

**Engineering Mechanics**  
**Prof. K. Ramesh**  
**Department of Applied Mechanics**  
**Indian Institute of Technology, Madras**

**Module – 02**  
**Dynamics**  
**Lecture – 09**

**Rotating frame of reference III - Choice of rotating frame of reference**

Module 2 Dynamics

Lecture 9 Rotating Frame of Reference III – Choice of rotating frame of reference

Concepts Covered

Recapitulation of expressions for velocity and acceleration in rotating frame, Discussion on what a person views from a fixed frame and rotating frame for the problems discussed in Lecture 8, Solving advanced problems involving Rotating Frames – Problem of a multi utility truck and Quick return mechanism of a shaper.

Keywords

Engineering Mechanics, Dynamics, Rotating frame of reference, Velocity and acceleration in rotating frame, Illustrative problems on rotating frame of reference.

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Let us continue our discussion on Rotating frame of reference.

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See we have looked at expression for velocity just to emphasize that we are dealing with rotating frame of reference. I am showing

this frame of reference rotating and I have angular velocity as well as angular acceleration. We do not put

Rotating Frame of Reference

**Velocity**

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$$

$$\mathbf{v}_{A|_{XY}} = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{A|_{xy}}$$

$$\mathbf{v}_{A|_{XY}} = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \dot{\mathbf{r}}|_{xy}$$

$$\mathbf{v}_{A|_{XY}} = \mathbf{v}_B|_{XY} + \dot{\mathbf{r}}|_{xy} \quad \left( \frac{d\mathbf{N}}{dt} \right)_{XY} = \left( \frac{d\mathbf{N}}{dt} \right)_{xy} + \boldsymbol{\omega} \times \mathbf{V}$$

$$\dot{\mathbf{r}}|_{XY} = \dot{\mathbf{r}}|_{xy} + \boldsymbol{\omega} \times \mathbf{r} \quad (\dot{\mathbf{V}})_{XY} = (\dot{\mathbf{V}})_{xy} + \boldsymbol{\omega} \times \mathbf{V}$$

any restriction on the rotating frame of reference. It can have velocity as well as acceleration and, in this animation, you know I have not said anything about point A it could be viewed in many perspectives ok. One perspective was I had shown that this is part of a

rigid body with a slot and the point A is moving in a slot ok.

We also saw other class of problems, where I had this axis rotating like this. I am also going to discuss these problems today fine. Certain aspects I am going to discuss about it and you know, I have the point A which is looked from a rotating frame of reference. I have a position vector  $r$  and this is  $r_A$  with respect to  $B$ . And I have the position vector  $r_A$  and for this I have an expression for velocity written down like this.

And the symbols  $A$  and  $B$  have specific meaning.  $B$  is the coordinate of the rotating frame of reference ok. That is the way we have labeled it and  $A$  is the point of interest. And if I have different points in the problem, we should be able to rewrite this expression appropriately and what is it that? We have said about this. It can also be looked at as

$$\mathbf{v}_{A|xy} = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{A|y} \quad \text{and} \quad \mathbf{v}_{A|y} \text{ is } V_{\text{rel}}.$$

See you take the advantage of a rotating frame of reference. Essentially, to get  $V_{Ax}$  or  $a_{Ax}$ ; that is the, that is the reason why you are looking for a rotating frame of reference. In fact, the first two problems, I have solved. They require that knowledge and it is easy to perceive, if you have a rotating frame of reference. We will also have a look at it again and this could be rewritten in this fashion.

I can look at this as  $\dot{\mathbf{r}}|_{xy}$  and this is what I have said that, the position vector is  $r$ . And I can also rewrite this as  $\mathbf{v}_{A|xy} = \mathbf{v}_{B|xy} + \dot{\mathbf{r}}|_{xy}$ . From this we also get the knowledge that  $\dot{\mathbf{r}}|_{xy} = \dot{\mathbf{r}}|_y + \boldsymbol{\omega} \times \mathbf{r}$ . So, what we discussed was, when I sit in the rotating frame of reference, it is easier for me to measure  $\dot{\mathbf{r}}|_{xy}$ .

And the rotating frame, a person moving with the rotate rotating frame misses the quantity  $\boldsymbol{\omega} \times r$  and this is what generalized. When we put this as any vector

$\left(\frac{dN}{dt}\right)_{xy} = \left(\frac{dN}{dt}\right)_y + \boldsymbol{\omega} \times \mathbf{V}$ . In fact, we utilize this very systematically, when we differentiated the velocity expression to get the acceleration. We use this repeatedly. This is very important is nothing, but if you look at this expression this can be viewed in this fashion ok.

So, that you have to be very clear about it and we use this important expression from a fixed frame of reference to the rotating frame of reference.

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We utilized it for the derivation of acceleration. So, I have this  $\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$  and I can also know that  $\mathbf{r} \cdot \dot{\mathbf{r}} = \mathbf{r} \cdot \dot{\mathbf{r}}_{XY}$ . Again, rewriting this

$\dot{\mathbf{r}}|_{xy} = \dot{\mathbf{r}}|_{xy} + \boldsymbol{\omega} \times \mathbf{r}$ . This is to emphasize the fact. And I can also write this expression as  $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ .  $\mathbf{v}_{A/B}$  has two

terms. Do not write this  $V_{rel}$  as  $V_{A/B}$  with respect to B; some students have done it. That is not the quantity.

You perceive that velocity, when you sit in a rotating frame of reference and move with it, but when, I say the relative velocity expression, this  $\mathbf{v}_{A/B}$  has two terms. Now,  $\boldsymbol{\omega} \times \mathbf{v}_{A|xy}$  and when you use this expression very carefully for differentiating this. I get the expression for acceleration as like this. I have a  $V_{rel}$  as well as  $a_{rel}$ ; this is the symbolism used in Meriam. So, I do not want to confuse you.

