Engineering Mechanics
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module - 01
Statics
Lecture - 03
Force Systems - I

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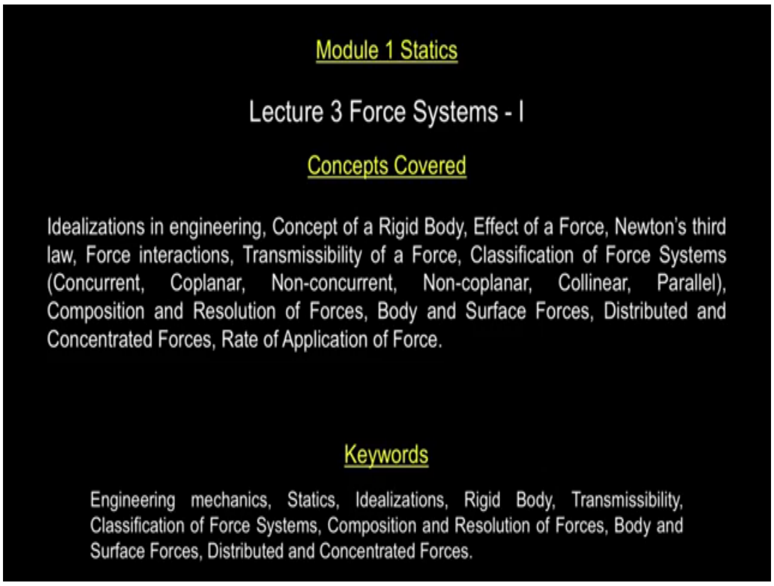

So, let us get on to the main body of the course and let me start with the first chapter on Force Systems.

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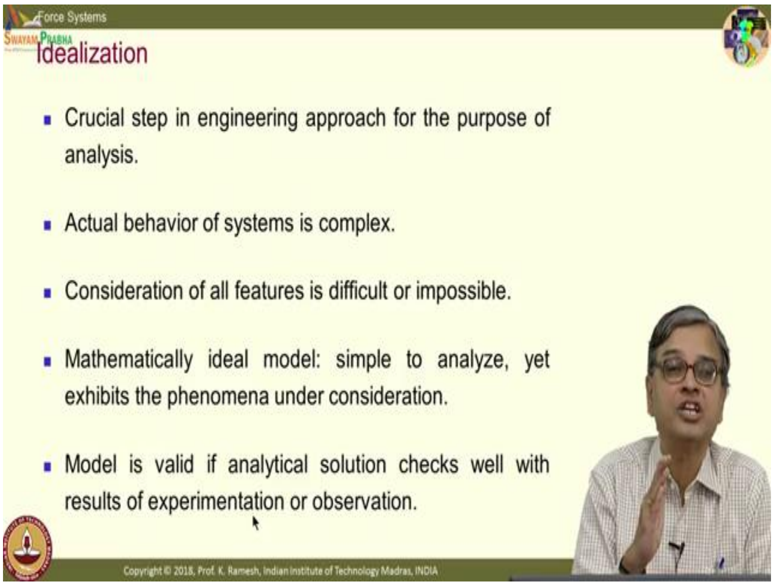
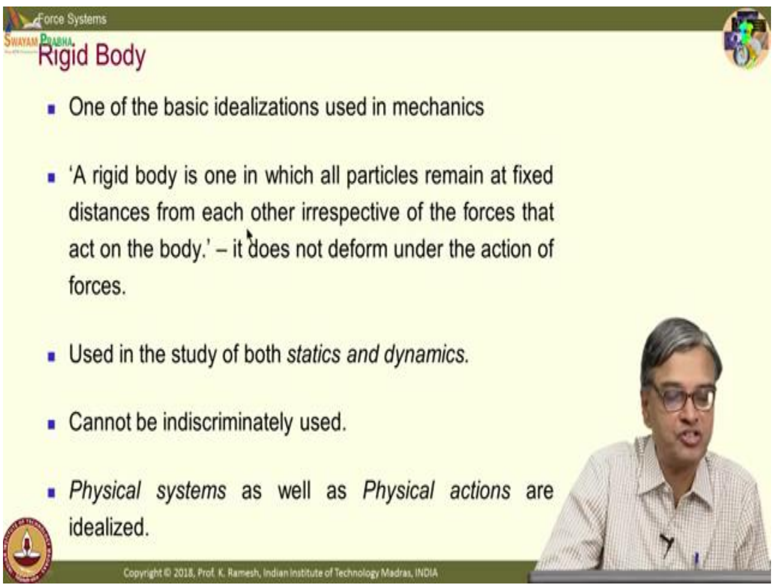


Idealization is a crucial step in engineering approach for the purpose of analysis. Why do we do that? This is because, actual behaviour of systems are generally complex. Consideration of all features is difficult or impossible in many cases. So, this is very important. You have to look at what is the phenomena that you want to analyze. Depending on the context, you have to make an appropriate approximation.

So, when I have a physical system, I extract the essence of it by a mathematical model. And what is a mathematical model? We choose a mathematical model, such that it is simple to analyze, yet exhibits the phenomena under consideration. So, the challenge lies in how to arrive at a mathematical model for a given physical system and how are you sure that whatever the mathematical model I have taken is good enough for the given application. The only way is, you verify it based on experimentation. So, experimentation is the ultimate key to verify a given mathematical model.

See, you have to look at all physical systems are generally nonlinear. We try to model it as linear for the purpose of analysis. Otherwise, the problem becomes too complex to analyze. See, if you can look at cosmology, they assume that mass is uniformly distributed. Whereas, you see there is a large distance between earth and the moon and earth and the sun, there is nothing is available, yet you make an idealization mass is uniformly distributed. This may be good to start with, when you want to analyze certain aspects of cosmology.

On the other hand, if you look at an atom, which is at a very small scale you have a nucleus and an electron cloud that are again separated. There the approximations would be different. So, depending on a given physical situation and length scales, certain approximations are acceptable for a given application. You may ask a question why not I encompass all the physical features. Though, it is desirable from a first look, it may be extremely difficult to implement.



So, the way to solve a given problem is idealize it appropriately. So that you can solve the problem based on mathematics, yet retain the essence of it is physical behaviour and that is how we have to attempt solving any given physical problem.

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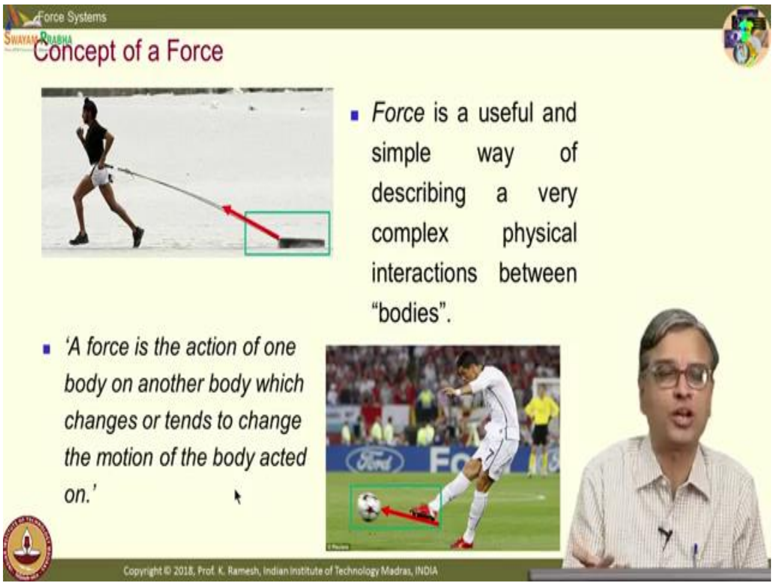
And one of the basic idealizations used in mechanics is, consider the body as rigid and we have already seen in the previous class, what is the actual definition of a rigid body. And the definition says a rigid body is one in which all particles remain at fixed distances from each other irrespective of the forces that act on the body. In essence, it does not deform under the action of forces. Is it true? Not strictly true in many instances.

But it is a very powerful idealization to get started and fortunately in this course, you will only come across rigid bodies. This idealization is used both in the study of statics as well as dynamics and you cannot use this idealization indiscriminately; you have to be very careful fine. And what you will have to notice is the idealized physical systems, as well as physical actions. When you say a body is rigid, you are idealizing the physical system and we will also represent a force by a simple vector indicating it is a concentrated force. Concentrated force is again an abstraction. It goes with rigid body idealization; you cannot think of a concentrated force without assuming the body as rigid.

Because in reality all bodies deform. You will only have a distributed force whenever there is a force interaction, if you are bound to accommodate that in your analysis, you will take years to solve a problem or for some instances you may not even have a first level solution.



