Engineering Mechanics
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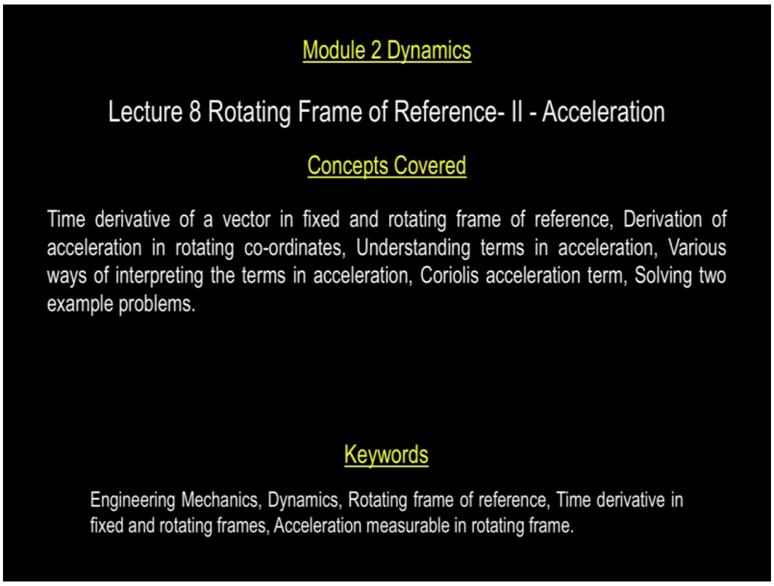
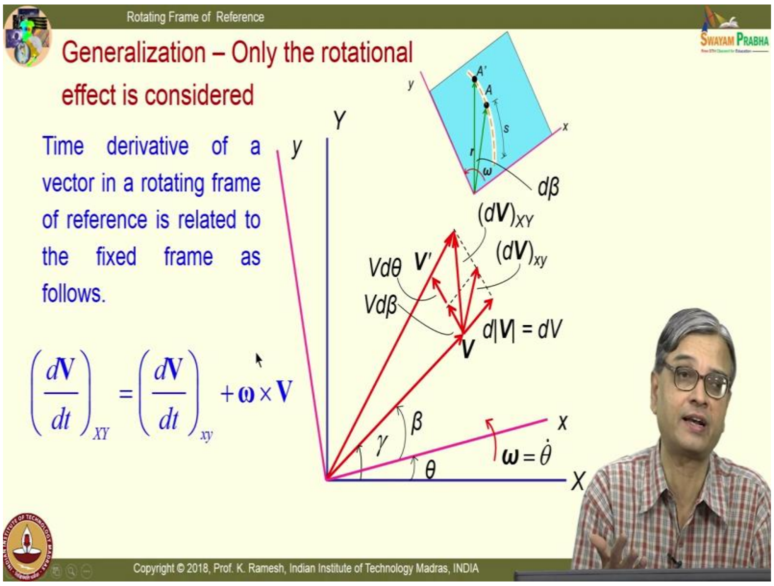
Module – 02
Dynamics
Lecture - 28
Rotating Frame of Reference- II- Acceleration





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Let us continue our discussion on rotating frame of reference. See the idea you have to keep in mind is a rigid body has a translation as well as a rotation. You are accustomed to attaching a non rotating translating axis from your earlier knowledge of particle dynamics. In fact in certain class of rigid body mechanics you would be able to determine only the relative quantities based on the construction of the mechanism and such problems lend themselves easily for analysis if we have a rotating frame of reference, with which you view the problem situation then you can solve them comfortably.

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And one of the concepts that we discussed in the previous class was how do you express the time derivative of a vector in a rotating frame of reference to a fixed frame of reference. And we have done the generalization only the rotational effect is considered, and the expression what we finally, got was if I have a generic vector V, expressed in the fixed frame of reference its time derivative is related to time derivative of that vector in the rotating frame of reference plus an additional component which is $\omega \times V$, this a rotating observer sitting on the rotating frame of reference fails to observe it.

So, you need to understand that there are advantages by rotating with the frame of reference you are able to see the velocity in that frame, acceleration in that frame, which are labeled as V_{rel} and a_{rel}, you can measure only when you rotate with the coordinate system. But in the process, you also miss out certain information, that is what is explained in terms of the vector. Vector in space is denoted like this you are able to get the components either with respect to the fixed frame of reference or to the rotating frame of reference.

So, in a time delta t or dt it has moved by a distance $d\beta$. So, the vector has a magnitude change as well as a directional change a rotating observer is able to observe only this change in the vector because of the directional change, whereas a fixed observer will also see the change because of the rotation $d\theta$. So, finally, you get the velocity vector V'. So, from a fixed reference the person will have to have this component dV_{XY}, from a rotating frame of reference he would observe only dV_{xy} small letters written here. And when you take the time derivative this is the final expression, this is a very important aspect no matter what vector you are looking at it could be a position vector, it could be a velocity vector.

So, this is like you get the new mathematical rule, when I have to find quantities with respect to a rotating frame with respect to a fixed frame of reference from a rotating frame of reference how do you write that. So, when I want to find out the acceleration, I would use this systematically, written down in another form this is nothing, but $\left(\frac{dV}{dt}\right)_{XY} = \left(\frac{dV}{dt}\right)_{xy} + \omega \times V$.

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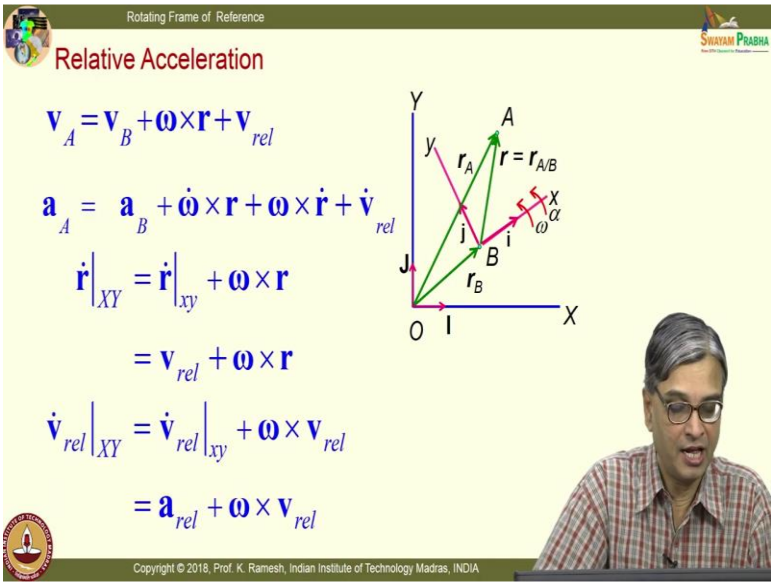
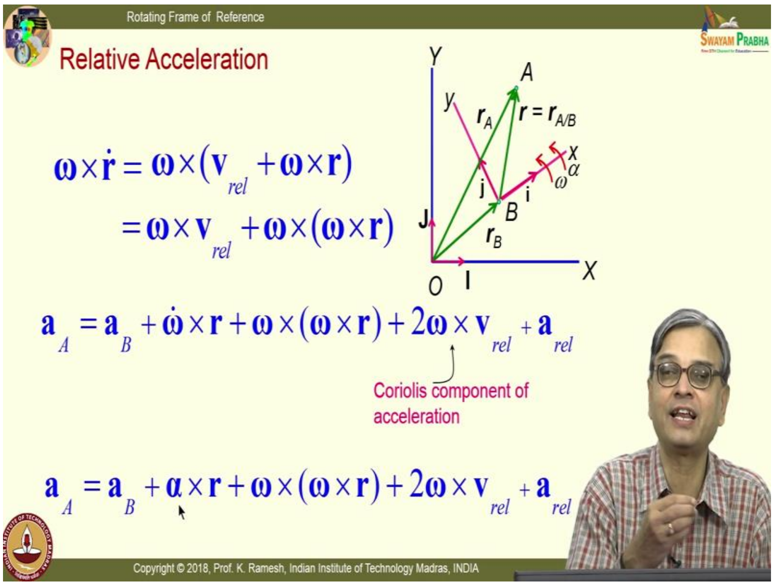
Let us now move for developing the acceleration, we start with the expression for velocity. We have $\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$. I have to essentially differentiate this and get the acceleration. So, when I differentiate this expression apply the product rule properly when I write it, I have $\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times \dot{\mathbf{r}} + \dot{\mathbf{v}}_{rel}$. See any one of these time derivatives you have to handle it carefully. we have just now learnt when I am sitting in a rotating frame of reference, I get one value, but if I have to refer it to fixed frame of reference, I should also have additional component of ω cross that vector.



