

Engineering Mechanics
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Module – 02
Dynamics
Lecture – 06
Relative Motion - III and Instantaneous Center

Module 2 Dynamics

Lecture 6 Relative Motion – III and Instantaneous center

Concepts Covered

Coincident points, Method of analysis using velocity polygon and vectorial analysis, Velocity analysis of Quick return mechanism – a practical application in Shaper, Instantaneous center of zero velocity, Example problems on rolling without slipping and analysis of Gear trains.

Keywords

Engineering mechanics, Dynamics, Rigid body, Coincident points, Quick return mechanism, Instantaneous center.

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Let us continue our discussion on Relative Motion.

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We will take up a very interesting engineering problem, we will start with a very simpler problem

then we move on to another interesting problem. And make a neat sketch of it, what I have here is I have a slotted member and I have a threaded collar and I have a pin

Relative Motion – Non-Rotating Axes

3. In many engineering applications one finds the use of slotted members. The motion of a slotted member is usually controlled by a pin moving in the slot. Threaded collar C containing the pin B moves down with a velocity of 0.3 m/s due to a suitable rotation of the power screw. Determine the angular velocity of the slotted arm when $\theta = 30^\circ$.

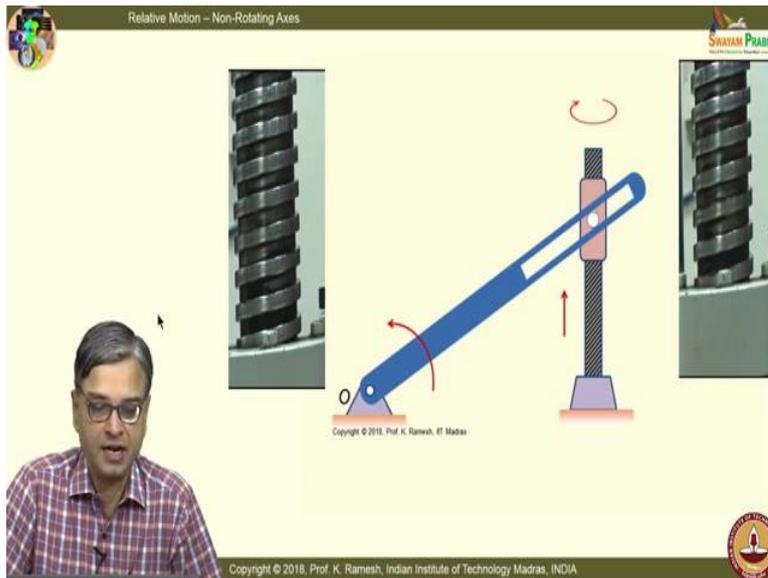
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attached to this threaded collar. And when the threads rotate in one direction it will move up, it will rotate in another direction it will come down.

So, the point B which is marked can move up or down, in this case the case

that you are going to analyze is when you it is moving down. And what you are given in the problem statement is the pin B moves down with a velocity of 0.3 m/s.

And the angular orientation of the slotted arm at this instance is 30° and you will soon find slotted members are used in many engineering applications and that gives a clue how to go about in solving problems of that nature. They could be solved from relative motion analysis; they could also be solved from fixing a rotating frame of reference that we will see later. And now we have to understand what happens when these relative motions take place fine.



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And we also have a very powerful method of approach where we bring in what is known as coincident points ok. So, this just shows that I have a power screw which is rotating in one direction it comes down, rotating in another direction it moves up. And what we would look at is I have a pin which is there on this collar, I would also imagine another point on this slotted member which is fictitious which is coinciding with this point fine.

And what we would look at is I have a pin which is there on this collar, I would also imagine another point on this slotted member which is fictitious which is coinciding with this point fine.

Coincident Points

- Useful in the analysis of constrained sliding contact between two links in a mechanism.
- The points A and B are chosen on each of the links but coincident at the instant considered.
- The two points are on different bodies and their distance change as a function of time.
- At the instant considered, $r_{A/B}$ is zero.
- The direction of relative velocity is dictated by the constrained sliding contact.
- It is essentially tangential to the slot.

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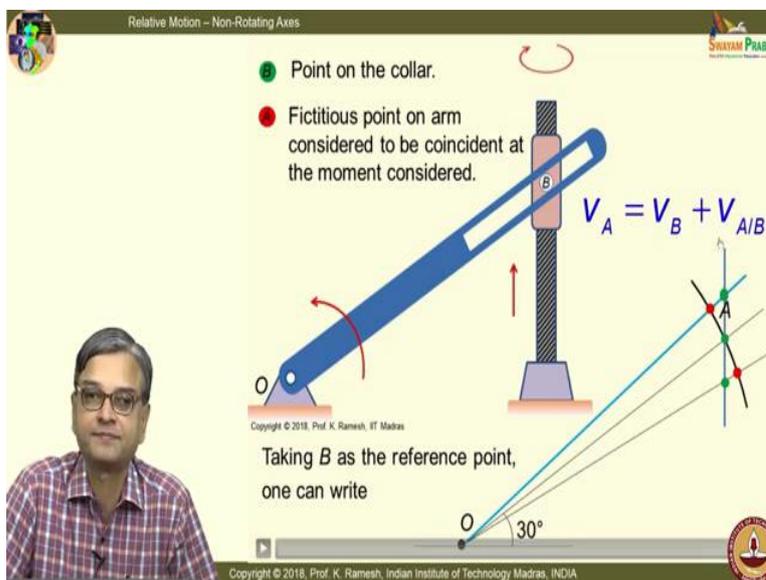
And we would see how the motion takes place fine.

