Engineering Mechanics
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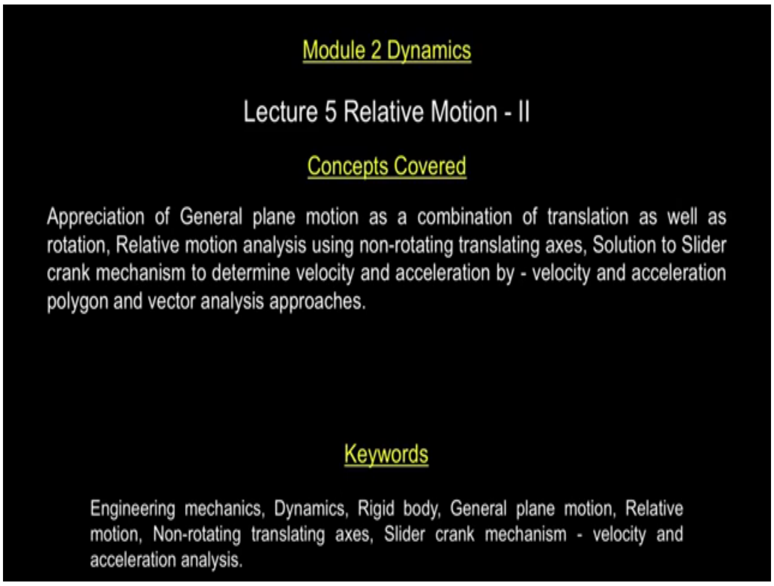
Module - 02
Dynamics
Lecture - 25
Relative Motion –II



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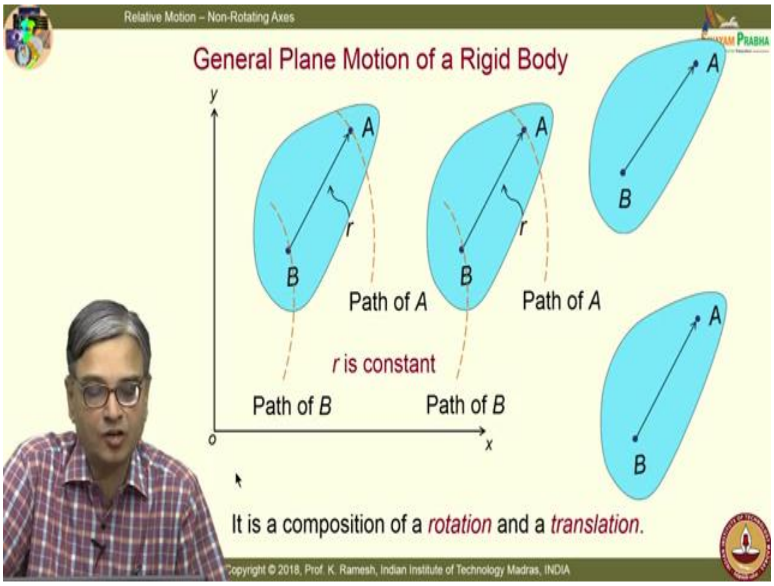
Let us continue our discussion on Relative Motion analysis. You know when you look at relative motion approach you really expand the scope of solving more class of problem. Absolute motion analysis restricted where is ratio motion analysis helps you to solve a larger class of problems.



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And one of the key points we learned in the last class was what constitutes general plane motion, here I have the body experience in general plane motion. This motion can be thought of as a combination of rotation and a translation, so, I rotate this first it is deviating from the path and I give a translation this comes to the path as what we have seen in this diagram. So, it is a very important concept.

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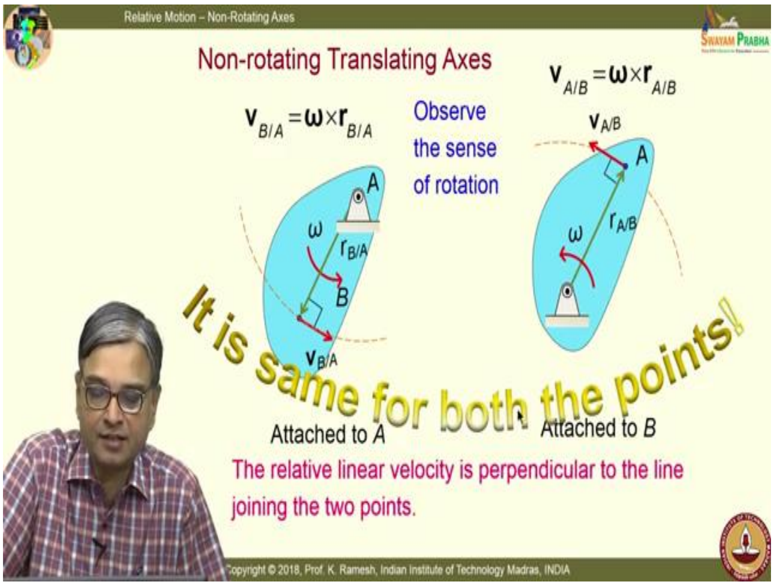
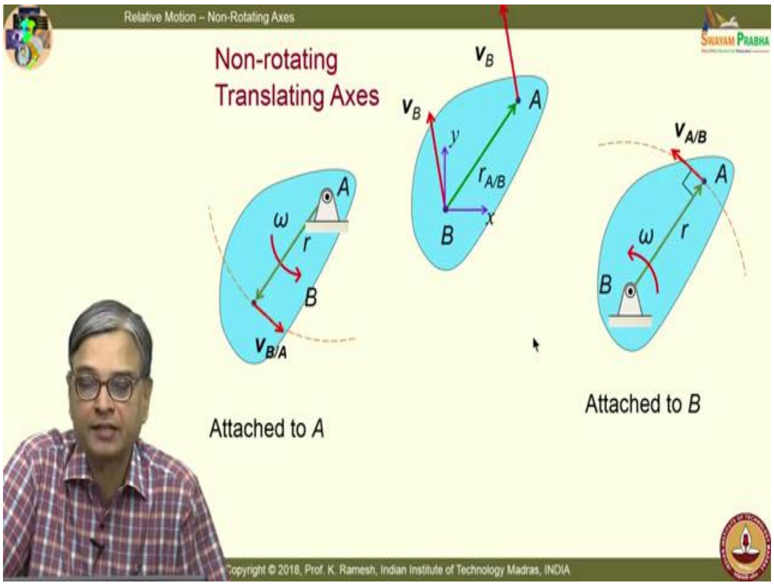


General plane motion is a composition of a rotation and a translation, it is a very subtle concept. And when I want to view the point *A* sitting at *B*; that means, I have attached a non-rotating translating axis at *B*. The axes can have velocity as well as acceleration, it is not an inertial frame of reference.

And when I have a rigid body this distance remains fixed; the relative linear velocity is perpendicular to this line joining these two points. And the body when I the point *A* when I view it from point *B* it rotates, observe the senses of rotation. Now, what we would see is? We would fix the axis at point *A* and view point *B*. Here again if we look at the sense of rotation, its one and the same. This is by virtue that you have been able to appreciate general plane motion consists of a rotation and a translation.



Now, we are translating with the body, when I view it from different points the mathematical details will differ, but the sense of rotation still remains the same. This is what is illustrated it says that the sense is same for both the points.

