Engineering Mechanics
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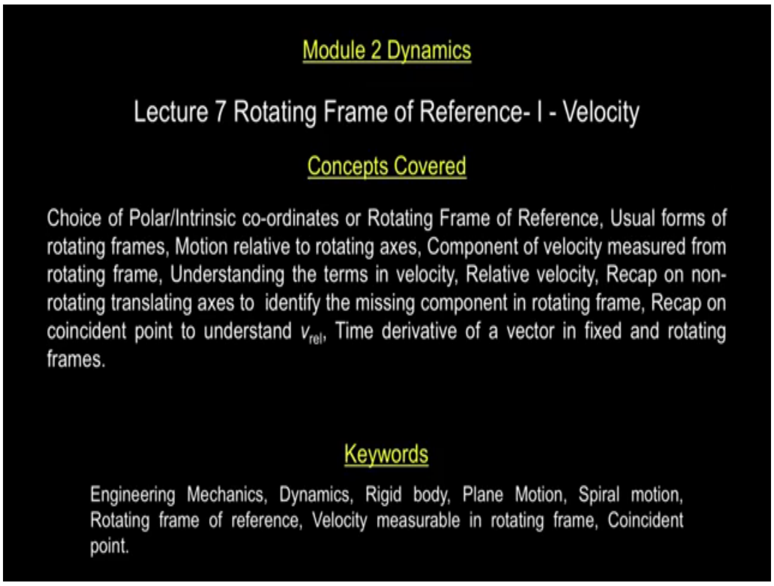
Module - 02
Dynamics
Lecture - 07
Rotating Frame of Reference- I – Velocity



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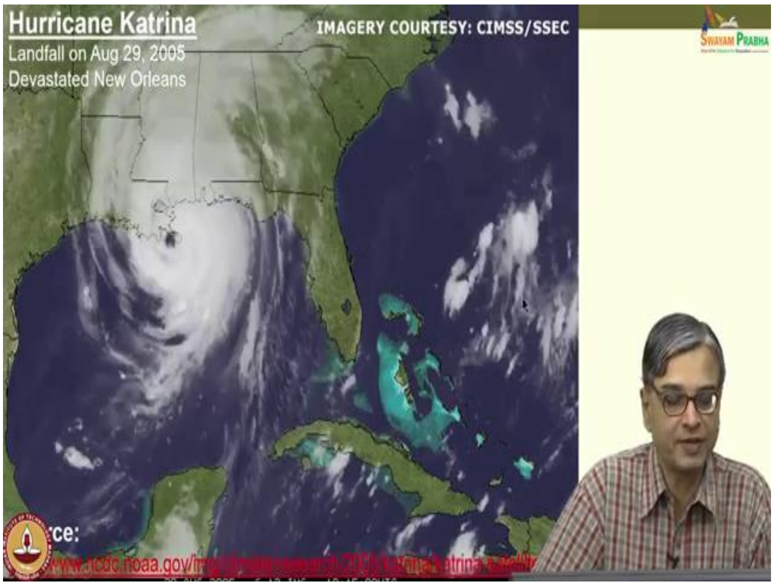

See a rigid body can have both translation and rotation. In the last few classes we had attached a non-rotating translating axes to the rigid body and analyzed relative motion using that knowledge, we also got the absolute velocity and absolute acceleration of certain key points in the body. Suppose I attach a rotating frame of reference to the rigid body what are the advantages? And what does it that I am able to measure easily and what is that I do not see this is needed for you to solve another class of problems, where rotating axes will be a natural choice for you to do the relative motion analysis.

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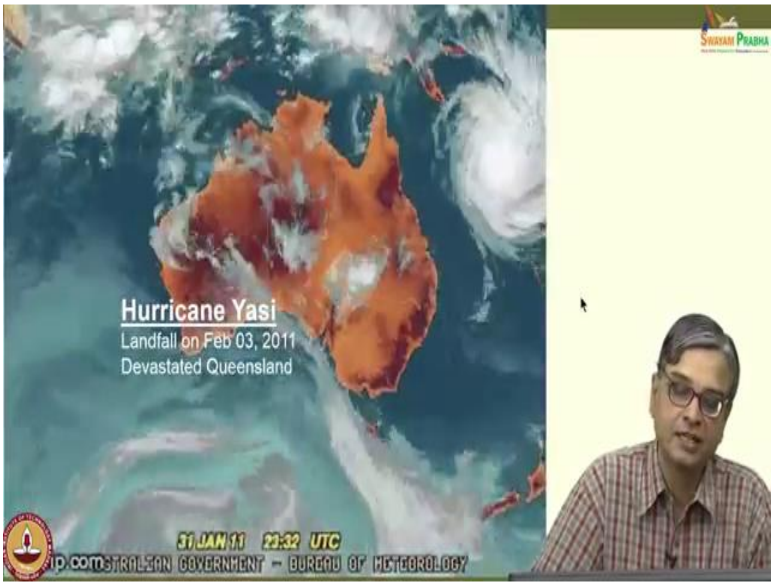
At the start of the course we have looked at this cyclone and this is in the northern hemisphere and you find that this is rotating in anticlockwise direction.



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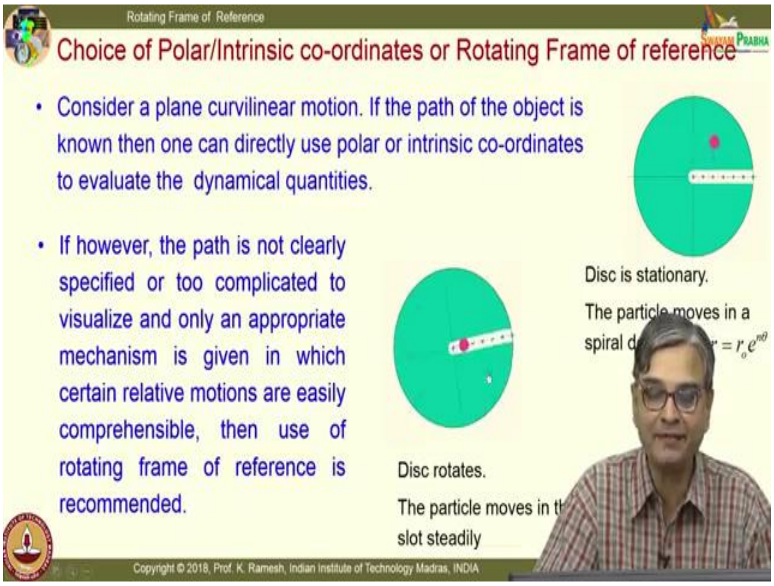
And we saw the southern hemisphere, this is rotating in clock wise direction; see for many problems of day to day importance we have just considered earth as a frame of reference. We felt everything is quite alright with the earth, we are not really considering whether the earth rotating or not, but when you observe some of these phenomena unless you recognize that earth is rotating it may be rotating slow, but it matters in certain understanding of these phenomena. So, there is a need for using your rotating frame of reference in the certain applications.



