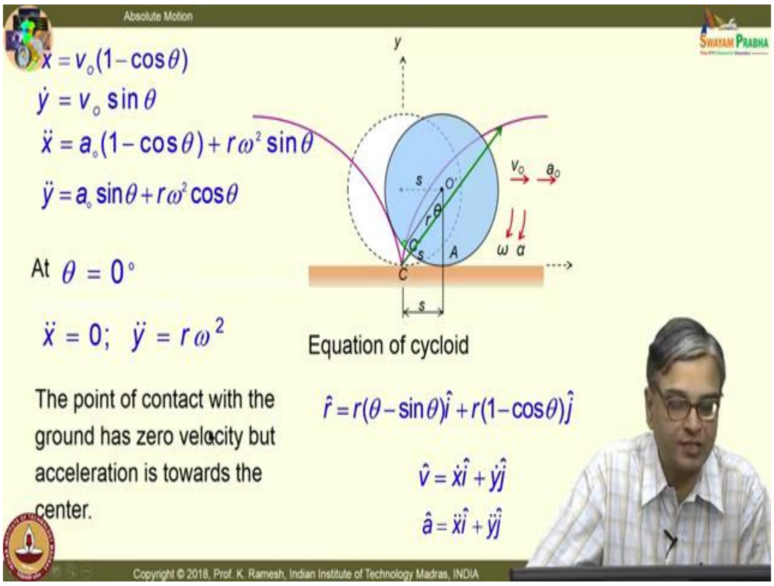
It rolls like this and this is nothing but if I monitor what is the path taken by a point on the rim, it would be the pink line what is shown here that is nothing but equation of a cycloid. And there are very interesting stories about who are all worked on cycloid people give credit to Galileo that he only coined the term and things like that and you know historians always have many names; so many people have contributed to the understanding of this, so it is very interesting to see.

And you also have many sites which provide you have a nice animation how when the wheel rolls you get the cycloid. So, I have the position vector r is simply given as $\hat{r} = r(\theta - \sin\theta)\hat{i} + r(1-\cos\theta)\hat{j}$. See one of the students came and ask me in my representation I simply put a cap to denote the vector. I am not distinguishing it as unit vector we put a cap, for any other vector we put an arrow.

You know with whatever the software that we are using for generating this equation it was easier to do. So, we have taken convenient route to represent this, I suppose this does not make much confusion in your mind in interpretation. So, I have



$$\dot{x} = v_o(1-\cos\theta)$$
$$\dot{y} = v_o \sin\theta$$
$$\ddot{x} = a_o(1-\cos\theta) + r\omega^2 \sin\theta$$
$$\ddot{y} = a_o \sin\theta + r\omega^2 \cos\theta$$

This was only a summary here; I hope I have the correct expressions here.

And we would like to see two important points; one point is when θ=0; I am looking at point C what happens at this point. An another one I would like to see what happens exactly oppose to the diameter and what I find here is I have this $\ddot{x} = 0;$, but I have acceleration $\ddot{y} = r\omega^2$ for this point. And you will also find that this velocity is 0, the point of contact with the ground has zero velocity.

It is again to emphasize when something is 0; do not jump on to conclusion everything associated with that point is 0 because these are all common things we extrapolate, we also label it as falsely as intuition; you will also have to be intelligent to get those intuitions, right intuitions. You cannot simply assume that when I have zero velocity, I have acceleration and acceleration is towards the centre that is what you get from this expression.