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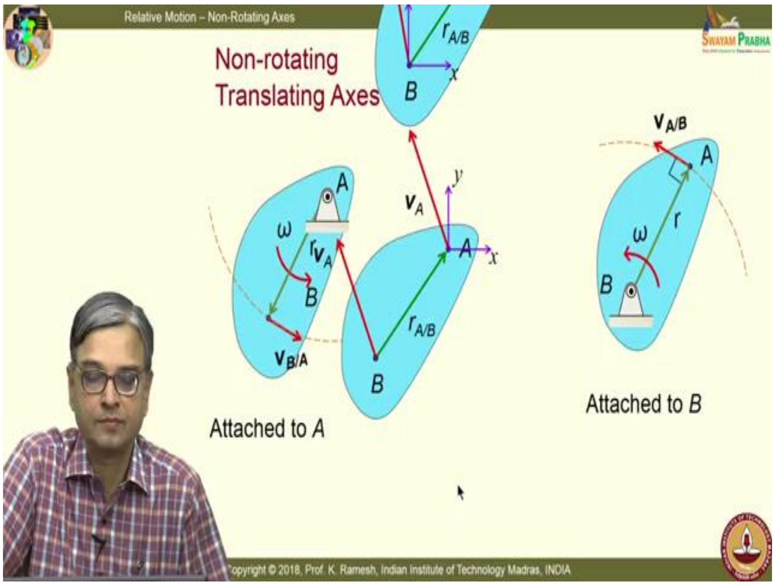

The idea of attaching a translating axis at B amount to translating with velocity V_B and you have a corresponding rotation here.

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If I attach the axis at *A* it amounts to travelling with velocity V_A and rotation in this way.

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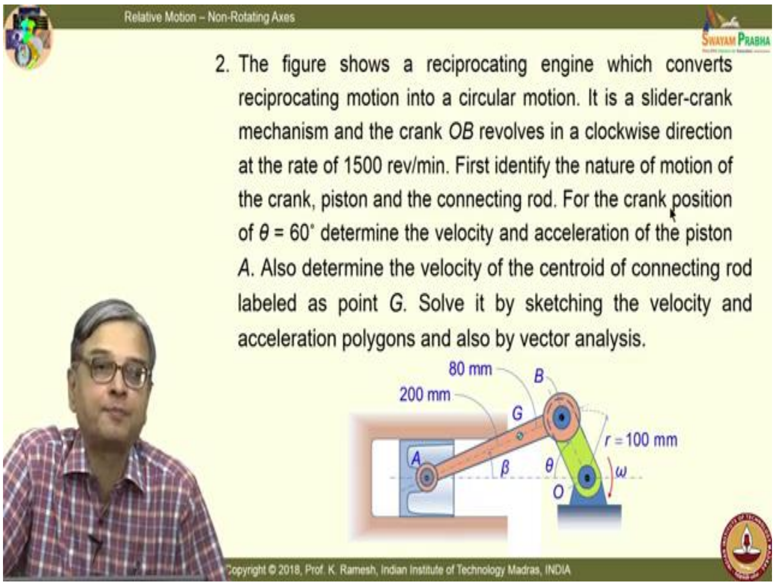
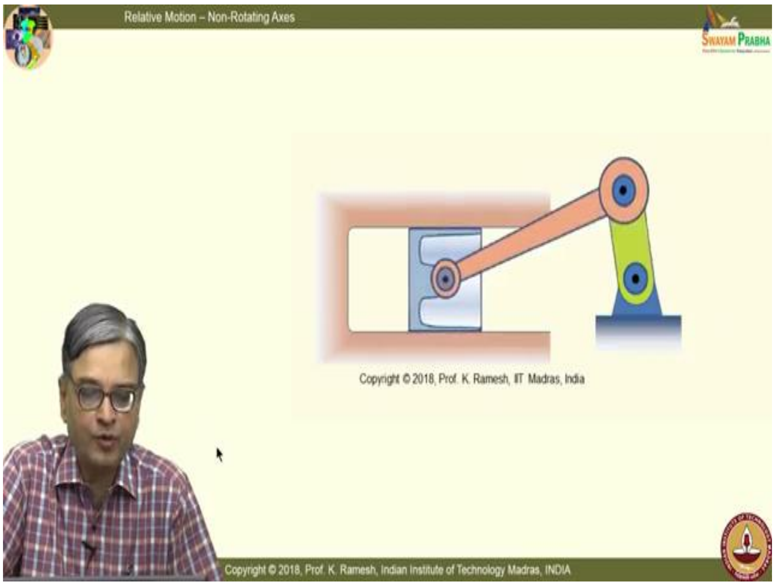


We will go and solve the very practical problem; you have a reciprocating engine. This is again example of four bar mechanism and it is called as slider crank mechanism and the crank *OB*, this is the crank *OB* revolves in a clockwise direction at the rate of 1500 rpm.

First identify the nature of motion of crank piston and the connecting rod and you have to determine certain quantities at a particular crank position. The crank position is a given as $\theta = 60^{o}$, you have to determine the velocity and acceleration of the piston A. You know when you start solving the problem; we have developed the equations based on two arbitrary points *A* and *B*. To make your life simple more or less these points are similar to what we have started with.

So, you do not have confusion on interpreting the equations first, once you get the grip of the equation, we can label it in any way and solve the problem. And you are also asked to determine the velocity of the centroid of labeled as point *G*. Solve it by sketching the velocity and acceleration polygons and also by vector analysis. Say the idea is if you draw the velocity and acceleration diagrams you get a better physical appreciation of the problem. On the other hand, when you get into vectorial analysis, it becomes totally mathematical in a sense it also will be slightly boring to do that.



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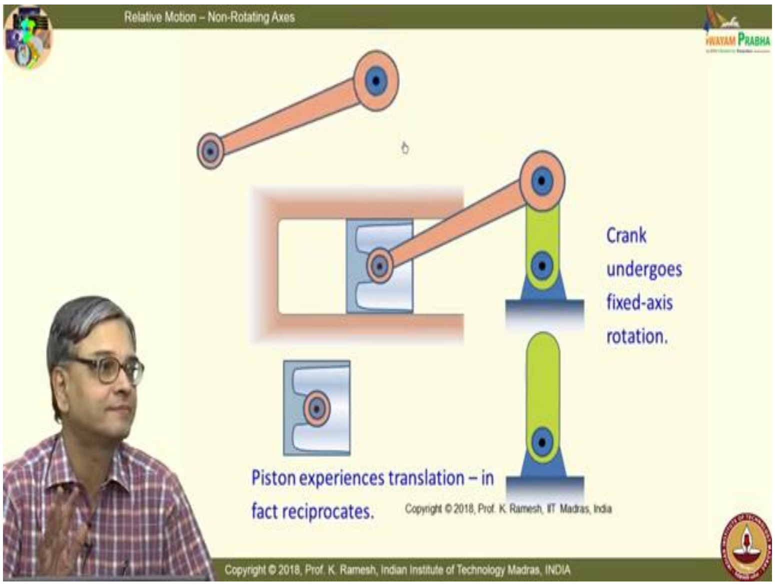

Let us understand the motion of this, so, this is how the whole system operates. Now, we will go and individually analyze what happens to the crank, what happens to the connecting rod and what happens to the piston.

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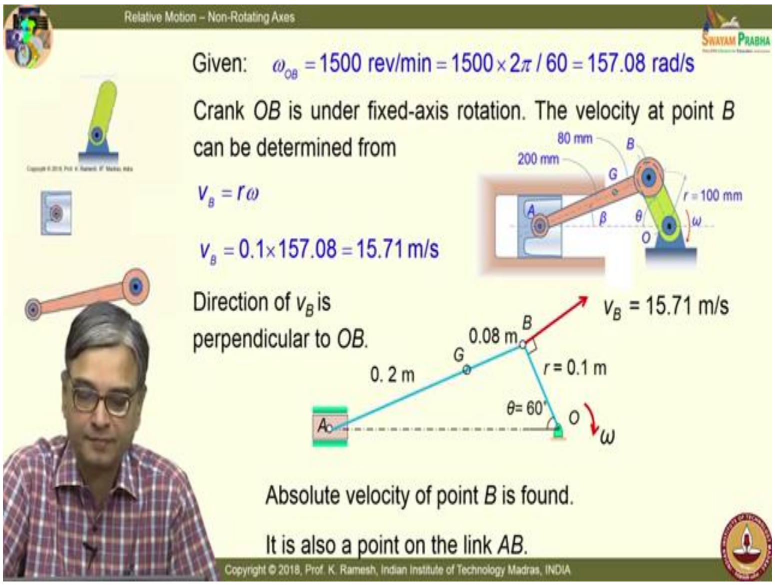
The connecting rod has I mean the crank has a fixed axis rotation and your piston has a reciprocating motion and the connecting rod has a general plane motion. That is what you have recognize; it is a very nice problem where you have the motion of all three varieties in one problem.



