

**Engineering Mechanics**  
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**Module - 02**  
**Dynamics**  
**Lecture - 10**  
**Rotating Frame of Reference IV - Crank and Slotted Bar**

Module 2 Dynamics

Lecture 10 Rotating Frame of Reference IV – Crank and Slotted Bar

Concepts Covered

Guidelines on how to identify a suitable choice of rotating frame of reference, Detailed discussions on solving crank and slotted bar problem, Evaluation of velocity using velocity polygon method, Using rotating frame of reference. Acceleration by rotating frame of reference, Visualisation of the mechanism from various standpoints through detailed animations, Relevance of using polar co-ordinates for solving the problem based on particle dynamics, Guideline to select the origin of rotating frame of reference.

Keywords

Engineering Mechanics, Dynamics, Rotating frame of reference, Choice of rotating frame of reference, Selection of origin of rotating frame of reference.

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Let us continue our discussion on rotating frame of reference. See, we have looked at the rotating frame of reference, can have both angular velocity and angular acceleration.

And for simplicity, we simply use the unit vectors  $i$  and  $j$ . With an explicit understanding

Rotating Frame of Reference

**Velocity and acceleration**

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$$

$$\mathbf{v}_A|_{xy} = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_A|_{xy}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_A|_{xy} + \mathbf{a}_A|_{xy}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

**Express all the vectors in Common Co-ordinate system Before summing them!**

we are using it for the rotating frame of reference. It makes our mathematical representation much simpler. They are not to be treated as Cartesian  $i$  and  $j$ . We have also looked at, that the rotating frame of reference can have a rotation like this. And we view the point A from a rotating frame of reference.

I have not specified any specific requirement here and we have the position vector  $r$ . And

we also have the position vector  $r_A$ . And viewing from the rotating frame of reference, it is possible for me to get  $\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$ . I said  $V_{rel}$  has a special meaning in this context. You should visualize this as the velocity perceived by an observer sitting on the rotating frame of reference, observes what happens to point A. And I can also go back to the basic relative velocity expression  $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ .

So,  $V_{AB}$  has two terms like what is shown there. And we have also looked at how to differentiate when I have a fixed frame of reference and rotating frame of reference. We have got the expression for acceleration. Here, again you have  $a_{rel}$  which is to be viewed as what is the acceleration that could be measured by an observer sitting and moving, rotating with the rotating frame of reference. That is very important, ok. And you can also have this as  $\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$ . And the important point to note is, express all the vectors in common co-ordinate system before summing them. This is a very important caution. And while doing that you also try to find out the simplest way to take the vectorial representation. In some instances that may be simpler to express it in terms of small  $i$  and

small  $j$ . In some instances, it may be simpler to express it in terms of capital  $I$  and capital  $J$ .

The slide is titled "Rotating Frame of Reference" and features a problem statement: "The crank OA revolves counter-clockwise with a constant angular velocity of 15 rad/s within a limited arc of its motion. For the position  $\theta = 30^\circ$ , determine the angular velocity of the slotted link CB and the acceleration of A as measured relative to the slot in CB using a suitable rotating frame of reference. Note the dependency of the angular motions of the crank OA and slotted link BC." The diagram shows a crank OA of length  $r = 150$  mm pivoted at O, and a slotted link CB pivoted at C. The angle between OA and the horizontal is  $\theta$ , and the angle between CB and the horizontal is  $2\theta$ . A video feed of Prof. K. Ramesh is overlaid on the bottom right of the slide.

So, if you look at the problem and find out what way you will mathematically solve. Have all this at the back of your mind so that you do not unnecessarily do

excessive computations.