So, that I retain the same symbolism and which can also be viewed as  $V_{A|xy}$  and  $a_{A|xy}$ . And I said, this is known as Coriolis acceleration. This was difficult to perceive, if you sit in a rotating frame of reference and try to find out the quantity. And we will also have a detailed discussion about this in one of the classes later. And here again I can write

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

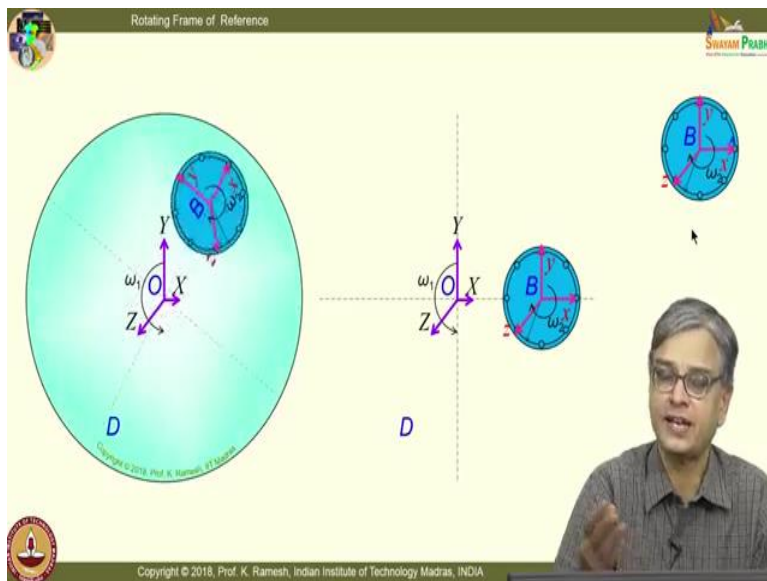
The acceleration of A with respect to B now has so many terms; four terms you have. So, what we have learnt is our ultimate objective is to get the absolute acceleration and absolute velocity. And we use the rotating frame of reference as an intermediary to do all

the computation. It actually simplifies your analysis of a physical problem and let us look

at the problems that we had seen in the last class.

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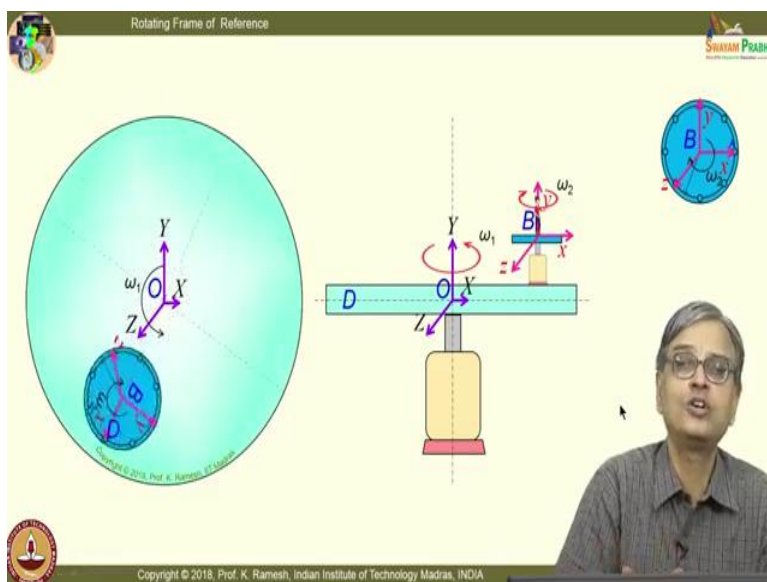
So, I had a problem like this, I had the fixed axis attached to this and I had the rotating axis attached to this. And I deliberately gave the problem that, the disk itself is rotating because the  $v_{rel}$  is easier



to compute from circular motion.

But the students what they had the confusion was, you have to recognize that this axis is rotating with angular velocity  $\omega_1$ . This axis fine and once I stand on point B; I would

perceive that this is rotating in a clockwise fashion. And look at I have A point marked; the point A is moving. So, this is the way that you have to understand. The relative; I mean not the relative velocity, the velocity from a rotating frame of reference. You can easily



calculate based on circular motion.

In this case the object is rotating in a circle, while we developed the theory; we took a rigid body with a slot, where the point A is moving within a slot. That is more

complicated motion. Here, the motion is far simpler; you do not need any mu equations, the equations you already know, fine. Let us also look at it from a different perspective.

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Now, what I am going to do is I am going to look at the side view of it. How do I make this appliance? I can think of this disk  $D$  is rotated by a motor; with some gearbox, so that I can adjust the speed fine.

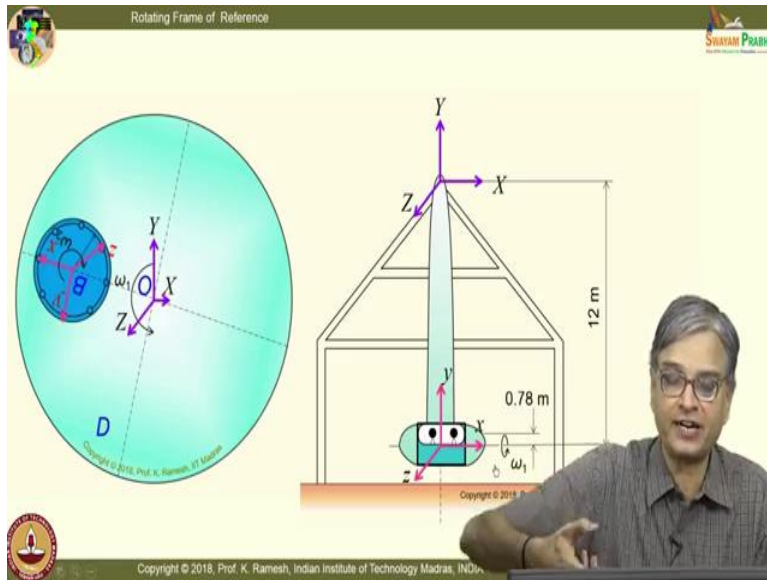
And if I want to have another rotating platform on top of it; I can imagine that I have another small motor, which rotates this. Is the idea clear? And I have the fixed axis here. And I have rotating axis here,  $x$   $y$  and  $z$  and this is rotating with an angular velocity  $\omega_2$ . And now, look at the picture; I have shown the picture as of this is stationary; only this is rotating.