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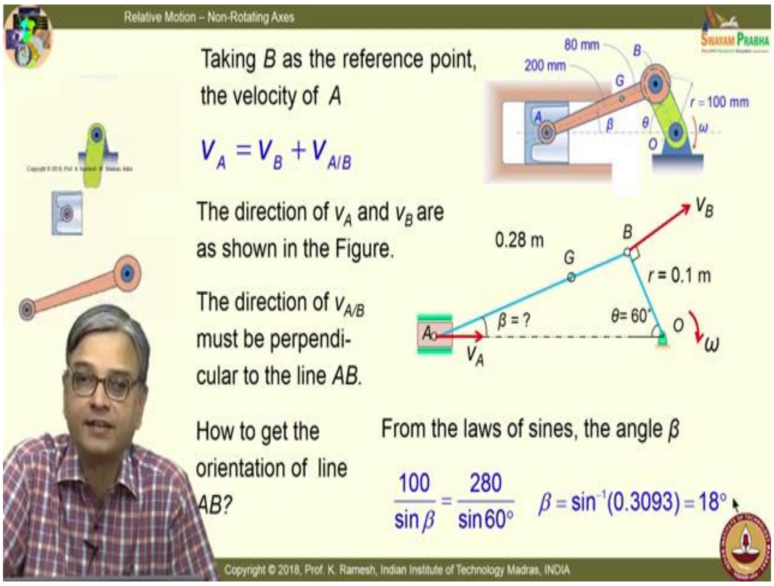
And what is given in the problem statement? The angular velocity of the crank is given in revolutions per minute; it is desirable that you convert it into rad/s. So, when you do that, I get data as 157.08 radians per second. See you have to interpret the engineering quantities properly. You should not ignore that conversion you have forgotten you have over stepped that all that you should not say, you should handle all these quantities with right units.

And we have already noted that crank *OB* is under fixed axis rotation, so, I have a fixed rotation like this. The velocity at point *B* can be determined directly from that, when I have a fixed axis rotation the calculations are very simple. This is what I told you earlier; though we say fixed axis rotation we have a mental block in looking at axis of rotation.

Even when I have a non rotating translating axes attached I still invoke fixed axis rotation equation when I want to look at what happens to general plane motion. Before that since we are analysis the crank which is having a fixed axis rotation. You can write the quantities VB is nothing but $r\omega$ and I get V_B as when I do that computation. I would also appreciate that you check these calculations using your calculator; this comes to be 15.71 meters per second.

And we also know what is the direction of V_B; it has to be perpendicular to the crank. So, that is what is indicated here, I have V_B is perpendicular to this; I have this as 15.71 m/s. And what we had determined we had determined the absolute velocity of point *B* and if you look at the connecting rod; point *B* is also part of the connecting rod. But for the point *B* we have already determined the absolute velocity, so, I can attach a translating axis to point *B* and look at the what happens at point *A*.



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Taking B as a reference point I can find out the velocity of A, we have already looked at that this is experience in general plane motion. If I attach a non rotating translating axis at *B*; what way I would perceive *A*? It would simply have a circular motion with respect to *B*. So, the velocity of *A* is $V_A = V_B + V_{A/B}$.

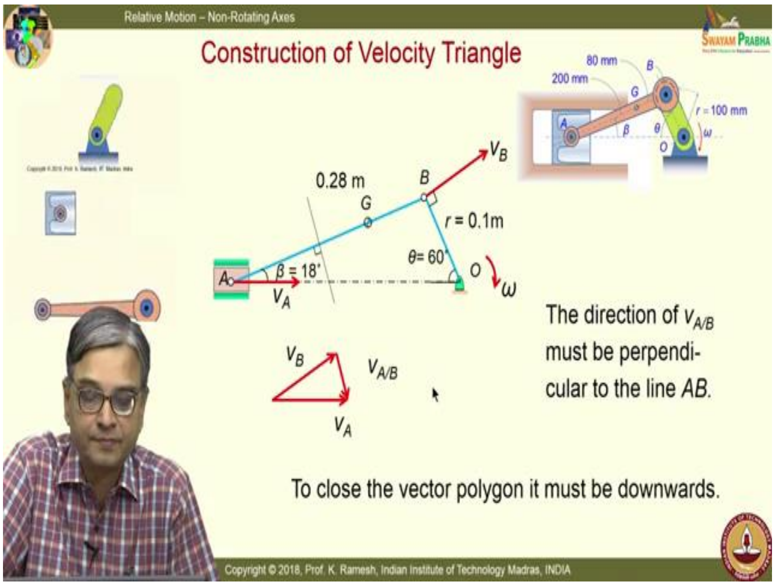
And by virtue of the piston having a reciprocating motion in this slot, I can say that this is the velocity of A; we have already determined what is the velocity of B. So, these are known and can you comment about what is direction of velocity of A with respect to B, you can do that from your general plane motion analysis.

We may not know the magnitude to start with, but we are in a position to find out its direction. The velocity direction would be perpendicular to the line jointing *B* and *A*, because I would perceive by sitting at *B* on a non rotating translating axis. The point *A* would have circular motion and I think yeah V_{AB} is perpendicular to the line *AB*.

And for us to proceed further I need to have certain geometric quantities; I need to know what is angle β when θ=60^o. You know I am doing all this calculation systematically, because when you have to solve the practical problem you have to bring back your old memories you have learn about properties of triangle, sine rule, properties of circle.

All that you have learn in your 11th and 12th standard you cannot forget, you must bring back goes memories. And I can get this from the law of sines, I can write $\frac{100}{\sin\beta}=\frac{280}{\sin 60^\circ}$. And this gives the value of β as $\beta=\sin^{-1}(0.3093)=18^\circ$.

So, I know how to put the direction of the relative velocity of *A* with respect to *B*, this is the key aspect that you have to recognize. This is what you spend time in understanding general claim motion which can be thought of as composition of a rotation and a translation when I attach a translating axis, I perceive only the rotation.



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So, I put the velocities I have V_B, I have V_A and I have to put V_{AB} perpendicular to this. And one can also guess which should be the direction of velocity of a with respect to B, because it has to close the polygon fine. I am essentially finding out what is V_A I know what is the direction of V_B, so, I can always say that the direction of V_A with respect to *B* should be like this.

So, this is what I am going to have, I have this perpendicular to this line joining *B* and *A* that happens to be the connecting rod what we have also shown as a line. So, I get this I close the polygon, now I know what is the velocity of *A* with respect to *B*.

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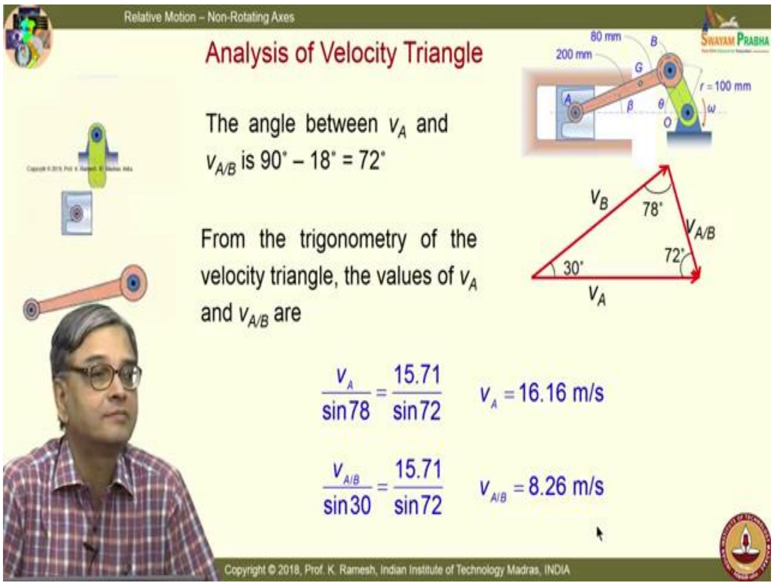
And we will look at the magnitudes I need some more angles for me to proceed, so, I need to get that by the appropriate properties. And we have already said that this is perpendicular to the line joining this and this line is at angle 18° from the horizontal, so, from the vertical this about 18°.