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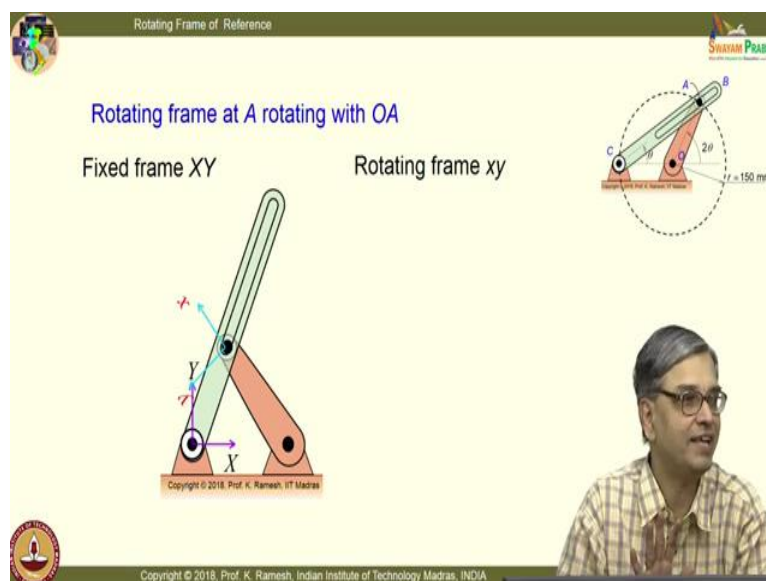
Let us take up a very interesting problem. It is simple enough, at the same time one can learn many aspects of the rotating frame of reference by looking at this. So, if I allow the crank to rotate it can have a complete sweep like this. And you have a crank  $OA$ . You have a slotted member  $CB$ . And the crank  $OA$  revolves counter-clockwise with a constant angular velocity of  $15 \text{ rad/s}$ . For the position  $\theta=30^\circ$ , determine the angular velocity of the slotted link  $CB$  and the acceleration of  $A$  as measured relative to the slot in  $CB$  using a suitable rotating frame of reference. So, I have to find out the relative acceleration which is asked as part of the problem. And you also have a very interesting aspect. I have this at angle  $\theta$ , whereas this link is at angle  $2\theta$ .

So, the problem statement says, note the dependency of the angular motions of the crank  $OA$  and slotted link  $BC$ . So, with the animation you have a fairly reasonable appreciation of what is the way the members are connected, how the point  $A$  moves. And one of the important steps in rotating frame of reference is to identify where will you have your fixed frame of reference and where will you have the rotating frame of reference. Here, I

have two members are rotating. I have a crank  $OA$  rotating and also the slotted member  $CB$  is oscillating.

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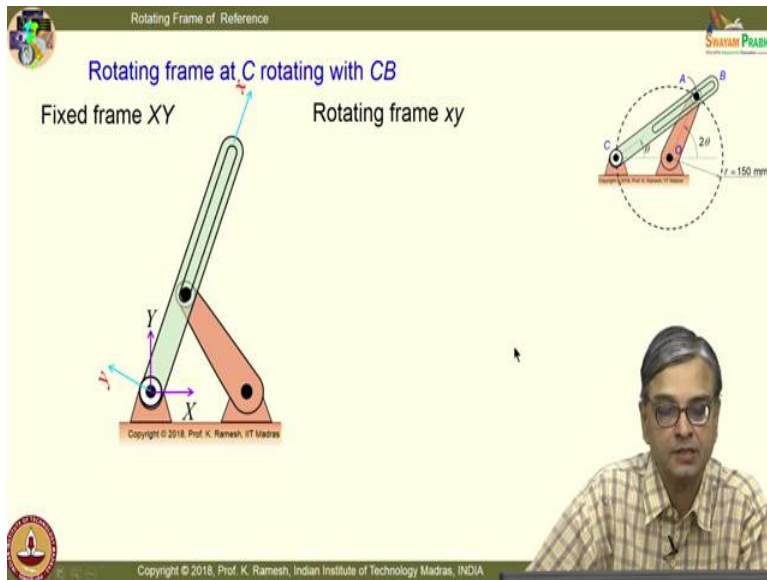
And can we have a rotating frame of reference attached to crank  $OA$  and then solve the problem. Because, I have crank  $A$ ,  $OA$  is



rotating. Why not I sit at crank  $OA$  and then see what is happening? I do not get any benefit out of this. It is not that when you have a rotating frame of reference, take a rotating frame of reference attached to any member that is rotating. Don't do that, fine?