See when you construct it, you have a motor, you have a speed reducer and then, you determine this speed. That you can easily measure. You can easily measure that. So, in my rotating frame of reference, I use quantities that are measurable fine. And then, what I am actually saying is when I have the point  $B$ , which is the origin of the frame of reference. You can imagine that there is a person sitting or standing at that point and view, what happens? When she views it, she will only see that this object is rotating because it is rotating with an angular velocity  $\omega_2$ . Now, it is very clear fine. There is no difficulty at all. What I have shown here is I have shown that this is static and this is, this alone is rotating.

Now, what you will have to visualize is; suppose, I also rotate this, but I still stand there. Even then, I would see the same rotation; I would not see anything else. See, you may say that I am too tall, so that I see, what happens in this disc, what happens the other disc. I can also shrink it because these are all visualization. You do not see anything outside this blue disc ok; you see only this disc. You will still perceive this disc is rotating. Is the idea clear?

You have to get this idea that is the reason why we use rotating frame of reference. See, here, I do not have the path of point  $A$ , from an absolute sense. From way the mechanism is constructed; I can determine this velocity. Even when this disc is rotating at  $\omega_1$ ; this

will stay how  $\omega_2$  with respect to that. So, I use quantities that are easily determinable and then, use it in my rotating frame of reference. That is why you use rotating, not for the

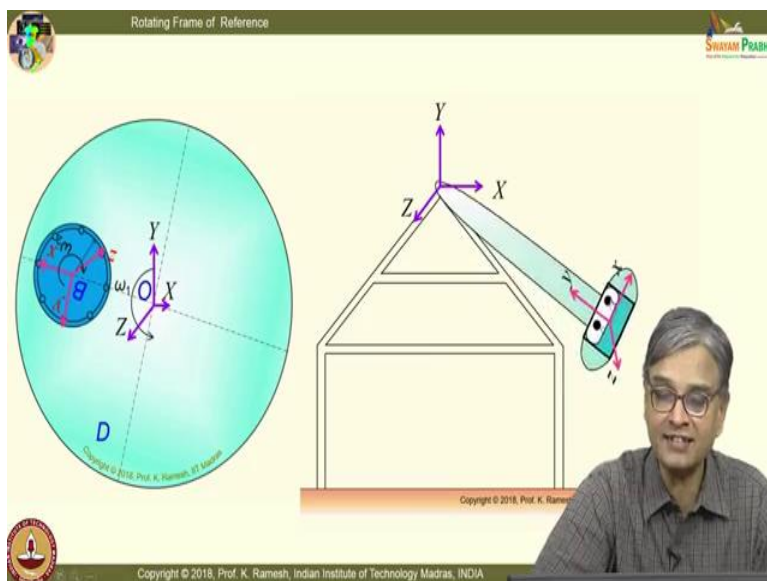


heck of it you are using rotating frame of reference. Certain quantities it is not possible for you to measure; otherwise.

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So, we had first two problems like this, one first problem is like this; where I have this and I use the

quantities that are easy for you to measure. And even in this problem only the rotation direction is different, it is rotating like this ok; this rotating like this.



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And here again this axis has a rotation of this angular velocity ok.

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We will solve other problems. Make a neat sketch of this and you know you have such gadgets available even in India now. If you want replace any one of these fittings, you have such motions available in the truck.

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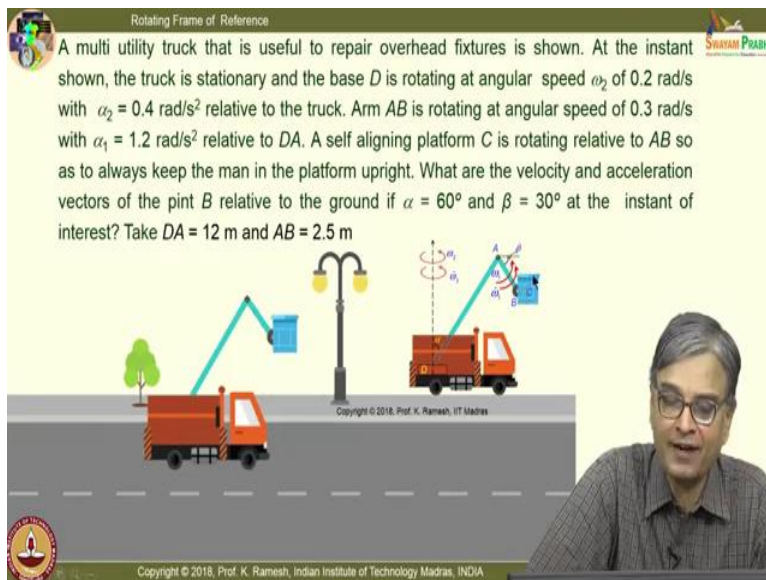


And the problem is like this. You have a multi utility truck; it is useful to repair overhead fixtures is shown. At the instant shown, the truck is stationary and the base  $D$  is rotating at an angular speed  $\omega_2$ . I have labeled the different part of this. Here, this is the base  $D$ . I



have a norm here, I have labeled this as A, I have labeled this as B and I have a platform attached at C. And the platform is such itself aligning, whenever the arm is having a motion like this it will self-align itself. So, that the person standing here will remain horizontal and he is in a position to do the job.