Useful in the analysis of constrained sliding contact between two links in a mechanism. So, I am going to imagine that I have a point here and this is the constrained motion. And you could visualize when

I have this point here, this point can move only in this slot, so the relative moment is understandable.

So, I have two points A and B we have seen that we have label B as the point on the collar and A as a fictitious point ok, which is on the slotted link. And you have to notice that these two points are on different bodies. And the important aspect is that distance change as a function of time, unlike what we had seen earlier at the instant considered the relative position from A to B is zero.

Because they are coincident what I am trying to say is in the slotted member I would have a point ok, coinciding with the point B . And what is stated here is the direction of



relative velocity is dictated by the constrained sliding contact, it is essentially tangential to the slot. In this case I have a straight slotted member, suppose I have a curved slot and this member is also of a different geometry then the velocity will be tangential to the slot.

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See let us understand the concept of fictitious point and that is what is depicted beautifully in this animation, please take your time to make a neat sketch out of it, I would repeat this. So, I have a point B on the collar, I have point A on the arm and you could see here I have shown the point B as green in colour, I have shown the point A as red in colour.

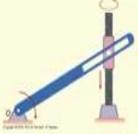
At this instant of time points A and B are coincident, I will again show the animation; and you could see the point A as a circular motion. Because this is dictated by the pinned slotted member the point A is considered as if this coincident with point B at this juncture

ok. So, point A has a circular motion, point B which is moving up and down on the threaded member that is shown as green dots here.

So, again have a look at it and make a neat sketch of this and you exploit this understanding in solving the problem ok. And what I do here, taking B as the reference point one can write the velocity of A is velocity of B plus velocity of A with respect to B , we would see the mathematics little while later ok.

Relative Motion – Non-Rotating Axes

Coincident Points



- Useful in the analysis of constrained sliding contact between two links in a mechanism.
- The points A and B are chosen on each of the links but coincident at the instant considered.
- The two points are on different bodies and their distance change as a function of time.
- At the instant considered, $r_{A/B}$ is zero.
- The direction of relative velocity is dictated by the constrained sliding contact.
- It is essentially tangential to the slot.

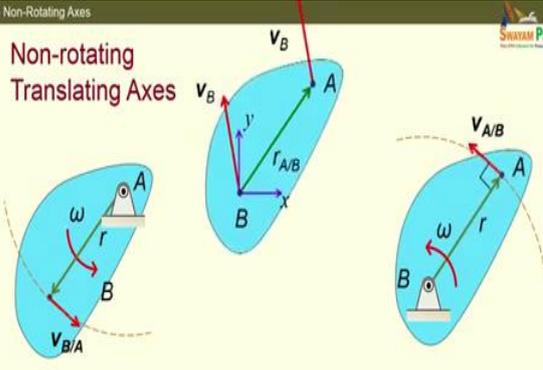
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And again, if I summarize those points, we will quickly review this, recognize that points A and B are chosen on each of the links, but coincident at the instant considered. And what we do is we have exploited the constrained motion behavior in writing out the relative velocity equation, what is that we have done earlier? We have done earlier when we looked at general plane motion.

Relative Motion – Non-Rotating Axes

Non-rotating Translating Axes



Attached to A

Attached to B

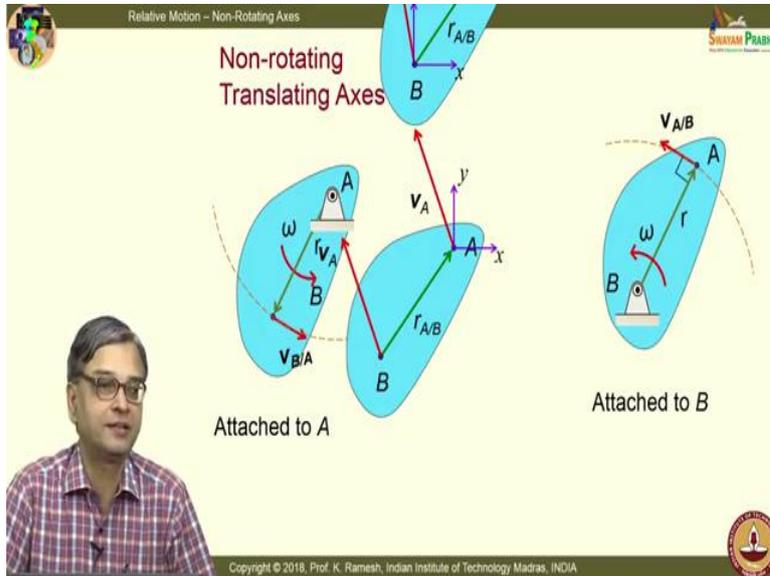
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We have identified that if I attach a non rotating translating axes at B , I would see the point A to move in the circular fashion. If I attach it with A

the point B would move it in a circular fashion, this understanding has helped us to write the relative velocity equation.