So, if I want to find out this angle that turns out to be 72° and this naturally comes to be 78 and from the trigonometry of the velocity triangle the values of V_A and V_A with respect to B are determined as follows. I have $\frac{V_A}{\sin 78} = \frac{15.71}{\sin 72} \quad V_A = 16.16 \text{ m/s}$



Similarly, I can also write $\frac{V_{A/B}}{\sin 30} = \frac{15.71}{\sin 72} \quad V_{A/B} = 8.26 \text{ m/s}$. So, I get the velocity of A with respect to B as 8.26 meters per second, is the idea clear. Can you find out with these quantities is what is the angular velocity of the connecting rod, you can do that fine.

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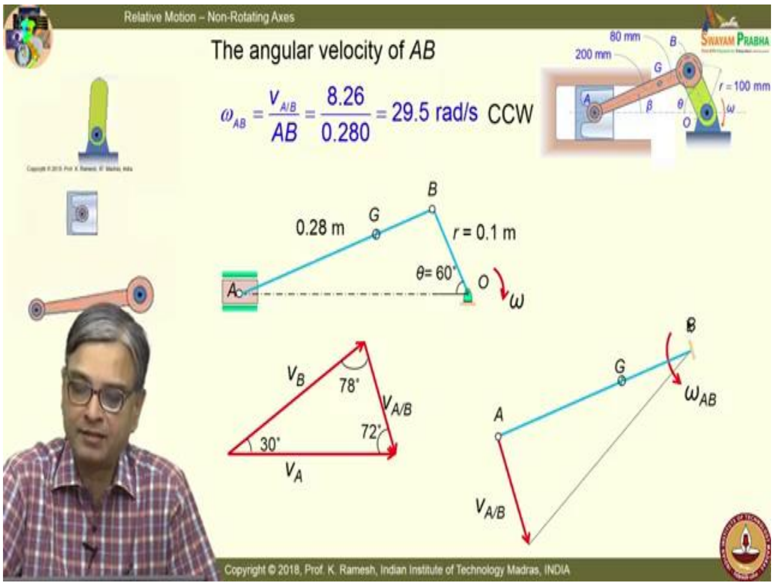
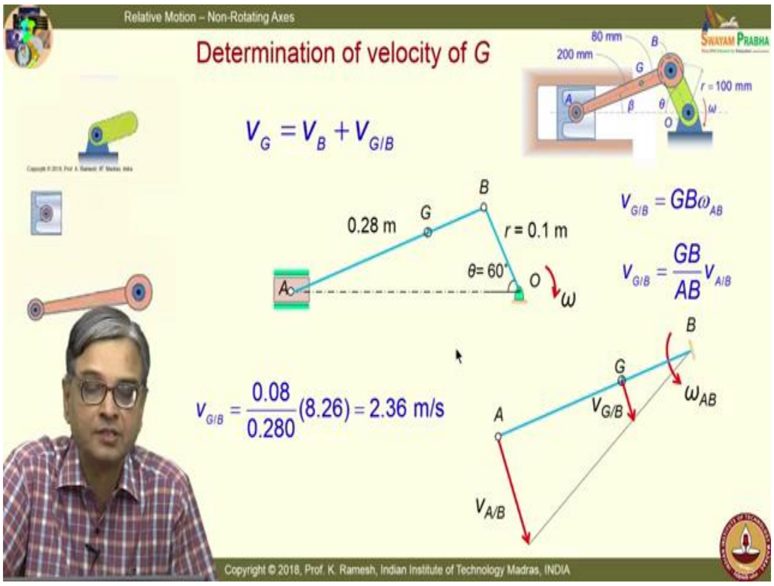


So, that is what we are going to do angular velocity of *AB* $\omega_{AB} = \frac{V_{A/B}}{AB} = \frac{8.26}{0.280} = 29.5 \text{ rad/s}$. So, let us look at the connecting rod and we have attached our non rotating translating axis at *B*. So, it is shown as fixed and I have perceived the velocity of *A* with respect to *B* like this.

We are also asked the question what is the velocity of point *G* and from this diagram you can easily say what is the relative velocity of *G* with respect to *B*. And before that you know we are also looking at from the since of $V_{A/B}$, I attach the angular velocity direction and I say it has to be counter clock wise all this is very important. You have to physically appreciate all aspects of the problem and whenever you make a statement there has to be a mathematical basis for it and it should fit with the physical reality.



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So, I have determined this angular velocity direction and we will have to find out velocity of *G* fairly simple and V_G can be written $V_G = V_B + V_{G/B}$. And we have already looked at velocity of a with respect to *B*, and we can say about a what should be relative velocity of velocity of *G* with respect to *B*.

I can simply put this as $V_{G/B} = GB\omega_{AB}$, which could be re written in this fashion. And when I do this, I get the value of velocity of *G* with respect to *B* as $V_{G/B} = \frac{0.08}{0.280}(8.26) = 2.36 \text{ m/s}$. And it can be easily drawn in the sketch as a line like this these two are parallel, because I see the whole connecting rod rotates with respect to point B. Once you know the relative velocity you already know what is V_B I can find easily that is V_G, that is no great big deal in getting that is what is done in the next slide.

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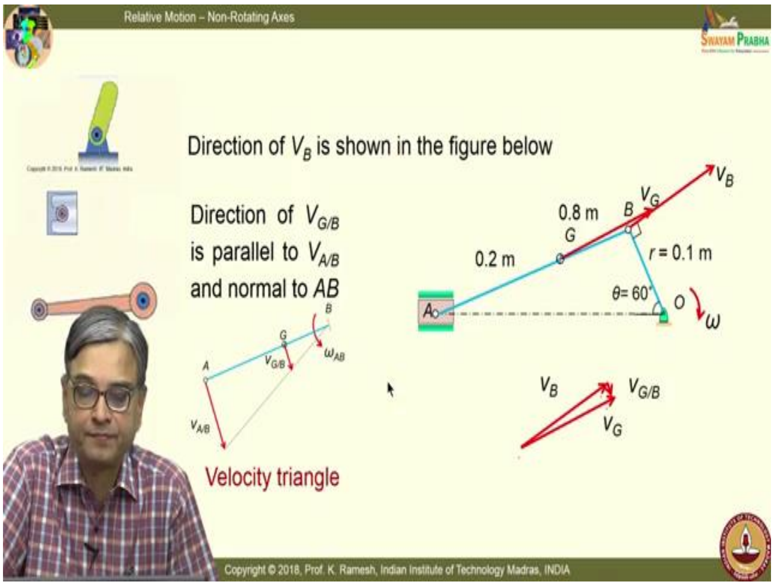
We already have the direction of V_B, you know I would like you to make neat sketches; you have to be trained as engineers to draw neat sketches. And what I have drawn also need sketches, I have not taken any geometric assistance to draw them these are freely drawn figures only.

So, I have velocity of V_B and we know velocity of G with respect to B which we have see in the previous sketch is in this direction. This perpendicular to this connecting rod I have velocity of G with respect to B. When I close the vector polygon, I get the velocity of G and that is what is shown there in the actual point, we can also determine the magnitude now.