So, that I have to systematically put it, if you do that final result is very straight forward and simple. I am going to do that systematically I am going to recognize $\dot{\mathbf{r}}\big|_{XY} = \dot{\mathbf{r}}\big|_{xy} + \boldsymbol{\omega} \times \mathbf{r}$. So, when I have this $\dot{\mathbf{r}}$, I am going to replace it like this and we have already looked at what is $\dot{\mathbf{r}}\big|_{xy}$ the other symbolism which we have used was V_{rel}.

So, I have this as $\mathbf{v}_{rel} + \boldsymbol{\omega} \times \mathbf{r}$, then I have to worry about what is $\dot{\mathbf{v}}_{rel}$. So, if I write $\dot{\mathbf{v}}_{rel}\big|_{XY} = \dot{\mathbf{v}}_{rel}\big|_{xy} + \boldsymbol{\omega} \times \mathbf{v}_{rel}$ you should not forget this. This is very important when you are having a rotating frame of reference. So, when I write this expression from the quantities which we known which we have the symbol I can write this as a_{rel} that is the acceleration perceived by an observer sitting in the rotating frame of reference and observes the particle.

When he sits, he is able to measure V_{rel}, he is also able to measure a_{rel}. And I would have $\omega \times V_{\mathrm{rel}}$ as an additional term that you have to do when you are handling a rotating frame of reference. Now what I am going to do is, I am going to substitute the terms in these expressions and get the final expression for the particle or the point *A*. So, this is the absolute acceleration of point *A*, this is the absolute acceleration of point *B* which is nothing, but the origin of the coordinate axis. Which is rotating with the frame of reference we are using that as attached to a rigid body or attached to any other particle also ok.



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So,

$$\boldsymbol{\omega}\times\dot{\mathbf{r}} = \boldsymbol{\omega}\times(\mathbf{v}_{rel} + \boldsymbol{\omega}\times\mathbf{r}) = \boldsymbol{\omega}\times\mathbf{v}_{rel} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}).$$

We have already got $\boldsymbol{\omega}\times\mathbf{v}_{rel}$. So, when I write the final expression, the final expression turns out to be like this $\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$. See if you go back to the history, I said people were making observations, people were able to find out intuitively when you are sitting in air tent and fire an artillery from north from the equator to the north pole it is not going straight, but it gets deflected.

Because you are sitting on an earth which is a rotating frame of reference for a very long-time people were not able to perceive it independently and the term $2\boldsymbol{\omega}\times\mathbf{v}_{rel}$ is known as the Coriolis component of acceleration. We will also have a detailed discussion later. This was very difficult to perceive and it was articulated by a French mathematician Coriolis. He was dealing with water wheels, that was the in thing at that point in time.

So, he said these are all supplementary forces that is how he could get it. And I can also rewrite this expression in terms of other known quantities I can replace ω dot as α and you need to remember this expression. See what is new here is I have $2\boldsymbol{\omega}\times\mathbf{v}_{rel}$ is new, rest of it is very simple and if you go back and look at from the point of view of your relative acceleration concept I have $\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$ whole lot of this quantities has to be relabeled as relative acceleration of A with respect to B it has many components. And we need to get an interpretation of these quantities that we will also do that.

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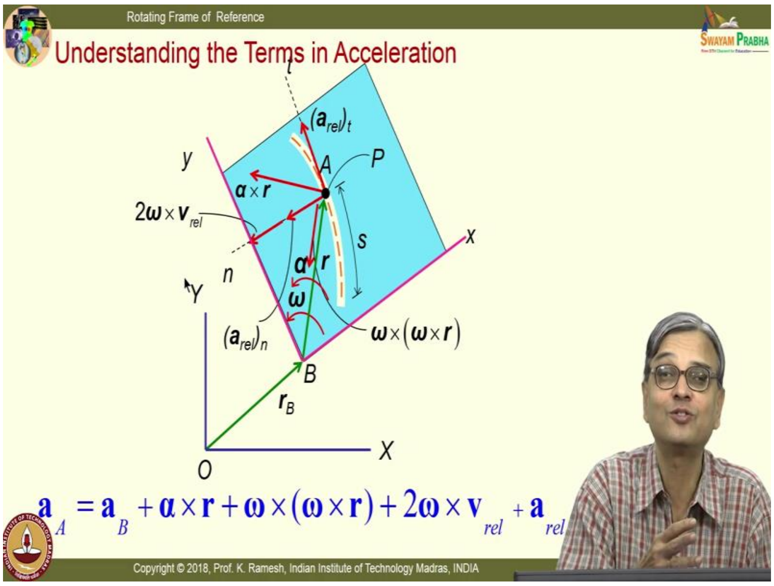
So, let us understand the Terms in Acceleration. So, here again I take a fixed frame of reference I have the origin of the rotating frame of reference as *B*, which is located at a position vector r_B. And then I have rotating frame of reference attached to this and I said in order to visualize what we are discussing we are also imagining an appropriate rigid body fine. A rigid body which has a slot that is the way we visualized for understanding velocity. A similar exercise we will do here and when it is convenient to us, we will make this as a non rotating translating axis and view what happens.



When it is necessary, we will consider that as a rotating frame of reference and find out what quantities I can determine. When you solve a problem, these ideas become clear. So, I imagine rigid body like this it has a slot the moment I say a rigid body if I attach a non rotating translating axis at a particular point all other points will appear to rotate about that point.

This is a very subtle concept which we have discussed at length that is what is new to you, that you have to bring in your understanding for discussing the result. So, I have a particle A which is moving in that slot. And I also consider a coincident point; it is a fictitious point which is there on the slot at this instant of time coincident with point A.

The path is depicted as follows, then I also have a position vector with respect to the rotating frame of reference like this, I have the position vector r and this is at a distance S from one end of the slot. You know for animation, I just shown ω it can also have acceleration α fine.

And I have a tangential and normal component and I can visualize this if I sit on the rotating frame and move with it. I can perceive a_{rel} tangential and normal component only, when I rotate with the frame of reference or the rotate with the object with its angular velocity and angular acceleration.