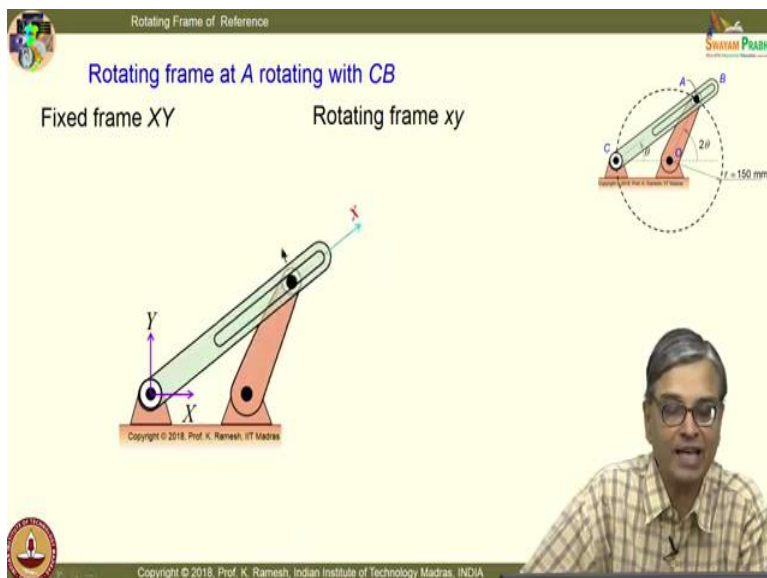
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On the other hand, suppose I have a rotating frame which is attached to the slotted member. I have some sort of a physical appreciation of the problem bit. If I move with this, what I would see is, I would see the point  $A$  is simply having a linear motion within the slot. And I am going to repeat this animation several times later also so that you get a



comprehensive appreciation of how we take the rotating frame of reference. And the same problem can also be solved from your simple polar coordinates. We will also see how we can solve it from simple polar coordinates, fine?

So, in this case what I have is, I have the rotating frame origin is kept at this point and this is rotating with the slotted arm  $CB$ .



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You know, when this problem was given to students in exam, some of them have also used another way of fixing the rotating frame of reference which I was unable to animate it completely. They have fixed the point

$B$  in our expression to the point what you have here, which is not a very good choice.

So, imagine that I have a, the  $Y$  axis is like this moving with this. Because, before you solve a problem you should find out which is the best rotating frame of reference. If you know that, then the problem is straightforward, there is nothing more to it ok. So, you need to at least solve one or two problems where you exhaust all possibilities. And then burn your fingers and find out which one you should not do. That is also a learning. It is

not that you are taught what to do always correctly. What not to do is also learning.

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**Rotating Frame of Reference**

**$v_{rel}$  by Velocity polygon**

**Given**  
 $OC = OA = 150 \text{ mm}$   
 $\omega_{OA} = 15 \text{ rad/s} = 15 \hat{k} \text{ rad/s}$

Let  $P$  be a point on slotted arm  $BC$  coincident with  $A$  on crank  $OA$ .

$v_A$  is completely known, direction  $\perp^t OA$

$v_A = 150 \times 15 = 2250 \text{ mm/s}$

$v_P$  is  $\perp^t CB$      $v_{A/P}$  is  $\perp^t v_P$

$v_P = 2250 \times \frac{\sqrt{3}}{2} = 1125\sqrt{3} \text{ mm/s}$

$v_{A/P} = v_{rel} = 2250 \times \frac{1}{2} = 1125 \text{ mm/s}$

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You know, when I have a problem like this, there is no restriction that I should find out the relative velocity only by a rotating

frame of reference. You could also bring in your velocity polygon approach and find out, the focus is that you try to understand what kind of motions happen in the problem that you are solving. The focus is on visualization first, not application of mathematics. Visualization is very important, that is why we take a very simple problem. And in this also we have followed a very nice symbolism. A slotted member is simply shown as a line with some slot. It is not exactly reproduced to the actual slot length or slot width or anything like that. It is a very nice way of representation. And we are asked to find out for a particular angle  $\theta = 30^\circ$ .

So, we have the relative position smart. And here you are able to see that you are really looking at the analysis when the point is coming down. That is very clear from the animation. That is the angle which we have to analyze. And what are the quantities that you know? You are given the angle of velocity of link  $OA$ . So, do you get the velocity of point  $A$  completely? Again, you go back to your circular motion. You know everything about point  $A$ , that is why you are not asked to find out what is the absolute acceleration of point  $A$ . You are asked to find out what is the relative acceleration in the slot. You look at the problem statement. In this class of problems, you have to find out by solving

the problem what is the value of the relative acceleration. That is, in the sense it is not the relative acceleration from the relative equation. What I mean is the  $a_{rel}$ ;  $a_{rel}$  is nothing but what is the acceleration a person sitting on the rotating frame would perceive.