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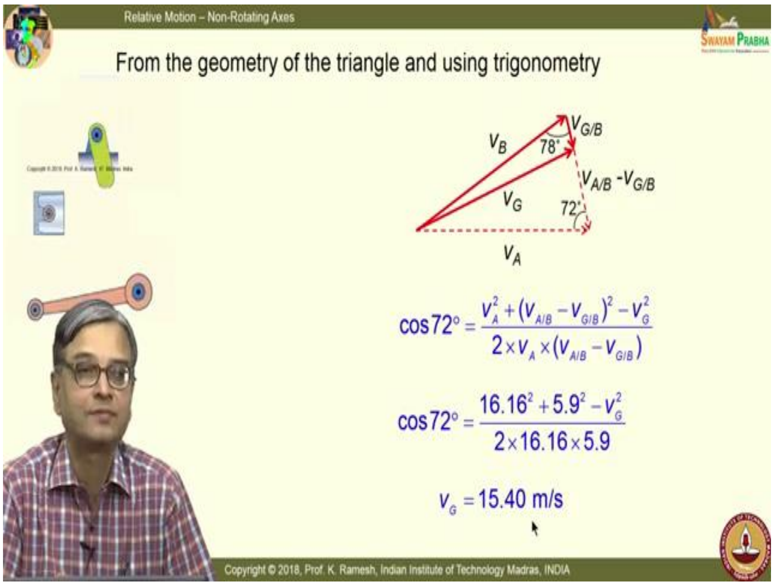
For that I again I need to do properties of the whole diagram. So, I need to find we already know the angle 72°. And I can write from knowledge of trigonometry cos 72° as a function like this, so, you have to go back and brush up your basic trigonometry properties of triangles properties of circles.



So, I can find out from this the quantity *VG* turns out to be 15.4 m/s, please verify my calculation using your calculator. The idea of solving this problem systematically is to bring about you cannot come and say I have for gotten the sine rule. Only a sine rule I forgotten all that you should not say, for solving a practical problem your old knowledge is required fine, so, brush up those fundamentals.

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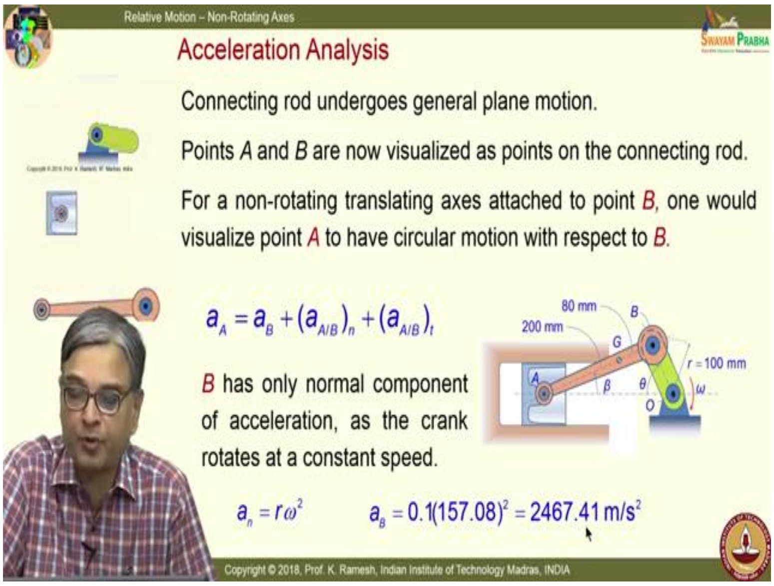


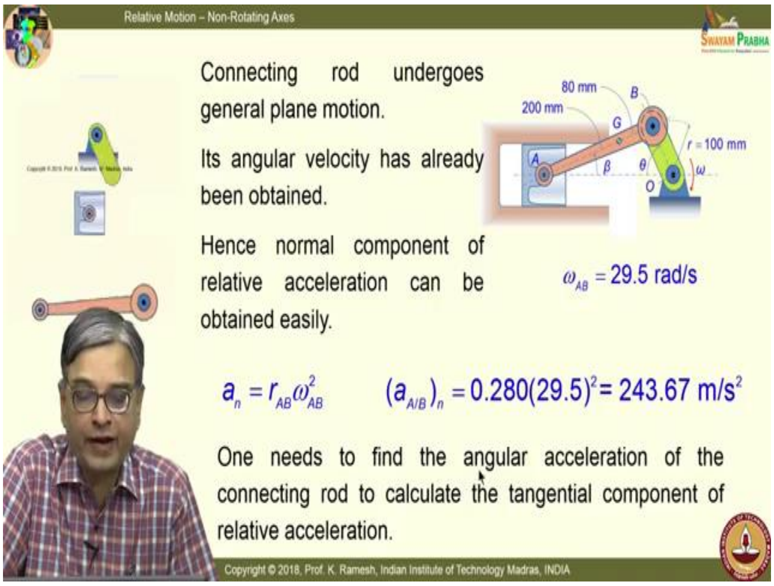
Now, let us move on to acceleration analysis, we have already recognized that connecting rod undergoes general plane motion. Points *A* and *B* are now visualized as points on the connecting rod, again the statement is emphasized. For a non rotating translating axes attached to point *B*, one would visualize point A to have circular motion with respect to *B*.

See the next chapter we are going to have an axis which will be rotating with the system. So, when I say non rotating translating axis you should appreciate in that context. We are making a very clear distinction; in one class of problems you simply attach a translating axis. In another class of problems your life become much simpler if you attach a rotating axis.

So, I can write the expression for the acceleration at point *A* $a_A = a_B + (a_{A/B})_n + (a_{A/B})_t$. These two quantities are essentially relative acceleration quantities and what all things that you know about acceleration at *A*. You can definitely say that it will be horizontal direction fine and about B you can find out everything, because it is part of the crank. So, I can find out what is absolute acceleration of point B. Once you know ω which we have already determine I can find out $a_{A/B}$, the only catch is I have to find out α $a_{A/B}$ that is what I have to find out ok.

And when you go to point *B* it is said that this crank rotates with constant angular velocity. So, I have only normal component of acceleration, there is no tangential component and I can write this as $a_n = r\omega^2$. So, the absolute acceleration of point *B* turns out to be this magnitude, I brought this as 2467.41 m/s^2.

See again and again you use the circular motion equations intelligently; you have to appreciate general plane motion is a composition of translation and rotation. And put an appropriate axis and then calculate all these quantities do not apply Newton's law in any one of this coordinate systems. We are determining the absolute acceleration of point A by using this intermediary. Once I get the absolute acceleration in that frame of references apply the Newton's law, that we have do the when I go to kinetics.



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We have already seen the connecting rod undergoes general plane motion; you have already determined the angular velocity. That is $\omega_{AB} = 29.5$ rad/s. So, I can find out the normal component or relative acceleration easily, that is nothing, but $a_n = r_{AB}\omega_{AB}^2$, so, I get this value as $(a_{A/B})_n = 0.280(29.5)^2 = 243.67$ m/s^2.

As I told you earlier one needs to find the angular acceleration of the connecting rod to calculate the tangential component of relative acceleration. So, we will take advantage of the acceleration diagram.

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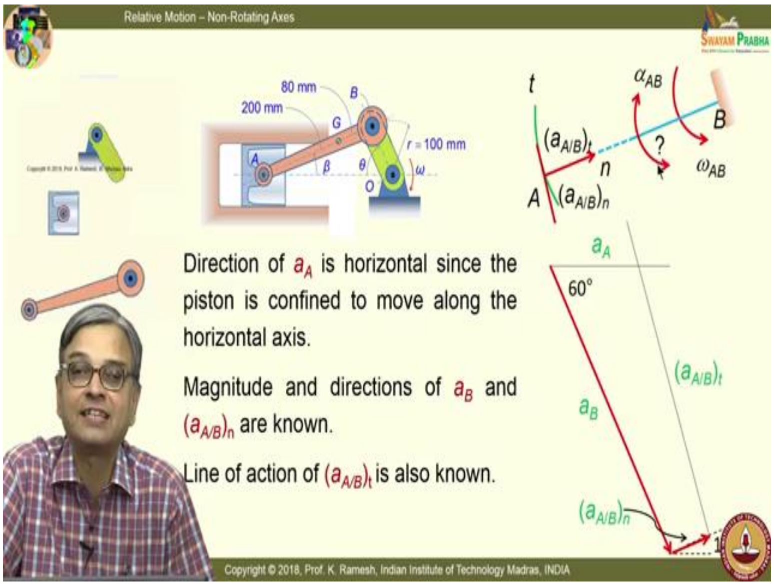
And let us look at what all quantities that we know, direction of a_A is horizontal that I have already told you, because its reciprocating like this. So, I can put this as a horizontal line and we know we absolute acceleration of point B. So, a_B is known and the relative acceleration of the normal component is also known, normal component is known like this and your tangential component will be on this direction ok.