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And just observe this and also make a need sketch, what I have is I have a particle having a motion like this and by some means if you know what is the description of this motion. Then I can use polar or intrinsic co-ordinates to evaluate the dynamical quantities.

So, question is are you in a position to get an expression and in this case, it is possible because this is developed by us and the particle is experiencing a spiral, most spirally and the equation is given as $r = r_o e^{n\theta}$. So, I have the motion described and once the motion is described I can use polar r intrinsic coordinates see in the case of statics, I said when I show you a load, I have always asked you to visualize how the load could have been applied.

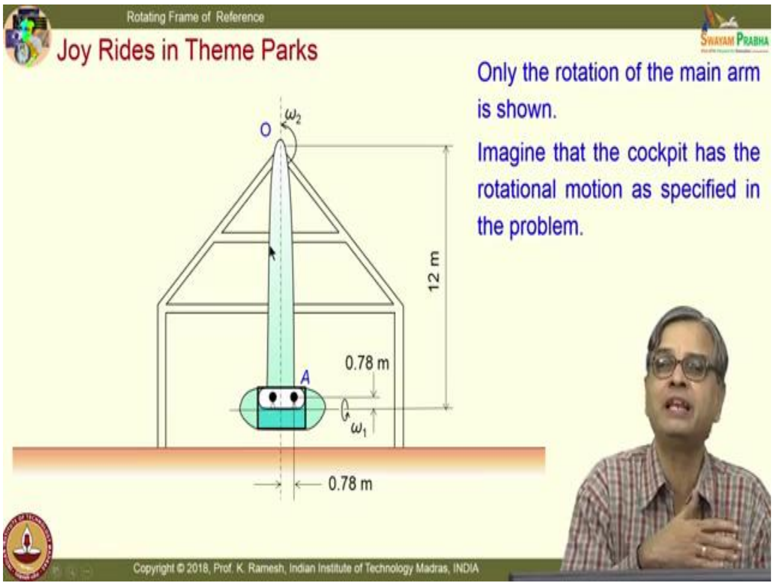
Because in a practical problem you see only a load application in a practical sense which you have to modulate for you to solve the problem and you also learned better engineering if you visualize how the load has been applied. It is easy to say that the particle is given a spiral motion, but how do you give this motion? You must also have a via media to do that is it not; and if you visualize that that makes you a better engineer and you appreciate problems on dynamics better. And look at the second animation what do you see here? Here I see the particle is moving in a spiral. Observe this closely there are two things that you can observe the first thing is that the disc is rotating that is visible because, you are not sitting on the disc and observing what is happening to the particle you are standing outside and visualize and the disc is rotating slowly.

So, you are able to see the rotation of the disc as well as how the particle is moving now? You can say that the particle is steadily moving in the slot. And if you look at what is the difference between these two, I only made the animation I made the disc to rotate in a synchronous fashion with the particle. I may not getting all together a different picture in comprehending how this particle moves and also if you say a finally, I want only a spiral motion of the particle for a given application how a spiral motion can be achieved by having a rotating disc and have your particle to move linearly in the slot; is the idea clear.

So, in many engineering applications what you will find is, only an appropriate mechanism is given in which certain relative motions are easily comprehensible. Many of the problems that come in engineering they are like that; we will also see one example a short while later. And in this case suppose I have not given this motion as spiral I have started with this then you would say that I have a disc is rotating and a particle is moving in the slot steadily that is what you have here, disc rotates the particle moves in the slot steadily.

So, this is what it says the path is not clearly specified suppose I have given only this problem without this picture, only an appropriate mechanism is given in which certain relative motions are easily comprehensible then use of rotating frame of reference is recommended see mind you, I am looking at this as an external observer from the top I am not rotating with the disc. Suppose I rotate with the disc what is it that I would observe? I would observe only the particle is moving in the slot steadily. I would not sense that I am rotating with the disc it is a subtle point and this is what happen people are looking at cosmological happenings around the earth and they were sitting on a rotating frame of reference.

So, rotating frame of reference becomes so important for you to comprehend for you to analyze another class of problems. So, look at this, the only difference between the two animations is here the particle is having its path; the particle has again a similar path in this animation also. But what I have done is I have made this disc to rotate synchronously with the particle that is the only difference if you look at, suppose I have a light emitting mechanism to the particle and I have a photographic film and then I find out exposure of this light and find out what is the path. In both the cases the particle would have only spiral motion fine.



So, on the other hand if I have to get spiral motion, I simply have a disc to rotate and make the particle to move, both are achievable I can make that particle to move linearly in this in many fashion I can have a screw thread and then it moves over it. So, physically it is achievable; it is physically achievable and look at another example.

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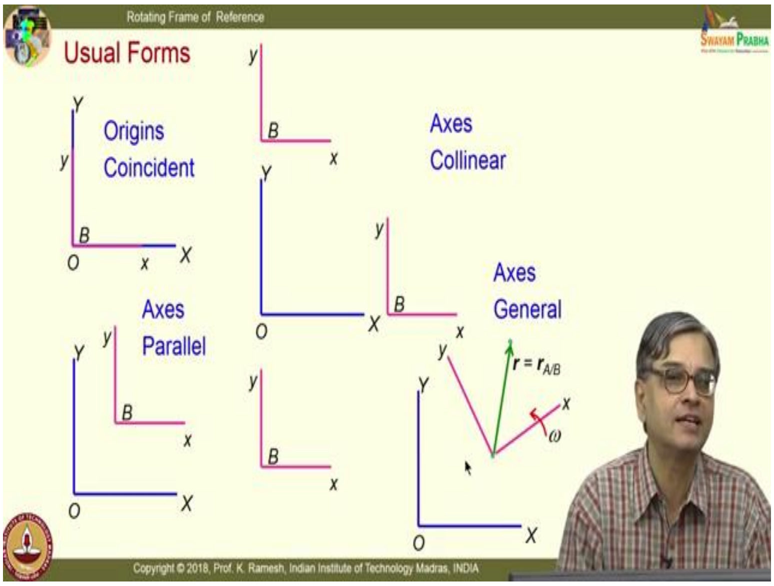
You know there are many theme parks are coming up when you have to be very careful before you select the theme park see that it is well maintained. And then he has used new equipment not imported equipment which they discarded abroad and then they are setting up a park and all this you have nice joy rides ok.

So, only the rotation on the main arm is shown and what is given on the problem, this cabin is rotating about itself in addition to the main arm rotating. So, in a problem like this I know only the relative motions easily, I know what is the speed of the motor that I have given it to the cockpit here; and that is fitted to this arm and this arm is rotated and people device many such variations to give you sort of happiness ok. But from an engineering perspective if you are designing a theme park ride, you are expected to calculate the accelerations a person who would experience when he sits in the cockpit and moves through it. It cannot exceed certain recommended levels given by organizations which do research on human safety.