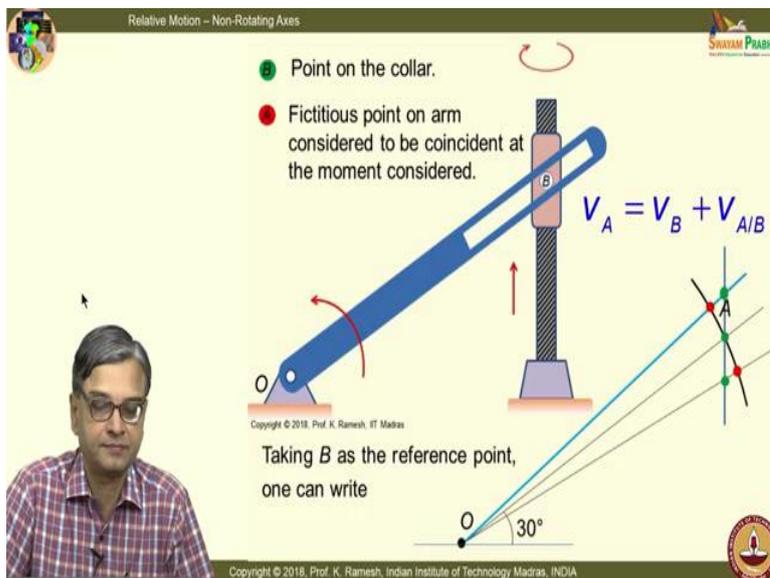
So, there it was different and the distance r remains constant, on the other hand in this case it is coincident at the instant of time. And it varies as a function of time nevertheless because of the strength of our understanding of the constrained motion we have been in a



position to write a relative velocity equation.

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So, the situation what we saw earlier was different and what we are going to use now is different. Ultimately, I am in a position to write the relative velocity equation.



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So, this is again depicted here for clarity, because it is a very subtle point and I have taken efforts to bring it out in a nice animation. Otherwise books say that left it to your imagination. So, you should recognize this even for getting out all the other expressions, even

if I have to do the vectorial analysis I have to visualize these movements only then I would be able to write the unit vectors properly.

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And we would solve the problem both by a graphical approach as well as a vectorial approach. Magnitude and

Relative Motion - Non-Rotating Axes

Magnitude and direction of v_B is known completely.

The direction of $v_{A/B}$ is along the slot.

From the simulation it is clear that it is away from O .

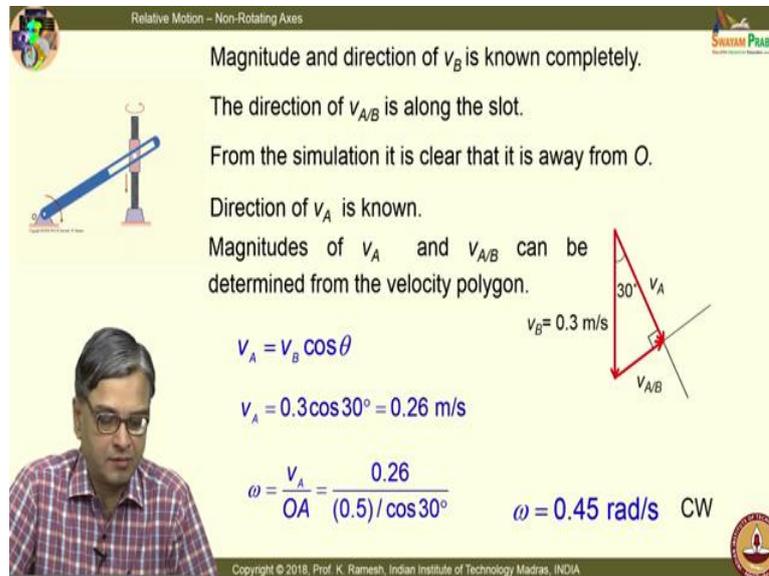
Direction of v_A is known.

Magnitudes of v_A and $v_{A/B}$ can be determined from the velocity polygon.

$v_A = v_B \cos \theta$

$v_A = 0.3 \cos 30^\circ = 0.26 \text{ m/s}$

$\omega = \frac{v_A}{OA} = \frac{0.26}{(0.5) / \cos 30^\circ} \quad \omega = 0.45 \text{ rad/s CW}$



direction of V_B is known completely, it is going to come down that is where the problem statement is, so it is coming down here.

So, I have V_B as 0.3 m/s and from the constrained motion you know what is the relative velocity direction, you also know

what is the relative velocity the absolute velocity direction of point A . Both are known the absolute velocity of point A will be perpendicular to the slotted link ok. And the relative velocity will be in the direction of the slot, because I have a straight slot.

So, the direction of relative velocity A with respect to B is along the slot. From the simulation it is clear that it is away from O , so, you can see all these from this I have this animation running. Direction of V_A is known, because it is perpendicular to that, so, I have this, this is the slot and this is perpendicular to that, because the slotted link is pinned.

So, it has a circular motion, with all these quantities the problem is solved its very simple and straightforward to do in a graphical approach. Magnitudes of V_A and V_{AB} can be determined from the velocity polygon, I get $v_A = v_B \cos \theta$. And this turns out to be 0.26 m/s and I can also find out the angular velocity of the slotted bar. So, that is nothing, but

