

**Engineering Mechanics**  
**Prof. K. Ramesh**  
**Department of Applied Mechanics**  
**Indian Institute of Technology, Madras**

**Module - 02**  
**Dynamics**  
**Lecture - 04**  
**Relative Motion - I**

Module 2 Dynamics

Lecture 4 Relative Motion - I

Concepts Covered

General plane motion, Animations to demonstrate that general plane motion is a combination of translation as well as rotation, Derivation of expressions for velocity and acceleration, Animations to visualise motion of a rigid body from a non-rotating translating axes, Discussion of solution approach by graphical and vector algebra approach, Solution to problem on rolling using relative motion concept.

Keywords

Engineering mechanics, Dynamics, Rigid body, General plane motion, Non-rotating translating axes, Relative velocity and acceleration.

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Let us continue our discussion on rigid body dynamics, you know we started with simple circular motion, then we moved on to absolute motion. And today we will take up relative motion using non-rotating translating axes. And we will find in this

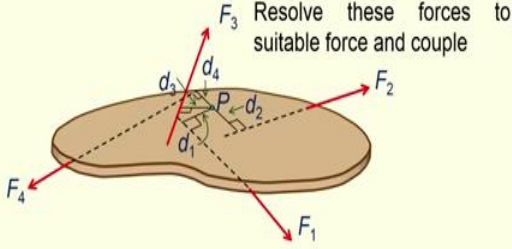
development we would still go back and then use the equations developed for circular motion very intelligently.

Relative Motion - Non-Rotating Axes

**Resultants of force system**

Move these forces to point P.

Resolve these forces to suitable force and couple



$$M = F_1d_1 + F_2d_2 - F_3d_3 + F_4d_4$$

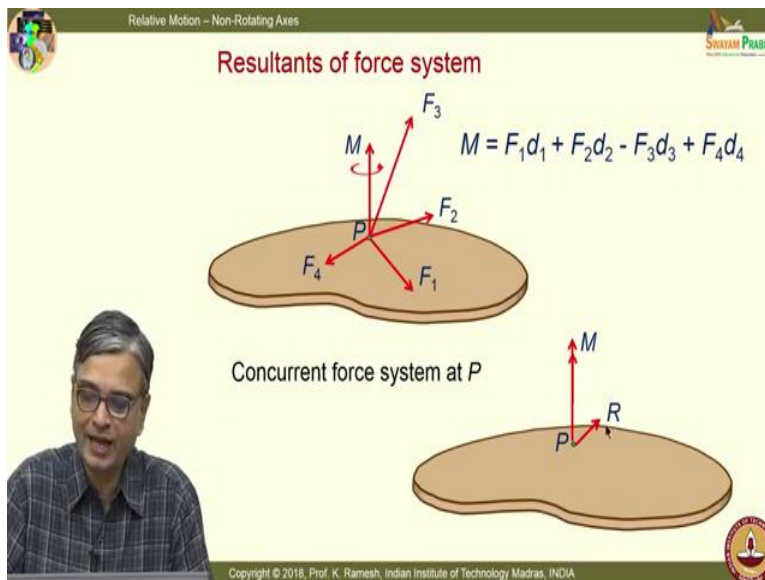
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So, whatever you have learnt in circular motion is not a waste, even though I said and cautioned you have looked at many of the practical applications. When you look at a circular motion you have a fixed axis of rotation. And

your mind is also clouded that rotation means you should have a fixed axis of rotation that you have to get out of it.

But nevertheless, we would use that very intelligently and now we go back to our review of statics when we wanted to find out the resultants of force system. Several forces are acting on this body, they are not concurrent and you make it concurrent by your understanding of moving a force from its line of action to another point by a force and a couple, so, this is what we had looked at.



So, any rigid body when you have multiple forces, in general the force system acting on the body can be reduced by moving these forces to the point of interest; by a force and an appropriate couple. And this expression for the couple is given here.

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So, then I make the force system as concurrent at point P, once I make this as concurrent

Relative Motion - Non-Rotating Axes

Force Systems

- Analysis of forces and determination of resultants of forces is common to both statics and dynamics.
- In statics, the body is said to be in equilibrium when,
 
$$\sum \vec{F} = 0; \sum \vec{M} = 0$$
- In dynamics, non-zero resultants of force systems and their effect is studied on motion of particles, a rigid body or a system of rigid bodies.
- The point P would be usually the mass point - not any arbitrary point.

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at point P its easier for me to get the resultant force, this is what we have seen earlier. So, what you find is when I have a general force system acting on body like this, it can always be reduced to a combination of a resultant force and a resultant moment. You may have

exceptions; you may have only resultant moment or you may have only resultant force. In a generic situation you will have only a combination of this.

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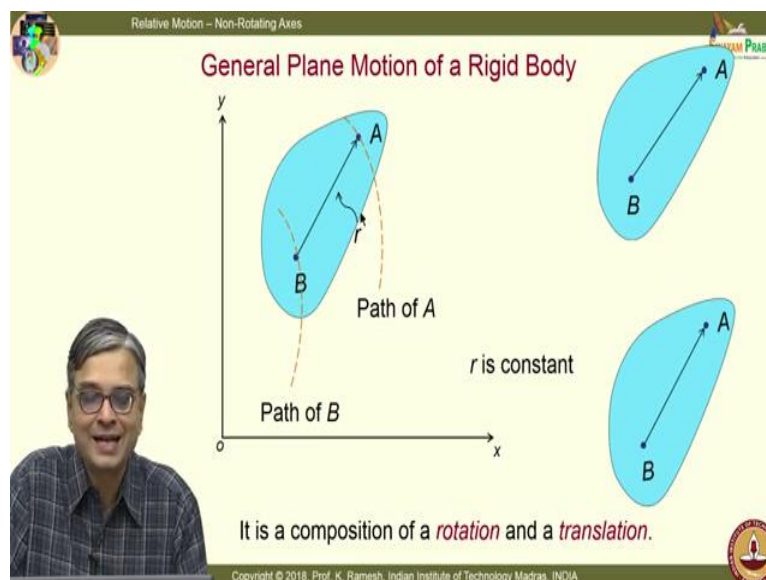
And what is the effect of these forces, and you should recognise analysis of forces and determination of resultants of forces is common to both statics and dynamics; there is no difference at all. In statics the body is said to be in equilibrium which you have seen at great length,  $\sum \vec{F} = 0; \sum \vec{M} = 0$ .