So, if I have to do that, I necessarily need a rotating frame of reference for me to start with and do the analysis.



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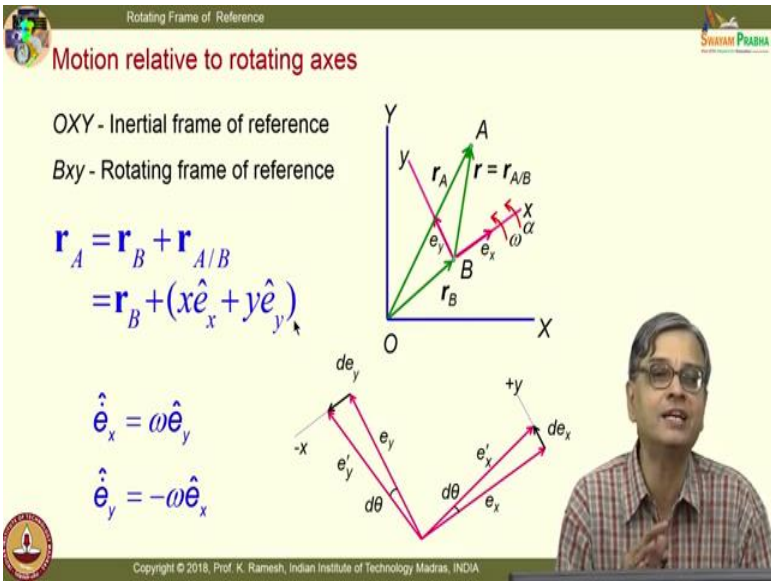
So, there is a class of problem that requires use of a rotating frame of reference and what are the usual forms I have given fixed axes like this. The simplest one is I have rotating axes whose origin coincides with this, it makes your life very simple the origins are coincident. So, the mathematics becomes lot simpler.

And I would like you to make a sketch please keep a note of it and I have another possibility I can have the axes collinear, I have them collinear and I can have any rotation, I can have a clock wise rotation, I can have a anticlockwise rotation, I can also have the speed of rotation changed.

That is why I have given all these possibilities because normally you get a mental block; we solve only a few problems whatever we have done in this problem is what you anticipate. In order to remove that mental block what I am trying to bring out is here, the axes are collinear the axes can have a clock wise rotation anticlockwise rotation as well as the speed can differ it can also have acceleration all that is possible.

Then I have another possibility I have these axes are parallel and the most general one is what you have here, I have this axis is oriented at some angle to start with. And if I develop the equations for this all the other things are easy for you to do. And what I have is I have the position vector; I have always used a position vector in green color and I have also followed a discipline in labeling this fixed axes and rotating axes. Rotating axes is in pink color and the stationary axes is a family of blue I have used different types of blue later, but it is a family of blue. So, you can easily identify which is the fixed axes of reference and which is the rotating frame of reference.



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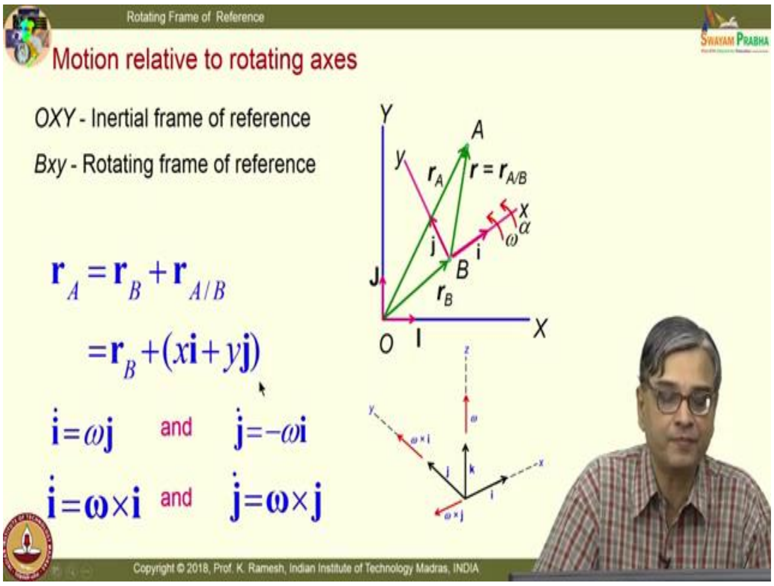
So, I call the frame of reference *O* capital *X* and capital *Y*. I call that as inertial frame of reference, because that is fixed and I have a rotating frame of reference which is labeled as B_{xy} and these x and y are small. These are capital so these are small. You emphasize that this is a rotating axis I have shown that this is rotating the origin of this rotating axes is labeled as capital B. And whatever the point that I have on the object I put it as A and I view this point from the rotating frame of reference by the position vector here.

There is r B which denotes as position vector of B with respect of fixed reference there is r A with respect to B, which is seen in the rotating frame of reference. And once I have a rotating frame of reference I have to be cautious that the unit vectors are not going to remain same they will change; to start with I put this as e_x and e_y and later on to make our life simple in writing I would simply replace it by small *i* and *j*, do not confuse this with Cartesian coordinates *i* and *j* you have to be very clear of that.

I use a symbol mainly because it is convenient for us to handle because we are going to handle it repeatedly with the explicit understanding this is for a rotating frame of reference; and you would attach that it will have *i* dot and *j* dot. You should not jump on to Cartesian coordinates and say *i* dot is 0, *j* dot is 0; do not get confused, this is to write your equations faster and easier to print books with that.

So, this rotating axis can have an angular velocity as well as angular acceleration we are not putting any restriction on it. In some problems you may have only angular velocity, in some problems you will have both angular velocity and angular acceleration we are not putting any restriction. We are only going to view with the rotating frame of reference and the moment I bring in the rotating frame of reference I must also bring in the variation of e dot.

So, $\dot{\hat{e}}_x = \omega\hat{e}_y$ and $\dot{\hat{e}}_y = -\omega\hat{e}_x$, which could also be written down in a different fashion. And I have the position vector for the point A is given as r_A and you have the relative position vector is r_A with respect to *B* make a neat sketch of this. I have a expression $\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$. So, I can write $\mathbf{r}_B + (x\hat{e}_x + y\hat{e}_y)$.



So, when I differentiate, I would recognize $\dot{\hat{e}}_x$ will be there $\dot{\hat{e}}_y$ will be there, but later on I would simply replace it by *i* and *j* for convenience. Because it is again nomenclature, we which we use, in this chapter we will use it like this. So, in the context you interpret *i* and *j* differently.