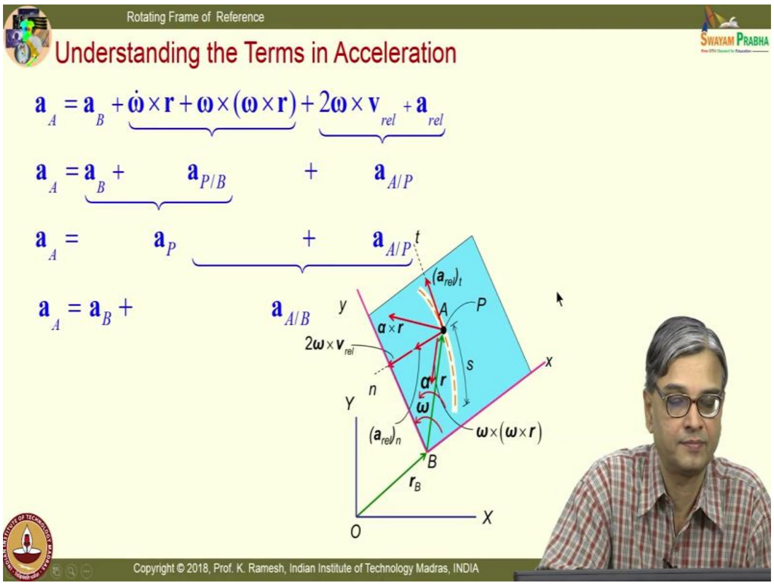
I cannot get this quantity otherwise because if you look at many appliances, they are constructed that way I would know only the values with respect to that frame of reference. I would not know from an absolute frame of reference. So, in such class of problems it is prudent to attach a rotating frame of reference to that axis, use those quantities and use your mathematics to get the absolute velocity and absolute acceleration.

Now, you know I also have this visualization of α, it is easy for me to put now. When I want to carry on the discussion further, I bring in I look at what happens to a coincident point *P*. if I have to find out the parameters for the coincident point, I imagine that I attach a non rotating translating axis at *B*. And you know very well that this will appear to rotate like this.

So, this will be the component of acceleration in this direction this is $\boldsymbol{\alpha} \times \mathbf{r}$ and you will have a component towards the point of rotation about which it rotates $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$. So, using this I can find out what is the absolute acceleration of point *A*, then from point *P*, I have to get the absolute velocity of point *P* as well as absolute acceleration of point *P*.

From this I can find out what is the relative velocity or relative acceleration of point A with respect to *P*. That is how we have developed the concept then you can understand all the terms that involve in the development of this we will again revisit this by term by term. And in addition, what a person would not be able to physically argue is another quantity, which you have this as $2\boldsymbol{\omega} \times \mathbf{v}_{rel}$. It was so simple when you followed the mathematical rule, you cannot miss that you do not have to do a circus to get the term

$2\boldsymbol{\omega}\times\mathbf{v}_{rel}$. It was very clear and straight forward, but this took quite a bit of effort from the scientist to comprehend this and give an explanation for this and you know people also



use this to discount the theory of heliocentricity, you look at people were so happy with geocentric frame of reference, I mean conception of universe and proponents of geocentric theory use this to discount heliocentric theory all that happened in history.

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So, let us look at the terms involved in this expression. And you should appreciate that this is the absolute acceleration of point *A* and a_B is your coordinate acceleration. If you understand it that way then when I change the symbols in my problem you would be able to interpret this expression correctly and you have to remember this expression there is no other go some amount of memory is also required and if you understand it easily.

So, like I discussed earlier you can easily understand that $\dot{\boldsymbol{\omega}}\times\mathbf{r}+\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})$ is what we could get if I attach a non rotating translating axis at B and view the fictitious point *P*, point is coincident with *A*. So, this is nothing, but the relative acceleration of point *P* with respect to *B*. And these 2 quantities refer what is it the relative acceleration between point *A* and point *P*.

You know if you look at the velocity, I had a_{rel} and this was also I mean V_{rel} and this was also V_A with respect to *P*. In addition to that I have another term $2\boldsymbol{\omega}\times\mathbf{v}_{rel}$. So, this is what you know made the scientist to very difficult for them to comprehend; because they were making the observation then trying to find out a mathematical expression for that. When you follow the mathematical steps without any difficulty this comes you cannot miss this term at all.

So, this is nothing, but relative acceleration of point *P* with respect to *B* which we could visualize if you attach a non rotating translating axis at *B* otherwise you will not be able visualize it which the rotating observer missed it fine. Rotating observer will be able to see only this a_{rel} nothing more than that. So, I can put this as a $\mathbf{a}_A = \mathbf{a}_P + \mathbf{a}_{A/P}$, this is one way of looking at it and then the whole of this quantities can be looked at $\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$.

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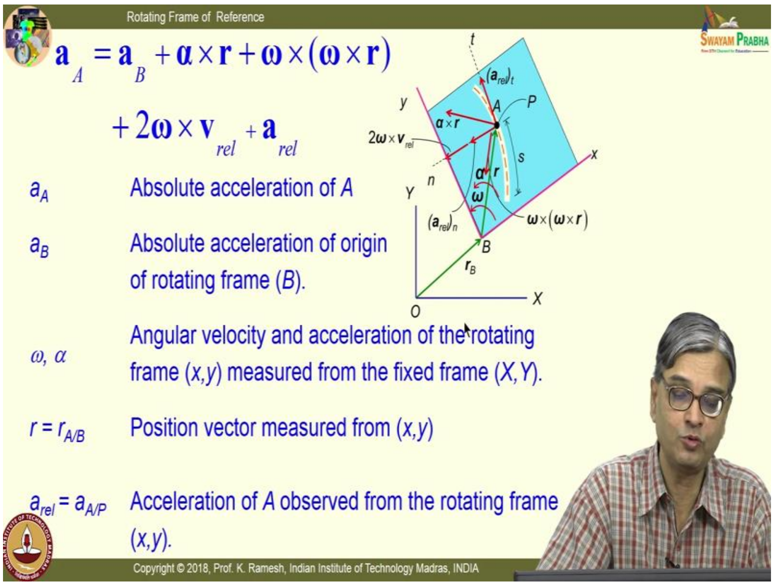
So, this is what you have here and we also have terminologies which explain whatever I have said. This is absolute acceleration of point *A*, and this is the origin of the rotating frame, absolute acceleration of origin of the rotating frame, and ω and α are the angular velocity and acceleration of the rotating frame measured from the fixed frame ok. You are able to see that from that and then I have the position vector in the rotating frame of reference, then I have a_{rel} which is the acceleration of *A* observed from the rotating frame. Here rotating observer will be able to measure only this.



So, even in some problems to get those quantity I have be having a rotating frame of reference and always look at this as coordinate acceleration ok. The coordinate is fixed at point B, suppose I call that as point *A* or point *C*, you should be able to interpret you should not see in a problem, I have *A* and *B* put at somewhere you will apply the equation directly for the points labeled as *A* and *B* that becomes absurd.

So, this you have to be careful and the second point which I have mentioned earlier certain quantities are referred with respect to the fixed frame, certain quantities are referred with respect to the rotating frame. If I have coordinate axis inclined like this then you have to take care of the unit vectors properly, you can also make a mistake.

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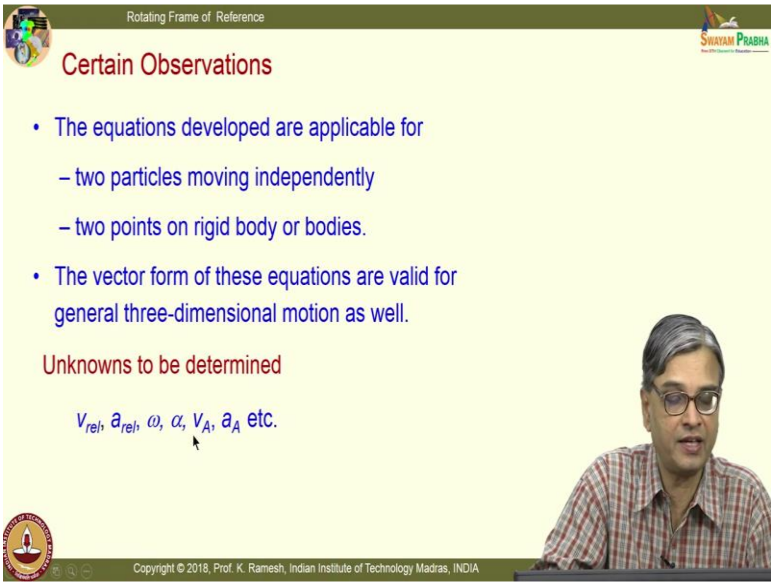
We would solve some problems and before that we will also have certain observations. See if you look at the way they have developed the equations they are applicable for; two particles moving independently, I can also do it for particle analysis. Two points on rigid body or bodies they can be in the same body or they can be in different bodies. And another saving grace is we have developed it using a vector notation for simplicity I have started with planar situation; I do not need to change anything if I go to three dimensions.