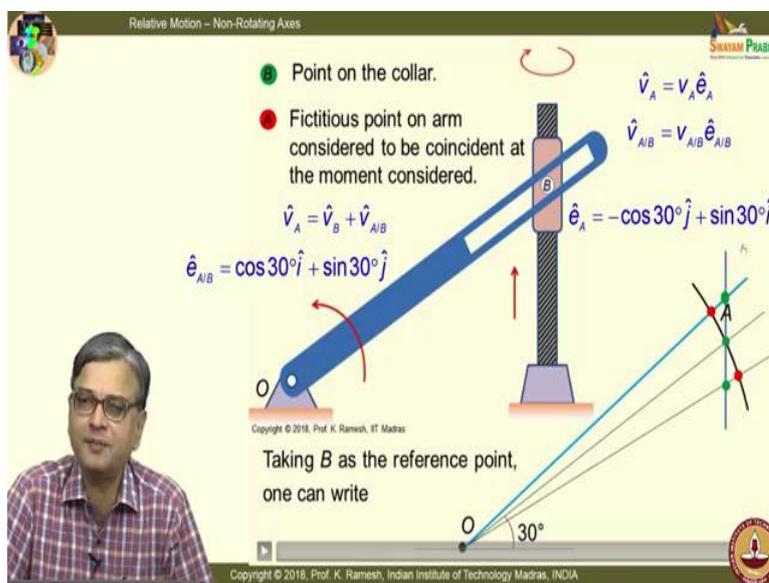
$$\omega = \frac{v_A}{OA} = \frac{0.26}{(0.5) / \cos 30^\circ} \text{ and you get } \omega = 0.45 \text{ rad/s}$$

You have been able to write the relative velocity equation primarily from the understanding of the constrained motion, and you have brought in a concept of coincident points. And in this instance the vectorial approach we will see later, but the

graphical approach is so simple in just two three steps you are in a position to get the quantities that; you want you would also solve it by vectorial calculus.

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And before we get into that we should again go and look at the motions, otherwise will not be in a position to write the vectors properly. So, I know the relative velocity equation as $\hat{v}_A = \hat{v}_B + \hat{v}_{A/B}$. Now, I need to know the unit vectors for the direction A and the relative motion direction.



So, I have to write

$\hat{v}_A = v_A \hat{e}_A$ and you have a depiction of the motion of point A and the motion of point B. And we will also

have to write $\hat{v}_{A/B} = v_{A/B} \hat{e}_{A/B}$.

Can you write the unit vectors for the e_{AB} and e_A ; e_{AB} is fairly straight forward. And for you to write e_A and you have to

visualize this motion without visualizing this motion you would not be in a position to write e_A properly.

Can you find out e_{AB} that is nothing but the direction of a slot ok, so, there is nothing but $\hat{e}_A = -\cos 30^\circ \hat{j} + \sin 30^\circ \hat{i}$. I have not shown the axis here, you can take the axis as horizontal and vertical here. And what is the unit vector for velocity A; that is tangential to this, it is having a circular motion like this or in other words you can say that this is perpendicular to the slot at this instance.

You have to recognize this so, in a vectorial approach also in this instance you have to visualize the motions only then you are in a position to assign the vectors, because you are exploiting the property of constrained motion. So, in a sense the vectorial approaches slightly involved and you have to be careful in handling the quantities.

Once you recognize that $\hat{e}_{A/B} = \cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}$, the problem is done. For you have to visualize this you have to go back and understand what happens to the coincident point. At this instance it was coinciding with the point of the collar, what happen an instance before, what would happen an instance later would give you how to identify the unit

vector for the fictitious point.

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Once you have to determine this, the problem is straight forward; so, you

are given $\hat{v}_B = -0.3\hat{j}$ and I have the reference axis listed here. And I have

$\hat{r}_B = \hat{r}_A = 500\hat{i} + 288.68\hat{j}$ these

are all simple arithmetic you can easily check these quantities you are given 500 mm you are given the angle, so, you can find out the other quantities.

Then we have written down this as $\hat{v}_A = v_A \hat{e}_A = -\cos 30^\circ v_A \hat{j} + \sin 30^\circ v_A \hat{i}$, we have already determined this and the relative velocity equation is like this. So, we also know what is the unit vector e_{AB} and again the usual procedure collect the i terms and j terms.

And then you will get sufficient number of equations for you to solve for the unknowns, so that is what we are going to do. The important point here is in many other previous problems when you approached vectorial calculus it is enough that you write the appropriate equation, because very little imagination was required.

But in this class of problems you will also have to picturize the relative motion for you to appreciate how to write the appropriate vectors. So, this reduces to this and grouping i terms I get one expression relating V_A and V_{AB} . And I get $v_{A/B} = 0.57 v_A$; please check the numerical calculations and verify whether these numbers are correct.

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So, grouping j terms I get this expression and from this in a position to get the velocity

Relative Motion – Non-Rotating Axes

Grouping j terms

$$-0.87v_A = -0.3 + 0.5v_{A/B}$$

$$-0.87v_A = -0.3 + 0.5 \times 0.57v_A$$

$$v_A = 0.26 \text{ m/s}$$

Angular velocity $\omega = \frac{v_A}{OA} = \frac{0.26}{(0.5)/\cos 30^\circ}$

$$\omega = 0.45 \text{ rad/s CW}$$

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$v_A = 0.26 \text{ m/s}$. I can also find out the angular velocity that is $\omega = 0.45 \text{ rad/s}$ clockwise. So, you have to take a decision whether to approach the problem using vector polygon I mean velocity polygon or vector calculus.

In some problems vector calculus may be simple; in some problems, velocity polygon may be simple. It is better that you learn both of these techniques; so, that you have an option to choose or if you solve the problem in one method verify the answer from other method all that you should do. Because I said doctors and engineers have to be very

confident of their decisions, you cannot be ambiguous.

Relative Motion – Non-Rotating Axes

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And this is the very nice application we will solve; we will also have a satisfaction that whatever you learn in the classroom is of use in some engineering application.

This is the shaper what you have to observe here is, you have to observe what way this tool goes and comes back. Do you find these two the forward and backward stroke are they of same time duration, observe that?