And what is shown here is the rotating axis have both velocity as well as acceleration both the cases. And this is rotating with angular velocity  $\omega_2$  and this is rotating with an



A multi utility truck that is useful to repair overhead fixtures is shown. At the instant shown, the truck is stationary and the base  $D$  is rotating at angular speed  $\omega_2$  of 0.2 rad/s with  $\alpha_2 = 0.4 \text{ rad/s}^2$  relative to the truck. Arm  $AB$  is rotating at angular speed of 0.3 rad/s with  $\alpha_1 = 1.2 \text{ rad/s}^2$  relative to  $DA$ . A self aligning platform  $C$  is rotating relative to  $AB$  so as to always keep the man in the platform upright. What are the velocity and acceleration vectors of the pint  $B$  relative to the ground if  $\alpha = 60^\circ$  and  $\beta = 30^\circ$  at the instant of interest? Take  $DA = 12 \text{ m}$  and  $AB = 2.5 \text{ m}$

angular acceleration  $\omega_2$  dot. And here this arm is pinned here and this is rotating with angular velocity  $\omega_1$ . This also have an angular acceleration  $\omega_1$  dot and here, I have deliberately modified the symbol. So, that you also get to know, how to write the expression? You are learnt in the class and how you convert it for the problem concern? Straightforward application of the equation that you have learned; there is nothing new here.

But you have to be systematic, you have to do the calculation systematically fine. So, the first issue here is, how do I determine the fixed axis? How do I determine rotating axis?

Rotating Frame of Reference

Platform C is rotating relative to AB so as to always keep the man upright.

Since 'C' is hinged at B to freely adjust to remain horizontal, all the points in the platform can be assumed to have same velocity and acceleration of point B.

Fix XYZ to truck      Fix xyz to point A

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And where do I fit them?

So, you are given this alpha is at  $60^\circ$  and  $\beta = 30^\circ$ .

You are asked to find out the velocity and acceleration vectors of point B. It is put as pint, its point B related to the ground and you are given this length of the member as DA equal 12 meters and

this length as 2.5 meters.

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So, you have to decide, where do I put the fixed frame of reference and rotating frame of reference? You know, I have the point D; this is rotating about this. So, it is easier for me to put the fixed frame of reference to the truck. So, I will put the axis like this; I have  $xy$  axis like this. And we have already seen from the problem statement that this arm is having an angular velocity about point A. So, it is better to have a rotating frame of reference attached to point A because I can easily calculate the value of the velocity of point B from the parameters given in the problem.

That is the way you look at it fine. So, it is easier for me to fix this  $xyz$  to point A. And with this choice you know, I have no confusion of capital J and capital I and small i and small j; they are parallel. So, I have no difficulty in handling the expression because I already said when I say  $\mathbf{V}_A = \mathbf{V}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{V}_{rel}$ , they are all referred to different coordinate system in a generic problem.

So, you have to use the vectors properly fine. And here no such confusion is there, but the only confusion is what we used it as point A as become point B? What we used it as point B has become point A? So, you have to know, how to write this expression? And



the motion is easy to visualize; there is no difficulty at all. See, ultimately in many

Rotating Frame of Reference

Given:

$$\alpha = 60^\circ \text{ and } \beta = 30^\circ \quad \omega_1 = 0.3 \text{ rad/s} \quad \dot{\omega}_1 = 1.2 \text{ rad/s}^2$$

$$\omega_2 = 0.2 \text{ rad/s} \quad \dot{\omega}_2 = 0.4 \text{ rad/s}^2$$

Position vector

$$\hat{r} = 2.5(\cos\beta\hat{i} - \sin\beta\hat{j})$$

$$\hat{r} = 2.5(\cos 30^\circ\hat{i} - \sin 30^\circ\hat{j})$$

$$\hat{r} = 2.16\hat{i} - 1.25\hat{j} \text{ m}$$

Continued.

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instances you again invoke only circular motion equations in different forms. Circular motion equations are very useful. Here, again because this is rotating like this. I will again use that kind of expression for me to find out, what is the velocity of  $B$ ?

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So, these are all the quantities given. You are given  $\alpha = 60^\circ$  and  $\beta = 30^\circ$   $\omega_1 = 0.3 \text{ rad/s}$   $\dot{\omega}_1 = 1.2 \text{ rad/s}^2$   $\omega_2 = 0.2 \text{ rad/s}$   $\dot{\omega}_2 = 0.4 \text{ rad/s}^2$  are given. So, let us determine the quantities; you have the position vector. So, this is nothing, but  $\hat{r} = 2.5(\cos\beta\hat{i} - \sin\beta\hat{j})$ . So, when we substitute the quantities and simplify, I get this expression.  $\hat{r} = 2.16\hat{i} - 1.25\hat{j} \text{ m}$ ; fairly, simple and straightforward.

See, one thumb rule is how do you select the rotating frame of reference? From the rotating frame of reference, I should be in a position to find out the velocity of the point of interest comfortably. That is the basis for you to select the rotating frame of reference and when I put this coordinate axis like this. I can also find out the coordinate acceleration and velocity based on the other part of the problem. So, you get identify the rotating frame of reference, where to fix it. That is the very important step in solving these problems.

You should select the fixed frame of reference and rotating frame of reference carefully. do not rush doing that. In friction problems I said you should determine, what is the direction of frictional force? Otherwise, you are solving a different problem all together. So, in these problems, if you select the coordinate axis properly; rest of it is just

application of the formula. While deriving the formula we have looked at the physics, but for applying it you do not anything of that is all. You should do the calculation systematically.

The motion of point B w.r.t the rotating frame of reference can be found easily.