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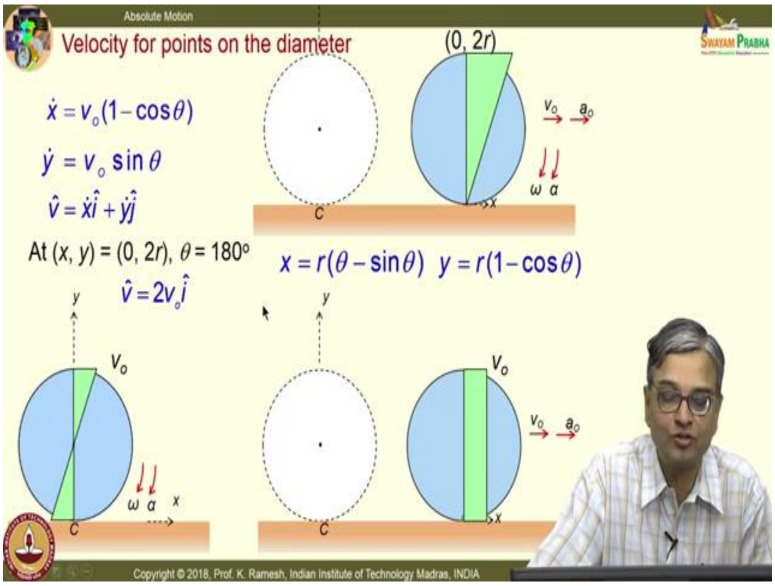
And we will also look at what way the velocity varies for a diameter like this, we have already said that this is the combination of a rotation and a translation. So, the body has rolled over to this point by half of its circumference and I have these equations repeated to facilitate discussion. And at $(x, y) = (0, 2r), \theta = 180°$, what happens to the velocity? When cos180^o what happens? So, I get this velocity at this point is twice the velocity of the mass center; I get this as $\hat{v} = 2v_o\hat{i}$.



And if I simply consider the objectives having a fixed axis rotation how do you anticipate the change of velocity from point to point? The central does not have any velocity, I have a rotation here, I have a translation here. In a rotation I would anticipate the velocity variation is like this fine and here translation every point will have identical velocity and when I add these two; I have a linear variation on the diameter and this is also a very special point. What is this special point?

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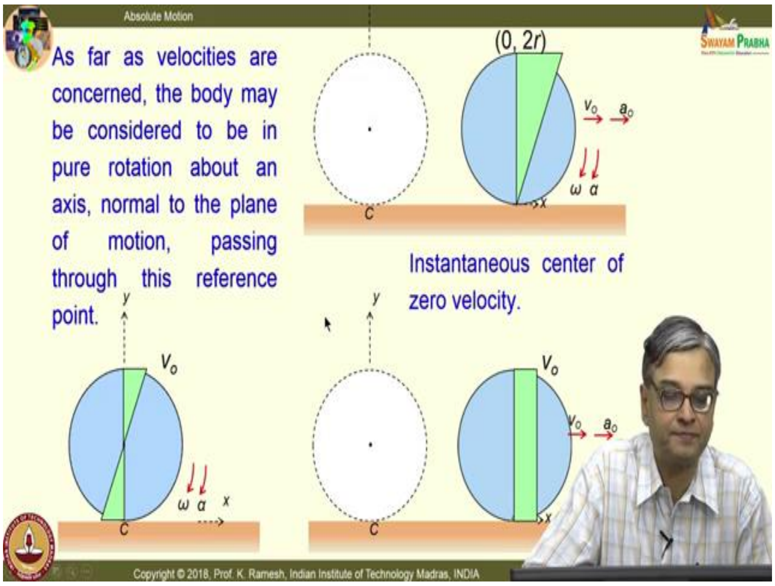
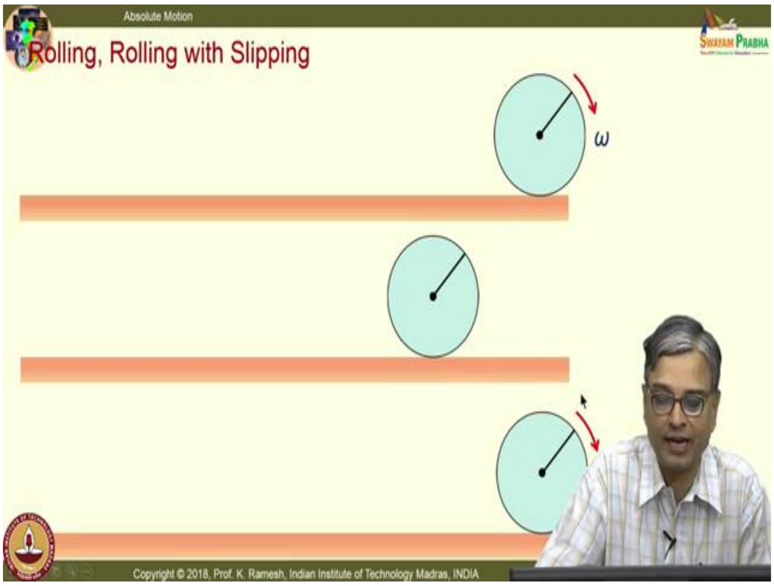
I can also look at and call this as the point c as instantaneous center of zero velocity.