See in the forward stroke it is actually doing the cutting, the machine the machine is shown you do not have the object put here ok. You have to imagine that it takes a longer time to move forward, so, it does the cutting operation. And after cutting operation you know industrial engineers will say you have to reduce time in manufacturing, how to they reduce the time?

When it is not cutting it can come back at a faster pace, and how this is achieved? This is achieved by a mechanism here. So, you have a slow cutting phase and fast return and this is achieved by a mechanism here. And that is a mechanism that we are going to analyze,



I have nice animations for you have to dwell upon it.

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So, what I have here is this is the mechanism that is shown here, this is what we are going to analyze. So, you have a norm that is oscillating like this ok and that pin joint is literally in a bath of oil well. Look at

the amount of lubrication that is required and also this slotted arm is quite sturdy. See it has to withstand the cutting forces; it is not very light; you could see from the construction that this is quite sturdy.

Because when it does the cutting operation it has to bear the cutting forces, so, all that will come here and if you watch very closely you also have a provision to change the length of the crank there are many adjustments are possible. You can adjust the stroke length; you can adjust the time duration all that you have control depending on the problem one hand. So, the idea is you minimize the time for a machining operation, so, you can improve productivity like that.

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So, the problem is like this, so, I have a slotted arm like this, I have a crank which is

The following quick-return mechanism which produces a slow cutting stroke of the tool (attached to D) and a rapid return stroke is used in a Shaper. If the driving crank AB 's angular velocity is 5 rad/s , determine the velocity of point C for the instant when $\theta = 30^\circ$.

rotating it. And the shaper tool post that was moving back and forth is just put as a sliding block like this and you have the geometric dimensions make a neat sketch of it.

And you have already seen that it takes a longer time while cutting the work whatever the job that is

kept there and takes a shorter time to comeback is also has a nice name this is called a quick return mechanism. And the machine that you saw was known as a shaper. And the problem statement is if the driving crank AB 's angular velocity is $5 \text{ radians per second}$, determine the velocity of point C for the instant when $\theta = 30^\circ$.

When $\theta = 30^\circ$ you have to find out what is the velocity of point C . So, this has a slotted member you have a crank which is rotating and this is used in an application involving the machine shaper. So, from that sense if you know how to solve it whatever you learnt has

some practical use in the field.

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And this is shown slightly differently in this sketch you know I had done the problem long time back, then I got the image of the shaper. So, it is not exactly the functionality of this the strokes are slightly opposite in direction, but it gives an impression that I can achieve if you look at this animation.

Method for velocity Calculations
 Let P be a point on OC coincident with B .
 Absolute velocity of B is $\hat{v}_B = \hat{v}_P + \hat{v}_{B/P}$
 The directions of \hat{v}_P and $\hat{v}_{B/P}$ are known.
 Their magnitudes can be found using vector addition.

$\hat{v}_{B/P}$ \hat{v}_B $\gamma = 60^\circ - \beta$
 β 60° \hat{v}_P

$OB = \sqrt{330^2 + 110^2} - 2 \times 330 \times 110 \times \cos 120^\circ$
 $= 396.61 \text{ mm}$

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You can see that it is coming back slowly and then moving forward fast whereas, it does the exactly opposite one. It is moving forward slowly coming back fast, here it is shown differently nevertheless you have a practical situation which is possible

by a mechanism fine. You have a mechanism which does that job and we would analyze this problem

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So, I have this diagram shown like this and let us identify point P be a coincident point with B . I have B attached to this pin which is also part of the crank, and I have a point P which is in this slot ok. You have to understand that they are coincident points to see them distinctly I put B and P like this.

So, they are the same point on this, I have a point B below which on the slotted member you have the fictitious point B , then I have seen this has an oscillatory motion. So, the velocity of C is simply perpendicular to this, for me to get this I should have the knowledge of the angular velocity of this. If I know the angular velocity of this then the problem is solved. So, you will also have to appreciate that I have determined this quantity by solving the problem.

So, we have absolute velocity of B is $\hat{v}_B = \hat{v}_P + \hat{v}_{B/P}$, this comes from the concept of coincident points. And can you find out the absolute velocity of B ; you have the absolute

velocity of v , because the crank is rotating at a particular angular velocity you know that completely.

And I can say what is the direction of V_P , I can also say what is the direction of this relative velocity component all that is known, so, the problem is amenable for solution. There is what is stated here, the directions of V_P and $V_{B/P}$ are known, say I am also using different label.

So, that you get accustomed to applying your equations no matter what way you have labeled it, I am doing it gradually I am not doing an abrupt change. It was in the rotating frame of reference usually we call the coincident point as P that is the reason why I have taken that as P . That is the way we will develop the argument and do the discussion of several terms.

So, their magnitudes can be found using vector addition. So, I have this relative velocity which is in the direction of the slot and I have this is perpendicular to this slot. So, I have this V_P and I know V_B completely, so the problem is solved. I am in a position to if I get the V_P , I can find out what is angular velocity of this. Then I can find out easily the velocity of point C , see the problem appear to be so complex, it is also a very interesting quick written mechanism. Once you understand the concept of coincident points it is very easy for you to attempt a solution.

So, you can find out this angle this is 60° and this is nothing but the angle β . So, I put $\gamma = 60^\circ - \beta$ and you can also get from your geometry what is the length OB or OP whatever it is ok. So, that turns out to be 396.61 mm. So, it is a very essential that you refresh your properties of triangles, sine rule, cos rule all the other simple calculations; they are needed you should be able to write them confidently without making a mistake.