So, all your robotic arm and other things which you can analyze again based on these equations they are valid for general three-dimensional motion as well what are the unknowns to be determined? I have V_{rel}, a_{rel}, ω, α, V_A, a_A etc. .

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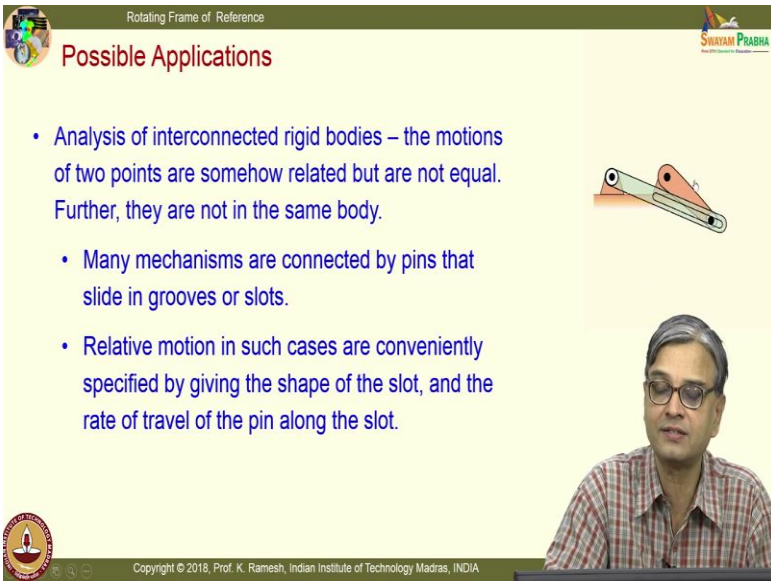
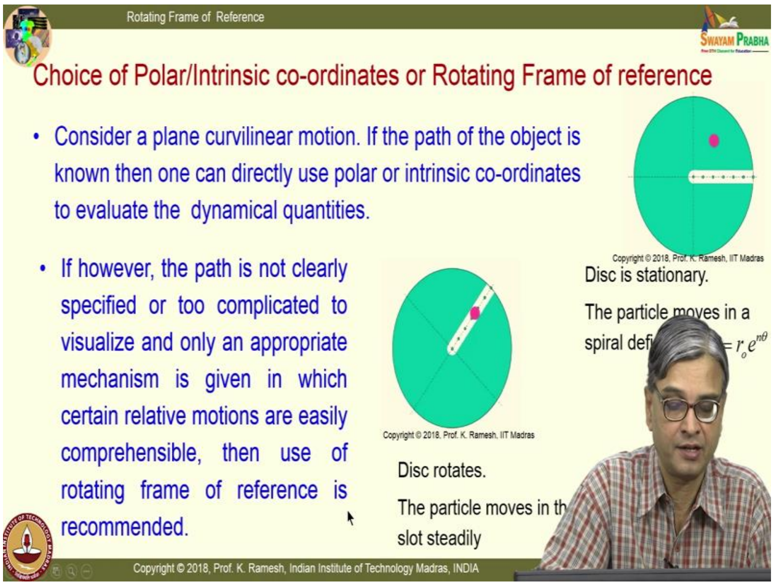


So, these are the quantities that need to be determined. And it is also useful for analysis of interconnected rigid bodies, I have interconnected rigid body like this. I have a slotted system and you see very well that things are rotating the motions of two points are somehow related but are not equal. Further, they are not in the same body ok.

Many mechanisms are connected by pins that slide in grooves or slots. So, one of the thumb rule is relative motion in such cases are conveniently specified by giving the shape of the slot, and the rate of travel of the pin along the slot. And attach a rotating frame of reference to the slotted member it is a thumb rule. Whether it is applicable for the problem under consideration you have to verify and use it.



So, what I needed to give is, what is the velocity with which it traverses in that slot and what is the rotation many mechanisms are constructed like this.

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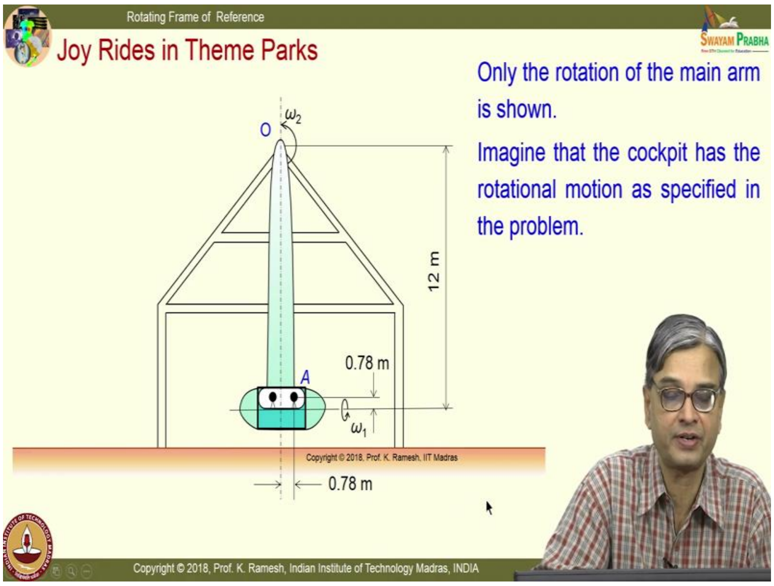

And again, we will revisit this see you are able to see that this is moving steadily in a slot whereas, the particle has a spiral motion which we have seen the previous class also. So, if I have to have a spiral motion, I may have to device my mechanism like this, I have to select the speed of the rotating platform and also the speed with which the particle has to move. So, the idea is the path is not clearly specified; that means, you do not have a mathematical expression or it is too complicated to visualize only an appropriate mechanism is given, then you can think of using a rotating frame of reference and do the analysis.

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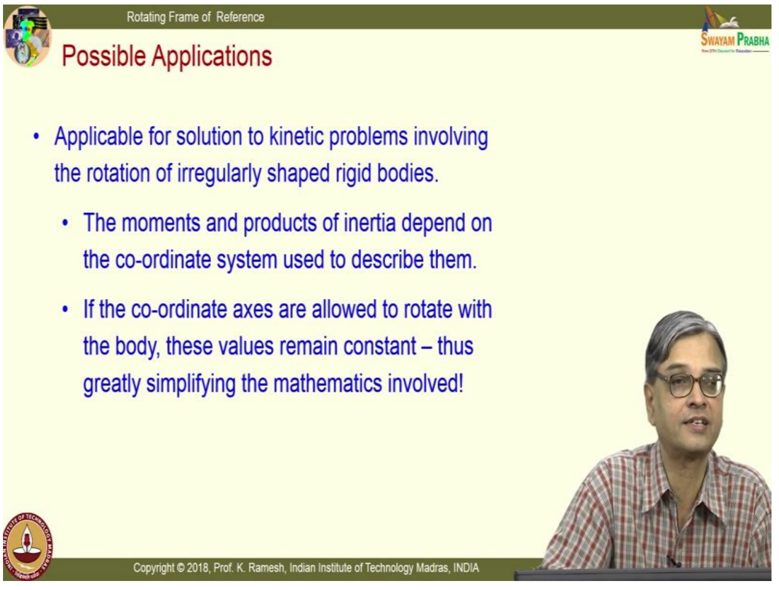
And we have also seen the disc problem earlier we will analyze these problems later. So, here again I have complicated motion and it becomes easier to handle such problem using rotating frame of reference.