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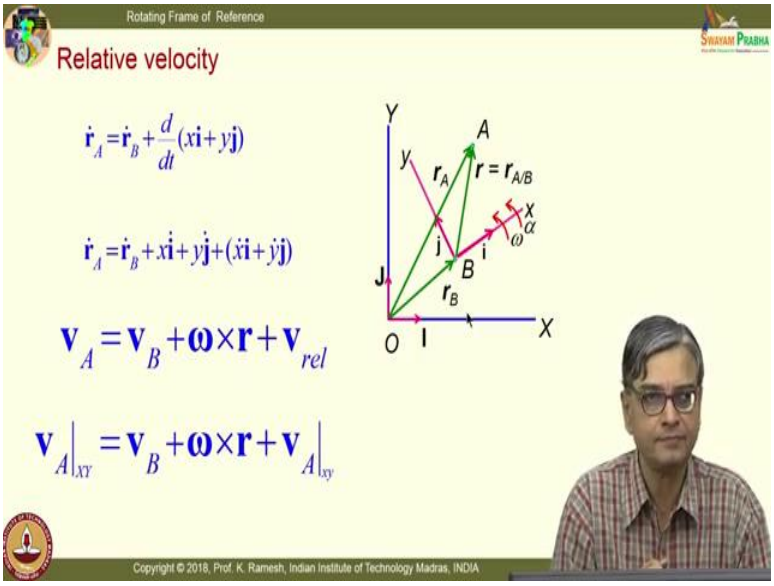
So, that is what I have said. So, when I go for this axes where I have used capital X and capital Y; I would use capital I and capital J as the unit vectors, when I use small x and small y for the rotating axes, for convenience we simply put it as *i* and *j*. Understand the word convenience every time I have to write e_x and then put a dot all that takes time and it is also very difficult to compose books with that fine. So, I have the fixed axes as capital I and capital J and mind you if I express these quantities with respect to i j you should recognize that these are different. So, I can have some quantities with respect to capital I some quantities with respect to capital J.

So, if I do any mathematical calculation you must have one consistent reference axes to which you put all the unit vectors appropriately, when you solve problems you will understand that statement better. And we have $\dot{\mathbf{i}}=\omega\mathbf{j}$ and $\dot{\mathbf{j}}=-\omega\mathbf{i}$, I can also write these quantities in a much better convenient fashion and it is also very easy to remember, can I write it differently as a cross product? When do I get ωj? $\dot{\mathbf{i}}=\boldsymbol{\omega}\times\mathbf{i}$ and $\dot{\mathbf{j}}=\boldsymbol{\omega}\times\mathbf{j}$.

So, I use this for all my mathematical simplification later, so put this axis and recognize that these all vectorial quantities. So, I have put the vector cross product and then $\dot{\mathbf{i}}=\boldsymbol{\omega}\times\mathbf{i}$ and $\dot{\mathbf{j}}=\boldsymbol{\omega}\times\mathbf{j}$ with an explicit understanding that we are in the chapter on rotating frame of reference, we would interpret *i* and *j* as unit vectors of the rotating frame for convenience. So, I have also replaced $\mathbf{r}_A=\mathbf{r}_B+\mathbf{r}_{A/B}$ that is $\mathbf{r}_B+(x\mathbf{i}+y\mathbf{j})$



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Now, the idea is to get the velocity, let me differentiate this and get the velocity see now mathematics is very well developed. So, when you follow the mathematics rule you will never make a mistake, but if you look at the development of these part of mechanics people make observations in those days. So, based on observation they develop the mathematic they did not have mathematics to assist them.

So, there was lot of discussion on what a person would observe and use it in a rotating frame of reference, there were lot of debates on that. So, we will also go through that from that perspective and see whether we could understand the concepts better. And you can write this straightforward I have $\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + x\dot{\mathbf{i}} + y\dot{\mathbf{j}} + (\dot{x}\mathbf{i} + \dot{y}\mathbf{j})$. And there are also multiple symbols people use to refer these quantities and we have adopted Meriam. So, he uses the terminology V_{rel} to denote $(\dot{x}\mathbf{i} + \dot{y}\mathbf{j})$.

We have already seen how I can write i dot and j dot on that strength; I could replace this as $\omega \times r$. It is very important to do this and we have not done anything great we have just applied the mathematics systematically step by step.

So, what we have got? Suppose I sit in a rotating frame of reference and I know what is the angular velocity of the rotating frame of reference? If I observe the point A, I can write the velocity of A with respect to the fixed frame of reference as $\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$. And what is this V_{rel}, how can we measure this V_{rel}?

Another interpretation to this is; this is the velocity with respect to the frame of reference *XY* and what you actually call as V_{rel} is what is the velocity a person who is sitting on the rotating frame would observe; that is a better terminology. Is the idea clear? Please write this expression it is not given in Meriam, but you should write this and if you see some other books, they use this kind of symbolism. There are multiple symbolisms you have in this context and you have to understand it what is best for you.

So, what I have is, this component of velocity is what one would observe if you are sitting in a rotating frame of reference; see in this development I have not really brought in a rigid body. I have just said I have a particle A which is moving in space and then I view that from another particle *B* which is where I have put a rotating form of reference. Now, let us give a feel to this, how I can achieve this kind of motion? Let me put a rigid body to this in a particular fashion and then handle it.

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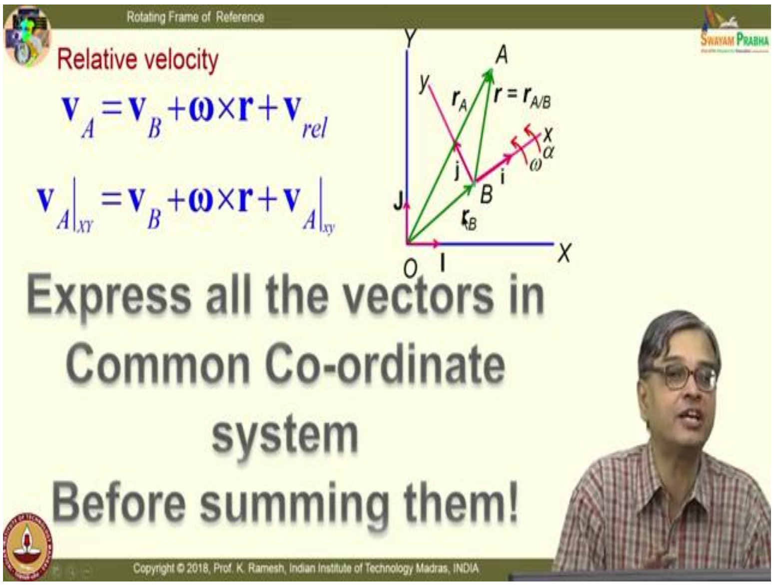
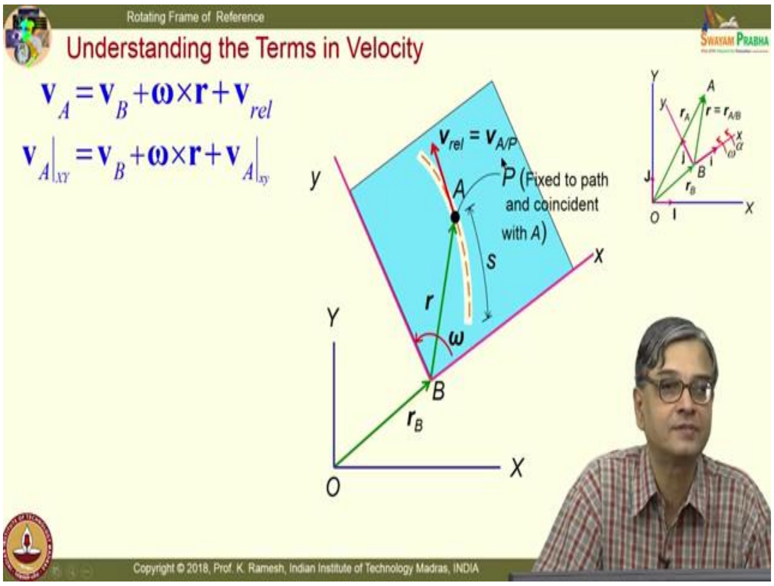
So, before I get into that I make a caution, I have a rotating frame of reference which is oriented at an angle i and j have to be expressed in terms of capital I or capital J or capital I and capital J should be expressed in terms of i and j for you to do mathematical simplification, do not rush to solve the problem.