What is the meaning of it? It is as if the object rotates about this point at that point in time; I have a variation of velocity like this; this is very similar to what I have done it from the centre to this, from the center to this I have put a triangular variation; this was the axis about which it was revolving.

Now, I have an instantaneous center of zero velocity very useful later we will learn several tricks of this one of it will be identifying instantaneous center of zero velocity and simplify your calculation. So, it is a pure rotation about an axis, normal to the plane of motion, passing through this reference point at that instant of time; that is what you have to keep in mind. At that instant of time it makes your computation much easier and straightforward.



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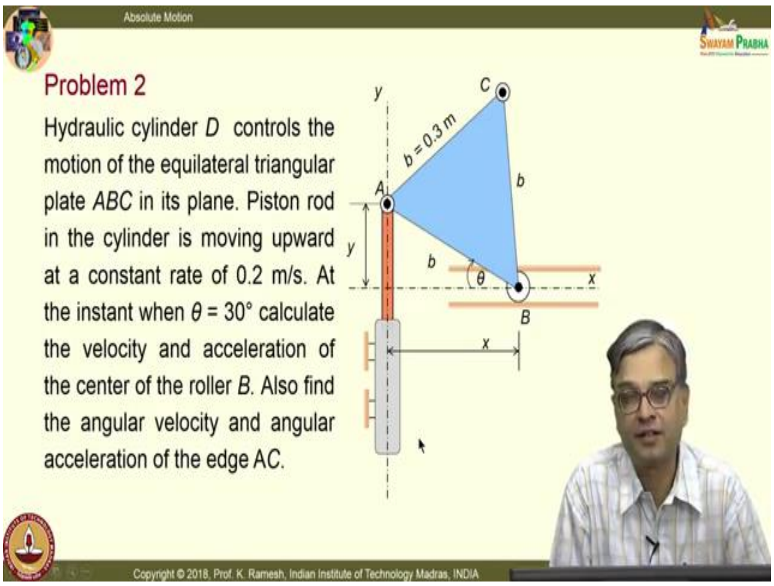
And let us look at what happens visually when there is rolling with slipping. I have a roll rolling of this disc which is a pure rolling, there is no slipping at all and look at the second disc how does it moves? So, you do have a rolling and slipping it is precipitated by the surface condition, if your path is not alright, if the floor is not alright; you could have slipping. And the moment there is slipping of the wheel, the kinematical conditions are not valid and this has a pure rolling.



So, there is a distinction between pure rolling and rolling with slipping and we make our life simple in many instances you would have noticed in actual life when you have oil that is where many accidents take place. If there is a some there was an accident and there was oil spillage; people do not have a stability, there will be sliding. So, if I have a slipping, I cannot apply the kinematical condition that we have developed earlier and I have to analyze it much more carefully.

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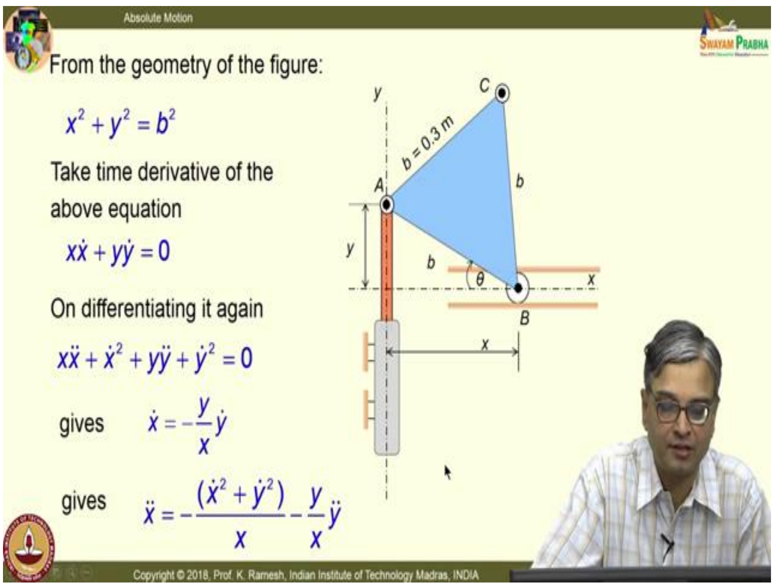
Let us move on to another problem make a neat sketch of this. I have a hydraulic cylinder which can move up and down and I have a triangular plate which has a roller here. And this roller is guided on the horizontal direction using this and geometrical details are given, it has a triangle of equal sides, I have lengths *b*, *b* and so on. And this also gives an animation which tells you what happens when the cylinder is moved up; watch the animation then the dimensions come.