So, I have drawn this a *B*, because a *B* I know completely from the problem statement, and this direction is known. So, I can also put that, see these animations are carefully done, so, that you have sufficient time to write it down. It is done slowly and also the sequences is done in such a manner even if I do not speak, when you see the animation the logical evaluation of concepts you can appreciate.

All this take a very long time do the animations, they are not very simple to do time consuming lot of effort has gone into all this. Line of action of the tangential acceleration is known, as of now I have no knowledge what is the sign of acceleration. I have determined the sign of angular velocity from the physics of a problem, knowing angular velocity does not tell you anything about the direction of acceleration this is the point I want to drive home fine.

Let us see what way the acceleration turns out to be in this problem, but I know this direction, so, I can complete the polygon like this. So, I will have this in this from this polygon I am also in a position to find out what is the final direction of the this, so, we will continue with this in next slide. So, the question what I have raised is whether it is clock wise or anti clock wise that question is unanswered at this stage.

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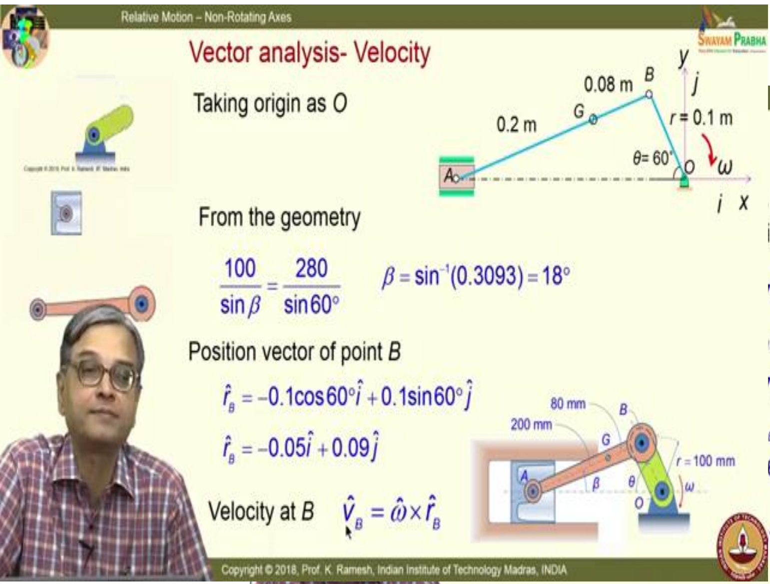

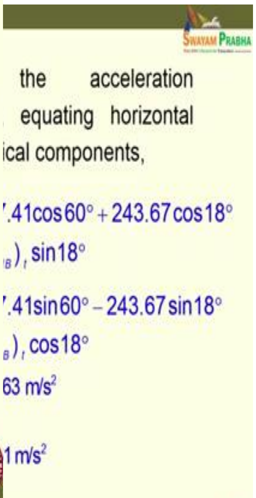

So, I have this from the acceleration polygon equating horizontal and vertical components I can do this a_A is so much. And I am in a position to calculate the tangential component that turns out to be 2167.63 m/s^2. And we already know what is the acceleration of A 795.61 m/s^2 so, the acceleration of a_A is smart.

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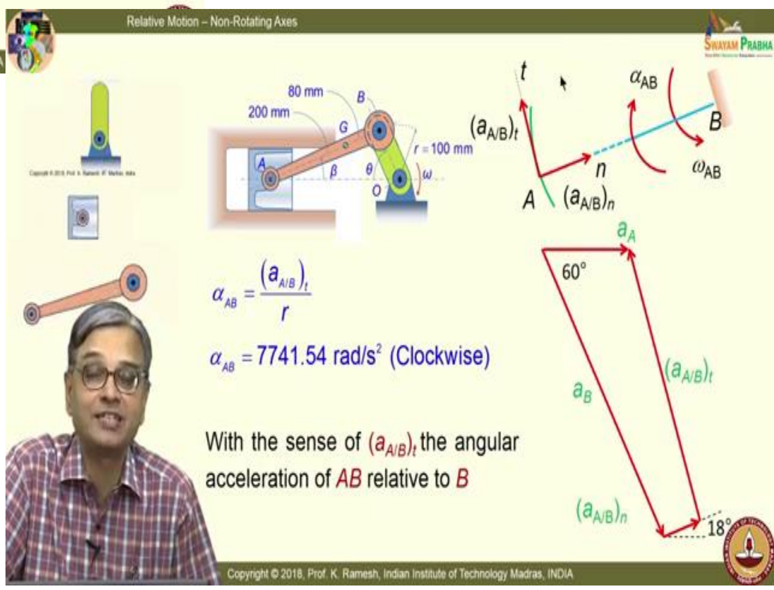

So, this fixes the direction of the tangential component of acceleration, once I know the tangential component of acceleration the direction of α can be fixed. So, $\alpha_{AB} = \frac{(a_{A/B})_t}{r}$ that turns out to 7741.54 radians per second square. So, can you tell me what is the direction of this acceleration? We know the direction of this with the sense of a_{AB} that angular acceleration of a B relative to B can be determine.

So, I get this as clockwise. See this problem very interesting that is why I choose this problem. With this problem I am able to bring out all the nuances that you can come in cross in a generic problem. I have combination of fixed axis rotation, translation and a general plane motion. And once I know the angular velocity, you should not jump to say angular acceleration is same direction. These are all the common mistakes students do, when you know one quantity as a scientist without assessing what is other quantity do not jump to easy conclusions fine.

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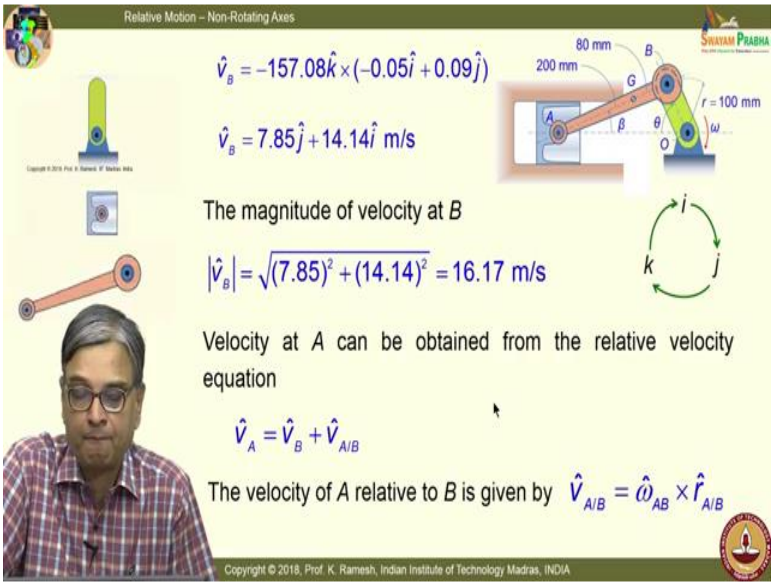
Now, let us again solve this problem through vector analysis. So, will take origin as at point *O*, I have put the positive *x* axis like this, positive *y* axis like this all the other quantity output. And you have to find out the position vector very carefully with respect to this coordinate system, whatever the coordinate system that you take.

And we have already determined this angle *β* from the previous analysis. So, the position



vector of point B you can find out and write very easily and that is nothing, but $\hat{r}_B = -0.1\cos 60°\hat{i} + 0.1\sin 60°\hat{j}$, when you simplify it takes a value like this. All this position vector you should do it very carefully with references to the coordinate system that you have taken, what is the velocity at *B*? Is nothing but $\hat{\omega} \times \hat{r}_B$, *ω* we have already determined, r_B we know, so, I can calculate what is V_B.