In dynamics non-zero resultants of force systems and their effect is studied on motion of particles, a rigid body or a system of rigid bodies. You have a force and a couple, a force will tend to have a linear motion, a couple will try to have a rotation. So, you will always have a combination of rectilinear motion and a rotation and we have also seen curvilinear translation all that is possible ok.

In statics you take any arbitrary point for you to do the computation, in dynamics the point  $P$  would be usually the mass point which we will see when we take up kinetics. You may not look at an arbitrary point, but there again when we want to develop the subject, we would also have the luxury of looking at from an arbitrary point. All that you

have to learn, you have to learn several tricks; so, that you have many methods available with you to solve a problem of interest.

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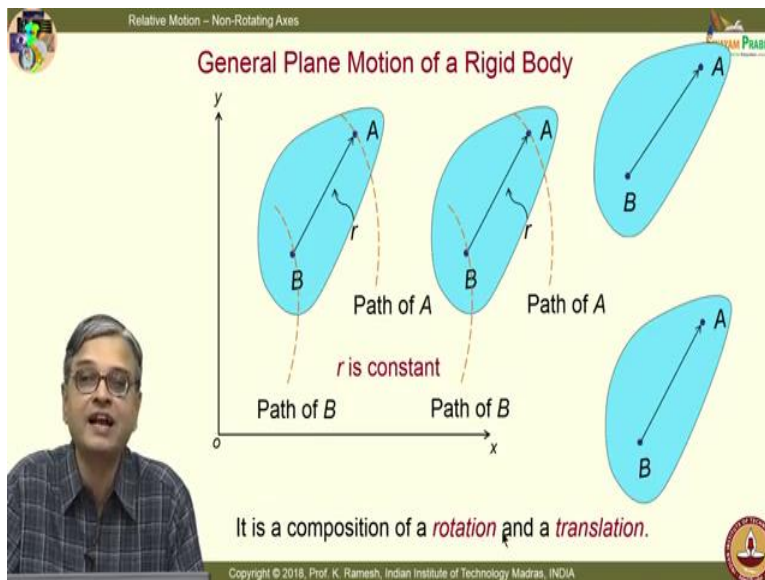
Now, make a neat sketch of this object the shape is arbitrary. So, what I have is, I have an object of rigid body, I have scribed a line

$AB$ ; and what I find is the rigid body has a complex motion. And if I monitor what is the motion of point  $B$ , it has a motion like this. And if I look at the point  $A$ , it has another curve of this nature. And what I am going to look at is develop the generic equations, so, I would consider general plane motion of this rigid body.

Let us look at the general motion and it moves to another position like this. And if you look at this motion, this can always be thought of as a combination of rotation and translation. And that is what is shown here, I would have this animation quite a few times. So, that you get the central idea better and even when you want to draw the sketch, you could take advantage of the multiple repetition of the same in various forms.

So, I have another important aspect I am having a rigid body. So, if I put point  $A$  and  $B$  the distance remains fixed,  $r$  is constant. So, you get an idea how we get into a circular motion, in a circular motion the radius does not change fine. We will see how we will link this to circular motion intelligently and make our life simple. And being a body in general plane motion, it has a rotation and a translation. And imagine that the rotation is something like this of a very small rotation. It is all done slowly, so, that you comprehend the combination of these motions.

So, I have a rotation, and I also have a small translation like this; combination of these two as resulted in the final position of this rigid body. So, we will look at this from another perspective, putting the rotation and translation on the side where does not



convince you whether the motion is a combination of these two.

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We will again go back and what I will do is I will have the original motion preserved in this part of the sketch. So, I make the screen smart whenever I

want. So, let me make the body to move, this is the motion it has reached; look at the point where it is located on this. And this is reasonably done these are all the animations I have done it, reasonably brings out the essence of it. You know microscopically you can say that it is exactly not same you can argue, but do not get into that argument.

Now, what I am going to do is, I am going to rotate this first; this is the kind of rotation I am giving it, let me rotate this first. When I rotate this first what happens, this is not the final position here; it is totally different it also gives an impression am I looking at the motion correctly.

Now, to this rotation, let me give the translation, I have a translation like this; and let me give the translation it is exactly identical. So, it is very convincing to see that a general plane motion can be thought of as a combination of rotation and translation. It is a very very subtle concept, if you look at the equations, they are very simple; but you have to get this concept to assist you in solving problems confidently.

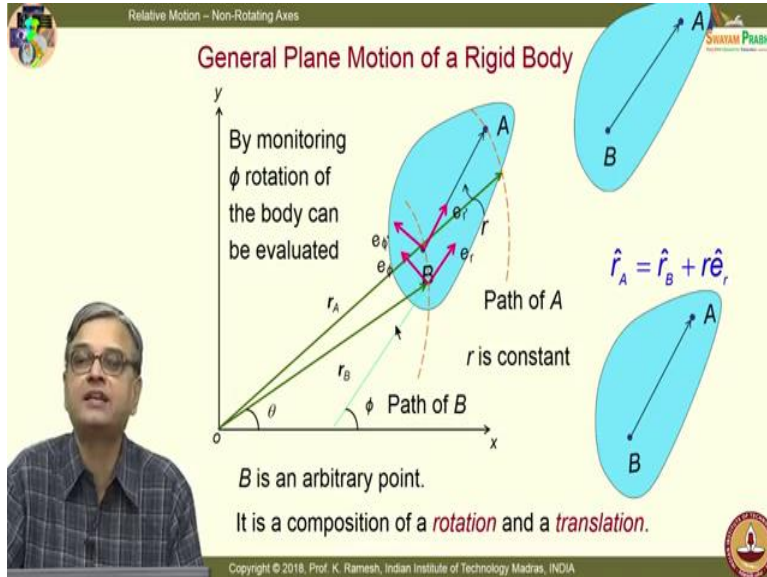
It is not that you have some equations, you know how to apply these equations; you do not feel how the body is getting this kind of motion. Now, let me ask a question, we have looked at I have applied rotation, I have applied translation in a particular sequence. In reality the body has a combination of this rotation and translation happening simultaneously.