So, I have this cylinder is moved up, it is moved up, this roll and then moves within that slot and you find that plate has a rotation. And in the problem, itself it is labeled as distance x here and this distance is taken as y and with respect to horizontal the orientation of the plate is θ. At the instant when θ=30^{o}; calculate the velocity and acceleration of the center of the roller B, this is one part of a question.



The second part of the question is; also find the angular velocity and angular acceleration of the edge AC. You know after listening to whatever the discussion we had about the angular velocity you should be able to say, suppose I ask the question what is the angular velocity of the triangular plate, how will you find out, do you have to do the computation again or this itself is the answer? This itself is the answer fine because I scribed the line and then monitor its motion then I get the angular velocity.

And the piston rod in the cylinder is moving upward at a constant rate to make our life simple, there is no acceleration, it is moving at a constant rate of 0.2 m/s. Fairly straightforward problem, it is easy to visualize how the motion takes place and we can write the geometrical relationship conveniently. Then simply differentiate get the velocity, then get the acceleration that is what I am going to do.

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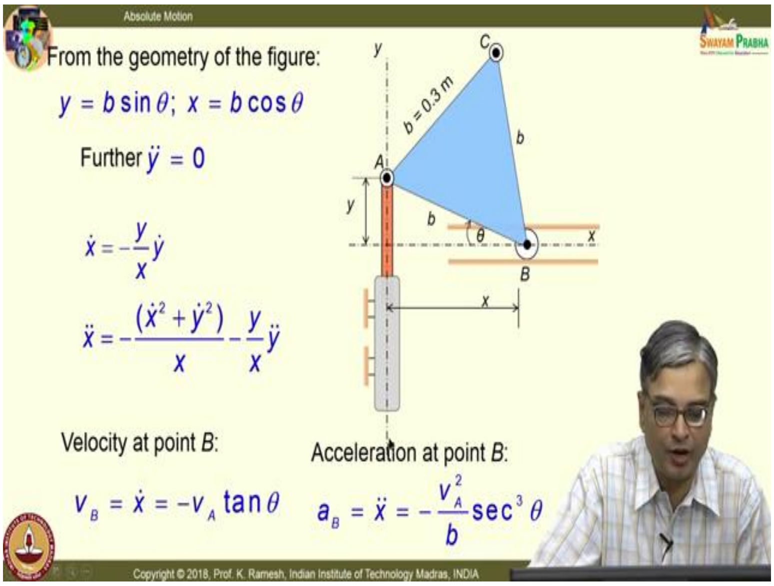
So, from the geometry of the figure I can write it shows the animation again and then the markings come which shows the dimensions and so on. So, I get an expression $x^2 + y^2 = b^2$, mind you that we are dealing with rigid body. That is why we have idealized that as a rigid body, if you considered that as an elastic body; under the action of forces it can have a minute deformation that will make our life difficult to do.

But even when you do deformable solids; we will keep our eyes half closed and then still do the same thing saying that deformations are very small fine; that is how engineering works, see engineers have to deliver answer with the available knowledge. So, you will make intelligent approximations and justify them also. It is not that you make approximations without any logical basis.

So, take the time derivative of the above equation; once this is written and you recognize that this is what is to be handle; rest of the procedure is fairly straightforward and simple mathematics. So, when I differentiate this, I get this as $x\dot{x} + y\dot{y} = 0$. On differentiating it again I get $x\ddot{x} + \dot{x}^2 + y\ddot{y} + \dot{y}^2 = 0$.



So, this gives me $\dot{x} = -\frac{y}{x}\dot{y}$ and you get the acceleration $\ddot{x} = -\frac{(\dot{x}^2 + \dot{y}^2)}{x} - \frac{y}{x}\ddot{y}$. Now, we have to go back to the problem statement; the problem statement clearly says there is no $\ddot{y}$; we have $\ddot{y}$ as 0. So, that is how you interpret and simplify substitute the quantities, you are also given what is y dot. So, you just substitute these quantities you are in a position to do that.