So, I express all the vectors in common coordinate system before summing them, otherwise this expression has no meaning. Because certain quantity is, I get it from rotating frame of reference; this quantity I get it from fixed frame of reference. As long as you have a coincident axes we started the first one like that ok, if you see the samples you may not have to do anything even when the axes are parallel or even when the axes are collinear; you have no problem only when they are inclined like this. In some problems you may require this also; in those problems you have to be careful in doing the mathematics; be sensitive to that.



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So, what I am going to do now is I am going to visualize how this motion could be seen if I have a rigid body ok. And my interest is I should be able to see certain quantities when I sit in the rigid body and rotate with it and we have also seen a slotted member earlier. So, this is the kind of idea that I am going to bring in.

So, I have a fixed frame of reference please make a neat sketch of it like, I have given a large segment here for you to write for me to draw the pictures you also apportion this because we are going to bill that the picture one after the other ok. So, what I am going to have is I am going to imagine a rigid body in the rotating frame of reference, I attach this point *B* to one corner of the rigid body.

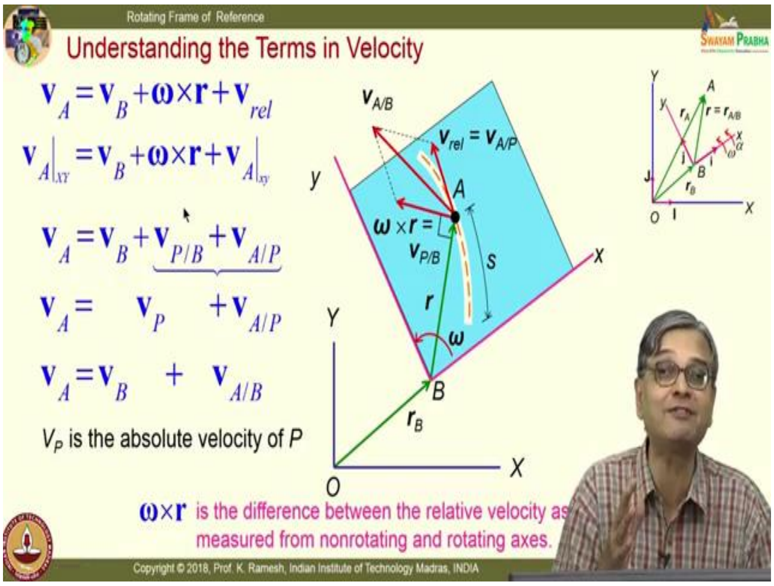
I have the point *A*, so the until this it is like what we have seen earlier now I bring in I have a rigid body and I have a slot in the rigid body in some fashion I have shown some generic curve; it can have any shape depending on the problem context. I am trying to give a life to this how this motion could be achieved in practice. Like in statics I said visualize how the loading has been applied. If you say there is a concentrated moment acting, I need to attach an arm and then put a couple force system, similar to that I have a particle A moving like this particle *B* moving like this I give a life to this for clarity.

And even before we start this, we have the expression $\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$ as given in Meriam. From another perspective it is velocity with respect to *xy*, because this is a fixed frame of reference this is your absolute velocity and I have this V_B as absolute velocity of B with respect to this. See in your non rotating frame of reference that is you have non-rotating translating axes if I say V_A I have $V_B + V_{A/B}$. That V_A with respect to B now has two components $\mathbf{v}_A\big|_{XY} = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_A\big|_{xy}$ is the idea clear? This is what I am going to explain it from different perspectives. Certain perspectives it will be clear for some students; other perspective will be clearer for a larger class of students.

Let us see what it is, now I have this point located at a distant s and we know very well that this is rotating with respect with an angular velocity ω. And mind you we have also learnt the concept of coincident points, it is a fictitious point; the point is not existence ok, but we imagine on the slot a point P which is identical to point A at this point in time. And I have the path is denoted as a dotted line and only when I setting this frame of reference and move with it.

I would be able to measure this; I would be able to get this $V_{A|xy}$ or V_{rel} which is also labeled as velocity of A with respect to P is the idea clear? We have already seen when I had the particle freely moving, I was not recognizing that it is moving linearly in a slot, but when I make the disc with the slot rotate synchronously with the particle I could perceive because the motions are slow where the particle is moving linearly on this slot.

Extending that logic; suppose I move and rotate with the body here; I would perceive the point A to move on the slot whichever the contour that it has and tangential to path is your velocity direction that is what you get it here.



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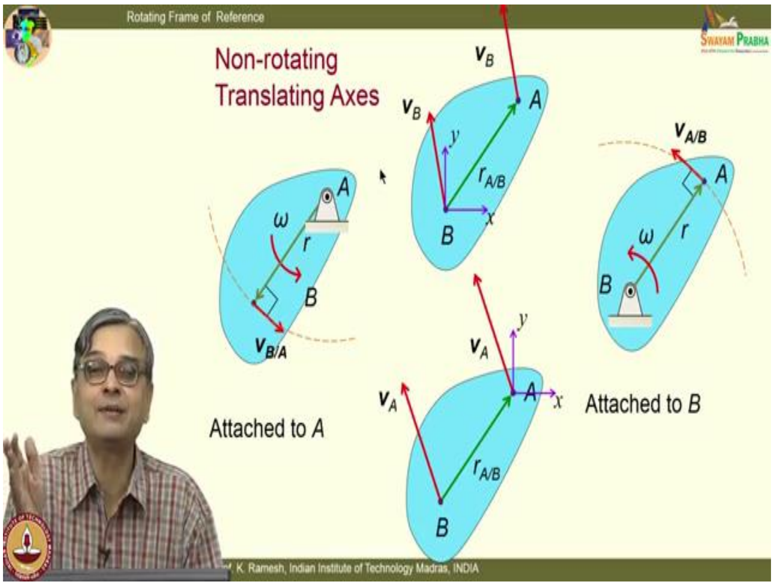
And this is to sensitize that this is not a fixed frame of reference, it is a moving frame of reference, only when it is a rotating frame, I am in a position to see and measure the component Vrel or $V_{A|xy}$ I will repeat it. I could see that some of you are not totally convinced; we will get back to it will again; and I have also written down one more thing I have put this as $\omega \times r$ I have put a direction perpendicular to that.