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And I have from sine rule I can get the angle $\beta=13.9^\circ$ and I have this velocity diagram written like this. So, I can also get $\hat{v}_P = v_B \cos \gamma$, so, I get $V_P = 381.37$ mm/s. So, once I know V_P I can find out the ω_{OB} , so, I can instead of finding out the ω I have directly written down this, so that is also fine.

Relative Motion – Non-Rotating Axes

From sine rule in ΔOAB

$$\frac{110}{\sin \beta} = \frac{396.61}{\sin 120^\circ}$$

$$\beta = 13.9^\circ$$

$\hat{v}_{B/P} = \hat{v}_B$

$\gamma = 60^\circ - \beta = 46.1^\circ$

$$\hat{v}_P = v_B \cos \gamma$$

$$= 550 \cos 46.1^\circ$$

$$= 381.37 \text{ mm/s}$$

$v_C = \frac{OC}{OB} v_P = 528.87 \text{ mm/s}$

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So, I get velocity C as $v_C = \frac{OC}{OB} v_P = 528.87$ mm/s, I have actually used vector polygon I mean velocity polygon. And you understand the relative motions very clearly here and in easy for you to handle a problem that

appear to be very complex when you look at the shaper as such.

So, it is a very useful way to solve the variety of engineering problems, coincident point

Relative Motion – Non-Rotating Axes

Instantaneous Center of Zero Velocity

- Instead of choosing a convenient reference point, here one chooses a unique reference point which has zero velocity.
- It acts like absolute center of rotation at the instant considered.
- It is not a fixed point in the body nor a fixed point in the plane.
- Can this point have acceleration?

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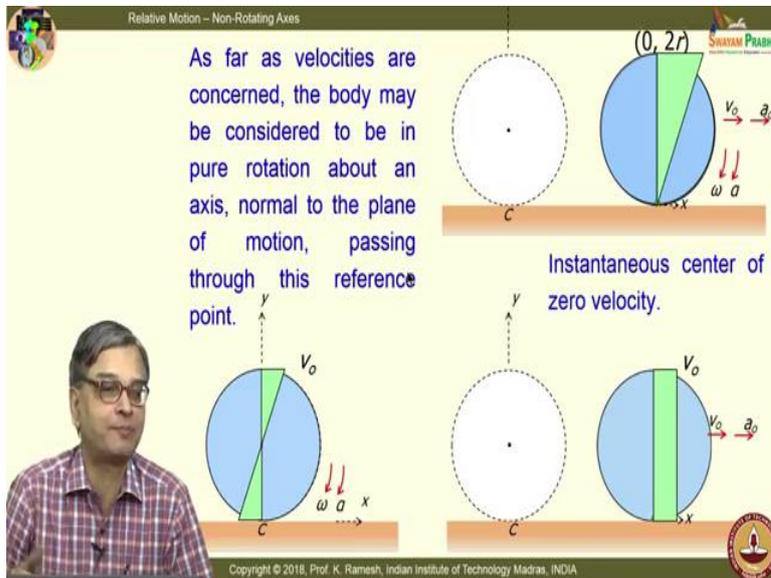
is a very useful concept. And you should recognize that I write the relative velocity equation based on constrained motion. There is the distinction between when you handle a general plane motion and a constrained motion.

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So, these are all various tricks to solve a given problem, another trick what we have is the concept of instantaneous centre of zero velocity. So, instead of choosing a convenient reference point, here one chooses a unique reference point which has zero velocity. Once

I find out this, I can find out the absolute velocity of any point in the rigid body in one short I can find out no difficulty at all.

So, it acts like absolute center of rotation at the instant considered you have to underline



that and you should also know it is not a fixed point in the body nor a fixed point in the plane. It can be anywhere depending on the complexity of the motion. See normally whenever you say something is zero; everything else associated with also is zero is the way you are normal intuition

works ok. So, I want to caution can this point have acceleration the question itself has an answer fine.

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And you have seen for a rolling wheel what happens at point C ? It as a zero velocity, that we learn it from the first principle that it has a rotation and a translation. So, I have the velocity of due to rotation is like this, velocity due to translation like this. When I superimpose, I have zero velocity here and then it increases to the maximum here and this is what you call it as instantaneous center of zero velocity.

So, the object can be visualized have a rotation about that point and you know from rolling that this point C what you have here as an acceleration. So, when this is zero velocity, it does not imply zero acceleration. So, instantaneous center of zero velocity can have an acceleration component dictated by the problem situation.

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And how do I find the instantaneous center, let us consider a rigid body having a plane motion. When I say plane motion it has a linear component of translation as well as a

Relative Motion – Non-Rotating Axes

Instantaneous Center of Zero Velocity

- Let the rigid body be having a plane motion.
- Absolute velocity V_A of a point is given as well as the rotation ω of the body.
- These two quantities V_A and ω define the velocity of all other points in the body.

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rotary motion. Absolute velocity V_A of a point is given as well as the rotation ω of the body is given. And if I know these two quantities, it defines a velocity of all other points in the body. Or in other words if I know the absolute velocity of any two points the rigid body, I

know the motion of the rigid body comfortably.

Relative Motion – Non-Rotating Axes

Instantaneous Center of Zero Velocity

- If for a rigid body, absolute velocities of any two points A and B are given
- Instantaneous center** can be found by drawing perpendiculars from these velocities.
- Angular velocity can be determined by

$$\omega = \frac{V_A}{r_A}$$

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I take two arbitrary points A and B and you know the absolute velocities of these points from your understanding of the motion ok. We will solve a problem then will appreciate how do I get this. So, I have the velocity V_B like this and velocity V_A

like this, and if they have to find out the instantaneous centre of zero velocity just draw perpendicular lines from this and they meet at a point and this is your instantaneous centre of rotation.