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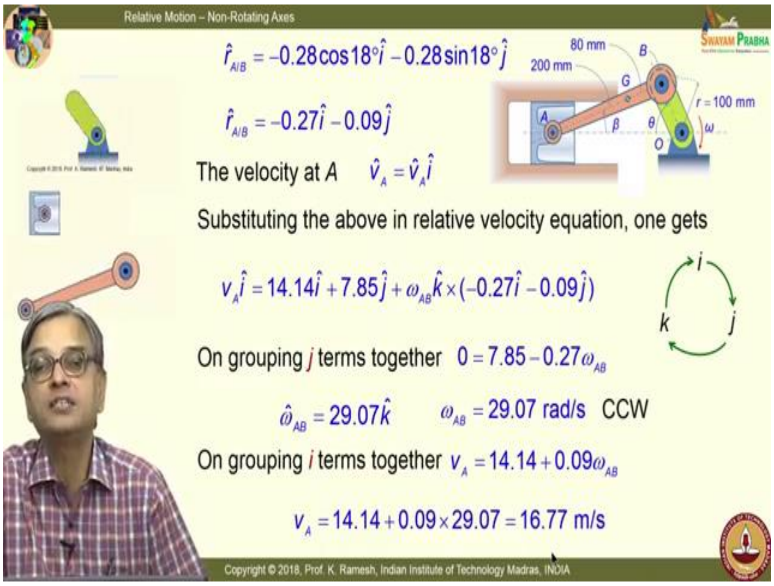
It is a simple mathematical operation, so, I have this and to a divided cross product I also put this i j k like this, so, that you keep track of the signs properly. So, finally, I get $\hat{V}_B = 7.85\hat{j} + 14.14\hat{i}$ m/s, see mind you would find small variations in values when you do a vector analysis. Primarily because when you are determining the position vectors, you are not taking all the digits fine, the round of errors can come.

So, that is very common you should also know in any engineering calculation you should take care of round of errors there may be small deviation from the numbers which you have got earlier, so, you have to keep track of. So, when you are using this, I have actually rounded of 2 digits, if I round it off to 3 digits may be, I will have improved estimations fine, so, that matters.

Whenever I put this vectorial calculation whether I stop at 2 digits or 3 digits that matters. So, the VB turns out to be $|\hat{v}_B| = \sqrt{(7.85)^2 + (14.14)^2} = 16.17 \text{ m/s}$, I think earlier we got it as 16.16, so, there are there will be some deviations ok. We have determined what is a velocity of point B and velocity of A can be obtained from relative velocity equation. The equation is like this $\hat{V}_A = \hat{V}_B + \hat{V}_{A/B}$. And velocity of *A* relative to *B* is given by we know from attaching the axis here, if I know the angular velocity, I can calculate that, so, I put that as $\hat{V}_{A/B} = \hat{\omega}_{AB} \times \hat{r}_{A/B}$.



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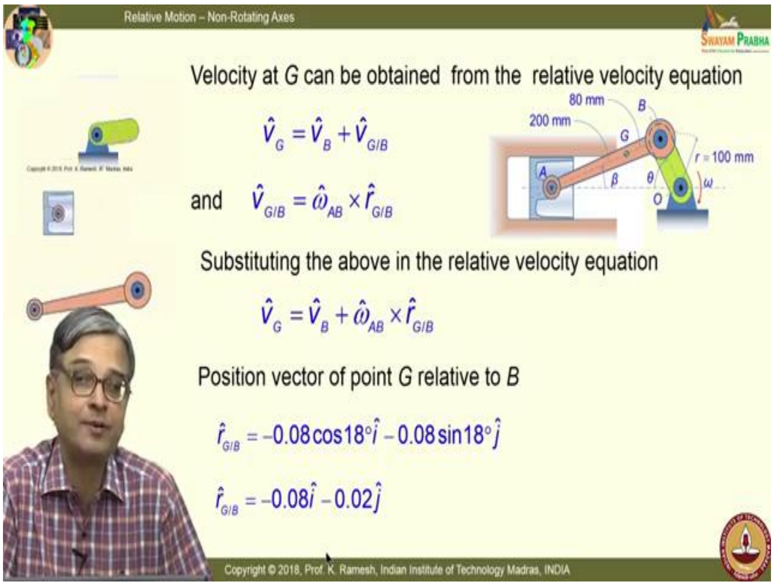
And we have r_{AB} write it with respect to reference axis, it is written in this fashion. And I find out the velocity at A you know that this is translating, so, the direction is known the direction is i, I do not know the magnitude V_A. So, I write the expression then you collect terms involving *i* terms involving *j* this is how you do simplification in your vectorial analysis, so, I write the expressions.

So, I write $v_A\hat{i} = 14.14\hat{i} + 7.85\hat{j} + \omega_{AB}\hat{k} \times (-0.27\hat{i} - 0.09\hat{j})$. So, on grouping j terms together, I get the value of ω_{AB} that is 29.07 radians per second. And this is I have this as k, so, this counter clockwise. In vectorial analysis the directions are determined from your mathematics itself straight forward.

And on grouping i terms together I am in a position to get the velocity A, I get $v_A = 14.14 + 0.09\omega_{AB}$. We have already determined ω_{AB} on sub situation I get the value as 16.77 m/s. See in a vectorial analysis you have to carefully write the position vector and also the appropriate angular velocity and write the equation and group the *i* terms *j* terms.

And once in a while you may probably peep in and see whether physically these numbers are alright directions are alright. On the other hand, when you do a vector polygon when you are writing the velocity polygon, you have to visualize and that aids your thinking and you can also appreciate the physics of the problem better and you become a better engineer.



So, in the initial stages you to try to visualize as much as possible also learn the trick of doing a vectorial analysis and vectorial analysis to an extent try ok.

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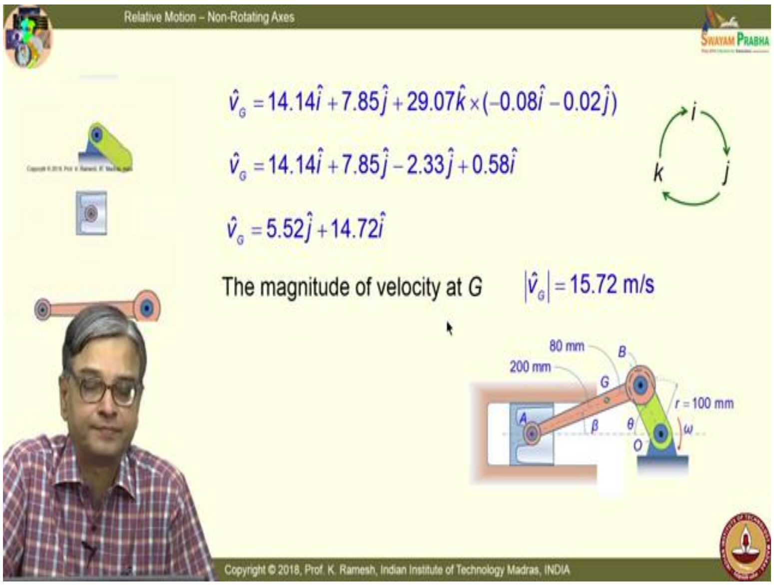
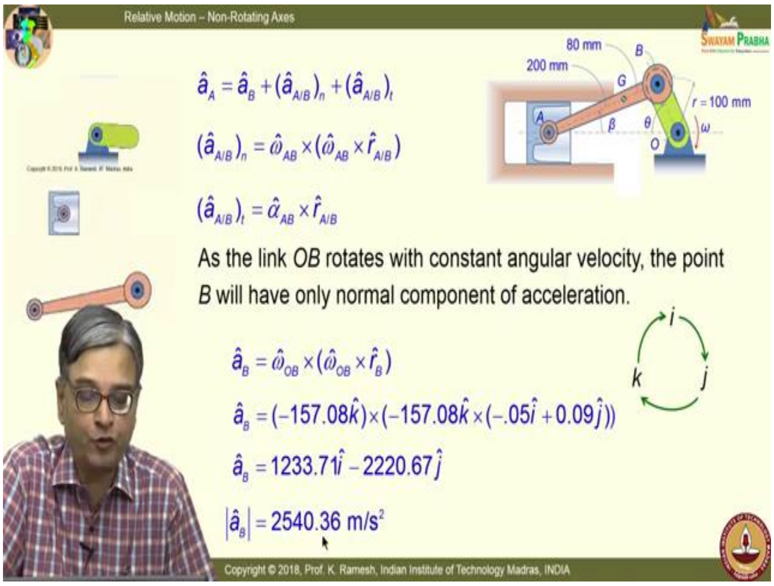
Having said that you know when I go to rotating axis I would completely well on vector analysis, because that becomes simpler to handle with. Whenever there is some opportunity to visualize better visualize that will help you to think better. And like we have determined the velocity A; we will also determine the velocity *G* write again the relative velocity equation.