Suppose I imagine at point B I attach a non-rotating translating axes it could be is again x and y or x prime y prime which is non rotating. Can I write this? Because I know any point on the rigid body will appear as if it is rotating with respect to the point, then I can find out the relative velocity, relative velocity is perpendicular to the direction that I label it as VP with respect to B, I can also calculate the magnitude as $\omega \times r$.

So, I rewrite this expression as $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{P/B} + \mathbf{v}_{A/P}$, this is what I have written it now. Then I can say $\mathbf{v}_B + \mathbf{v}_{P/B}$ simply V_P where V_P is the absolute velocity of the fictitious point; is the idea clear? See my ultimate objective is to find out what are the quantities I get from a rotating frame of reference. And what quantities I do not see both are my interest now. From a mathematical perspective we have already got the correct expression what I would have to use, when I use a rotating frame of reference attached to point *B*.

Now, the question is how do I rationalize these terms from a physical perspective? VP is the absolute velocity of P and we have looked at $\mathbf{v}_{P/B}+\mathbf{v}_{A/P}$ fine. And then I can write this as $\mathbf{v}_A=\mathbf{v}_B + \mathbf{v}_{A/B}$ which is what I started with, I have an expression for velocity here, if I have an expression from a non rotating translating axes I would only have $\mathbf{v}_A=\mathbf{v}_B + \mathbf{v}_{A/B}$ and what is that $\mathbf{v}_{A/B}$ is nothing, but $\omega \times r + V_{A|xy}$.

So, what is the difference between a translating axis and a rotating axis? That is nothing, but $\omega \times r$ the persons sitting on the rotating frame misses this quantity. See whenever I use something there are advantages; there are also disadvantages. The advantages a person sitting on the rotating frame would be able to observe and measure this $V_{A|xy}$ comfortably, he would be able to measure this comfortably otherwise it is very difficult to do it.



On the other hand, because he is rotating with the object; he does not recognize this, you have to have this in your expression when you want to do it only then your mathematics will be correct ok. Let me go and explain it in steps we will spend some time on it this is the crux of rotating frame of reference, if you understand this then doing acceleration is simple the mathematics will help you and you can comprehend it better.

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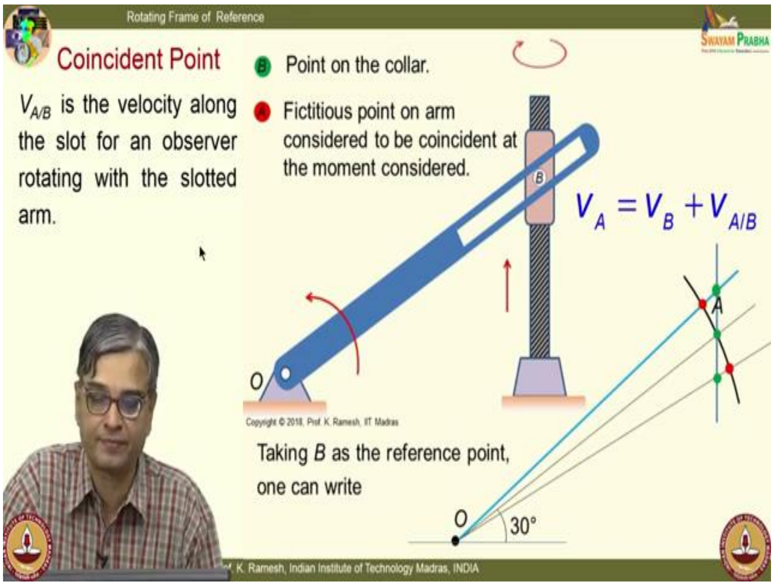
So, I explain it in stages. I want to bring back your old memories when I attach a non-rotating translating axes to a rigid body, I see every other point on the body rotates about it you might have forgotten that ok. So, again bring back that old memories if I attach a non rotating axes translating axes to a rigid body, I would see every other point to rotate above that point; this is one knowledge you have got.



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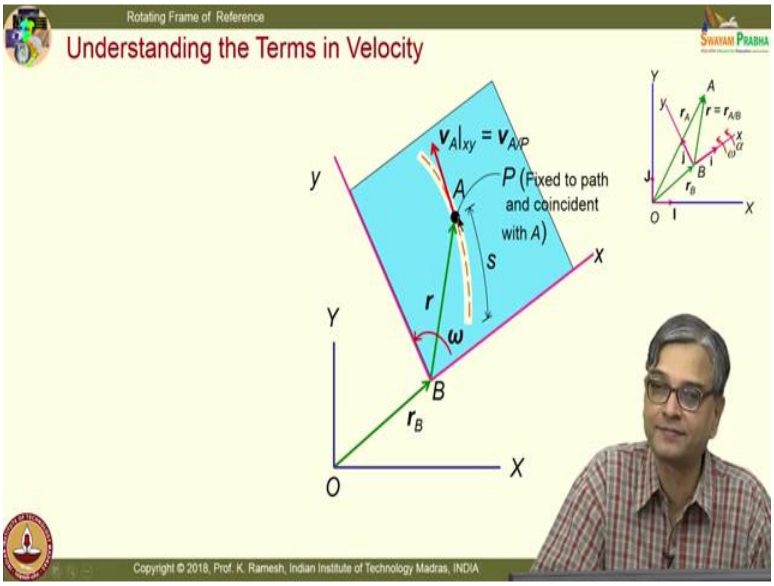
Another knowledge what is that you have got independently, I had concept of coincident point introduced in a very simple system where I could figure out what is the motion of A? And what is the motion of point B And I said point A is coincident with point B this is what I said and we understood from the physical motion of this the relative velocity will be along the slot; you got that idea earlier; isn't it; that you would see only when you rotate along with the slotted member, I would be able to see that from a coincident point perspective.



So, when I have a slot when I have a member that is rotating the knowledge you get from coincident point discussion is, the relative velocity will be along this slot. This is what you have got I could write in this problem $V_A = V_B + V_{A/B}$. Now, we come back to this $V_{A/B}$ is the velocity along with slot for an observer rotating with the slotted arm.