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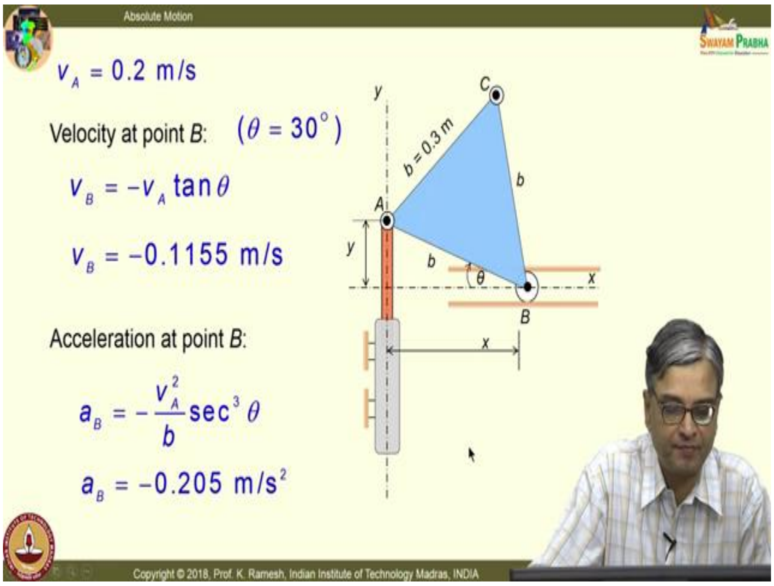
And we also know $y = b\sin\theta;\ x = b\cos\theta$ and as I said the problem statement says $\ddot{y} = 0$; this is moving at a constant velocity upwards and we have this interrelationship. So, that helps me to find out what is the velocity of point *B*, V_B is nothing but when I apply this expression and use this, I get $v_B = \dot{x} = -v_A\tan\theta$.

And we are asked to find out at θ=30° and the expression, for $\ddot{x}$ rewritten here substitute these values and I get the acceleration at point *B* as $a_B = \ddot{x} = -\frac{v_A^2}{b}\sec^3\theta$. You are given θ=30° and b is 0.3 meters and V_A is I think it is 0.2 m/s, so when you substitute, I get the numbers.



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So, velocity of point B $v_B = -0.1155$ m/s.

Acceleration at point *B* is given this expression when it substitutes the quantities, I get a *B* is $a_B = -0.205$ m/s^2; fairly straightforward problem. Now, we have to go back and find out what is the angular motion of the edge *AC*.

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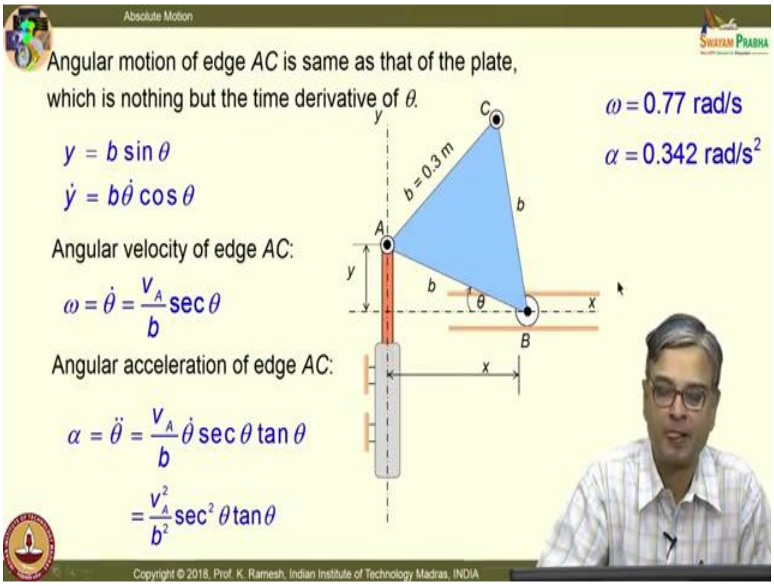
So, we are given $y = b\sin\theta$ from geometry and when you differentiate, I get this $\dot{y} = b\dot{\theta}\cos\theta$, so I can find out θ dot. So, I get $\omega = \dot{\theta} = \frac{v_A}{b}\sec\theta$. So, when I substitute, I get this as I have the values also, but before that I have also written down the expression for alpha; differentiate this again $\alpha = \ddot{\theta} = \frac{v_A}{b}\dot{\theta}\sec\theta\tan\theta$. And when I substitute the values; I get $\omega = 0.77$ rad/s and $\alpha = 0.342$ rad/s^2.