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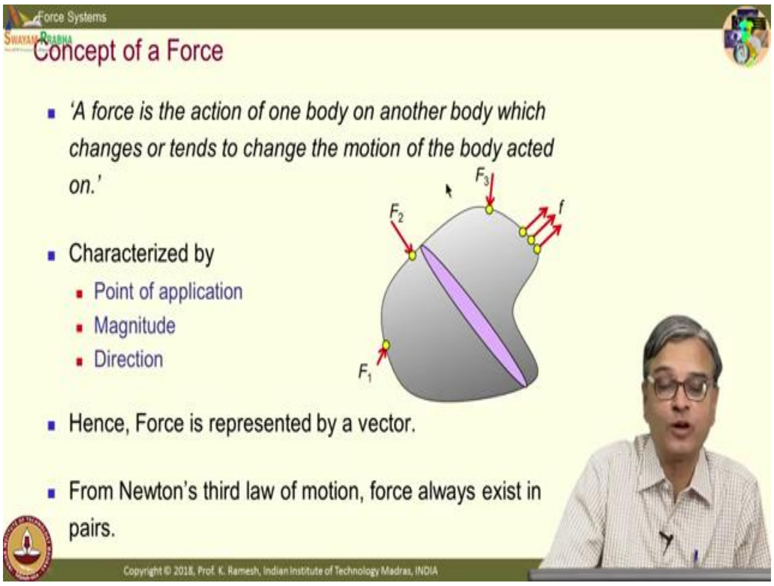
And what is the concept of a force? You find a person is pulling something in the field and another one is footballer is hitting the ball. So, you have some kind of an interaction takes between the footballer foot and the ball. Now, you would like to do the analysis.

How do I get started with the analysis? It is a very simple way of describing complex physical interactions between bodies. What happens physically, you model it as a force. And what is the effect of this force? The force changes or tends to change the motion of the body acted on. In order to understand it very clearly, I have taken the example of a footballer because, you could see what happens to the ball and you should be trained in handling the football, so that you hit a goal ok.



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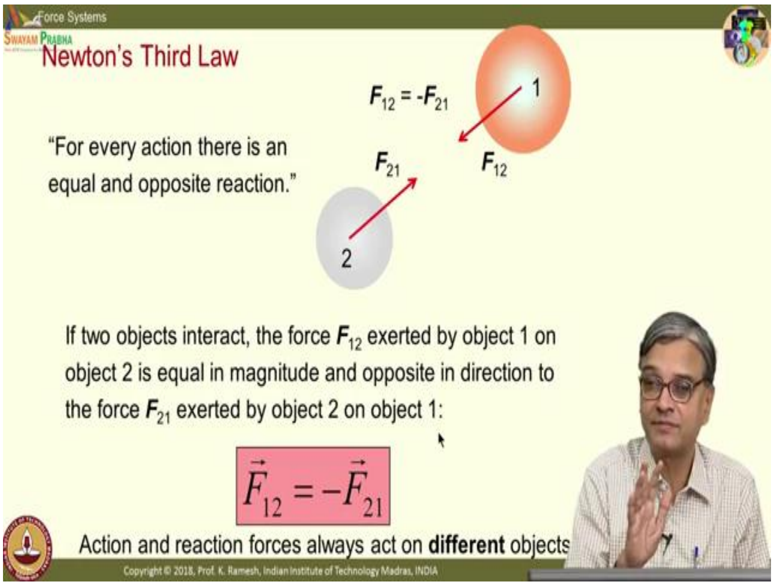
And what is it characterized by? Force is characterized by point of application. I have a generic body here, I could have multiple points where there could be force and when I say a force it is characterized by it is magnitude, as well as a direction. So, when you have a quantity which has a magnitude and direction, how will you label that? You can label that as a vector provided it also satisfies the commutative law. If it does not satisfy the commutative law, it cannot become a vector. We will see later finite rotation does not come under the purview of a vectorial quantity whereas, infinitesimal rotation can be shown as a vector quantity.

So, I could represent the force by a line like this, showing the magnitude as well as the direction. So, it could be concentrated or distributed forces like this. I have already told you in reality; whenever there is the interaction of one body with the other it will always be distributed. But, in many instances to make our life simple, we will indicate this by the resultant and show that as a concentrated force. Concentrated force is also an abstraction goes hand in hand with rigid body idealization. From Newton's third law of motion, force always exists in pairs; I cannot have just one force indicated like this.



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For which I have to look at what is the Newton's third law. For every action there is an equal and opposite reaction. I have just shown two bodies 1 and 2. So, the body 1 will exert the force on 2, body 2 will exert the force on 1 and I could show them as vectors like this, F_{12} and F_{21}. So, when I say force, it indicates this pair and they are equal and opposite. Very obvious and you should also remember and understand it very clearly; action and reaction forces always act on different objects. They do not act on the same object.

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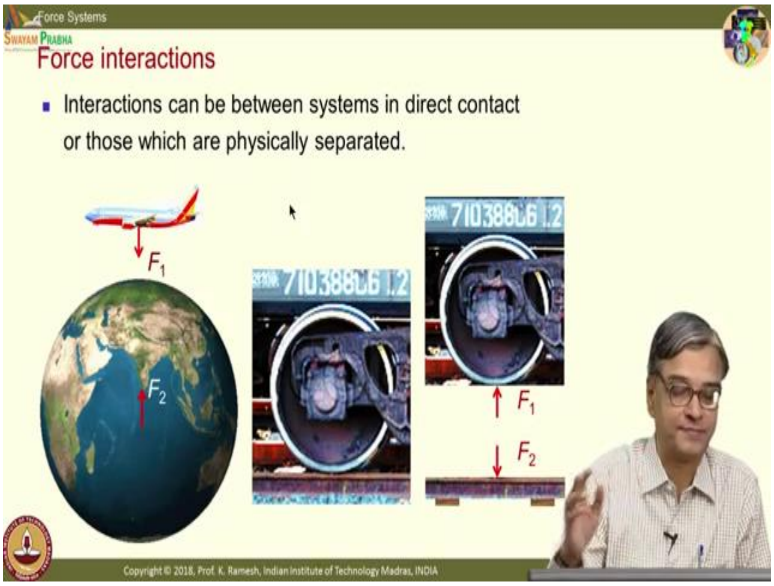
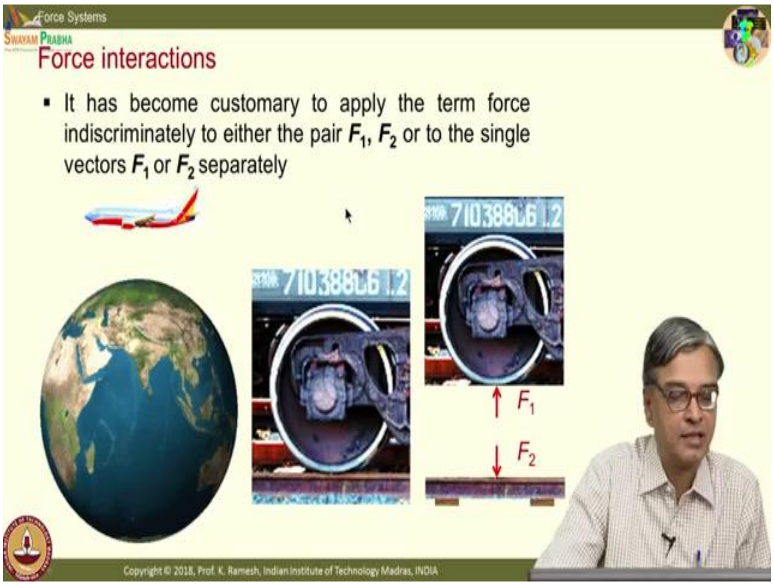
And what kind of force interactions you can come across. Interactions can be between systems in direct contact or those which are physically separated. So, I have a rail wheel



on rails and there is a direct physical contact between the wheel and the rail and how do I represent this interaction? I separate these two bodies and I have force F_1 acting on the wheel and F_2 acting on the rails. This is by direct contact. I would like you to make a sketch and then put it. These concepts will look very simple and too obvious but, your notes should be complete fine because, we have to build a strong foundation.



I can also think of an airplane flying. It does have an interaction between the earth. There is no direct physical contact and that is why gravitation was so difficult to comprehend. So, whenever you are going to draw a free body diagram, you will have to look at force that happen in direct contact and force interaction that act without a direct contact. We would see them as surface forces and body forces.