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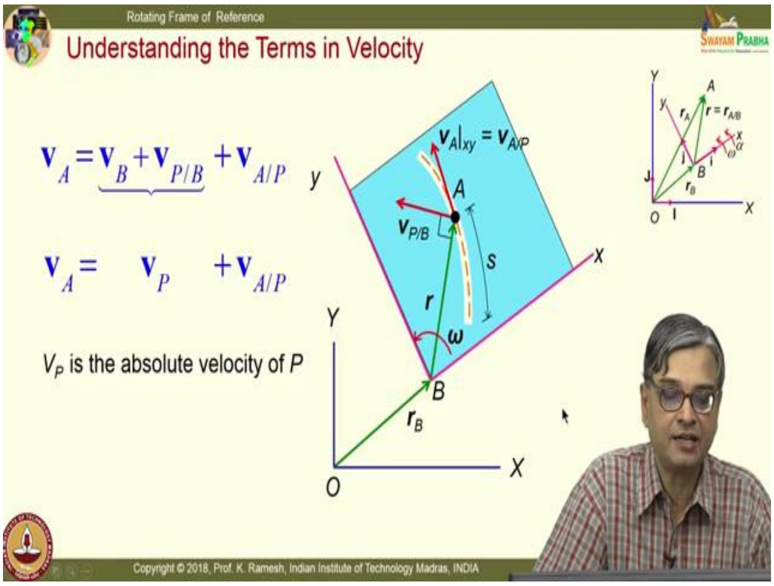
Now, let us understand what is happening here. Now, let us see what I am going to look at this is, I am going to look at the fictitious point *P*. I know very well that this object rotates with an angular velocity ω, I can attach a non-rotating translating axes at *B* and see what happens to this point somehow have to get $V_{A/P}$ that we get it from a rotating frame of reference ok.



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But if I want to do it for the point for the point P, I would say that $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{P/B} + \mathbf{v}_{A/P}$, can you find out what is $V_{P|B}$?

You can find out from your non-rotating translating axes attached to the rigid body; I can always say $V_{P|B}$ is nothing but $\omega \times r$ that I can confidently say is the idea clear? As our original problem demands V_{AP} I have also shown this and we also have the knowledge that V_{AP} is nothing, but what is the velocity perceived in the rotating frame x y that idea you get from the coincident point when we took a very simple problem of a slot. And this I write it as $\mathbf{v}_A = \mathbf{v}_P + \mathbf{v}_{A/P}$; then let me go and see the whole again.

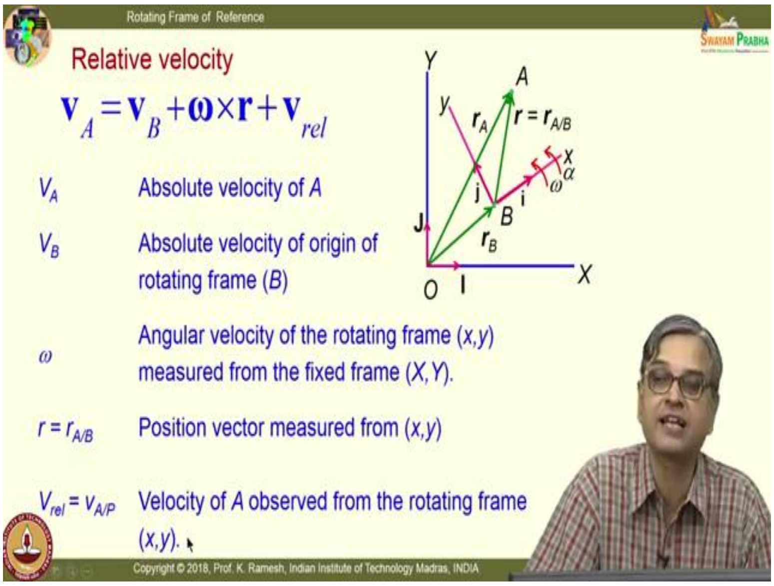
This is nothing, but what we had seen earlier. We are revisiting the same discussion again; and I have this and if I have to get the velocity V_{AP} I can get it only when I sit and rotate with the object. Otherwise I cannot measure V_{AP}, if the problem statement gives this then you do not have to do it, but if I have to get that I have to rotate with it, but this I can visualize from a non-rotating axis attached to it.

So, what we are trying to give is a physical meaning to an expression like this, which we got without any difficulty by applying the mathematic systematically. Getting the expression mathematically straight forward, but understanding it from a physical perspective it is little complicated and little tricky, this is what I am trying to explain it to you to the extent possible.

So, whenever it is convenient you consider the axes *XY* as a rotating axis, whenever it is convenient you see that as a non-rotating translating axes understand the term $\omega \times r$ from a non-rotating translating axes measure the term V_{rel} from a rotating axis.

So, we give life to this and finally, the message here is the discussion the final message of the discussion is $\omega \times r$, is the difference between the relative velocity as measured from non rotating and non rotating axes; it is a caution. That $\omega \times r$ a person who is rotating with the object does not perceive this; that is the message. If you understand that this is the message then you can handle it carefully how do I comprehend what we discussed for the slotted member, when I take coincident point; what is the distance between point *A* and point *B* when you say coincident what is the value of *r*? That is 0.

So, when I have $\omega \times r$ plus that this becomes 0 and you see only the relative velocity is the idea clear?



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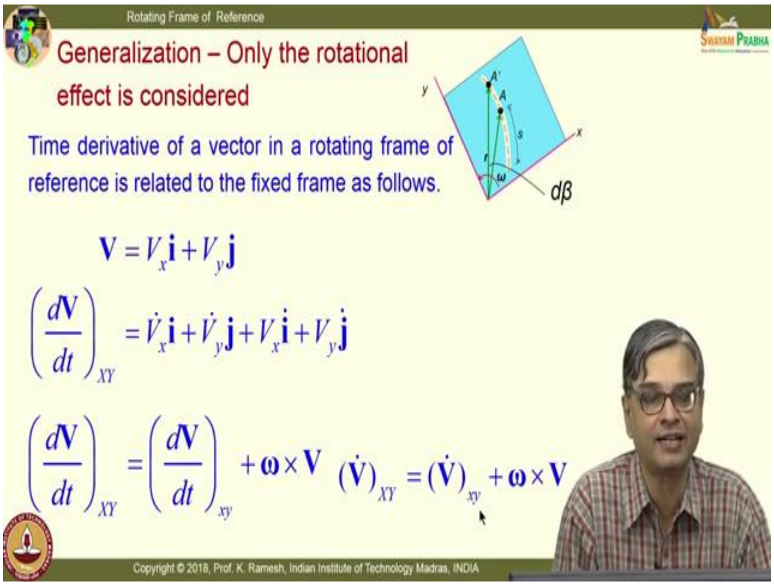
And this can be generalized this result can be generalized; and before we go into that we would also label this in a convenient fashion please write down step by step. I am going to label them V_A is the absolute velocity of *A* and V_B is the absolute velocity of origin of rotating frame, see in this context I have labeled it as *B* in some other problem it is labeled with some other symbol d or e or f. You should know how to apply it. So, this is nothing but coordinate velocity V_B is nothing, but absolute velocity of origin of rotating frame. So, it is a better way of remembering the terms in this expression.