But the only then you can have your gazettes working in particularly in packaging industry they need many such motions. It is a very interesting application where you have to have different types of complicated motions to achieve your packaging requirement.

Can I ask a question; suppose I reverse this order we have seen rotation followed by a translation. Now, let us look at translation followed by a rotation, do you think it will remain same or there will be difference, let us see what it is. The central idea is general plane motion is a composition of a rotation and a translation. It is a very subtle point if you understand it you can enjoy solving problems. Here again I have this general plane motion for this rigid body, and this we would initially have translation like this.

When I translate it is not same like what I have here, it is different. And when I give a rotation, like what I see here, it is precisely the same like what you have got earlier. This again reconfirms the central idea; general plane motion can be thought of as a composition of a rotation and a translation. You know this is qualitative appreciation, you know this is not going to help us; we need to develop the mathematics.

So, I need to put in the unit vectors, and look at what it is, and you have already learnt; if I have to recognise what is the amount of rotation for a rigid body simply scribe a line monitor its motion. You have come out of the mental block that I should look for a fixed



axis above which rotation takes place, that is what you have learnt in the last class.

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So, now let us apply all of these now, so, I have position vector for the point B is taken as  $r_B$ , and this is at an angle  $\theta$ . And I

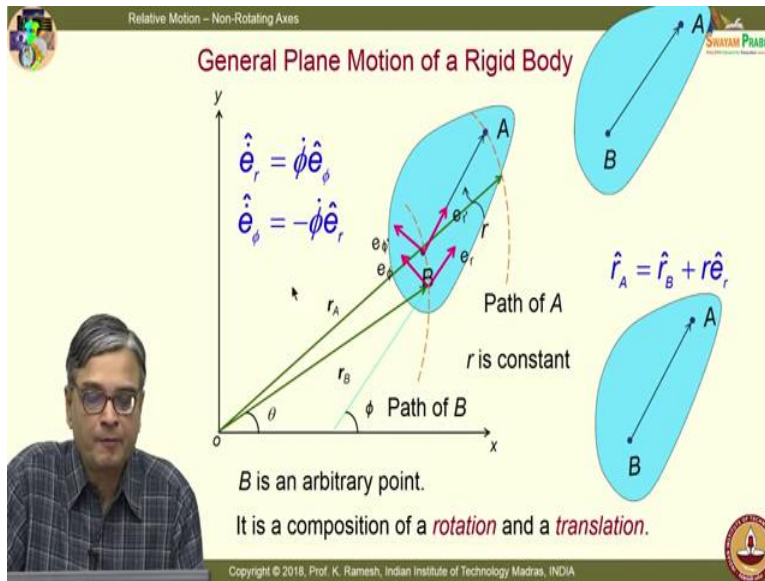
have a position vector for point A please make a neat sketch in your notes. And we have repeatedly seen that we have considered this as rigid body. So, the vector  $r$  remains constant that is a very important information from the way we have idealised the body to make our life simple and comprehend what is it.

Suppose I want to find out rotation of this body, what is that I should do, I already have a line  $AB$  scribed on it; just find out its orientation from a fixed reference. And then monitor what happens to that; so, let me put that as a line like this and put the angle  $\phi$ .

Now, I call it this as the unit vector  $e_r$  and I have another vector perpendicular to it I call it as  $e_\phi$ , to distinguish it from  $\theta$ ; I have a  $r$  and  $\phi$  coordinate system ok. I have an  $r$  and  $\phi$  coordinate system, and you should know here that this is a  $B$  is an arbitrary point. There is no special selection of point  $B$ , and as I mentioned you earlier by monitoring  $\phi$  rotation of the body can be evaluated.

And you can also write this interrelationship  $\hat{r}_A = \hat{r}_B + r\hat{e}_r$ . And mind you that  $r$  is a constant here I can also put this as  $r_{AB}$   $r_{BA}$  and so on. And I have already told you to indicate the vectors I just put a cap not making distinction between a unit vector and a normal vector, so, you can interpret these expressions correctly.

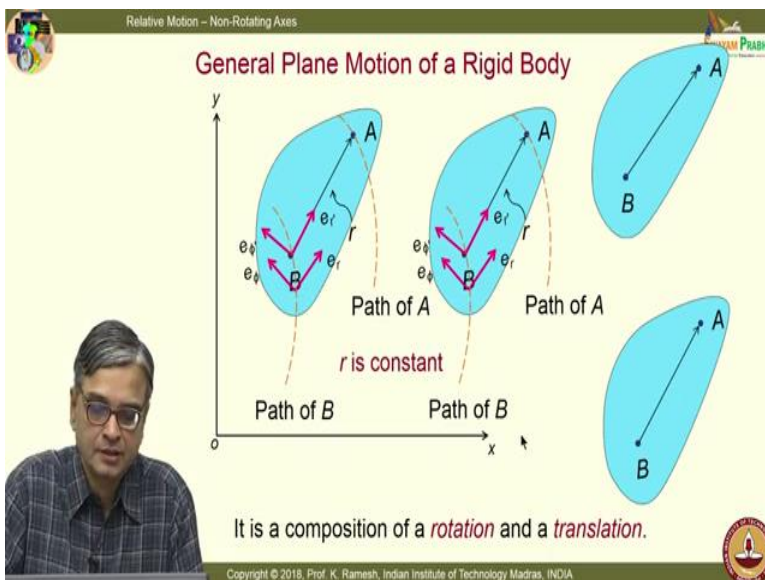
So, this has moved to another position here, and what you should recognise is, when I put the new unit vectors at the new position. They are not same as what was original, it is  $e_{r'}$  and  $e_{\phi'}$ ; please make a neat sketch of this, I would also repeat this from different perspectives again. So, you will have sufficient time to make this sketch as neatly as possible. We will again go back and see these unit vectors in stages.