Point B has a rotation w.r.t A

$$\hat{r} = 2.16\hat{i} - 1.25\hat{j} \text{ m}$$

$$\hat{v}_{rel} = \hat{\omega}_1 \times \hat{r}$$

$$\hat{v}_{rel} = (0.3\hat{k}) \times (2.16\hat{i} - 1.25\hat{j})$$

$$\hat{v}_{rel} = 0.65\hat{j} + 0.375\hat{i} \text{ m/s.}$$

Diagram details: A rotating frame with origin A. Point B is at a distance of 2.5 m from A. The frame rotates with angular velocity  $\omega_1 = 0.3 \text{ rad/s}$ . Point B rotates with angular velocity  $\omega_2 = 1.2 \text{ rad/s}^2$ . The angle between the x-axis and the line AB is  $\beta = 30^\circ$ . A small inset shows a truck on a road.

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The motion of point B with respect to the rotating frame of reference can be found easily. That is a basis, why you select the rotating frame of reference at this point. And from the problem statement it has rotation with respect to A.

So, I have this as  $\hat{v}_{rel} = \hat{\omega}_1 \times \hat{r}$ ; fairly, straight forward. And you are given

The acceleration of point B in the rotating frame of reference can be found easily.

$$\hat{a}_{rel} = \hat{\omega}_1 \times \hat{r} + \omega_1 \times (\omega_1 \times \hat{r})$$

tangential acc.      normal acc.

$$= 1.2\hat{k} \times (2.16\hat{i} - 1.25\hat{j}) + 0.3\hat{k} \times (0.65\hat{j} + 0.375\hat{i})$$

$$\hat{a}_{rel} = 1.305\hat{i} + 2.7\hat{j} \text{ m/s}^2$$

Diagram details: Same as the previous slide, showing the rotating frame and point B.

$$\hat{v}_{rel} = (0.3\hat{k}) \times (2.16\hat{i} - 1.25\hat{j})$$

So, when I do this, I get the velocity with respect to rotating frame of reference as  $\hat{v}_{rel} = 0.65\hat{j} + 0.375\hat{i} \text{ m/s}$ .

I have calculated  $V_{rel}$ , I will also calculate  $a_{rel}$  because in my calculations, I need  $V_{rel}$  and  $a_{rel}$  that is all I need; rest of it I need to

have the quantities that is specified in the problem.  $a_{rel}$  is also you can easily do it because there is a circular motion.

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And what I will have, because I have  $\omega_1$  dot as well as  $\omega_1$  I have two components for the acceleration. So, you have a tangential acceleration, as well as normal acceleration. The previous problems, we did not have  $\omega_1$  dot, we had only the angular velocity. Angular acceleration was 0 that is the kind of problems you have done earlier. So, it is fairly, straight forward.

You have to be careful in substituting the values properly and then do the computation. And here, what is done? I have directly put the velocity; then I can put this as  $\omega \times V$ , so, it simplifies, my calculations. So, I get the acceleration with respect to the rotating frame of reference as  $\hat{a}_{rel} = 1.305\hat{i} + 2.7\hat{j} \text{ m/s}^2$ . So, it is fairly straight forward. So, I have these two quantities. Now, I should apply my basic equations properly.

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So, there is no harm you write the expression what way you have learnt it in the class,

but rewrite this expression specific to the problem of interest because A and B positions are interchanged. So, I would now rewrite this as I have to get velocity of B with respect to capital XYZ. And replace this term as coordinate velocity. There is a better way of understanding it. So, I am

really looking at the coordinate velocity.

Then I have  $\omega \times r$  and this  $\omega$  is nothing but what is the angular velocity of the rotating frame. So, do not confuse this with  $\omega_1$ . The rotating frame of reference has only  $\omega_2$  as the it is rotating like this, it is rotating like this; that is where, students are not understood when I give problems, which has rotary motion.

You get confused, which rotary motion you are talking about. Now;  $\omega_2$  is this. So, I should write this then, rest of it is fairly, straightforward. There is no difficulty at all. You are much better than, doing this calculations than me. So, understand the concepts and get the quantities for this expression. So, coordinate velocity is nothing but I have this at a distance 12 meters. so,  $\hat{\omega}_2 \times \overline{DA}$ . It is turning out to be  $(6\hat{i} + 10.392\hat{j})$ .

Please verify these calculations. If there are any typographical errors; please bring it to my attention. And I have  $\hat{\omega}_2 \times \overline{DA} = 0.2\hat{j} \times (6\hat{i} + 10.392\hat{j}) = -1.2\hat{k}$  m/s and you know it is a good practice to write the quantities like this. And it is also easier for us to check your answers in a large class of students; if you write them in steps. At least we would be in a better position to give you partial marks.

So, have this habit of writing down these terms one after the other. So,  $\omega_2 \times r$  is like this and  $\hat{V}_{rel} = 0.65\hat{j} + 3.75\hat{i}$ . And so, I have all the quantities for me to calculate  $V_B$ . So, I get  $\hat{V}_B|_{XYZ} = 0.375\hat{i} + 0.65\hat{j} - 1.63\hat{k}$  m/s. See, what will have to keep in mind is I have chosen this rotating frame of reference because I can easily compute  $V_{rel}$ . That is one of the basis, that gives you a clue, how to select a rotating frame of reference and where would I fix

its origins?