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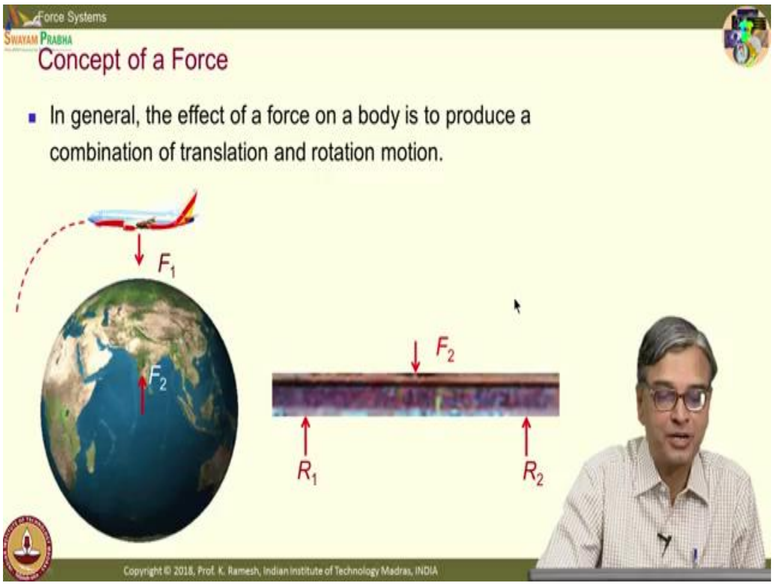
It has become customary to apply the term force indiscriminately to either the pair F_1, F_2 or to the single vectors F_1 or F_2 separately. So, both the usages are seen and we simply represent F_1 as a force and be done with it and what is the effect of these forces. See, I have an ordinary screen and I draw a line as a vector. I have not shown what would happen to the body. Suppose, I use the facilities of animation and try to show you, if I put a force on a body what would happen to it, we would also see that in the next slide ok.



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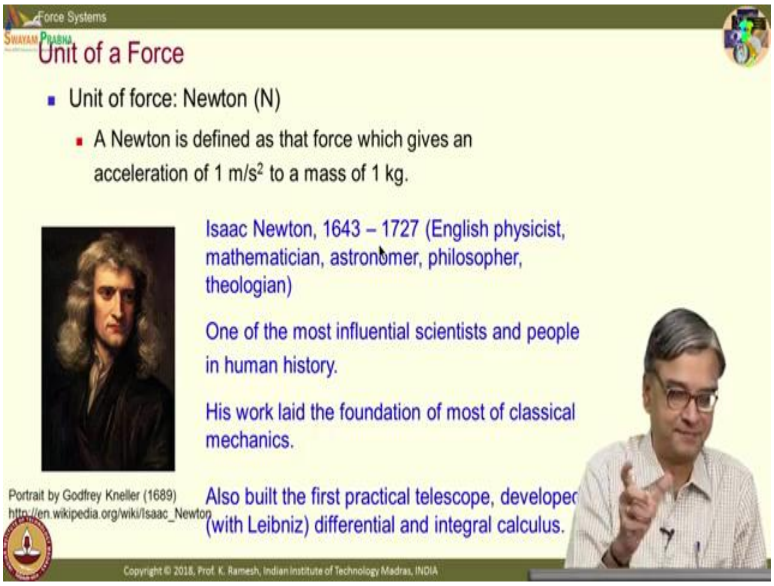
I have a force F_2 acting on this, what would happen to this? It could translate. Fine, you are not having a smart screen, which understand mechanics and when you make a drawing, you put a force and it shows what is the motion. Some of you should develop that for the Google because; they are making all new developments with the artificial intelligence. You can also make a smart screen which understands mechanics. Unless this force is resisted by the supports, which develop reactions R_1 and R_2, the rails will not remain in that place.

So, in general the effect of a force on a body is to produce a combination of translation and rotation motion. Here, I have cleverly taken an example, where this force F_2 passes through the central of gravity of the object. So, when I apply the force, it has only linear translation and you could feel this force if somebody pushes you, depending on where he puts his hand on your chest, if it is at the centre you will translate. If it is on one of the edges, you would translate as well as tend to rotate. But you might resist and then stand that is the different issue but if you allow yourself to be acted by the force, you should feel any force that acts on a body, in general produces a translation and a rotation. In exceptional cases, you may have only rotation or you may have only translation but, generic effect is combination of translation and rotation. And in the case of an air craft what happens. Unless the air exerts the force, the aircraft would fly off from the earth. Ok. So, it follows a particular trajectory.



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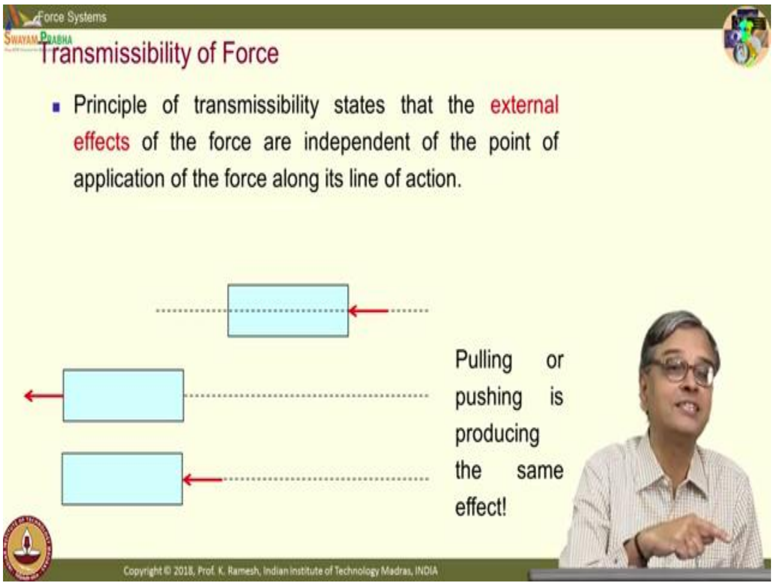
And let us also understand what is the unit of a force? Unit of a force is given the name in honor of Newton you call it as a Newton. A Newton is defined as the force which gives an acceleration of 1 meter per second square to a mass of 1 kilogram and is a very nice portrait of Newton, drawn in 1689. We have already learnt something about

Newton, we have some more data. He is an English physicist, mathematician, astronomer, philosopher and theologian. I also find these dates are different.

See if you go to history, scientific historians you find some dispute on some of these dates. The earlier slide I had this as 1642, it shows as 1643 and undoubtedly, he was one of the most influential scientists and people in human history. His work laid the foundation of most of classical mechanics. Also built the first practical telescope. Observations are very important. That is what I said. You have a mathematical model.

How do you verify the mathematical model? Only based on experimentation and in those days, their first choice was to look at stars and find out predict their orbits and then verify their understanding of mechanics. And he is also credited in developing differential and integral calculus along with Leibniz.



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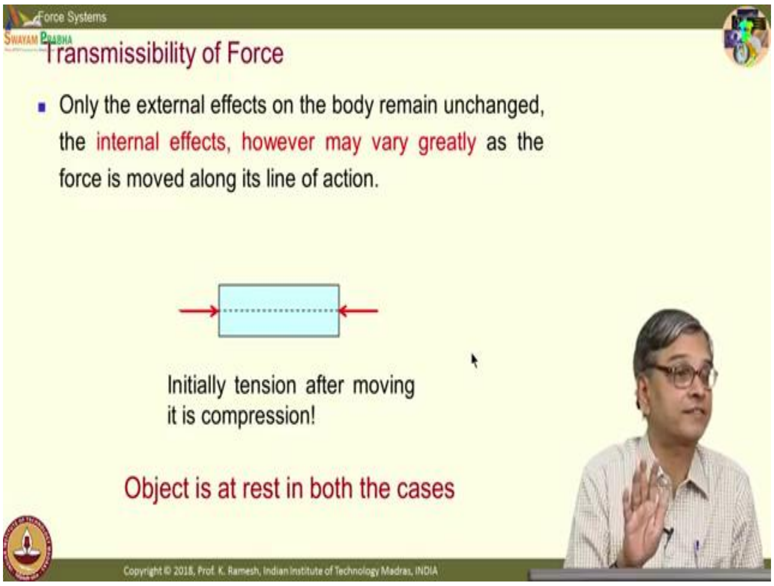
And another subtle concept of force is transmissibility of force, which we will be repeatedly using in this course. There is a justification; Principle of transmissibility states, that the external effects of the force are independent of the point of application of the force along its line of action.

What you have to pay attention is external effects. We are not talking about internal effects. Internal effects would be different but, external effects remain unaltered if I move the force along the line of action and I have a nice animation to show this, I have force acting on this. Now, let me have a body with this force now, I make the board as smart, it understands mechanics. Because of the force, the body will not remain at rest, it will move and I have taken a snap shot up to this point ok.