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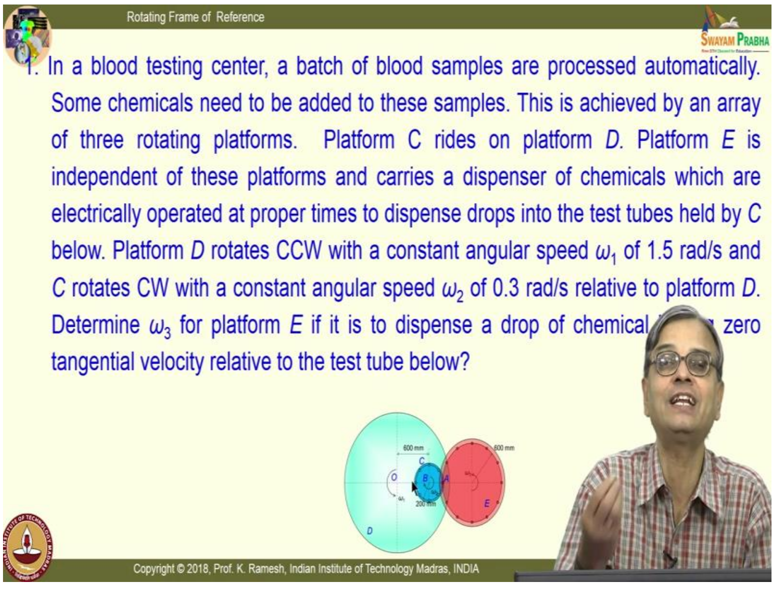

And you should also recognize this is also applicable for solution to kinetic problems involving the rotation of irregularly shaped rigid bodies. If time permits, we will spend time on it in later part of the course. The moments and products of inertia depend on the coordinate system used to describe them. If the coordinate axes are allowed to rotate with the body, these values remain constant thus greatly simplifying the mathematics involved.

So, there is a definite advantage when you learn rotating frame of reference, particularly in 3D kinetics rigid body kinetics it is very useful. And let us solve this problem.

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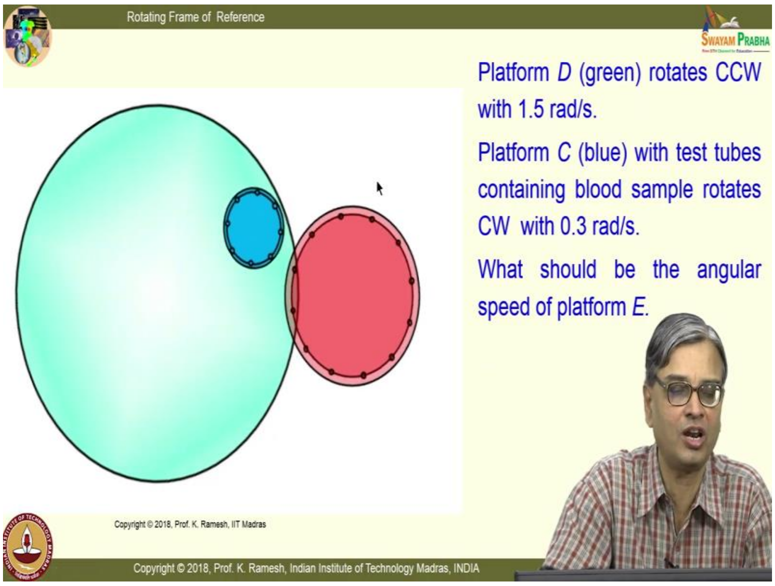
So, I have a very interesting problem easy for you to visualize and I said that initially you do not have to look at rotation as if you should look for centre of rotation; then later on I said when I go to a rigid body it has a general plane motion, when I attach a non rotating translating axis you would see the body to rotate. And finally, I said we would use whatever the equation developed for simple rotary motion intelligently in many problems. So, I start with that kind of a problem. So, what I have here is, I have 3 platforms I have this is rotating in 1 way, I have this small platform rotating in another way I have a 3rd platform they are stacked one above the other ok.

And some you know interesting information is given about what kind of problem is this. The idea is these all rotating the relative I mean the angular velocities are given ω_1 is 1.5 rad/s, ω_2 is 0.3 rad/s. When you physically build this how will you do it when I have to rotate this platform, I would just go on put a motor with an appropriate gear box to rotate at the angular velocity.



So, I know only the angular velocity with respect to the frame you get the idea. I do not get the absolute value, when even the method of construction gives you only that information. So, when you have only that information you will have to use it appropriately ok.

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And this shows a very nice animation my student has done it. He was Iqbal Baig, he was very good in all these animations. And you could cash I mean carefully see that this is rotating anticlockwise. And this is rotating clockwise you can see; we have seen the values as 1.5 rad/s and 0.3.

So, this is rotating at a smaller speed and you have a 3rd platform, the question here is what should be the angular speed of platform *E*? That is what is the question. Because when I drop something from platform *E*, it should go to the cubes fit in other platform without any spill over; that means, the velocity should be identical. So, that I drop it at the instant when I want to deliver it fine. These are all based on some time circuit which we do not have to worry about it. It is a very interesting problem that way.

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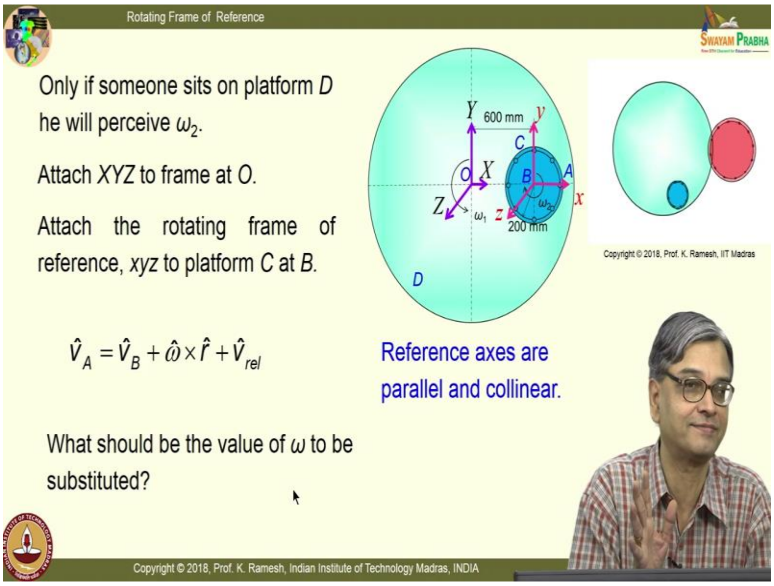
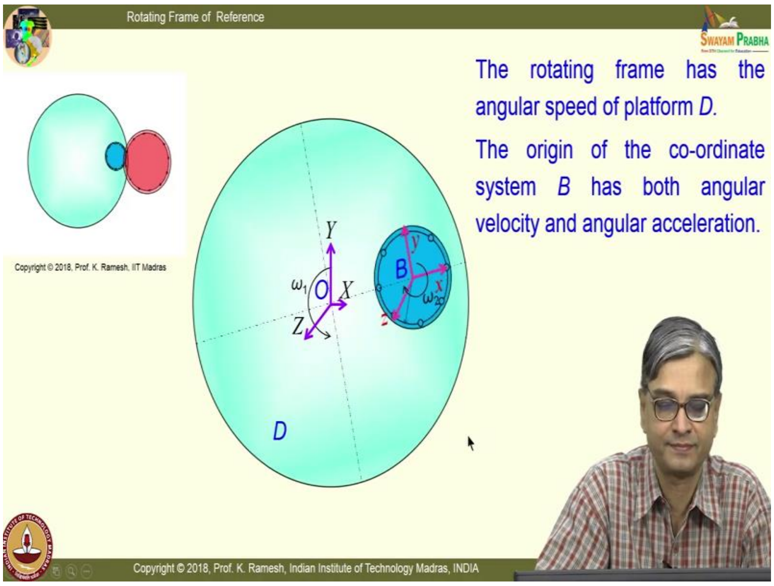
Let us understand what way we will go and solve the problem. So, only if someone sits on platform D, he will pursue ω_2. He cannot get ω_2 otherwise ok. Attach the frame XYZ to frame at O that is the fixed frame of reference. And then I would attach for convenience you know this is also I have done it for convenience I have not put it with the same equal length because if I draw it bigger it will go and hit this diagram.