And it is very clear for you because it rotates about this, so, the velocity is perpendicular to that. So, you use that property to find out what is the instantaneous centre of rotation, and I could think of different orientation of these velocities a variety of them. So, that

you have a recipe how to get the instantaneous center of rotation. The prerequisite is you should know the absolute velocity at two different points. So, this is one instance we have seen, we can also have before we go into that you know you also have the information that angular velocity can be easily determined simply as $\omega = \frac{V_A}{r_A}$ or V_B/r_B

whichever way you want to calculate.

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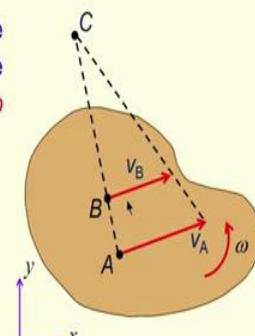
Suppose I have a velocity are parallel and of non equal magnitudes, how do a find the instantaneous center. So, you have to draw a line joining the tip and the base ok, they will intersect and form the

Relative Motion – Non-Rotating Axes

Instantaneous Center of Zero Velocity

If the velocities are parallel

- Lines joining the tip and the base intersect at the *instantaneous center of zero velocity*.
- When the parallel velocities become equal in magnitude, the instantaneous center pushes further and approaches infinity resulting in a pure translation motion.



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instantaneous center of rotation. And suppose I have these velocities are parallel; obviously, the body does not have a rotation it means at infinity it has only a translatory motion.

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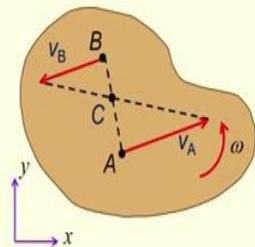
And suppose I have these velocities are parallel, but in opposite direction that possibilities also not ruled out. I have velocity which are parallel they are in

Relative Motion – Non-Rotating Axes

Instantaneous Center of Zero Velocity

If the velocities are parallel but opposite in directions

- Lines joining the tip and the base intersect at the *instantaneous center of zero velocity*.



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opposite direction. Again, follow the same recipe join the tips and the base I get the instantaneous center of rotation. I get the instantaneous center of rotation and it is a very useful concept to solve a class of problems ok.

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And we will again visit this rolling of a wheel you have this as instantaneous center of rotation, so, the point C in general can have an acceleration. When I say instantaneous

Relative Motion – Non-Rotating Axes

Instantaneous Center of Zero Velocity

- C can in general have acceleration.
- On similar lines one can also anticipate an *instantaneous center of zero acceleration*.
- It is a specialised topic in mechanism kinematics and beyond the scope of this course.

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center of zero velocity on similar lines one can also anticipate an instantaneous center of zero acceleration, which is used in mechanism kinematics which is the specialized topic. And happily, this is beyond the scope of this course ok, at least you know that is when I have

this, I have a parallel concept. You may have an opportunity to learn if you stick to one of the mechanical engineering-based branches.

Relative Motion – Non-Rotating Axes

4. A wheel of radius $r = 400$ mm rolls to the right without slipping. It is measured that the center O has a velocity of 4 m/s. Locate the instantaneous center of zero velocity. For the position indicated, determine the velocity of point A using the knowledge of the instantaneous center.

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I have very simple problem the idea is I have always been saying, I take problems to teach you a method I am not interested in getting only the answer. I am using the problems as a vehicle to teach a methodology and you know how to calculate whatever the quantities related to rolling very comfortably.

So, you can verify your answer and pat your back that you have learnt the method correctly. And what is important here is it is rolling without slipping its very important only when it rolls without slipping, I can use the kinematical relationship that you know

So, you can verify your answer and pat your back that you have learnt the method correctly. And what is important here is it is rolling without slipping its very important only when it rolls without slipping, I can use the kinematical relationship that you know

for a rolling motion. Whenever rolling comes you can invoke that and solve your problem, they very fairly simple problem you have this as instantaneous center of rotation, and you have to find out the velocity at point A we have also done it earlier.

Relative Motion – Non-Rotating Axes

The velocity of point A needs to be calculated using the concept of instantaneous center.

C is the instantaneous center.

$$AC = \sqrt{0.4^2 + 0.3^2 - 2(0.4)(0.3)\cos 120^\circ}$$

$$AC = 0.608 \text{ m}$$

From the property of a rolling wheel without slipping

$$V_o = r\omega \quad V_o = 4 \text{ m/s} \quad \omega = 10 \text{ rad/s}$$

$$V_A = r_{AC}\omega = 0.608 \times 10 = 6.08 \text{ m/s}$$

V_A is perpendicular to AC

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And so, I have this as instantaneous center, so, I determine the distance AC from the geometry this comes to be 120° and I have to get the distance

AC. So, use the trigonometry and get the distance, that turns out to be 0.608 meters from the property of a rolling wheel without slipping that is very important I know $V_o = r\omega$.

Relative Motion – Non-Rotating Axes

5. The gear A (teeth not shown) rotates clockwise about O with a constant angular velocity of 5 rad/sec. DOE is a 90° sector, which is mounted on an independent shaft at O. Each of the small gears at D and E meshes with gear A. If the sector has a counterclockwise angular velocity of 6 rad/sec at the instant represented, determine the corresponding angular velocity of each of the small gears.