Now, what I do is, I move this force to the rear of the body which I can do easily and I look at what will happen to this body, when I apply the force what will happen to this body? This body also will move forward. So, what do you find here? I can move the body by pulling or I can also move the body by pushing. It does not make a difference. It is a very useful principle that we will repeatedly use, when we have a force system to be simplified. And we have already idealized the body to be rigid. This concept also goes with rigid body idealization.



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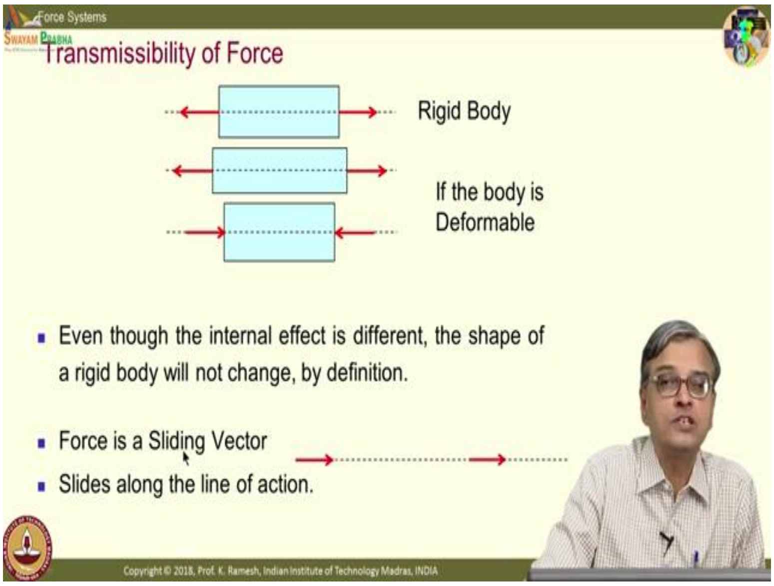
Now, let us look at, what kind of an internal effect possible. Now, I do not have one force but, I have another force to balance it and this is like you know pulling the body so the body is under tension. If I move them along the line of action, what happens; the body is under compression. And in both instances what happens? Object is at rest. That is the external effect. We have taken the body to be rigid, so the distance between the particles does not vary. In one case I apply tension, in another case I apply compression, and the external effect is same.



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And suppose I make the body deformable and visualize what could happen to the body, a rigid body will remain like this. If the body is deformable you will find when I apply tension it will elongate like this, when I apply compression it would get compressed from its original dimensions. Even though the internal effect is different, the shape of a rigid body will not change by definition. Look at the word definition. In physical reality it does happen, it does change shape may be minutely; if you make a very high magnification you will see.

If my interest is to find out the external forces this approximation is a convenient way to carry on with the analysis and we have also seen that force is a sliding vector, that should be used with care, that is what is emphasized and it slides along the line of action, that is what you see here. Force is a vector but, we also qualify that force is a sliding vector and this has to be used judiciously, when you are looking at the external effects of the force.

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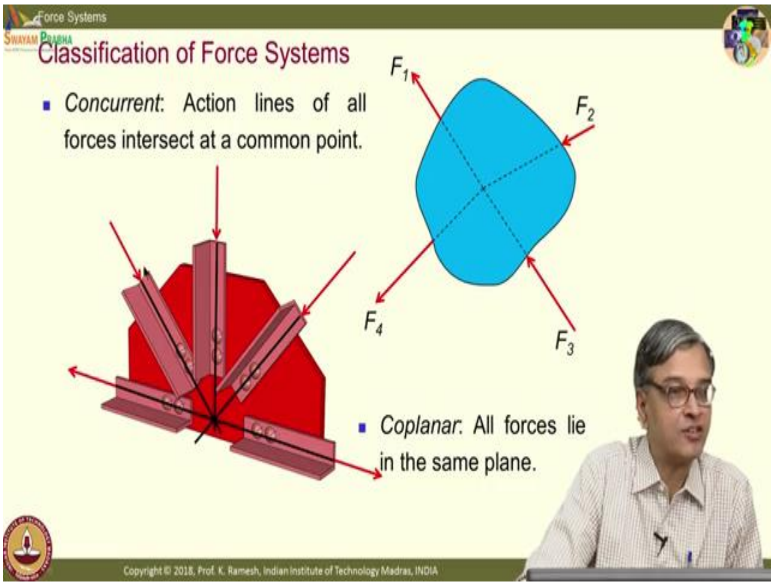
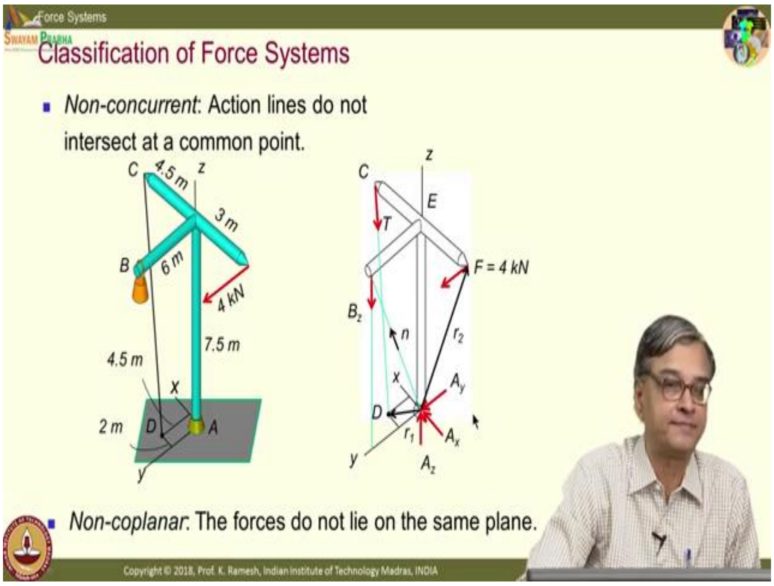


So, what you learn is force is a sliding vector and what are the classifications of force systems? If you understand different force systems, then you can easily find out the resultant based on it is properties and one of the simplest force systems you can think of is a concurrent force system. Action lines of all these forces meet at a point and I also use this is to introduce certain engineering concepts. Soon we are going to learn what are known as trusses and this is how you will have a joint in a truss. Make a neat sketch of it.

And this is a classic example, where you have a concurrent force system. What you have this as a plate is known as a gusset plate, on which the members forming the truss are connected. They have to be connected in a manner that it forms a concurrent force system, only then you can analyze the structure as a truss that we will see later. When I have these members, these members may be subjected to compression or tension and if you find out what is the line of action of these forces they would meet at a point.

Or in other words, when you are constructing a truss and connecting the members forming that truss to a gusset plate, you must ensure that the line of action of these forces pass through a point, it should not be offset. If it is offset, it is going to disturb the idealization, that you would model it for a truss. I would in addition also have a moment, which we do not want to have.

And is very clear from this diagram, you can also have a system which is concurrent as well as coplanar. All forces lie in the same plane and which you could see very easily, in the case of a gusset plate and make an neat sketch on this and you are in the first semester, you know if you are exposed to engineering drawing, one of the first training you are given is how to sketch? You may not become an artist but, you should know how to sketch physical systems from an engineering perspective and this is an isometric view of the system for you to get the details. Try your hand on it to the extent possible.



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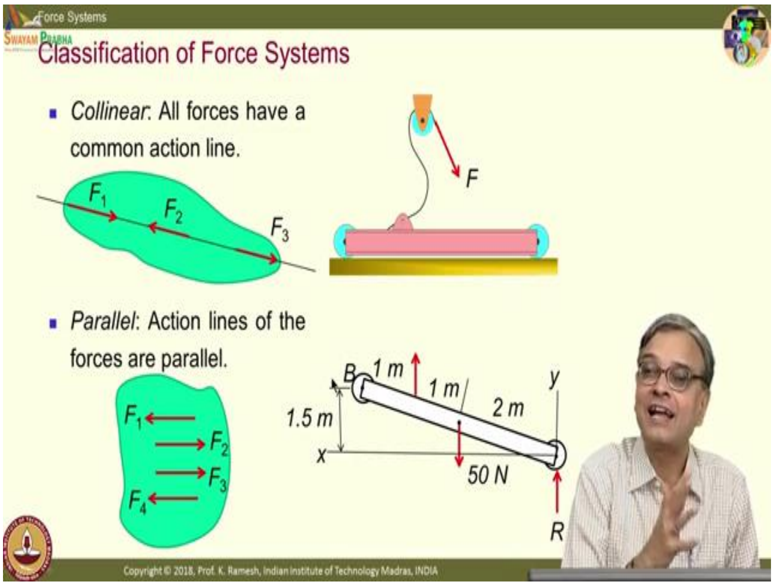
Then I can also have a system which is in general non-concurrent. Action lines do not intersect at a common point. I have a tall mast and you find a line is connecting this and this is called a guy-wire. See, if you look at any transmitting tower; For example, radio transmitting tower if you find, it will be a thin column and they will anchor it in its place by steel wires, they are called guy-wires to take care of the wind loads and the tension would be adjusted by a turn buckle at one of the ends.