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And once I have the unit vectors are changing, I can also bring back whatever we had derived for the polar coordinate system, here I use  $r$  and  $\phi$ . So,  $\hat{e}_r = \dot{\phi} \hat{e}_\phi$ ,  $\hat{e}_\phi = -\dot{\phi} \hat{e}_r$ . We will see this from a different perspective, what I have is,

I have this like what I had shown earlier and we will have rotation and translation, this is how the body has moved.



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And I have  $e_{r'}$  and  $e_{\phi'}$ , now I would have a rotation; we have already looked at rotation and translation can be interchanged. So, when I rotate, I have these changes, essentially to emphasize that  $e_r$  direction changes because of the

general plane motion. Now, I give a translation, so, you can very well see that  $e_{r'}$  and  $e_{\phi'}$  are different than  $e_r$  and  $e_\phi$ . Just to emphasize that you are looking at it from different perspective to convince yourself the unit vectors change.

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And now we go back to the mathematics, so, we have learnt that  $e_r$  and  $e_\phi$  changes from

polar coordinates we know how to write  $\hat{e}_r = \dot{\phi} \hat{e}_\phi$ ,  $\hat{e}_\phi = -\dot{\phi} \hat{e}_r$ . And from simple mathematics you can write what is  $\hat{r}_A = \hat{r}_B + r \hat{e}_r$ . And rest of it is straightforward, you simply have to do the differentiation get the velocity; again, differentiate that and get

the acceleration apply the product rule systematically; do not swallow some other terms ok.

And apply the interrelationships when I have  $\hat{e}_r$ , how it is related to  $e_\phi$  and  $\hat{e}_\phi$ , how it is related to  $e_r$  use it in your expression. So, this is going to be a similar expression like what I have got for the polar coordinate system which we are derived earlier, there is no difference at all. I have  $\hat{v}_A = \hat{r}_A = \hat{r}_B + r \hat{e}_r + r \dot{\hat{e}}_r$ . I am careful in applying the differentiation I have two terms  $r \dot{\hat{e}}_r$  and  $\hat{e}_r$ .

Then I do the acceleration; I have simplified it right here, because we know very well it is on a rigid body the distance between any two arbitrary points which is labelled as  $r$  here remains a constant. We invoke that, so,  $r' = 0$ ; so, I have this as, I have also labelled this as  $\hat{v}_B + r \phi \dot{\hat{e}}_\phi$ . And this  $r$  is replaced, because writing it in general may be confusing in a complex problem.

So, this is nothing but the vector  $r_{AB}$ , so, I have  $V_A = \hat{v}_B + r_{A/B} \phi \dot{\hat{e}}_\phi$ . So, it is very straightforward; do you see that the equation of circular motion is coming into this. I

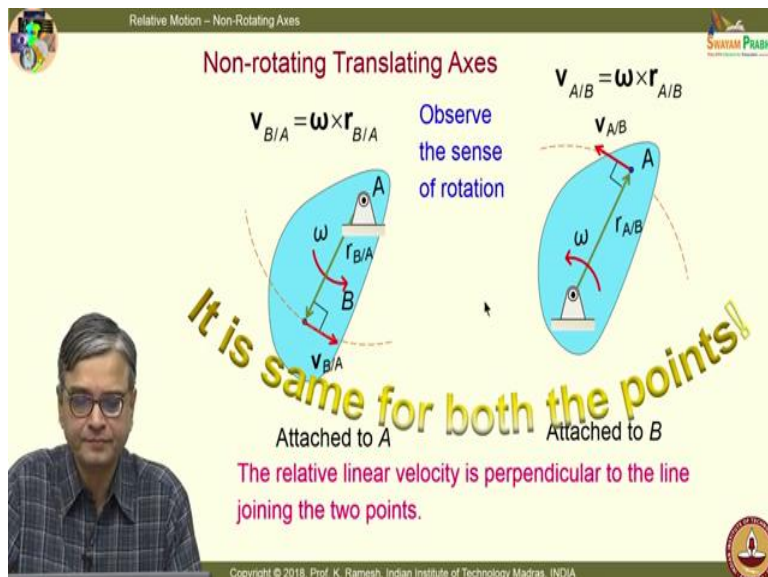


have a complex motion, but this complex motion is now simplified and we are going towards that, we will have detailed discussion on that fine.

So, I have this first expression, and you differentiate this I get a  $\hat{a}_A = \hat{a}_B + (\ddot{r} - r\dot{\phi}^2)\hat{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{e}_\phi$ . Here again I can invoke that  $r$  is a constant and knock off terms which will go to 0. So, I get this as  $\hat{a}_A = \hat{a}_B - r\dot{\phi}^2\hat{e}_r + r\ddot{\phi}\hat{e}_\phi$ , and this  $r$  can be replaced for clarity  $\hat{a}_A = \hat{a}_B - r_{A/B}\dot{\phi}^2\hat{e}_r + r_{A/B}\ddot{\phi}\hat{e}_\phi$ .

See, in all this discussion we have taken two points A and B, the idea is to get the quantities velocity and acceleration with respect to a fixed reference for point A. The intermediary is we find out what is  $V_B$  and  $a_B$  plus some other quantities. And this is how we would develop the equations for using a rotating axis.