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You are expected to use this kinematical relationship when the problem statement says it rolls without slipping. So, you have to remember the property of rolling, $V_o = 4 \text{ m/s}$ and $\omega = 10 \text{ rad/s}$ I get ok. And I get $V_A = r_{AC}\omega = 0.608 \times 10 = 6.08 \text{ m/s}$,

so, it is fairly simple V_A is perpendicular to AC. So, it is a fairly simple and straightforward problem it is a known territory for you, then we move on to another interesting problem.

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This is dealing with gear, so, I thought that I would also show you the animation of the gear. And you have to understand what this gear is all about, how it works? See essentially these are all spur gears with involute teeth like what you have here, but I have taken only the mean radius of that. And what I have here is the gear A which is shown in blue rotates clockwise about O with a constant angular velocity of 5 rad/s .

And what you see as grey in colour is a thick member, it is like a plate, it is a sector OED which is a 90° sector which is mounted on an independent shaft at O . It is very clearly given that is what is shown as a black circle here; this sector can rotate independently independent of the blue wheel.

And you have two small gears each of the small gears at D and E meshes with gear A ; see such aspects are required in some of the gear boxes ok. We are analyzing only a portion of a gearbox; if the sector has a counterclockwise angular velocity of 6 rad/s at the instant represented, determine the corresponding angular velocity of each of the smaller gears is the problem statement clear.

Relative Motion - Non-Rotating Axes

Absolute velocity of any two points completely defines the motion of the rigid body.

$$v_B = OB\omega_{OB} = (0.16)5 = 0.8 \text{ m/s}$$

$$v_D = OD\omega_{OD}$$

$$v_D = (0.2)6 = 1.2 \text{ m/s}$$

6 rad/s

40 mm

160 mm

5 rad/s

$\omega = ?$

C is instantaneous centre!

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I have a blue gear which is rotating in clockwise direction, I have a sector gear this sector is rotated independently when I rotate this independently these gears will roll on this base gear you are asked to find out what is the angular velocity of the smaller gears. And this is the good

candidate to analyze it from instantaneous center of rotation. See I teach this chapter and I solve this problem; it is easy solve to figure out I can apply instantaneous center of rotation. The real challenge is when you phase a physical reality even identifying which methodology to solve requires some to prior training.

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Let us see how we will apply instantaneous centre of rotation to do it. So, this is depicted very nicely here pictorially; I have this rotating at 5 rad/s. And when I show this the line OD on that it is put, so, you have to recognize that I have a sector I think I have that figure also put here for clarity ok.

I have this rotating at anticlockwise 6 rad/s, the question ask is what is the angular velocity of the smaller gear? You know I have already said I should be in a position to get absolute velocity at any two points on the object, the motion of the body is completely understood ok. Let me set the goal as I have to find out the angular velocity of this gear D . My focus is to find out absolute velocities of this gear at two different points, which are the points I can find out the absolute velocity from the given problem statement?

You have a blue gear, you have a sector, which all the points can I find out? Suppose I say that these two mesh I thing have label it as B let me see. I have label it as B , I also give you the clue that I have to find out the absolute velocity at B and absolute velocity at the center of the disk D .

Can I estimate it from the given problem statement, if I know the absolute velocity of two points on the object D ; the motion of D is completely specified. Do I have the luxury to find out what is the absolute velocity at B , yes, I have fine, that is nothing but $v_B = OB\omega_{OB}$; fine. I know the absolute velocity at point B and can I find out the absolute velocity at point D .

So, for you to recognize I have brought in that main picture, you have to recognize that I can rotate this sector this sector itself is rotated at anticlockwise. So, I can independently find out what is the velocity at point D . So, what is the expression is idea clear, do you follow what I am trying to say; for B I can do it from the gear A for point D I will look at the sector. And this as a distance OD and I need to know what is the angular velocity of the sector or the line OD whichever way you say, so, I can also write this as $v_D = OD\omega_{OD}$.

So, I know the velocity at D which is 1.2 m/s and from the sense of rotation I can also put V_B is like this and what way you will have V_D , V_D is like this. So, on the smaller gear

D, I have been able to find out absolute velocities of two points, and they are parallel in

Relative Motion – Non-Rotating Axes

Let ω be the angular velocity of the small gear.

$$\omega CB = V_B$$

$$\omega(CB + CD) = (V_B + V_D)$$

$$\omega = \frac{0.8 + 1.2}{0.4} = 5 \text{ rad/s (ccw)}$$

$\omega = ?$

6 rad/s

40 mm

160 mm

5 rad/s

6 rad/s

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opposite direction. So, it is easy to find out the instantaneous center of rotation. This is the instantaneous center of rotation I join these two tips and the base; I have this as instantaneous center of rotation. Now, I can easily find out the angular velocity of D its

now big deal.

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So, let ω be the angular velocity of the smaller gear, I get $\omega CB = V_B$. So, from the

geometry I get an expression like this, so, I get $\omega = \frac{0.8 + 1.2}{0.4} = 5 \text{ rad/s}$ counterclockwise. So, in this class we have learnt one more way of writing down the relative velocity equation.

We have used the constrained motion we brought in the concept of coincident points and we found that it could be applied to practical problem like analyzing a shaper mechanism. Then we also moved on to learn, what is the concept of instantaneous center of rotation for a class of problems; you can easily find out this instantaneous center of rotation and use it as a bases to calculate certain parameters concerning the problem.

Thank you.