And this is a system, which has forces acting on the guy-wire; there is weight acting on this. They do not meet at a point; they are very general and I have also replaced what is the kind of interaction by a system of forces. Soon we will learn, how to idealize supports, focus now here is we have idealized the body as rigid, then we are trying to understand different force systems.

In the process, I show you certain idealizations indirectly but, we will also have a formal discussion on how do I idealize a wire connected to a structure or how do I analyze a fixed end, that we would see sooner. And you can also see, the force system is non-coplanar, the forces do not lie on the same plane. So, it is very general.

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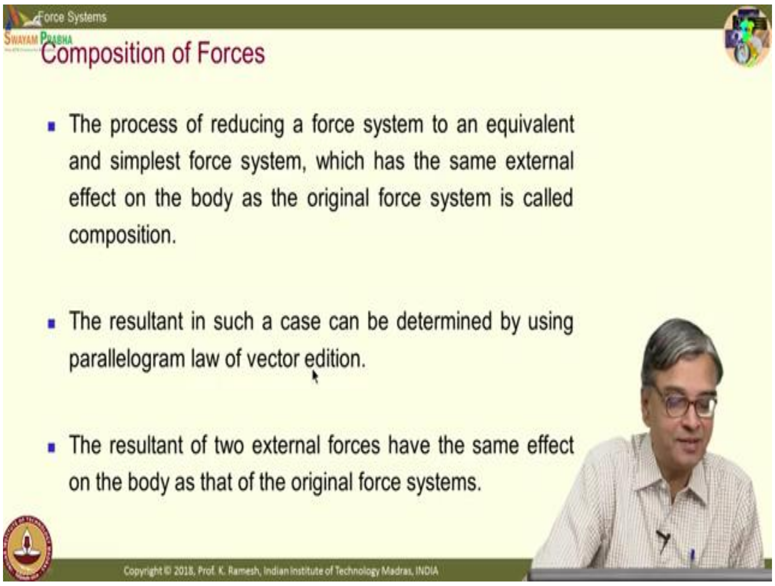

I can have another force system, where the forces are collinear. See, you can make any arbitrary shape that is convenient to you, the important point here is, whatever the force acting on the body, lies on a particular line of interest. I have force F_1, F_2 and F_3 acting at different places. And I could also have another force system, where action lines of the forces are parallel. You may ask where this kind of a force system can come into play.

I have a simple example and the simple example is like this. I have a member with rollers on two ends on a floor. I have a pin joint here and I have a wire connected to this a sort of a rope and I go and pull the rope. That is what I am going to do. When I pull the rope, I would draw the free body diagram of this member, this roller is lifted from the floor, this roller interacts with the floor, that interaction is replaced by a concentrated force.

And the interaction of the rope is put as a force like this and weight of the object is acting downwards like this. This forms a parallel force system. So, you could have in applications, systems having parallel forces, collinear forces, concurrent forces and non-concurrent forces, they can be coplanar or non-coplanar, all these combinations are possible.

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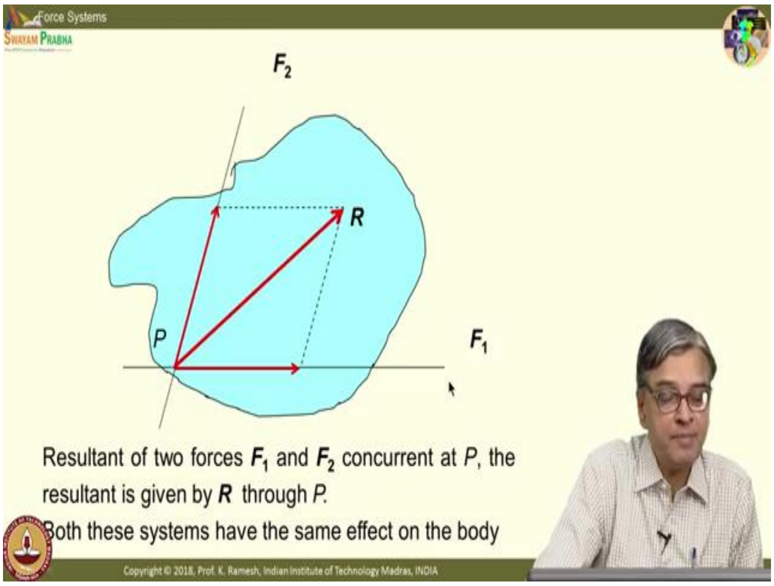
Once we have looked at the force system, we would like to simplify them ok. The process of reducing a force system to an equivalent and simplest force system, which has the same external effect. That is what you have to underline. We are only looking at external effects, on the body as the original force system is called composition.

For us to do this we would use the property of the force. We have seen transmissibility of a force. Once I move the force to a convenient point, I could find the resultant by using



parallelogram law of vector addition; vector addition it is put as edition here please correct it. The resultant of two external forces has the same effect on the body as the original force system that is a key point.

When I have multiple force interactions, reduce local interactions by simpler force system is desirable to analyze the body. For me to reduce that, I use the concept of composition of forces, which you all know indirectly in your earlier learning, we are only revisiting the same old concept. So, that you do not make a mistake in future.

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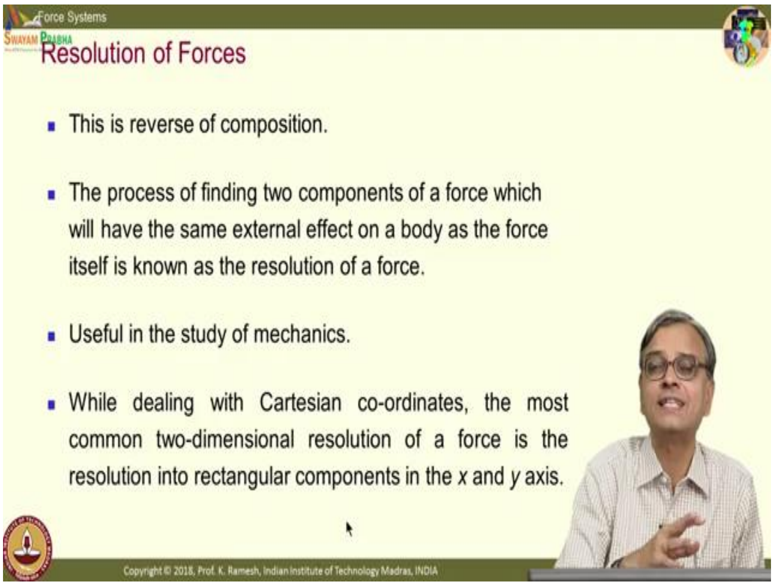
I can have a body like this, I have forces F_1 and F_2 acting on it and we have also shown what is the line of action and these two line of action intersect at point P and if I have to find out the resultant, it is easier for me to move these forces to the point P.

I use the concept of transmissibility of force and then take a resultant, very simple which you have done many numbers of times in your earlier learning. But it is better to recollect that and associate that you have been able to do this because, you are looking at the external effects number 1 and you are employing the property of transmissibility of force. You may not have paid attention on transmissibility of force in your earlier learning. By rote practice you would simply move and then take a resultant.

Now, you have a principle behind that. I have a resultant; this has the same effect as the individual forces. Both these systems have the same external effect on the body. Once you have looked at composition, you should also look at resolution of forces because, we need both tricks, depending on what is the problem on hand, I would do resolution or I will do composition.



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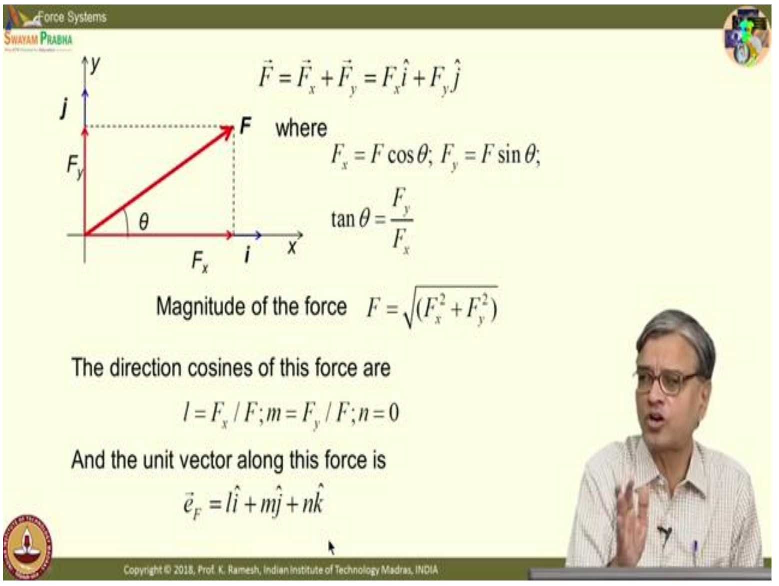
Resolution is reverse of composition, so I get the force into the components. So, the process of finding two components of a force, which will have the same external effect on a body, as the force itself is known as a resolution of a force. Here I have put two components because, it is easy to look at two-dimensional analysis first, if I do a threedimensional analysis, then I should get three components. It is useful in the study of mechanics.