So, this terminology B and A you should perceive it like this, when I want to apply Newton's law, I need to do it only in a inertial frame of reference; until then I cannot apply it. But to calculate the acceleration velocity, we employ different tricks to make our life simple and get the absolute motion as a combination of two different aspects.



So, what I am going to look at is, when I say non rotating axes, I would attach a non rotating axes at point B; that means, I will sit on the object and translate with it not rotate with it and observe what happens to the body which you can visualise physically and you can see that mathematically right

here. Mathematically this is nothing but you are sitting on B, and then you are viewing the other point moving in a circle, let us elaborate on it subsequently.

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So, I have a non-rotating translating axes; this is very important all these terms are very important. I have only a translating axis do not confuse it with what Galileo has done, Galileo said moving in a straight line or being stationary straight line at constant velocity is one and the same. That is how we develop the inertial frame of reference; we are not saying anything about that here.

We are only saying I am having a non-rotating translating axes, the translating axes can have both velocity as well as acceleration please understand that. I have not put any restriction on it, and I cannot apply Newton's law directly on this, I have to get the absolute values only then I can do that.

So, now we will visualise that this translating axis is attached to point B, how would I view the point A sitting at B; that means, I move along with the body it is you have to do the imagination little difficult to do. So, I will help you in doing the animation and then help you to do that.

So, I have this position vector here, this is nothing but  $r_{AB}$ , if I see the point A, the body is in general plane motion please understand that. But I translate I attach a non-rotating translating axes at B which has both velocity of B and acceleration of B; it is not having only velocity; it is not having a constant velocity.

I would see the point a moving in a direction this line of action, perpendicular to this line of action; that is v A with respect to B, is the idea clear. And I can get  $\mathbf{v}_{B/A} = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$ , the equation from your circular motion.

If I calculate the velocity like this, I can also calculate the acceleration in a similar fashion. And if you visualise that this is having an anticlockwise motion that is how we have taken it. You would see sitting at B, the point A would appear to move in this fashion; please make a neat sketch of it.

And observe the sense of rotation observe the sense of rotation observe the sense of rotation; if you understand this clearly then you will be very confident in solving

problem. There are many interesting aspects you get out of it; you know the direction of the relative velocity.

This is perpendicular to the line joining  $B$  and  $A$ , and that would be very useful in solving a problem practical problem. You know the direction you may not know only the magnitude. So, you can write a vector diagram graphical sketch where you can use this information to calculate the relevant quantities.

So, you need to understand the mechanics of it, if I attach a non-rotating translating axes at point  $B$ ; I would see point rotating with respect to  $B$  like this. Suppose I do a different aspect, I do not take point  $B$  as my reference; I attach an axis at point  $A$ , what way would I perceive the motion of point  $B$  looking from  $A$ , this is question number 1.

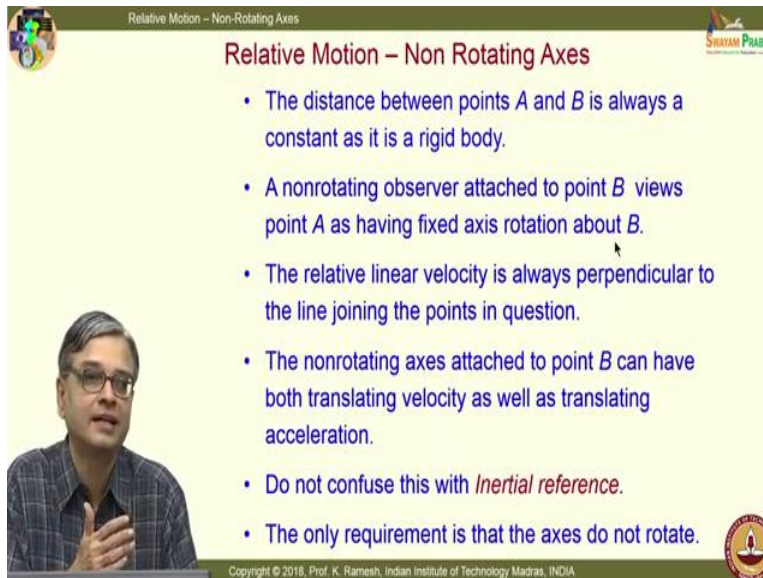
The question number 2 is, how you have seen the rotation here? The rotation is anticlockwise, how will you see the rotation sitting at point  $A$  and then look at what happens at point  $B$ ? These are the two questions you have to get clarity, see the body as such has a general plane motion. Our interest is to get the quantities of particles lying on the body to find out their absolute velocity and absolute acceleration this is our goal.

Intermediary what you are doing is, you are attaching the axes to one of the points and make your computation in stages. For one if it you might know the quantities from some other aspect or the physics of the problem. So, I get that information for the other point you intelligently use the equations and get it.

Now, the question is, if I want to view from point  $A$  to point  $B$ , how will this happen. I have attached an axis at this point; I put essentially as a pin joint, because pin joint you know very well it allows rotation. When I say that it allows rotation that is why the pin joint symbolism is used. While you solve the problem, you will not introduce it like this, you will just put the axes and label it as non-rotating translating axes this is to aid your visualisation.

And you can calculate the velocity  $\mathbf{v}_{B/A} = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$ , and this is again perpendicular to the line joining  $A$  and  $B$ . And look at how does this rotate; what is the sense of rotation, what was the sense of rotation earlier? It was anticlockwise; what is the sense of rotation now? It is again anticlockwise.

It does not become clock wise, it is again anticlockwise and the body as a whole has only anticlockwise motion. Is the idea clear? We will also understand it with further animations that helps and it is important to note the sense of rotation is same for both the points. Whether I view it from  $B$  or view it from  $A$ , the body as a whole has one sense of rotation which agrees with the physical reality.