So, it is put short for the neat looking attach the rotating frame of reference at point B to platform C, I have this as platform D, this as platform C and I have the point B and A. So, in the first problem I have decided that I will use the same symbols what we have used for developing the equations. So, that you understand how to handle the equations later on worry about to generalize it ok.

So, we have the point B and point A. So, I need to know what is the coordinate velocity of point B, then I can find out what is the velocity of point A. So, I can directly use the expressions whatever we have learnt it closing your eyes. And if you look at the axes, they are parallel and collinear. So, I have also not brought in any difficulty in handling your unit vector, they can be one and the same. So, it's a very simple problem to start with and the question is we have to find out the velocity and we know the expression $\hat{V}_A = \hat{V}_B + \hat{\omega}\times\hat{r} + \hat{V}_{rel}$.

Now, you have to find out what is the ω that I should use, how do I evaluate velocity of point B, how do I evaluate velocity which is labeled as V_{rel} or this is the velocity of point A, seen from a rotating frame of reference ok. And I have sufficient number of ω. So, you will have to be very careful and attentive in applying and selecting the correct ω for correct quantity.



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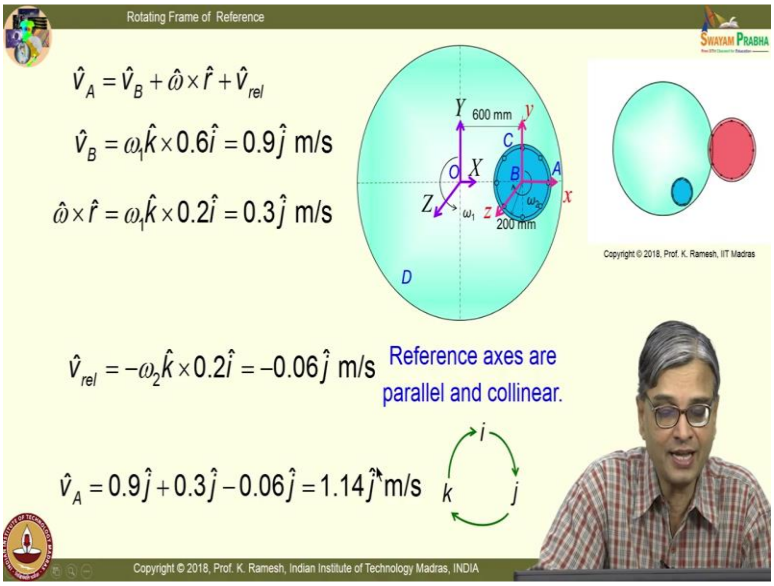

So, that is what I have also summarized it here what should be the value of ω to be substituted. That is again visualize you know in this animation I am not in a position to rotate the small platform you imagine that it is getting rotated at the other platform. This is just to make you visualize; what is the kind of motion that rotating frame of reference has, unless you visualize this you would not be able to select the ω, what should be the value of ω that, I should select for the rotating frame of reference same as the ω of platform D, because it is rotating with the platform.

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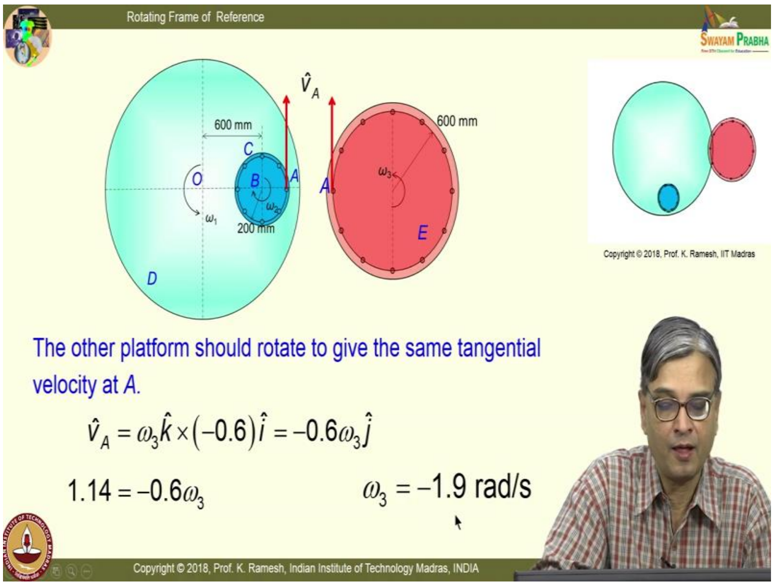
So, I have repeated this expression the reason why I repeated again and again is you should be able to remember this and you should be able to write it very carefully fine ok. So, I have all the quantities come in 1 shot, I wanted this to come one after the other.

So, I have $\hat{v}_B = \omega_1 \hat{k} \times 0.6\hat{i} = 0.9\hat{j}$ m/s. I have velocity of B you have this ω_1, because you have this rotating with the platform and this also, I use same ω_1 I get this as $\hat{\omega} \times \hat{r} = \omega_1 \hat{k} \times 0.2\hat{i} = 0.3\hat{j}$ m/s. And then the V_{rel} see I am in this platform, I am looking and observing what this is. This is also rotating ok.

It is also rotating I perceive the velocity as ω_2 and because it is simply having simple rotation like this, I get this as this is clockwise that is why I put this as $\hat{v}_{rel} = -\omega_2 \hat{k} \times 0.2\hat{i} = -0.06\hat{j}$ m/s. And when I add all of these quantities, I have V_A turns out to be $\hat{v}_A = 0.9\hat{j} + 0.3\hat{j} - 0.06\hat{j} = 1.14\hat{j}$ m/s. See please note the platform *C* which is rotating clockwise.



So, one would expect the velocity to be downwards, but the platform *D* on which this is mounted is rotating at *A* higher speed in anticlockwise direction. So, if you look at what is the absolute velocity of point *A*, that turns out to be in the positive direction of j. See for calculating individual quantities you do not require any additional equations you all know we are again using the equation for circular motion we are not using any new other equations, but you have to be very careful in selecting which ω to be used in which place is idea clear. So, now, I have to find out the angular velocity of this platform so, that I have the same velocity.