While dealing with Cartesian co-ordinates, the most common two-dimensional resolution of a force is the resolution into rectangular components in x and y axis. You have different coordinate system I have polar coordinate system; I have n-t coordinate system. So, depending on which coordinate system you are looking at, we would like to see the components and that is known as resolution of forces.

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I have taken the Cartesian co-ordinate system, I have a force *F*, all this is very well known to you but, for the purpose of completeness let me list out the steps. It is oriented at angle theta, more of recollection of your early learning. I could write the force in terms of the x component and y component. I have

$\vec{F} = \vec{F}_x + \vec{F}_y = F_x\hat{i} + F_y\hat{j}$ where i and j are the unit vectors in these directions.

So, I have this force magnitude F_x and F_y, then I can write many of this. What is F_x and F_y?

$$F_x = F\cos\theta;\ F_y = F\sin\theta;$$

what is angle theta? It is;

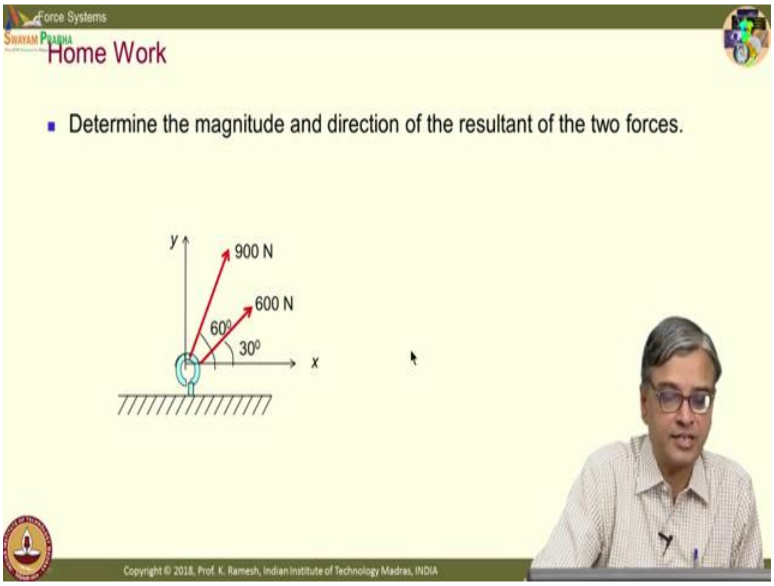
$$\tan\theta = \frac{F_y}{F_x}$$

what is the magnitude of force?

$$F = \sqrt{(F_x^2 + F_y^2)}$$

What are the direction cosines of this force? I can put them as *l*, *m* and *n*

$$l = F_x / F; m = F_y / F; n = 0$$



and what is the unit vector representing the force. I could put that as

$$\vec{e}_F = l\hat{i} + m\hat{j} + n\hat{k}$$

All this is taught to you earlier ok; we are just having a recapitulation.

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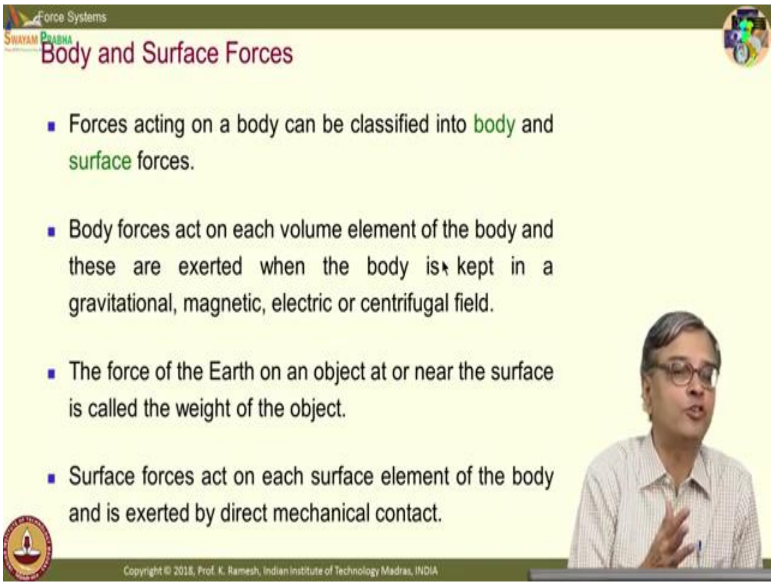
And you can also do a simple homework, if you want to do this, I have a hook, which is having a force two forces at 30° and 60° find out the resultant, I am not going to solve it; just for you to have some practice.



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I have told you earlier, you have forces that happen, when two bodies interact directly and there are also forces that act at a distance. We could classify them as body and surface forces and what are body forces and what are surface forces? By definition, body forces act on each volume element of the body and one of the commonest examples of body force is the force due to gravitation and you call that as a weight ok. I could also have this interaction from other sources; it could be from magnetic field, electric field or even centrifugal field.

And one of the very important aspects, when we draw the free body diagram later, you must consciously account for the body force interaction, there could be mistakes, if you do not accommodate this because, there is no direct physical contact. And this is what is explained again here, the force of earth on an object at or near the surface is called the weight of the object. On the other hand, surface forces act on each surface element of the body and is exerted by direct mechanical contact.

So, when you draw the free body diagram by definition you cut from the surroundings and replace the interaction of the surroundings by forces, you draw the free body diagram, you will normally not miss the surface forces but, body forces you may miss. But cleverly what we will do is in many engineering applications involving mechanical and aerospace, the effect of weight is usually neglected to make our life simple.

On the other hand, if I have to analyze a civil structure for example, if I have to analyze the beam, it is so heavy. I cannot neglect the weight of the beam in analysis of civil structures. So, what we do is, we do idealizations very cleverly to make our life simple and mind you the solution you get is not exact, it is always approximate, good enough for a given application, that is a way you have to look at it.

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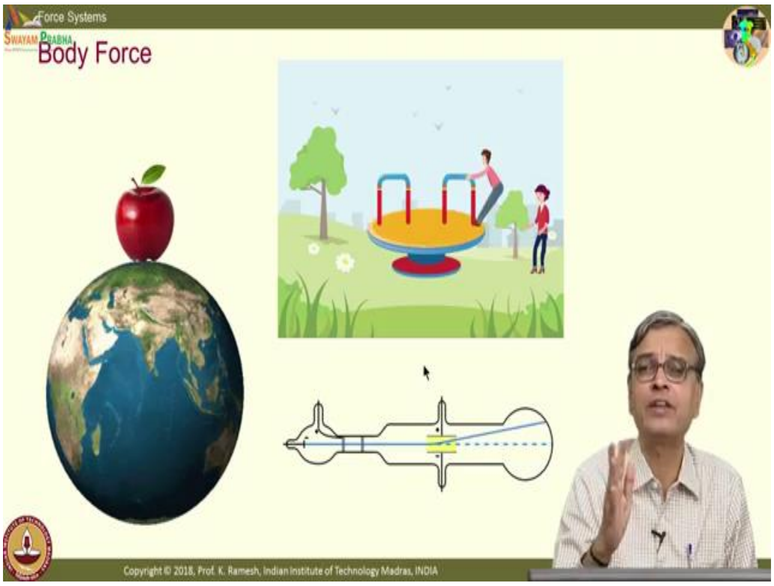
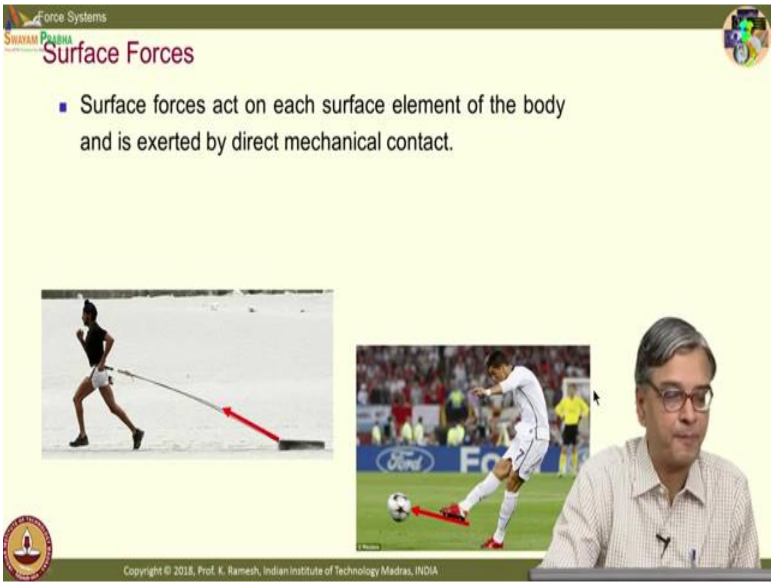


And one of the body forces that you can think of is the attraction of gravity, you see the apple falling and you have in this electrical charge. So, the electron beam gets deflected depending on the charge, that is again a body force and we have already seen a rotating platform, when you sit and enjoy the merry go round, you do a feel the force of centrifugal effect and they are all body forces.