Relative Motion – Non Rotating Axes

- The distance between points  $A$  and  $B$  is always a constant as it is a rigid body.
- A nonrotating observer attached to point  $B$  views point  $A$  as having fixed axis rotation about  $B$ .
- The relative linear velocity is always perpendicular to the line joining the points in question.
- The nonrotating axes attached to point  $B$  can have both translating velocity as well as translating acceleration.
- Do not confuse this with *Inertial reference*.
- The only requirement is that the axes do not rotate.

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And let us summarise the points, relative motion, non-rotating axes. The distance between points  $A$  and  $B$  is always a constant as it is a rigid body. If it is a deformable body what will happen? There may be

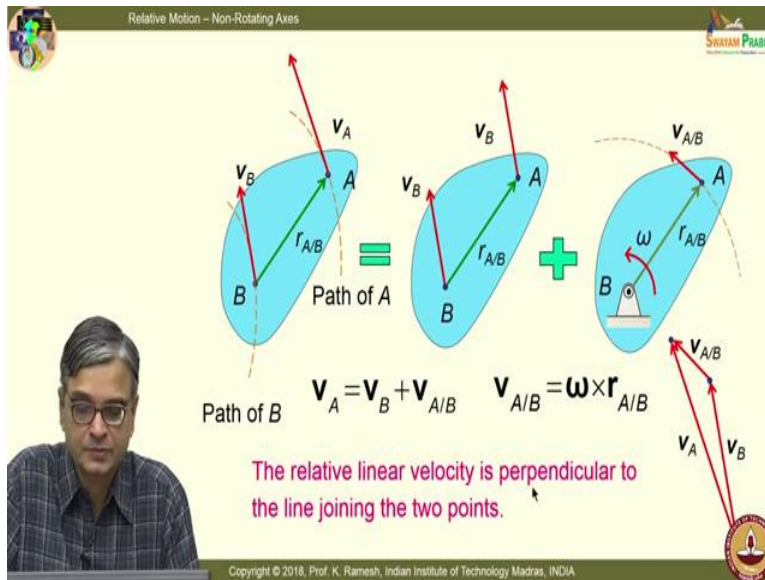
some small change in the distance. But I told you earlier engineers model the problem like this, even if you consult deformation you say it is the it is small. I can still work with initial coordinate system, all that tricks we do.

But here you do not have to do a trick; we have given an idealization that this is the rigid body. A non rotating observer attached to point  $B$  views point  $A$  as having fixed axis rotation about  $B$ , which is what we have seen with very nice set of animations ok. It is a central aspect of this discussion.

And once you know that this has a fixed axis rotation about this point, the relative linear velocities, please underline the term relative linear velocity. I am not getting the absolute velocity; the relative linear velocity is always perpendicular to the line joining the points in question.

This property we would exploit while solving problems in graphical methods, I am going to teach you graphical method as well as vector algebra. Because I would like as engineers, you should visualise things first. They are very important just learning the mathematics and blindly closing your eyes is not going to help you.

The non rotating axes attached to point B can have both translating velocity as well as translating acceleration, I am emphasising this. Do not take the reference frame attached to B as inertial reference frame; it is not an inertial reference frame. It is only an intermediate step for you to calculate the absolute acceleration at point A, I cannot apply Newton's law in this axis attached to point B. The only requirement is that the axes do not rotate.



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We will see and understand it better now, because you know the path, I can easily depict the acceleration, the velocity as tangential to the path. I have velocity  $V_B$  at point B and I have some other

velocity  $V_A$  at point A. My interest is; now try to visualise, suppose I attach an axis at B what velocity it would translate and how do I visualise the motion fine.

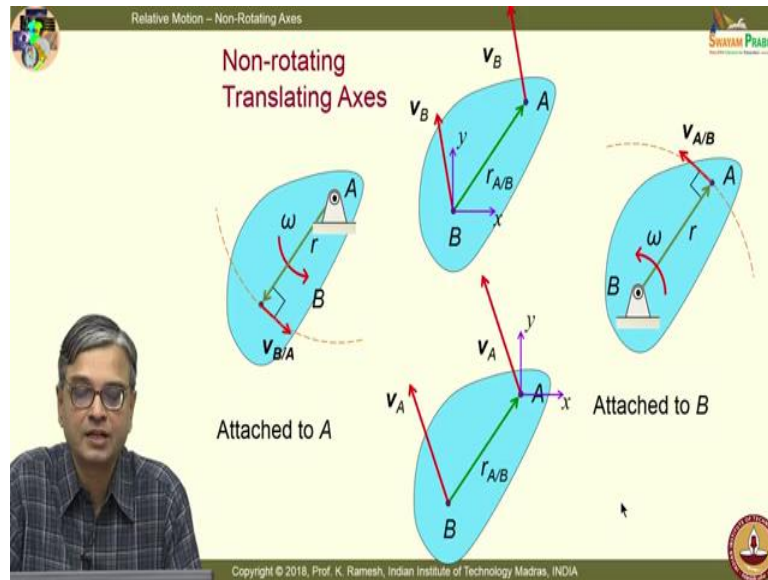
So, I have  $\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{A/B}$  and mind you;  $V_A$  is the absolute velocity which is measured with respect to the fixed reference  $V_B$  is also from the fixed reference, but  $V_{AB}$  is relative quantity ok. So, I have both the points have velocity  $V_B$ , because I have a translating axis attached to this; that means, for the observer he will have velocity  $V_B$  in this direction.

And this is nothing but a general plane motion is nothing but a combination of a linear motion like this plus a rotary motion like this which we have seen. And we have convinced ourselves, I could appreciate a general plane motion as a combination of translation and rotation, not withstanding whatever I have made the statement.

If you look at the mathematics, the mathematics tell you that this is what happens, but you do not appreciate the mathematics clearly until you see an animation to do that ok. We will also animate this and then reconvince our self. We will see what happens from

point  $B$  as well as point  $A$ . If I say from point  $A$ , then I would replace the velocity as  $V_A$  and so on.