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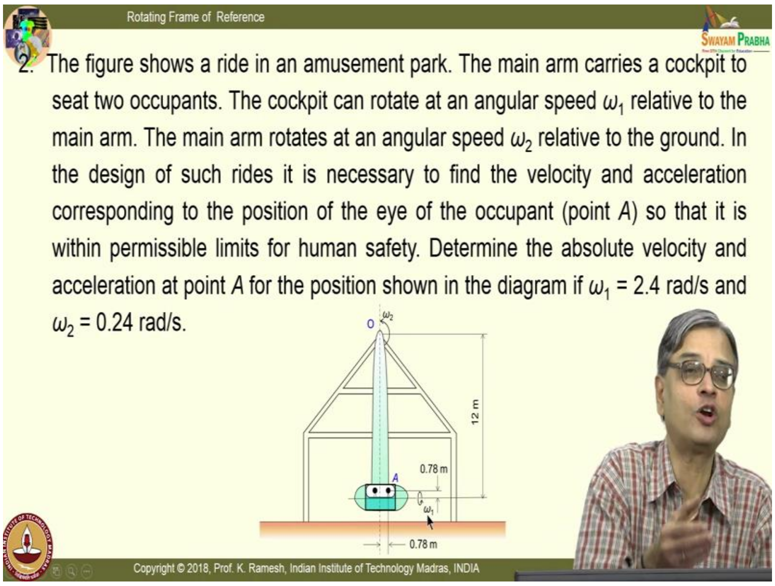
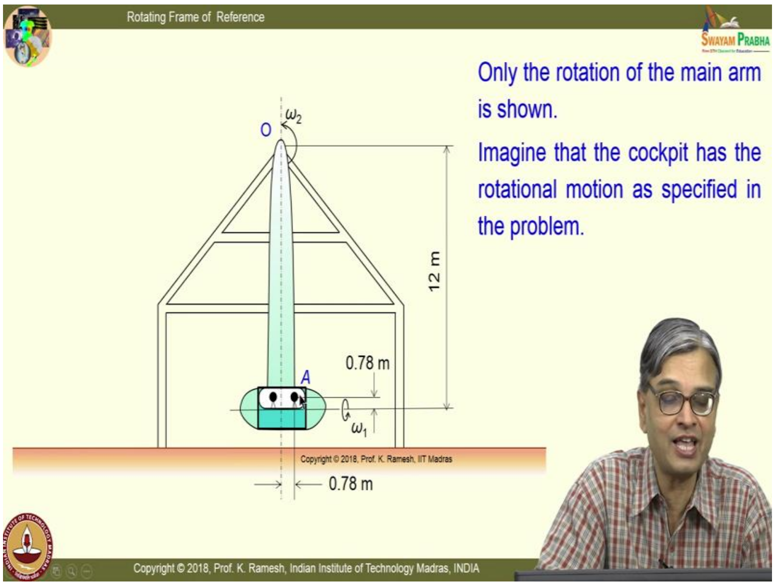
So, that the chemical is dropped without any spilling. So, I have this as the absolute velocity of point *A*, and the other platform should rotate to give the same tangential velocity at *A*.



So, I have this as ω_3 and then I have shown in my diagram in a static diagram I shown this as anticlockwise, because any unknown quantity you start with the positive, your mathematics will give you the symbol, but in my animation I have cleverly put this as clockwise ok. So, do not get confused on that, this is just to illustrate whatever you have learnt in statics how to handle the mathematics similar logic works here also. So, I get this ω_3 as 1.9 rad/s, this is put as minus because this has to rotate in a clockwise direction. So, the problem is very simple and straight forward very simple problem to start with ok.



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Let us move on to the other problem which we have set in a theme park. So, this shows a ride in an amusement park, and what I have here is I have a main arm to that you have a cockpit attached and this cockpit has a circular motion of ω_1. So, when I physically construct what I will do I will have an arm I have a motor with a gear box attached to this and I would be able to independently measure what is the value of ω_1. From the way I have constructed it, but the whole arm with the cockpit rotates ok. So, this also gives an idea to which you will attach the rotating frame of reference, how do you attach the fixed frame of reference and so on.

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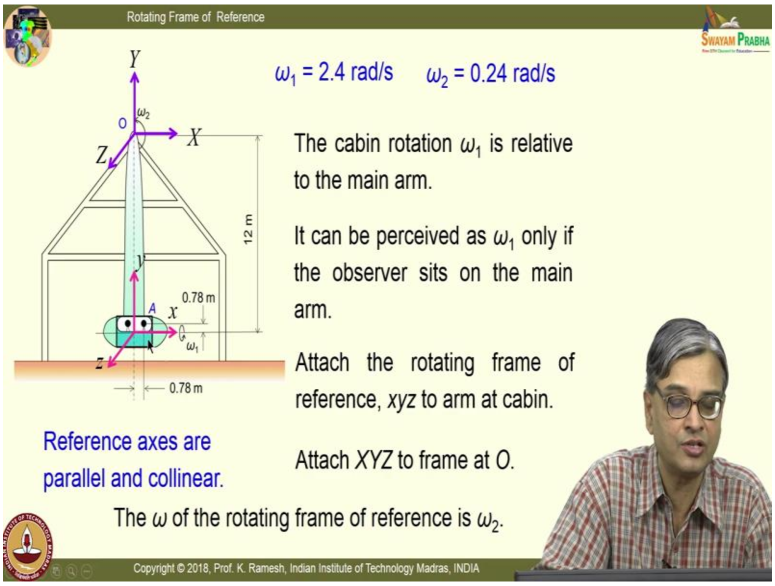
So, in order to visualize the whole mechanism, I also have a nice animation. So, in this I have said that this arm is rotating and imagine that this is also rotating which is not animated in this. And I am sure you would have seen some such you know rides in some its available in some other theme parks such rides are available people go crazy, you know and each individual's capacity to withstand acceleration differ. What is suitable for you may not be suitable for me fine, that is the reason why as designers we have to calculate what is the acceleration and velocity at the level of eye of the person would experience is a question.



And whatever the value I get? I should compare it with the bio medical people who have the recommendation what should be the limits. If it is within the limits, a large class of people will not have problem there will exceptions. If that is not suitable to you, you can have dizziness and you can have lot of problems, in fact I had one of my students had a great difficulty after getting into a simple giant wheel; young boy and that is the reason why I said you have to be careful when you go for such rides whether somebody is doing it not you should select and do it, you should whether you are comfortable with it.

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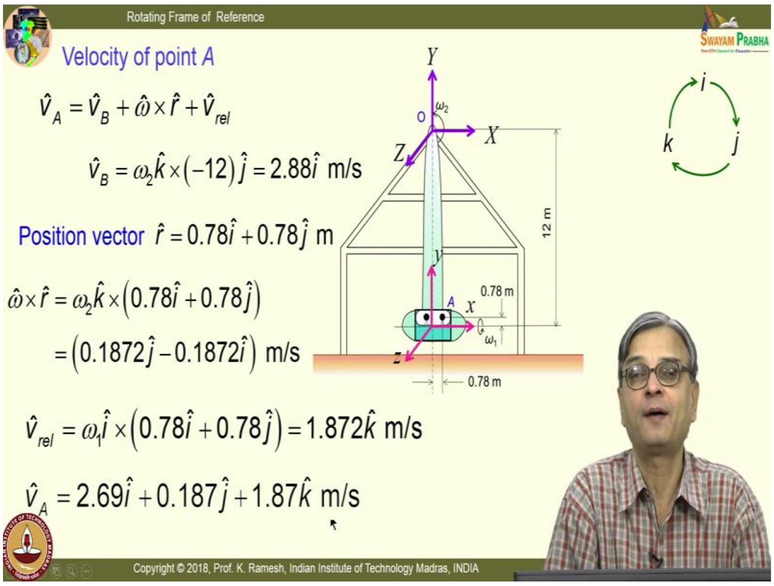
So, you are given in the problem situation ω_1= 2.4 rad/s, and ω_2 is 0.24 radians per second. And here again, the axes are simple it can be perceived as ω_1 only if the observer sits on the main arm, only if the observer sits on the main arm, he will observe this as ω_1. That is how it is constructed. So, I have the rotating frame of reference attached to this cockpit at this point I have the fixed frame of reference attached here it is just to make our mathematics simple fine, you can also attach this from here. Here and then do the computation, but if I attach the rotating frame right on the cockpit many of my calculations become simple and straight forward.