Particularly when you are analyzing a turbine blade, rotating at a very high speed, you have to take into account the effect of centrifugal effects in your analysis. If you do not do that, your results will be far away from your experimentation. And when we go to surface forces, it is very clear the direct mechanical contact is very clear.



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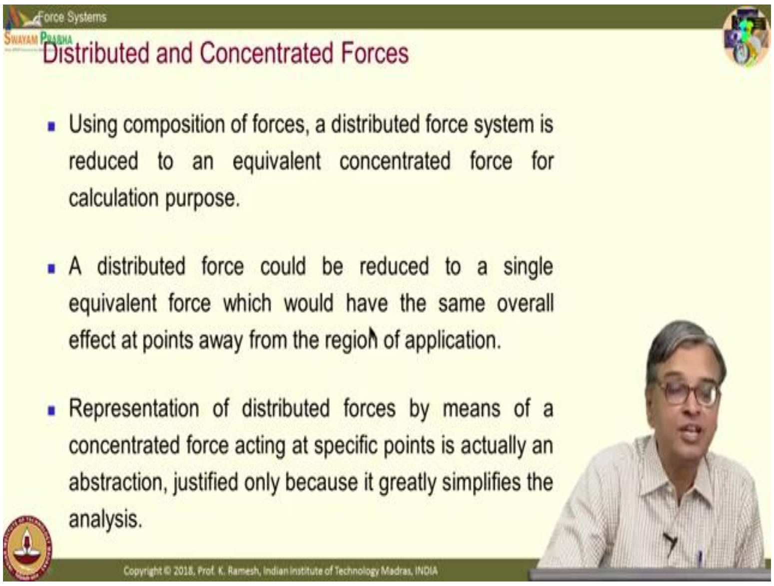

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And we will also look at, when I have a distributed force; I said all bodies are deformable. In general, you have any force interaction as only a distribution but, there are also some instances where we had seen parallel force system. So, we have to find out the composition of it fine. That is what we will look at it now. A distributed force could be reduced to a single equivalent force which would have the same overall effect at points away from the region of application. This is again you have to keep note of which engineers use.

When the body deforms at the actual interaction, what happens physically, what happens in a mathematical model can be different. Leaving those points of application, away from the point of application, if I have the force system statically equivalent, it will have the same external effect. That is very important. Representation of distributed forces, by means of a concentrated force, acting at specific points is actually an abstraction justified only because, it greatly simplifies the analysis.

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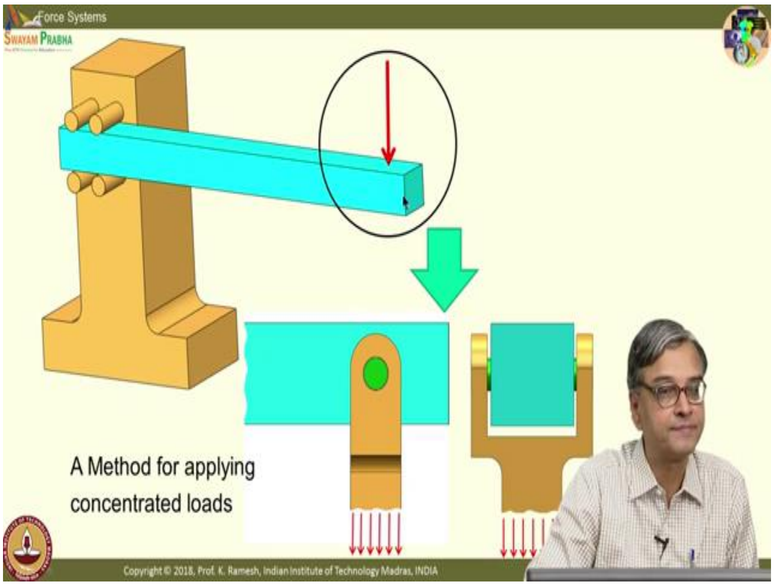
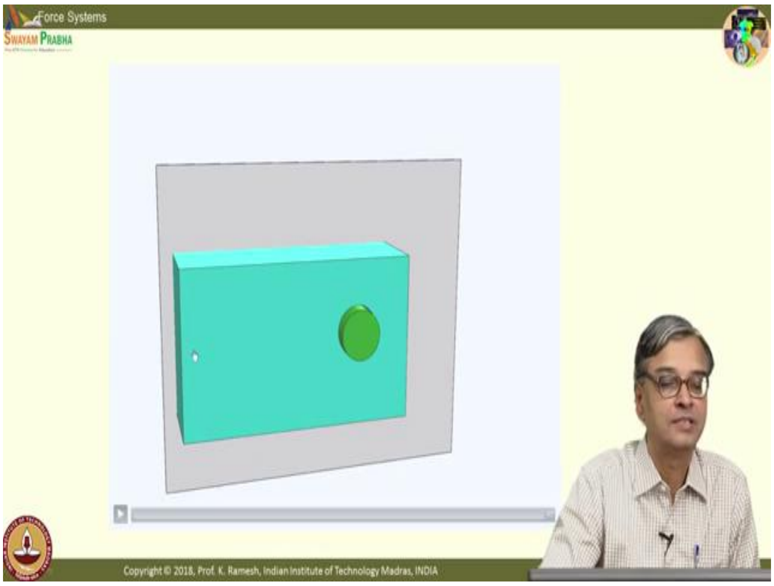


And I have an example, which shows a cantilever beam, say I am also blurting out engineering terminologies. So, the idea is when we take up beams, you understand what is a cantilever? It is better to get those jargons to start with and one simple representation is this cantilever is subjected to a concentrated force at the end. How do I get this concentrated force physically? There could be multiple ways to exert this force. It is a very nice example which brings into focus, how to comprehend a physical system fit enough for analysis.



One of the ways of applying this force could be, I could have a pin inserted into a hole at this point. I look at the front view here I look at the side view, I have a pin running through the hole, I have a fork. On the fork I could have a weighing pan and then put some weight there and if I analyze only the fork, I could replace this as a uniformly distributed force and I would have interaction of a pin with the hole on this member. Fine.

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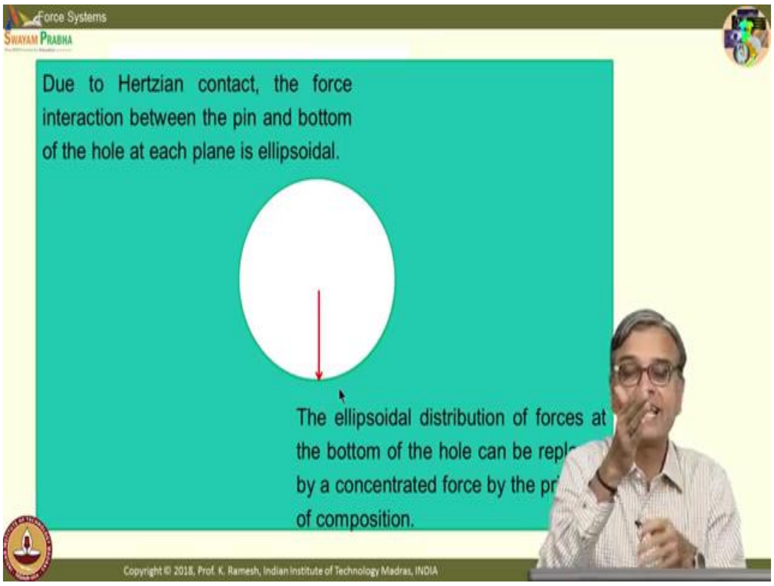

Let me show this, what I would do is, I would pass an imaginary plane that cut across this member. What way would I see the interaction between the pin and the hole.