So, you are given  $OC = OA = 150 \text{ mm}$  and  $\omega_{OA} = 15 \text{ rad/s} = 15 \hat{k} \text{ rad/s}$ . And whenever I have a slotted member, it is always better to visualize a coincident point. We have always been labeling the coincident point as  $P$ . So, the point  $P$  is coincident with point  $A$ . And from the givens problem statement, the velocity of  $A$  is completely known. I know the magnitude and the direction is perpendicular to  $OA$ . So, you have that information. And what is the magnitude? The magnitude is  $r\omega$ . We are given  $r$  as  $150 \text{ mm}$  and  $\omega$  is  $15 \text{ rad/s}$ .

So, I get the velocity magnitude is  $2250 \text{ mm/s}$ . And I have  $V_A$  is completely known and what all the other things that you know? What is  $V_P$ ? That direction is also known. It is perpendicular to  $CB$ . And what is the velocity of  $A$  with respect to  $P$ ? That is perpendicular to  $V_P$ . So, it is actually along the slot, fine? So, you have to visualize this. You know, you have to cultivate the habit of visualizing the problem. Do not memorize the solution procedure for a problem because it differs from problem to problem. You must physically argue, what is it happening? How I am in a position to take it, fine? And I have  $v_p = 2250 \times \frac{\sqrt{3}}{2} = 1125\sqrt{3} \text{ mm/s}$ . And velocity of  $A$  with respect to  $P$  simply turns out to be  $1125 \text{ mm/s}$ .

This is straightforward. I am also going to solve this from rotating frame of reference. The idea is, when we are discussing rotating from a reference, do not close your eyes that you will not find velocity by any other method because such mental blocks students get develop. I can also use instantaneous center of rotation and find out the velocity. I can also use velocity polygon. I can also use rotating frame of reference to find out the quantities. You must be adept in switching back and forth in any of these methods. And you know we also need to find out what is the angular velocity of the slotted member.

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And let us attach a rotating frame of reference to the slotted member CB like this. So, whatever the quantity that I have got and it is very clear. The point is coming down, it is very clear.

Rotating Frame of Reference

Attaching a rotating frame  $xy$  to arm  $CB$  as shown.

From geometry

$$PC = 2 \times 150 \times \frac{\sqrt{3}}{2} = 150\sqrt{3} \text{ mm}$$

$$\omega_{CB} = \frac{V_p}{PC} = \frac{1125\sqrt{3}}{150\sqrt{3}} = 7.5 \text{ rad/s}$$

$$\hat{\omega}_{CB} = 7.5 \hat{k} \text{ rad/s}$$

$$\hat{V}_{rel} = -1125 \hat{i} \text{ mm/s}$$

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So, if I have  $x$  is positive this way, when it is coming down  $\hat{V}_{rel} = -1125 \hat{i} \text{ mm/s}$ . And from geometry, we have already said that point  $P$  and  $A$  are coincident. So, I can find out  $PC$

$$PC = 2 \times 150 \times \frac{\sqrt{3}}{2} = 150\sqrt{3} \text{ mm}$$

And we have already got what is  $V_p$ . We know the length

$PC$ . So, it is a child's play to get what is the angular velocity of this. Even before doing it like this, you already had a clue in the problem statement. You had  $\theta$  and  $2\theta$ . There was a, problem statement was very clear if you observe all this. It has to be only like this. So, I get the angular velocity of the slotted member as  $\hat{\omega}_{CB} = 7.5 \hat{k} \text{ rad/s}$ . And  $\hat{V}_{rel} = -1125 \hat{i} \text{ mm/s}$ .

Rotating Frame of Reference

$V_{rel}$  by Rotating Frame of Reference

Attaching a rotating frame  $xy$  to arm  $CB$  as shown.