Rotating Frame of Reference

Acceleration of B

$$\hat{a}_A = \hat{a}_B + \hat{\omega} \times \hat{r} + \hat{\omega} \times (\hat{\omega} \times \hat{r}) + 2\hat{\omega} \times \hat{V}_{rel} + \hat{a}_{rel}$$

$$\hat{a}_B|_{XYZ} = \text{Co-ordinate acceleration} + \hat{\omega}_2 \times \hat{r} + \hat{\omega}_2 \times (\hat{\omega}_2 \times \hat{r}) + 2\hat{\omega}_2 \times \hat{V}_{rel} + \hat{a}_{rel}$$

Co-ordinate acceleration:

$$\hat{\omega}_2 \times \overline{DA} + \hat{\omega}_2 \times (\hat{\omega}_2 \times \overline{DA})$$

$$= (0.4\hat{j}) \times (6\hat{i} + 10.392\hat{j}) + 0.2\hat{j} \times (-1.2\hat{k})$$

$$= -2.4\hat{k} - 0.24\hat{i} \text{ m/s}^2$$

$\omega_2 = 0.2 \text{ rad/s}$   
 $\dot{\omega}_2 = 0.4 \text{ rad/s}^2$

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How will I put the axis? All the decision you can do by computing  $V_{rel}$  and  $a_{rel}$ . am I in a position to compute  $V_{rel}$  and  $a_{rel}$  comfortably. So, we have calculated the velocity then, we move on to acceleration.

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Here, again I write the expression as learnt in the class. This is not the expression that is to be used directly. We have to interchange the symbols and also substitute, what is  $\omega$  dot,  $\omega$  and so on. We have already calculated  $V_{rel}$  as well as  $a_{rel}$ . And this  $\omega$  is nothing, but the rotating frame of reference, that is  $\omega_2$  and this will be  $\omega_2$  dot.

So, here again I can write this as the point of interest is

$$\hat{a}_B|_{XYZ} = \text{Co-ordinate acceleration} + \hat{\omega}_2 \times \hat{r} + \hat{\omega}_2 \times (\hat{\omega}_2 \times \hat{r}) + 2\hat{\omega}_2 \times \hat{v}_{rel} + \hat{a}_{rel}$$

Here again simply substitute the values you get the final expression. So, the coordinate acceleration is  $\hat{\omega}_2 \times \overline{DA} + \hat{\omega}_2 \times (\hat{\omega}_2 \times \overline{DA})$ . And that turns out to be simplified to  $-2.4\hat{k} - 0.24\hat{i} \text{ m/s}^2$ .

See when I have coordinate acceleration you have one more extra term. The computation become little longer, but the methodologies still remains the same. You can easily comprehend the whole thing, but it is involved and you have to be alert and do your computation systematically.

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Rotating Frame of Reference

$$\hat{\omega}_2 \times \hat{r} = (0.4\hat{j}) \times (2.16\hat{i} - 1.25\hat{j})$$

$$\hat{\omega}_2 \times (\hat{\omega}_2 \times \hat{r}) = (0.2\hat{j}) \times [(0.2\hat{j}) \times (2.16\hat{i} - 1.25\hat{j})]$$

$$2\hat{\omega}_2 \times \hat{v}_{rel} = 2(0.2\hat{j}) \times (0.65\hat{j} + 0.375\hat{i})$$

$$\hat{a}_B|_{XYZ} = 0.98\hat{i} + 2.7\hat{j} - 3.41\hat{k} \text{ m/s}^2$$

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And here, I follow this practice individual terms I right; so, that you can also verify them, when you want to check your answers. And I have not simplified them. I have finally, obtained the acceleration as

$$\hat{a}_B|_{XYZ} = 0.98\hat{i} + 2.7\hat{j} - 3.41\hat{k} \text{ m/s}^2$$

Again, a straight forward problem; the idea is you are not using the same symbols in this problem; that is one difference. And the coordinates had in addition to angular velocity they also had acceleration. The idea of solving a variety of problem is if you solve a



variety of problem more number of problems you also get to know, how to attack a new problem; which you have not earlier, fine.



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We will also solve another interesting problem, which we have already seen in connection with coincident points. You have a shaper mechanism. You can very clearly see the forward motion is slow and the backward motion is fast.

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So, while cutting the specimen; while cutting the specimen it goes slowly; when it comes



out, it comes out fast and this is the mechanism that we are going to analyze. This is the mechanism that you have and I have also said lubrication is very very important. And say, actually in a bath of lubricating oil.

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So, we have already written down this problem earlier. Here, we are interested in finding out the and finding out the angular acceleration of link  $OC$ . For the angular velocity is 5 radians, determine the velocity of point  $C$  for instant, when  $\theta=30^\circ$ ; if the angular acceleration of the crank is 10 rad/s determine the angular acceleration of link  $OC$ .

So, you have asked to find out both velocity and acceleration. And you know one of the aspects is when problems are given in a book, they leave many things for your imagination. See, the position  $\theta$  could be during the motion of the point, where the block is moving from bottom to top or it is coming from top to bottom. It could be either

way, but when we have an animation, we have an understanding what is that we are looking at a.

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So, I have this and this shows very well that you have a quick return. So, we are going to analyze a situation, where the block is sliding from bottom to

The following quick-return mechanism which produces a slow cutting stroke of the tool (attached to  $D$ ) and a rapid return stroke is used in a Shaper. If the driving crank  $AB$ 's angular velocity is  $5 \text{ rad/s}$ , determine the velocity of point  $C$  for the instant when  $\theta = 30^\circ$ . If the angular acceleration of the crank is  $10 \text{ rad/s}^2$  determine the angular acceleration of link  $OC$ .

Diagram details: Crank  $AB$  length is 110 mm. Link  $BC$  length is 550 mm. The vertical distance from the pivot  $O$  to the pivot  $A$  is 330 mm. The angle  $\theta$  is measured from the vertical line through  $O$  to the link  $BC$ . Point  $D$  is a block on a horizontal surface.

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top. And it has reached the position  $\theta$  because the problem can be interpreted in either

way. Either it should be straighter at the problem statement that the motion is like this. And then, you are asked to find out. Because the final answer should be different; if I am analyzing the motion of the block coming down or going up. Is the idea clear? See, these are all the

Diagram details: Same as the previous diagram, showing the mechanism with dimensions 110 mm, 550 mm, and 330 mm, and angle  $\theta$ .