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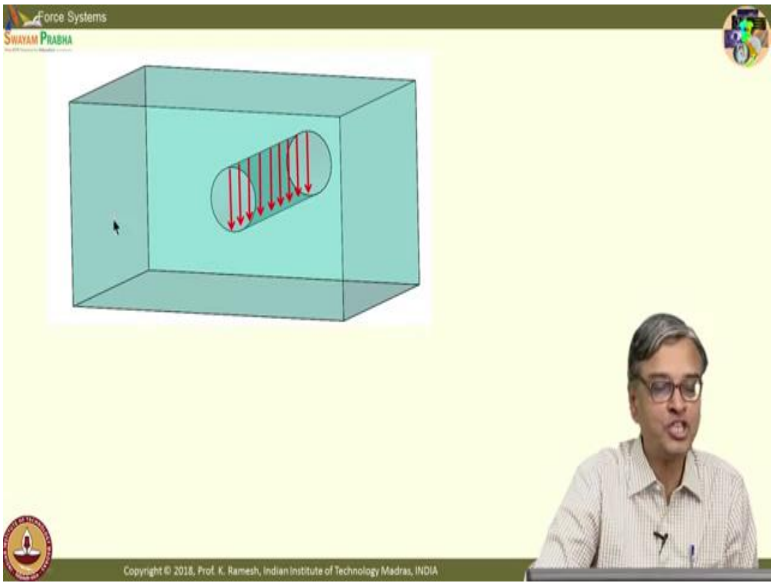

I would have a clearance, which is not visible in a small diagram. I will magnify it. I have a clearance and the pin would be resting on the hole like this and if you really look at what is the kind of interaction here, the force would be distributed and if you go for higher studies you will learn what is known as contact stress that also specifies, what kind of distribution is there. This distribution is elliptical and you idealize this as a Hertzian contact. I would in fact have a distribution like this. Imagine if I have a body which has interaction with surrounding, each one of this interaction is so complex, if I want to retain all of them and try to do the analysis, it will be too difficult to handle.

I would in fact replace this by a simpler force system, by a concentrated force. I can find out the resultant, I simply replace it as a concentrated force and you have already seen, I have put an imaginary plane passing through the member. I have looked at what happens at that plane. Physically the pin is occupying a finite distance. So, the same thing would get replicated at each one of the planes. Let us see how it happens.

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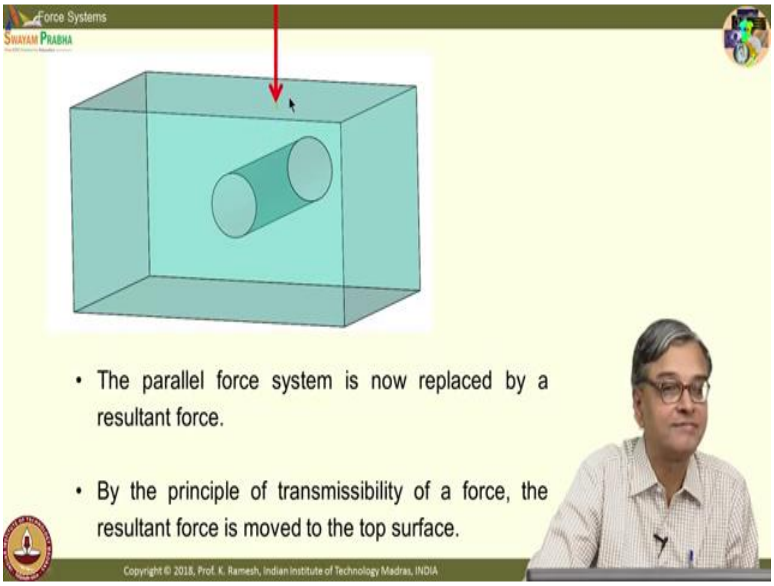
So, I have shown it like this. When I replicate that I would have a parallel force system, that would be existing between the pin and the hole and by composition I can replace the parallel force by its resultant.



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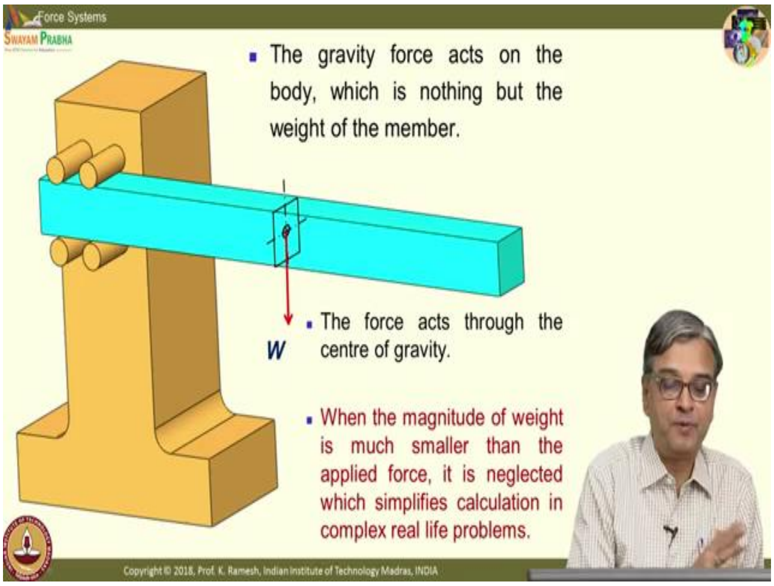
And by concept of transmissibility of force, I can move this force to this end. So, what you have seen is, in a simple picture you put a concentrated load, it is not actually that simple.

If you really look at a physical equivalence of it, many interactions are simplified and you have used the concept of transmissibility of force, composition of force and then replaced it by a single concentrated force to represent this complex interaction.



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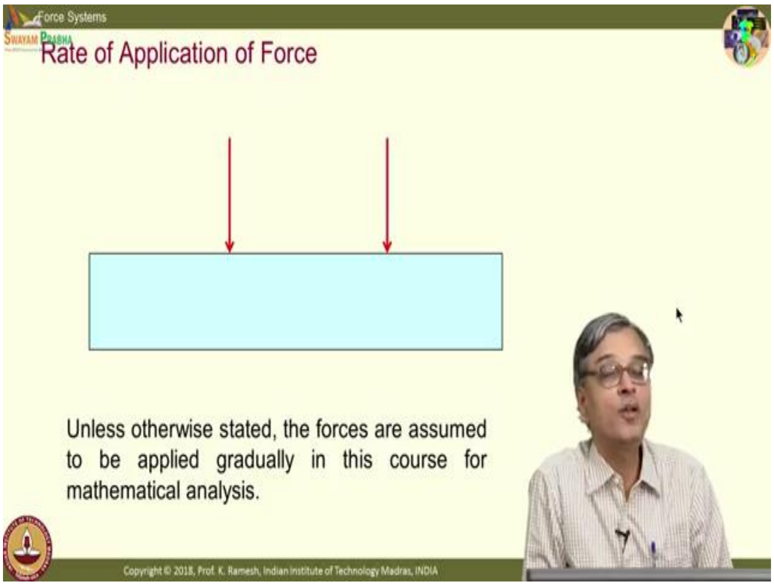
We have now looked at for the surface force. We would also look at the body force. By definition gravity acts on every volume element of the body and that is where you find the mass center and then you replace it by the weight. And you have to see with the caution, when the magnitude of weight is much smaller than the applied force it is neglected, which simplifies calculation in complex real-life problems, which we usually do in many of the mechanical and aerospace systems, not in civil engineering. Ok.



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And we will also see one more concept, we have been looking at apple falling, apple falling and we also show forces, which has a magnitude by length of the vector. How these forces are applied? It is a very important aspect. You know if the apple is falling like this, we are not going to analyze it in our course, where what happens?

At time t 0 there was no force, at time t just after that, you have a full magnitude of force, we are not going to analyze such problems. Whenever we say that there is a force, we always perceive that force has been applied gradually. So, in this course unless otherwise stated, the forces are assumed to be applied gradually for mathematical analysis, if I have a force, which is having an impact the approach has to be different. For convenience, we show that as a vector, we keep our eyes close.

How these forces are actually applied? I have shown a nice example, when there is a concentrated force on a cantilever, it is not that simple. Physical appreciation of this requires multiple elements, multiple force interactions and bodies are also deformable, we make it as rigid for analysis purpose, reduce the force into a concentrated force and do the analysis of the problem.

So, in this class, we have looked at in broad perspective, what is the concept of a force and we have learnt transmissibility of force, we have also looked at force is a sliding vector and we said that we would idealize the physical interactions physical body as rigid for the purpose of analysis and I also pointed out, a good engineer makes very intelligent approximations. Without approximations there is no engineering. You are a good engineer, if you are able to solve a practical problem with the essential features necessary for the given application.

Thank you.