And if somebody ask what is the angular velocity of the triangular plate you can confidently say now. I have said, if I have to find out the rotation of a rigid body simply scribe a line; here the shape itself is triangular, one edge itself is a line, if I find out for that line; that line whatever happens to that line is what happens to the rigid body also.

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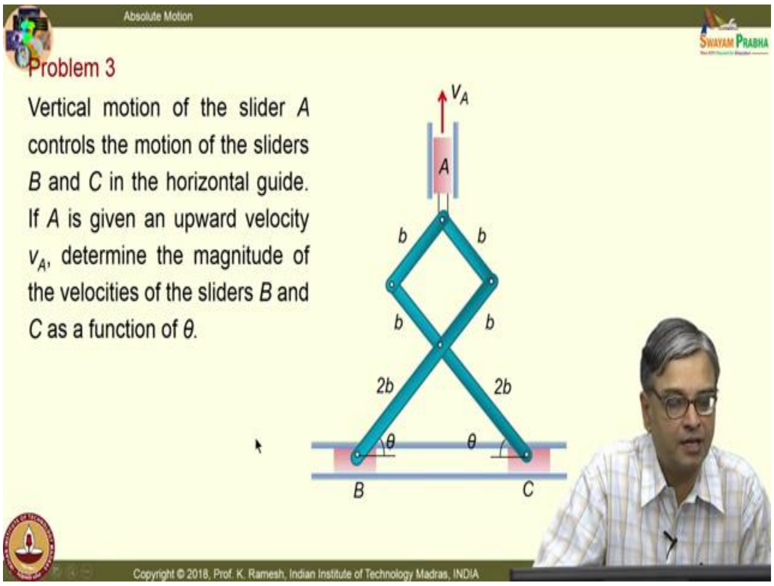
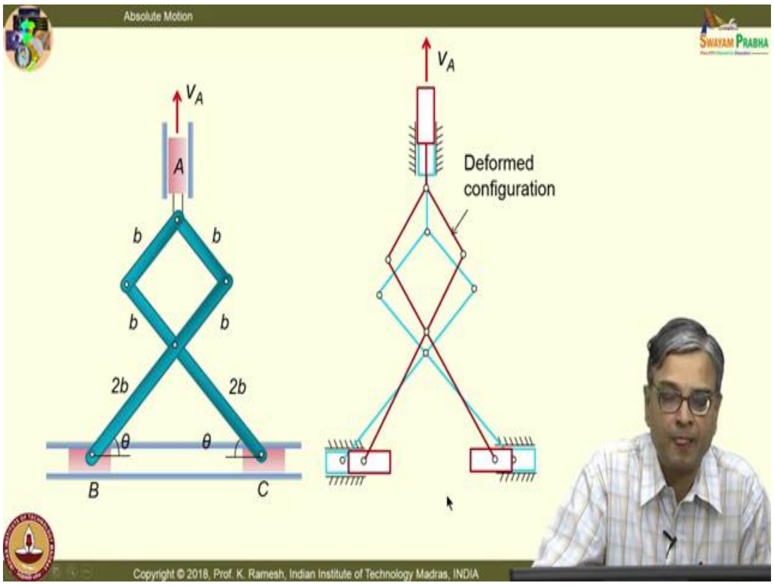


Then, we move on to the next problem which we had already looked at when we started thinking about analyzing the problems using absolute motion. Fairly straight forward the idea is to make it simple and get the concepts and make a neat sketch of this problem statement and you can easily find out the interrelationship what is *x* and what is *y* and the job is done and it is also very easy to visualize when I move this up *B* and *C* will come closer because of the geometry.



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And that is what we have shown in the deformed configuration these are all pin jointed members, very easy to visualize the deformed configuration and we get on to the mathematics.