So, in all these problems your first decision is, where do I attach the fixed frame of reference? Where do I attach the rotating frame of reference? If you do that carefully rest of the problem is only simple mathematics, which you will have to take care of it. The thinking part comes in selecting the fixed coordinate system and rotating coordinate system where they should be located in the system under consideration, you have to spend time on that do not rush in that step visualize from the given problem what quantities you know what quantities you do not know. So, here again the reference axes are parallel and collinear, it makes your life rather simple.



And also, so the ω of the rotating frame of reference is ω_2. Here ω_2 is given as the arm is rotating, so that is the ω of the rotating frame of reference.

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So, when I write this expression, I know what the ω that I should use and I can also find out what is the velocity of point *B*. So, that is nothing, but $\hat{v}_B = \omega_2 \hat{k} \times (-12)\hat{j} = 2.88\hat{i}$ m/s. So, for facilitating all the product rules I have this. This is the usual way to find out the sign, do not make a mistake in your sign. And then I have the position vector which is from the rotating frame of reference I have the *x* and *y* directions, so that is easy to get. And this is again rotating so finding out V_{rel} is again simple for you I have to use ω_1 for this.

So, I get $\omega \times r$ in this expression as ω_2 k and it turns out to be like this. And V_{rel} it is rotating with angular velocity ω_1. So, I get $\hat{v}_{rel} = \omega_1 \hat{i} \times (0.78\hat{i} + 0.78\hat{j}) = 1.872\hat{k}$ m/s and then I get velocity of A is $\underline{\hat{v}}_A = 2.69\hat{i} + 0.187\hat{j} + 1.87\hat{k}$ m/s.

So, finding out the absolute velocity is straight forward. And here again I have selected a problem in such a manner equation are developed for point A and B is my origin of the rotating frame of reference. You do not have to modify the expression in any manner, once you have done it for velocity let us move for acceleration terms.

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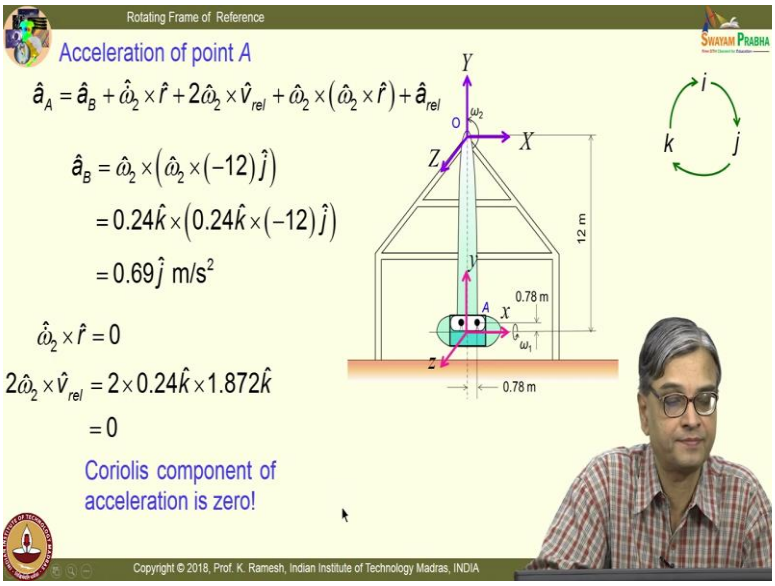
And you should first write this expression, now you should learn this whichever way you can remember this learn your art of doing it. And what I have also done here is I have also replaced the appropriate ω in the expression directly. If you can do that will also simplify your job or you write the expression with general ω, then substitute what ω to be put, whichever is convenient to you practice that rest of it is just putting the terms properly and adding them and then do the algebra.

Here I have the axes are parallel and collinear; I do not have to worry about the unit vectors; that also makes your life simple. So, I have



$$\hat{a}_B = \hat{\omega}_2 \times \left(\hat{\omega}_2 \times (-12)\hat{j}\right)$$
$$= 0.24\hat{k} \times \left(0.24\hat{k} \times (-12)\hat{j}\right)$$
$$= 0.69\hat{j} \text{ m/s}^2$$

And then I have this rotating with a constant velocity. So, I do not have an angular acceleration. So, that goes to 0 and then the Coriolis component acceleration; $2\hat{\omega}_2 \times \hat{v}_{rel} = 2 \times 0.24\hat{k} \times 1.872\hat{k} = 0$ and I have to get a_{rel} and I have to get this quantity.

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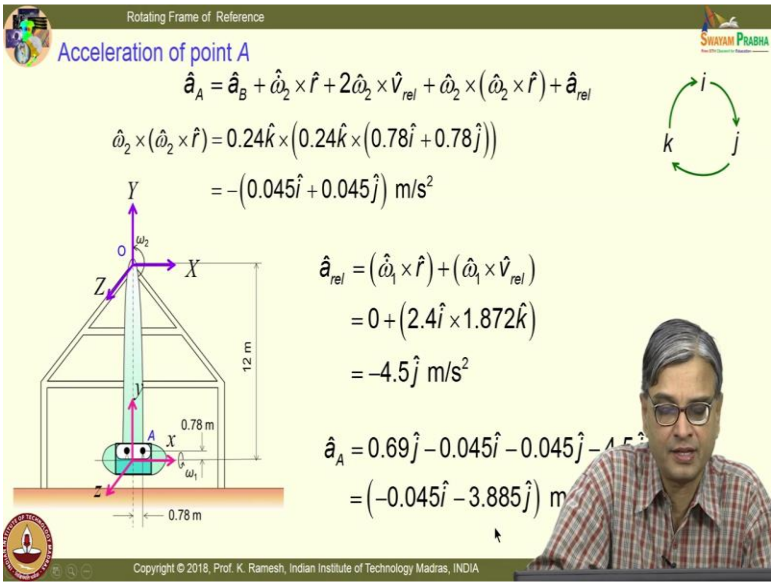
So, I have the expression rewritten here, and I have this $\hat{\omega}_2 \times (\hat{\omega}_2 \times \hat{r})$. So, this is what I said earlier if you have a vector triple product we have learnt a recipe, you do not have to multiply it from first principle, you can directly write the final vector negative of that final vector and product of all these numbers that is what is written here.



So, that is how you save your time then I also get the acceleration viewed from the rotating frame of reference, it is given in the problem $\dot{\omega}_1$ dot is 0. If ω 1 dot is present, I will have this component as well as this component. So, you have the normal and tangential components; only the normal components exist, tangential component is 0 in this case.

So, I have determined a_{rel} so then finding out this is straight forward. So, I get acceleration of point a as $\left(-0.045\hat{i}-3.885\hat{j}\right)$ m/s^2. So, this you may find out the magnitude and verify it with the bio medical tables which list out the acceleration for human safety, I do not have the data at the moment, but that is the reason why you have to compute this. So, this is fairly straight forward, direct applications of equations so there is no difficulty at all.

So, in this class we have developed how to determine absolute acceleration if I use a rotating frame of reference. We also saw that mathematical steps are simple and straight forward, to easily get the Coriolis component of acceleration and I mention visualization of this from an observation point of view was very difficult many scientists have contributed to that. And we have also looked at 2 simple problems; in one of which we determine only the velocity, in another case we determine the velocity as well as acceleration.

Thank you.