Make a neat sketch of this write down these expressions. So, we intelligently bring back our equations related to circular motion and you use them. So, I have a  $V_A$  like this,



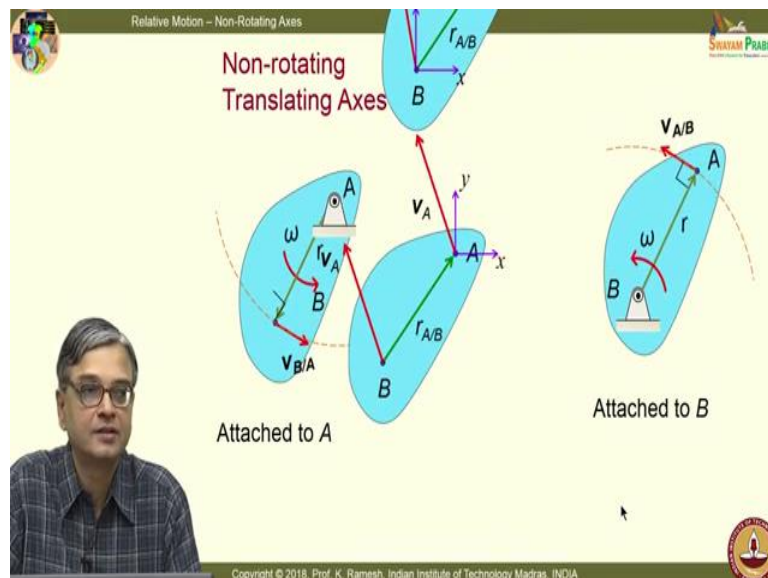
velocity  $B$  is like this, and I have the relative velocity I have velocity diagram complete.

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And we would see physically what happens I have  $V_B$ ; that means the object will have a translation and a rotation,

and let me make the board smart. So, it will show for a short while. So, attached to B, I will have a translation like this and a rotation like this. So, the combination of these two

explains the general plane motion of the original rigid body.

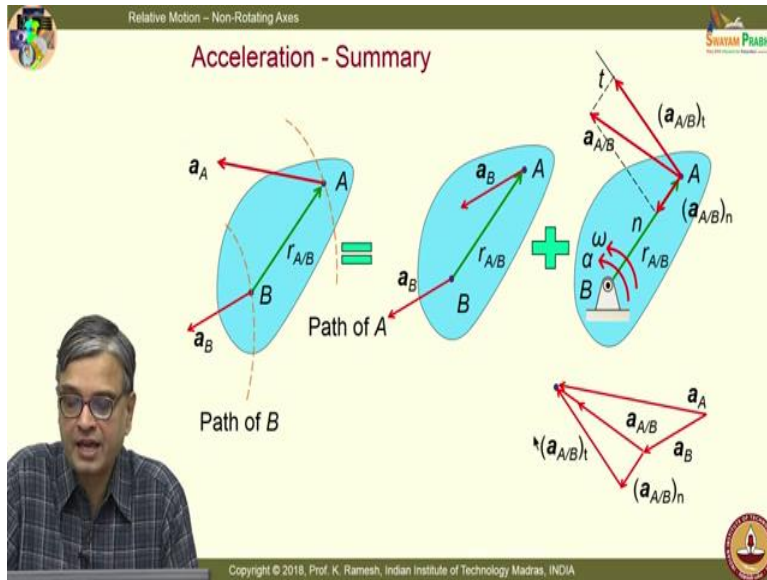


Now, we would look at if I have axes attached to point A, it would have a velocity  $V_A$  and the body will rotate like this, you can see this animation again. So, you have this velocity is different and you have a

rotation like this and which is what is shown simultaneously here, so, you get a clarity of what is happening.

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So, you can think of a general plane motion as a combination of translation and rotation. We have looked at the velocity, now we look at acceleration.

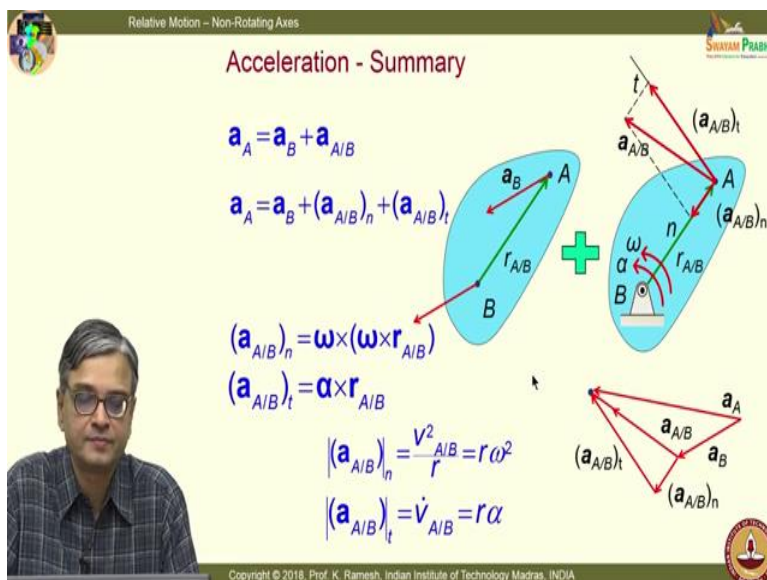


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See when I say the acceleration it will have a generic direction like this, I do not know the direction I have given an arbitrary direction like this. I have acceleration at point B and I have acceleration at point A and this is nothing but it is accelerating with AB

plus a rotation about B. And once you recognise this it is easier for you to put the relevant quantities and put the acceleration diagram. So, I have relative velocity is A,

acceleration is  $a_{A/B}$ , this has two components ok.



I have put this relative acceleration; it will have a; I can look at this as AB +  $a_A$  I get this. But for me to get  $a_{A/B}$ , I will have two components one along the radial direction or towards the centre of rotation I have this as  $(a_{A/B})_n$ , and I have a tangential component;

I put this component here. I have  $(a_{A/B})_n$  and I have  $(a_{A/B})_t$ , I can get the relative acceleration.