And what is ω? Angular velocity of the rotating frame X,Y measured from the fixed frame, usually the problems are constructed and you are in a position to make that device only by attaching a motor you know what is the rotation of the motor. So, you essentially get only the relative quantity so this is the angular velocity of the rotating frame.

And what is *r*; *r* is a position vector of the point seen in the rotating frame of reference, measured from *X*,*Y*. You should know that this is small x,y you know in some slides I will changed the font for you to easily write because, I am using a power point slides I could show a distinction between a small x comma y and capital X,Y they appear very similar ok. And $V_{rel} = V_{A/P}$ I also said V_{rel} is one type of nomenclature; a better nomenclature is velocity as measured in the rotating frame of reference x,y, where a person rotating with the object will measure only this component of velocity that is not the absolute velocity. So, you have to have additional terms or you to get the absolute velocity.



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So, whatever the understanding that we have gained in this can be extended to any vector. So, only the rotational effect is considered, we are not looking at the translation part of it. I have two positions *A* and *A* prime; please make it and this is different by an angle dβ ok. Time derivative of a vector in a rotating frame of reference is related to the fixed frame as follows.

So, we are writing it in a very generic fashion, I have any vector *V* its not necessarily the velocity vector it is put as that is why it put with the capital *V* it can have a component Vx and Vy, $\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j}$. And when you differentiate it you get it like this we have already done this for the position vector r, the same mathematics and what we perceive this is, if I want to find out the time derivative of a vector in fixed frame of reference, I should know the time derivative of the vector in the rotating frame of reference, plus $\omega \times V$ this is the very important aspect.

This a rotating frame of observer, if you do all the calculation based on your observation; that overall is not sufficient observation you miss it; somebody has to tell you; you have missed it; and you have to have that in your analysis, but mathematics are developed very well now; so from the mathematical rule you have no difficulty, when these concepts were developed they were not having parallel development in mathematics.

So, they had to bank on observation and then people predicted that if you have a canon ball from a auxiliary it will move when you do it on a north it will move eastward so on and so forth. People figured it out from a different perspective and ultimately again as I said many of the scientific development all clean each other whether you like it or not so they were worried about this.

So, they had made observation then they understood from a mathematics, now you have the advantage and luxury of mathematics to help you. So, whenever I have a rotating frame of reference if I am getting the time derivative, if I have to do it for a six frame of reference, I must have the quantity $\omega \times V$, this is also pictorially depicted in the next slide ok. So, this is $(\dot{\mathbf{V}})_{XY} = (\dot{\mathbf{V}})_{xy} + \boldsymbol{\omega} \times \mathbf{V}$. Here I have changed the font. So, that you can easily write in your handwriting you can write this kind of a font; just for clarity I have written it like this, but in all the other slides I have put only the aerial font.

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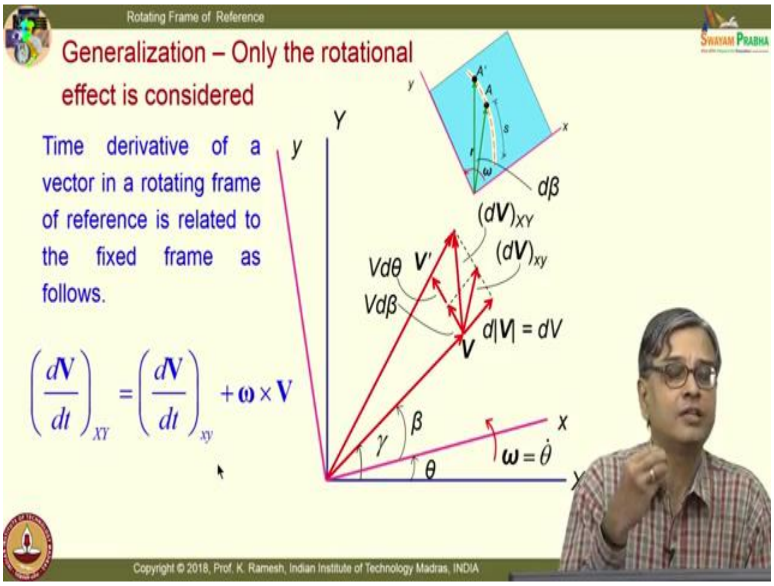
So, let us look at what happens when a small movement takes place dβ, I have this as an expression when I graphically represent, I should recognize that this is rotating with an angular velocity ω I denote this position as θ. So, $\omega = \dot{\theta}$ and I have at this instant of time, see the vector in space there is no difficulty at all I have a vector in space I view it from different axis then I have all this mathematical requirement; what happens to the vector can be perceived from different frames. When I do it from a fixed frame, I get the final expression that is what we will use it for our later computations.



So, I have this with respect to the rotating frame of reference suppose this is at an angle β and in a small interval of time this has moved by a distance dβ; and the point has moved from *A* to *A*' I have given a kinematical representation for this. So, the vector magnitude changes as well as direction changes; is the idea clear? So, what is the change in the magnitude? I have a small change in the magnitude, I have put that as *dV* and change in the orientation would be perceived by a observer sitting in the rotating frame in one way, observer sitting in a fixed frame in a different panel this is what you will have to appreciate, because I have a fixed observer will see this at an angle gamma.

So, it will go by an angle d gamma, it has gone by an angle dβ now I will go for dβ; *V* dβ and I have this, this is what the observer sitting in the rotating frame will observe as the change dV/dt_{xy}. He has a component as a function of as a magnitude variation as a orientation variation. In addition to this a person sitting here would also see $Vd\theta$ and the final quantity that this person will observe is dV_{xy}, so this is the *V*'.

So, the take today is if I have $\left(\frac{d\mathbf{N}}{dt}\right)_{XY} = \left(\frac{d\mathbf{N}}{dt}\right)_{xy} + \boldsymbol{\omega} \times \mathbf{V}$. In fact, we would exploit this when we want to derive the acceleration terms and when we do that without any difficulty you will also get the Coriolis component of acceleration; no difficulty at all. So, to understand it from a physical perspective it took generations for people to do it.

So, in this class we have said that a rigid body has a translation as well as a rotation, in the previous chapter we have looked at if I attach a non-rotating translating axes; what do you perceive? Suppose I attach a rotating axes to a rigid body what does that you perceive and what is that you do not perceive both are there both are necessary so that you can have a proper mathematics and in a class of problems use of rotating frame of reference is very elegant to solve.

Thank you.