From geometry

$$CA = 2 \times 150 \times \frac{\sqrt{3}}{2} = 150\sqrt{3} \hat{i} \text{ mm}$$

From Rotating Frame Approach

$$\hat{V}_A = \hat{V}_C + \hat{\omega}_{CB} \times \hat{r}_{CA} + \hat{V}_{rel}$$

$$v_A = 150 \times 15 = 2250 \text{ mm/s} \quad \hat{V}_A = 2250(-\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j})$$

$$\hat{V}_{rel} = V_{rel} \hat{i}$$

$$\hat{\omega}_{CB} \times \hat{r}_{CA} = \hat{\omega}_{CB} \times 150\sqrt{3} \hat{j}$$

$$\hat{V}_{rel} = -1125 \hat{i} \text{ mm/s}$$

$$\omega_{CB} = \frac{1125\sqrt{3}}{150\sqrt{3}} = 7.5 \text{ rad/s}$$

$$\hat{\omega}_{CB} = 7.5 \hat{k} \text{ rad/s}$$

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Now, let us go and solve this from a rotating frame of reference. We have taken the rotating frame of reference like this. Then, I write the expression. Before that, I have the vectorial expression for  $CA$  as

$CA = 2 \times 150 \times \frac{\sqrt{3}}{2} = 150\sqrt{3} \hat{i} \text{ mm}$ . And you can write this expression comfortably. See, I consciously mix the symbols. The idea is, you must get out of the symbols and

physically look at the problem statement. I am looking at point A. So, the velocity of A is my interest.  $\hat{V}_A = \hat{V}_C + \hat{\omega}_{CB} \times \hat{r}_{CA} + \hat{V}_{rel}$ .

And what you should understand is, in this expression you already know what is it to be written for  $V_C$ . I have taken C as a fixed point. What is the velocity of that point? 0 also you can have. So, that goes off. And I can know from the physics of the problem,  $V_A$  completely. I know its direction as well as its magnitude. I can express it in terms of the coordinate system that I choose which is easier for me to solve. Here, it will be easier to handle it in small  $i$  and small  $j$ .

So, essentially what you do not have in this expression is, I do not know what is the velocity of  $\omega_{CB}$ . I do not know what is the value of  $v_{rel}$ . So, I essentially generate two equations out of this by collating the  $i$  and  $j$  terms. I have 2 unknowns; I can solve for 2 of them. We know what is  $V_A$ , this is completely known. And I can also express  $V_A$  because I know the geometry of the system and the relative orientation of the rotating frame with respect to that. So,  $\hat{V}_A = 2250(-\sin 30^\circ \hat{i} + \cos 30^\circ \hat{j})$ . Please verify some of these, there could be some small typographical errors. So, I have yeah, this is a very important point. I have, I thought that I should discuss and then write. What will you write as  $V_{rel}$ ? See, a thumb rule is, if I have a slotted member attach your rotating frame to the slotted point member. And if a person is sitting on the slotted member and then rotate with it, he would perceive the particle to move along the slot. That is why we choose a rotating frame of reference. I have just made a statement. After solving all of this, I will go back to the animations again.

Because, I want you to help you to visualize. The visualization is very important. Then you will be very confident in handling rotating frame of reference. If you solve one or two simple problems, any other problem you can tackle ok. So, one of the important learning here is, the moment I have a slotted member, because in this  $k$  the slotted member is along one of the coordinate axes. It is just a straight line. I can comfortably write what is the component of velocity a person sitting on the rotating frame would perceive, which is labeled as  $V_{rel}$ . It is not the complete relative velocity. We have seen  $V_{A/B}$  has two components,  $\omega \times r$  and  $V_{rel}$ . So, keep that in mind. And I have

$\hat{\omega}_{CB} \times \hat{r}_{CA} = \omega_{CB} \times 150\sqrt{3} \hat{j}$ .



Because, I have A cross i, I get this as j. So, from this, equating the i terms, I get  $\hat{v}_{rel} = -1125\hat{i}$  mm/s and then I get  $\hat{\omega}_{CB} = 7.5\hat{k}$  rad/s. Same as what we have got it by using the velocity polygon. There again, we have to do it in two stages. Here, again I should do it in two stages. Only thing is, here the computation is vectorial. There, there was physical appreciation of how do you handle the problem. There is a greatest advantage. Whichever way I calculate velocity, we have always seen, it is always better to use a

vectorial approach to calculate the accelerations.

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So, we will get on to calculating the acceleration. So, as before, you have the x y axis attached to CB and I have this distance is

**Rotating Frame of Reference**

**Acceleration**

Attaching a rotating frame  $xy$  to arm  $CB$  as shown.