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And we would also write the expressions; ultimately, we need to settle down with the expressions. So, I have given this  $AB$  and want to rewrite the same thing, I need to get the absolute acceleration of point  $A$  that is nothing but absolute acceleration of point  $B$  and relative acceleration of point  $a$  with respect to  $B$ .

And my computation is largely simplified; because of a recognition that relative acceleration I can easily get it from circular motion equations that is the central core of it. The motion as such is very complex since we have looked at this as addition of translation and rotation, we are in a position to write these expressions.

So, I have  $a_{AB}$ , let us put here this is nothing but combination of a normal component and a tangential component. You can view it in either way as a radial or  $\theta$  direction and I have this as normal component, and I have the tangential component this way.

And the normal component can be obtained from your circular motion as

Relative Motion - Non-Rotating Axes

### Solution Approach

- Always sketch the vector polygon – clear enough to reveal the physical relationships involved.
- First sketch known vectors. Use this as a basis to proceed further.
- From this sketch, scalar component equations can be written by projecting the vectors along convenient directions.
- Alternately use vector algebra to construct equations.
- From this by collating the terms corresponding to say  $i$  and  $j$ , construct scalar equations.

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$(\mathbf{a}_{A/B})_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})$ . And your tangential component is nothing but  $\alpha \times r$ , and in long hand I can write

$$(\mathbf{a}_{A/B})_n = \frac{v_{A/B}^2}{r} = r\omega^2$$

$$(\mathbf{a}_{A/B})_t = \dot{v}_{A/B} = r\alpha$$

I have already cautioned you, when you write it as vectorial form you always call it as  $\alpha \times r$ . Otherwise

you simply say as  $r\alpha$  which is convenient; so, you should understand the distinction between the two.

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So, what is the solution approach, always sketch the vector polygon clear enough to reveal the physical relationships involved. So, this is where the sketching is very important for engineers. So, sketch known vectors use this as a basis to proceed further, from this sketch scalar component equations can be written by projecting the vectors along convenient directions.

So, this is how you do it in a graphical approach is not that you are going to take a graph sheet and then solve it. You can sketch it that is good enough for you to do it, alternately use vector algebra to construct equations from this by collating the terms corresponding to say  $i$  and  $j$  construct scalar equations. So, what we will do is we will replace  $e_r$  and  $e_\phi$

as simply as  $i$  and  $j$  by representing the axes we will do that.

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And take up a very simple problem which you all seem to have a better exposure on rolling. A wheel of radius  $r = 400$  mm rolls to the right without slipping that is a

very important aspect. It is measured that the centre  $O$  has a velocity of  $4$  m/s using relative motion analysis calculate the velocity of point  $A$  on the wheel for this instant. I have already told you in dynamics, we are learning methods through the vehicle of problem. So, the method is more important than the answer for the problem. So, I want you to solve it only by relative motion analysis.

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It is a very simple problem there are multiple ways to solve it. So, we look at what way we will handle it. Taking O as the origin of non rotating axes, from relative motion

Relative Motion – Non-Rotating Axes

Taking O as the origin of non-rotating axes, from relative motion approach one gets,

$$V_A = V_o + V_{A/o}$$

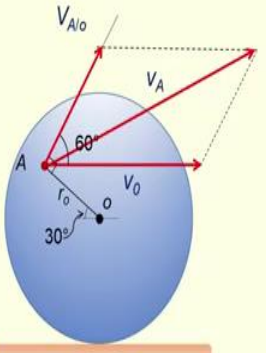
From the rolling motion basics

$$V_o = r\omega$$

$v_o$  is given as 4 m/s  $\omega = 10$  rad/s

$$V_{A/o} = r_A\omega = 0.3 \times 10 = 3 \text{ m/s}$$

$$V_A^2 = 4^2 + 3^2 + 2(4)(3)\cos 60^\circ = 37 \text{ (m/s)}^2$$

$$V_A = 6.08 \text{ m/s}$$


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approach one gets

$V_A = V_o + V_{A/o}$ . So, I have this distance is fixed, the angle is given as  $30^\circ$ . And I am given the velocity  $V_o$  in the problem statement that is horizontal like this.

And you know very well from our discussion on relative motion the relative

velocity when I attach a translating axis at O has to be perpendicular to this. And mind you while developing the equation we have put A and B. So, in a physical problem, you should find out what corresponds to B and what corresponds to A, so, do not confuse these two issues.

So, you should know how to apply the equations properly, suppose I call this as B and then A, you should be able to apply the same equations appropriately fine. It is only a labelling requirement, but you should understand the motion correctly. And since it is said in the problem statement that this is rolling, I bring in the kinematic relationship  $V_o = r\omega$ .

So, I get this  $\omega$  as 10 rad/s, it is given in the problem its 4 metres per second. So, now, I can find out what is  $V_{A/o} = r_A\omega = 0.3 \times 10 = 3 \text{ m/s}$ . So, I write this, then it is a child's play to get the resultant.

So, it is a very simple problem to start with, and we have got the value of  $V_A^2$ , please check the arithmetic. This turns out to be; it is also wrongly written here; I have to get this final value; this is put as  $V_A = 6.08 \text{ m/s}$ . I was initially thinking that this is  $(\text{m/s})^2$ , but I am calculating  $V^2$ . So,  $V_A = 6.08 \text{ m/s}$ , fairly a straightforward problem.

So, in this class we have looked at what constitutes general plane motion. I have said that it is a combination of rotation and translation whichever way you do the sequence; the final motion remains same. Even though the motion is little more complicated we have been able to bring in the circular motion equations intelligently to handle the problem like this.

And once you learned how to do relative motion analysis, you can solve a wide variety of problems which was not possible only with absolute motion.

Thank you.