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ambiguities that you have in a textbook problem.

When you do the animation at least I am in a position to say, that I have come down I have gone up. I am trying to analyze the problem, when it has moved up. That is a way I am going to look at it. And whenever I have a slotted member, the thumb rule is you

attach the rotating frame of reference to the slotted member. Because in a slotted member, the path of the particle is known; it may be easier to calculate, it may be difficult to calculate at least the path is known. In this case you know, this is simply sliding in this.

Rotating Frame of Reference

Attach the rotating coordinate system  $x-y$  to slotted link and fixed co-ordinate system  $X-Y$  as shown.

$$r_{OB} = \sqrt{330^2 + 110^2 - 2 \times 330 \times 110 \times \cos 120^\circ}$$

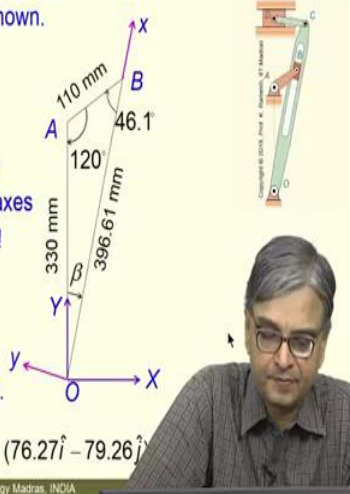
$$= 396.61 \text{ mm}$$

From sine rule in  $\triangle OAB$  The origins are coincident but axes are at an angle!

$$\frac{110}{\sin \beta} = \frac{396.61}{\sin 120^\circ}$$

$$\beta = 13.9^\circ$$

It would be convenient to use the rotating frame of reference for expressing all vectors.



$$\hat{r}_{BA} = (110 \cos 46.1^\circ \hat{i} - 110 \sin 46.1^\circ \hat{j}) \text{ mm} = (76.27 \hat{i} - 79.26 \hat{j}) \text{ mm}$$

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So, I am going to have a linear velocity here ok. And this member is having an angular motion like this. So, that is the way you were rotating frame of reference ok. It is oscillating actually. In this case it is but anyway we are going to analyze it only for a particular position of

the crank position.

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So, in all these problems you have to decide, what is the fixed axis and the rotating axis? So, fix the axis at point O and then, have the axis aligned to the member. You have small  $x$  and small  $y$ , along the slotted member and perpendicular to that. Then, you have to calculate some of these quantities using geometry. So, I can find out  $r_{OB}$  as 396.6 mm. And here what you have is? I have the axis like this, but the rotating axis they are not aligned. So, you have to be careful in handling the capital I and capital J and small  $i$  and small  $j$ . So, instead what I will do is I will find out everything with respect to  $xy$  small  $xy$ .

So, we need to get certain other quantities. So, I get the angle  $\beta = 13.9^\circ$ . So, you have to bring in your sin rule, property of triangles for solving a problem you need that as well. It would be convenient to use rotating frame reference for expressing all vectors. So, I should have some discipline because when you have that expression. This is referred with respect to different axis. If they are collinear or parallel or coincident; then, there is

no difficulty. If they are inclined then, you have to be very careful in handling your vectors.

So, I will have  $\hat{r}_{BA} = (110\cos 46.1\hat{i} - 110\sin 46.1\hat{j}) \text{ mm} = (76.27\hat{i} - 79.26\hat{j}) \text{ mm}$ . So, that is reduced to this quantity.

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Rotating Frame of Reference

$$\hat{v}_B = \hat{v}_O + \hat{\omega}_{OC} \times \hat{r}_{OB} + \hat{v}_{rel} \quad (1)$$

$$\hat{v}_B = \hat{\omega}_{AB} \times \hat{r}_{BA} = (396.3\hat{i} + 381.35\hat{j}) \text{ mm/s}$$

$$\hat{v}_{rel} = \dot{x}\hat{i} \quad \hat{\omega}_{OC} \times \hat{r}_{OB} = 396.61\omega_{OC}\hat{j} \quad \hat{\omega}_{OC} \times \hat{r}_{OB}$$

Substituting in Eq. (1),

$$396.3\hat{i} + 381.35\hat{j} = 396.61\omega_{OC}\hat{j} + \dot{x}\hat{i}$$

Equating the terms in x and y direction,

$$\dot{x} = 396.3 \text{ mm/s}$$

$$\omega_{OC} = 0.9615 \text{ rad/s}$$

$$V_C = \omega_{OC} \times r_{OC} = 528.83 \text{ mm/s}$$

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And you know I have the basic expression. In fact, I have written down for this specific problem. So, we are interested in finding out the velocity of B that is

$$\hat{v}_B = \hat{v}_O + \hat{\omega}_{OC} \times \hat{r}_{OB} + \hat{v}_{rel}$$

And because point O is fixed, the velocity of O is 0 ok.

So, if you graduate then, you can directly write the expression appropriate to the problem. Whichever way you do it I have no difficulty ok. You can write, what is expression thought in the class and then, convert that to the first specific problem of interest. And write each of these quantities  $V_B$  is nothing, but  $\hat{v}_B = \hat{\omega}_{AB} \times \hat{r}_{BA} = (396.3\hat{i} + 381.35\hat{j}) \text{ mm/s}$ . You should recognize that this is part of the crank. So, I get the absolute velocity directly.

So, here the left-hand side is known. I do not know, what is the angular velocity? That is what I am supposed to calculate from this. And then, your  $V_{rel}$  is nothing, but  $x \dot{i}$ . So, this is the convenience. If I attach the rotating frame of reference; then the  $V_{rel}$  is very simple to write. It is nothing, but  $x \dot{i}$ . And substituting in equation 1; so, I put this equating the terms in x and y direction. I get  $\dot{x} = 396.3 \text{ mm/s}$ ;  $\omega_{OC} = 0.9615 \text{ rad/s}$ . And we are asked to find out, what is the velocity of point C? So, from circular motion I can write  $V_C = \omega_{OC} \times r_{OC} = 528.83 \text{ mm/s}$ ; fairly, straight forward.