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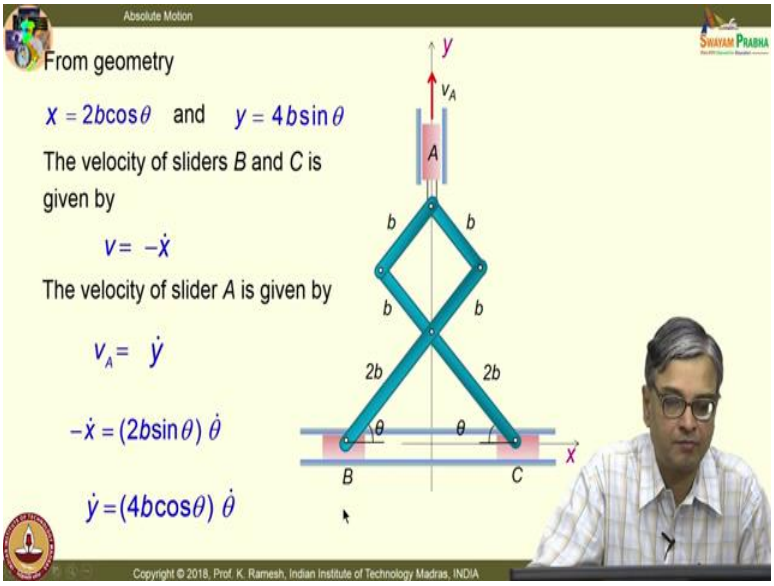
We can write x as to 2bcosθ I can write this as $x = 2b\cos\theta$ and $y = 4b\sin\theta$; the job is done. The velocity of sliders *B* and *C* is given by $v = -\dot{x}$ because we know very well that it is going to when I move up *C* will be in the negative direction; *C* will move in the negative direction of the axis $-\dot{x}$ and we have a relationship $x = 2b\cos\theta$. So, the velocity of slider A is given by $-\dot{x} = (2b\sin\theta)\,\dot{\theta}$ and V_A is y dot and $\dot{y} = (4b\cos\theta)\,\dot{\theta}$ from this expression.



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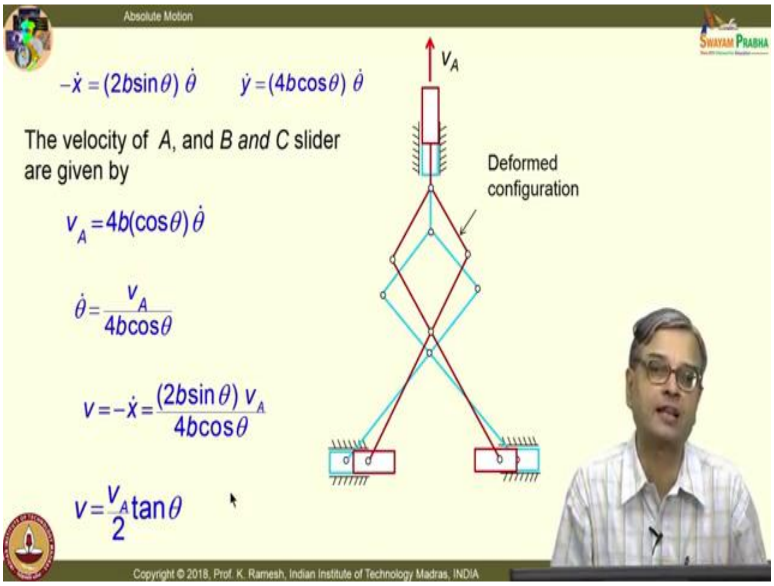

So, I have expression $-\dot{x} = (2b\sin\theta)\,\dot{\theta}$ $\dot{y} = (4b\cos\theta)\,\dot{\theta}$ Velocity of A, B and C slider are given by I have $v_A = 4b(\cos\theta)\,\dot{\theta}$ and then $\dot{\theta} = \frac{v_A}{4b\cos\theta}$ and $v = -\dot{x} = \frac{(2b\sin\theta)\,v_A}{4b\cos\theta}$. So, I can simplify this finally, this as $v = \frac{v_A}{2}\tan\theta$.

So, when I move this up; this comes down here I have written only the values here and this is fairly a straightforward problem for you to comprehend the motion and you are able to do an absolute motion analysis. So, in this class we have looked at what are the kind of motions a rigid body can have, easy to identify rectilinear motion or a fixed axis rotation.

What is subtle is curvilinear translation of a rigid body; even a general plane motion you can easily comprehend, curvilinear translation is a slightly tricky idea for you to appreciate. Then we moved on to solving a problem of rolling of a wheel and we recognized rolling is a combination of rotation and translation; we have seen nice animations depicting that, determined the kinematical relationship.

And I cautioned these kinematic relationships are applicable only when there is no slipping, if there is slipping those relations are not valid anymore. And finally, we solved two example problems to illustrate you can do absolute motion analysis when you are looking at rigid body dynamics.

Thank you.