You are able to write this relative velocity equation on the strength of understanding general plane motion. We have attached a non rotating translating axis at *B*, so we are in a position to write this relative velocity equation, otherwise it not possible to write. So, $\hat{v}_{G/B} = \hat{\omega}_{AB} \times \hat{r}_{G/B}$ and when you substitute these quantities in the relative velocity equation. I also get the final values and we need to get the position vector of point G.



So, that is written in this fashion, so, $\hat{r}_{G/B} = -0.08\hat{i} - 0.02\hat{j}$. If you do not handle these quantities properly to appropriate number of which is that would be variation in your numbers, because of round of errors. And all engineers have to appreciate round of errors in major calculations, so, you have to accommodate your design to take care of such errors in your analysis.



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So, I have the expression for V_G, we have already determined what is ω_{AB}, so, I have substituted that. And to aid your thinking is *i k j* is put like this, so, I get $\hat{v}_G = 5.52\hat{j} + 14.72\hat{i}$. And we will also find out the magnitude, the magnitude turns out to be 15.72 m/s.

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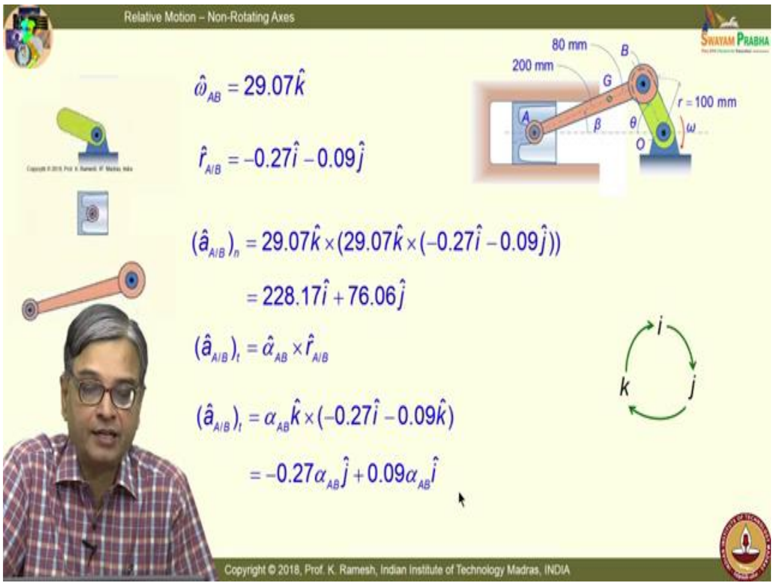
And I have to find out the acceleration, again you write the relative velocity expression instead of doing it from polygon of the acceleration you are directly handling the vector quantities here. Even for that I started with this equation and then wrote down the polygon. And the normal component is $\omega \times r$, we are finding out what is the relative acceleration a with respect to B.

So, I put the appropriate ω, these are all the places where you can make mistake in if you do not keep track of figure ω properly. You may use ω of the crank to this place on all your calculation will go for a 6 ok, so, handle the mathematics also systematically. And I have the tangential component as $\alpha \times r_{AB}$ and this is again brought your attention the point B has only normal component of acceleration.

So, I can write the normal component like this $\hat{\mathbf{a}}_B = \hat{\omega}_{OB} \times (\hat{\omega}_{OB} \times \hat{r}_B)$, so, that turns out to be we have already learned the trick. If I have a vector triple product how do I write the



final quantity, you do not have to go through the conventional approach directly write the numbers and the vector is same as this, but the opposite sign.

So, you can take advantage of all this fine, because you are going to do this repeatedly there is no point in doing these mistakes. And finally, arrive at this vector representation, and I can also find out the magnitude; the magnitude is 2540.36 m/s^2.

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When I move on to determining the other quantities, so, I have this normal component of accelerations like this. Here again use the simplification which we learn earlier for that vector triple product and find out the value as

$$(\hat{a}_{A/B})_n = 29.07\hat{k} \times (29.07\hat{k} \times (-0.27\hat{i} - 0.09\hat{j}))$$
$$= 228.17\hat{i} + 76.06\hat{j}$$

And the tangential component of acceleration is $(\hat{a}_{A/B})_t = \hat{\alpha}_{AB} \times \hat{r}_{A/B}$. So, I have this expression I do not know the value of α_{AB} ok and I will write the complete expression.

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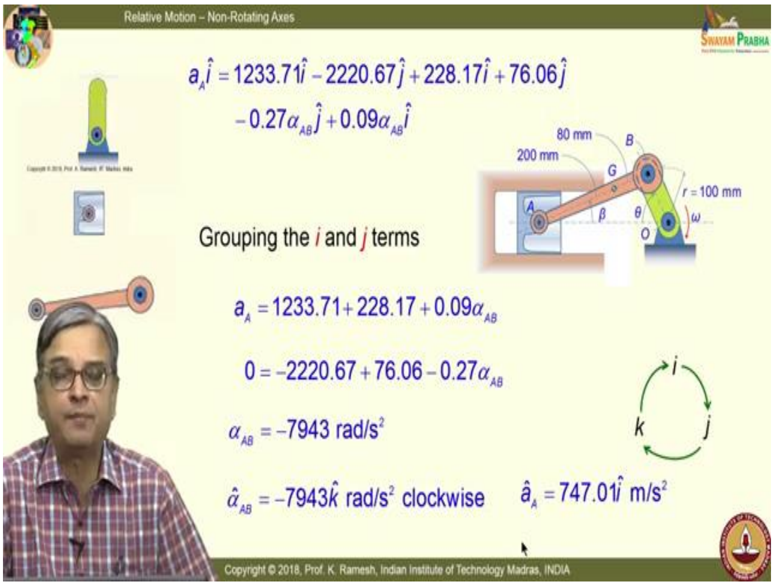
And from this by grouping the terms, I am in a position to calculate what is the value of acceleration at A as well as the angular acceleration of the connecting rod. So, grouping i and j terms I get one expression like this, I have another equation, I have 2 unknowns, and I have 2 simultaneous equations and it is possible for me to find out the value.

So, α_{AB} terms out to be -7943 rad/s^2, so, this directly tells me that this is clock wise and I can also find out what is the acceleration of point A. You know the idea of solving the problem by both graphical approach as well as vector approach is to appreciate what are the aspects you should pay in your graphical approach, what are the aspect that you should pay in your vector approach.



And in your exam problem I may specify the method I want you to be proficient in both of these methods. So, in this class we have taken up a very interesting problem dealing with slider crank mechanism is very widely used in many applications. From the data given in the problem you have been able to find out what is the acceleration of the piston and also the rotation of the connecting rod.

And we recognize the connecting rod as a general plane motion and we also have to appreciate by attaching a non rotating translating axis to a rigid body. You see other points in the rigid body having a circular motion with respect to the attached axis.

Thank you.