You have to be careful in selecting the fixed frame of reference and rotating frame of reference. Then, apply the equations systematically, handle the vectors properly.

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So, now I have to find out the acceleration that is also written for this specific problem. And here, again you will have

$$\hat{a}_B = \hat{a}_O^0 + \hat{\omega}_{OC} \times \hat{r}_{OB} + \hat{\omega}_{OC} \times (\hat{\omega}_{OC} \times \hat{r}_{OB}) + 2\hat{\omega}_{OC} \times \hat{v}_{rel} + \hat{a}_{rel}$$

Rotating Frame of Reference

$$\hat{a}_B = \hat{a}_O^0 + \hat{\omega}_{OC} \times \hat{r}_{OB} + \hat{\omega}_{OC} \times (\hat{\omega}_{OC} \times \hat{r}_{OB}) + 2\hat{\omega}_{OC} \times \hat{v}_{rel} + \hat{a}_{rel} \quad (2)$$

$$\hat{a}_B = \hat{\omega}_{AB} \times \hat{r}_{BA} + \hat{\omega}_{AB} \times (\hat{\omega}_{AB} \times \hat{r}_{BA})$$

$$= (-1114.15\hat{i} + 2744.2\hat{j}) \text{ mm/s}^2$$

$$\hat{\omega}_{OC} \times \hat{r}_{OB} = 396.61\hat{\omega}_{OC}\hat{j}$$

$$\hat{\omega}_{OC} \times (\hat{\omega}_{OC} \times \hat{r}_{OB}) = -365.52\hat{i}$$

$$2\hat{\omega}_{OC} \times \hat{v}_{rel} = 760.9\hat{j} \text{ mm/s}^2$$

$$\hat{a}_{rel} = \ddot{x}\hat{i}$$

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And you have the expression for this and we have been able to calculate this.

So, I have this Coriolis component, can you just find out what is the direction of this? Direction of Coriolis components is this. Please make a sketch and note this and you find

this perpendicular to the slot. We will discuss about it little by later. You will always find this is perpendicular to the slot, may have a slotted member. The sense is determined by the angular velocity and one of the important aspects in Coriolis acceleration, what is the direction of Coriolis acceleration?

This is one of the important parameters that you need to find out. And then, I have

$\hat{a}_{rel} = \ddot{x}\hat{i}$ . So, I can group the  $i$  terms and  $j$  terms and you can determine the quantities that are required to be found.

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So, substituting in equation 2; I get this. Equating the  $i$  and  $j$  components, I get  $\ddot{x} = -748.63 \text{ mm/s}^2$ . And  $\dot{\omega}_{OC} = 5 \text{ rad/s}^2$ . And we have already seen that  $\omega_{OC} = 0.9615 \text{ rad/s}$  and  $V_C = \omega_{OC} \times r_{OC} = 528.83 \text{ mm/s}$ .

So, it is again straightforward problem. You have a satisfaction that whatever that is the

Rotating Frame of Reference

Substituting in Eq. (2),

$$-1114.15\hat{i} + 2744.2\hat{j} = 396.61\dot{\omega}_{OC}\hat{j} - 365.52\hat{i} + 760.9\hat{j} + \ddot{x}\hat{i}$$

Equating the  $i$  and  $j$  components,

$$\ddot{x} = -748.63 \text{ mm/s}^2$$

$$\dot{\omega}_{OC} = 5 \text{ rad/s}^2$$

$$\omega_{OC} = 0.9615 \text{ rad/s}$$

$$V_C = \omega_{OC} \times r_{OC} = 528.83 \text{ mm/s}$$

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mechanism, that is used in a shaper which is an important machining operation in the machine shop. And you know how to handle it from your knowledge of rotating frame of reference. In fact, when we discuss the coincident point; I took this for you to appreciate, what is the velocity? Fine.

And in all these problems what you find; this problem is slightly different. I do not know the  $a_{rel}$  and  $V_{rel}$ . I determine  $a_{rel}$  and  $V_{rel}$  by solving the problem, but I know very well that, a slotted member; this member can have only a linear motion like this. I use that as a basis to select the rotating frame of reference. See, the earlier problems you are able to calculate  $a_{rel}$  and  $V_{rel}$  comfortably. Here, I do not know, how to compute them initially? But I know how these quantities are fine.

So, the problem could be coined on anyone, the left-hand side is known or right-hand side is known. So, you can equate the two and then find out the rest. So, in this class we have reviewed how we determine the velocity and acceleration when I have a rotating frame of reference? A person sitting on a rotating frame, he will be able to find out  $V_{rel}$  and  $a_{rel}$  comfortably, but he misses out  $\omega \times r$ .

In a generic fashion,  $\omega \times V$  and that is used, when you want to differentiate the velocity expression to get the acceleration systematically. And once you finally, get this

expression can be written down as  $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$  and  $\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$ . These relative quantities can have more number of terms depending on, whether you are finding out the velocity of the acceleration. And we also reviewed the two problems we have done earlier, so, that you get a clarity what is happening to the rotating frame of reference; what is that you perceive. And in all the problems I said that the challenge is, how to identify the rotating frame of reference and how to identify the fixed frame reference?

So, for determining this; please, pay attention, take your time to fix them. Then rest of the problem is straight forward, mere application of the formula with careful attention on the mathematics. Because this is written down in different frame of reference; so, the capital I capital J small i small j, you have to handle it properly. We have not looked at any problems, where it is too complicated. They are all very simple as of now. You can also have problems were these are little involved. There you do not make a mistake. So, handle the vector equations properly.

Thank you.