From geometry

$$PC = 2 \times 150 \times \frac{\sqrt{3}}{2} = 150\sqrt{3} \text{ mm}$$

From Rotating Frame Approach

$$\hat{a}_A = \hat{a}_C + \hat{\omega} \times \hat{r} + \hat{\omega} \times (\hat{\omega} \times \hat{r}) + 2\hat{\omega} \times \hat{v}_{rel} + \hat{a}_{rel}$$

Angular velocity of rotating frame  $\hat{\omega} = \hat{\omega}_{CB} = 7.5\hat{k}$  rad/s

It is known that  $OA$  has constant angular velocity and since angular motion of  $OA$  and  $CB$  are related  $\alpha_{OA} = 0 = \alpha_{CB}$

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$PC = 2 \times 150 \times \frac{\sqrt{3}}{2} = 150\sqrt{3}$  mm. And

you should be comfortable to write this expression completely for the context of the problem. I have deliberately changed this. We had this as AB fine. And in this problem, we are actually having the origin of the rotating frame of reference located at C. You should get out of this A and B. You should interpret it in terms of what is the coordinate velocity, coordinate acceleration so on. And what is the symbol  $\omega$  that you will attach to? It is all related to  $\omega_{CB}$ . And in this problem, you have a luxury, that it this is rotating at the constant angular velocity.

So, I will have,  $\dot{\omega}$  will go to 0. And we have already determined  $V_{rel}$ . I would like you to ponder about what is it that you know on  $a_{rel}$ ; think about it. I want you to think about it now. As we discuss, I will ask a question. Let me see whether you answer, fine. We have already discussed about  $V_{rel}$ . And you have the angular velocity rotating frame is  $\hat{\omega} = \hat{\omega}_{CB} = 7.5\hat{k}$  rad/s. And we also have the luxury that  $\alpha_{OA} = 0$  which also implies  $\alpha_{CB}$  is 0 because they are connected beautifully.

So, whatever happens to the link  $OA$  is reflected upon link  $CB$  because the way they are constructed. So, when I do not have an angular acceleration for the link  $OA$ , I also do not have an angular acceleration for the link  $CB$ . That, you learned from the problem statement.

Rotating Frame of Reference

Since point C is fixed,  $\hat{a}_C = 0$ ;  $\hat{\omega}_{CB} = 0$  hence  $\hat{\omega} \times \hat{r} = 0$

Position vector  $\hat{r} = CP\hat{i} = 150\sqrt{3}\hat{i}$  mm

Coriolis acceleration

$\hat{\omega} \times (\hat{\omega} \times \hat{r}) = 7.5\hat{k} \times (7.5\hat{k} \times 150\sqrt{3}\hat{i}) = -14614.2\hat{i}$  mm/s<sup>2</sup>

$2\hat{\omega} \times \hat{v}_{rel} = 2 \times 7.5\hat{k} \times -1125\hat{i} = -16875\hat{j}$  mm/s<sup>2</sup>

$\hat{a}_{rel}$  direction is along the slot hence  $\hat{a}_{rel} = a_{rel}\hat{j}$

$\hat{a}_A$  is known and it has only normal component of acceleration

Unit vector  $AO = -\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}$

$|\hat{a}_A| = OA \times \omega_{OA}^2 = 150 \times 15^2 = 33750$  mm/s<sup>2</sup>

$\hat{a}_A = 33750(-0.866\hat{i} - 0.5\hat{j}) = (-29228.4\hat{i} - 16875\hat{j})$  mm/s<sup>2</sup>

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And your conscious that the point C is fixed. So, I have a C is 0. And then, I have already discussed where having a constant angular velocity. So,  $\hat{\omega}_{CB} = 0$  hence  $\hat{\omega} \times \hat{r} = 0$ . And you have the position

vector  $\hat{r} = CP\hat{i} = 150\sqrt{3}\hat{i}$  mm. We have seen this computation several times earlier. And we have  $\hat{\omega} \times (\hat{\omega} \times \hat{r}) = 7.5\hat{k} \times (7.5\hat{k} \times 150\sqrt{3}\hat{i}) = -14614.2\hat{i}$  mm/s<sup>2</sup>. See, what you should recognize here is, I have a very long expression for acceleration, fine. We are fitting in each and every component. Some other components when we discuss, a person sitting on the rotating frame of reference fails to observe, but mathematics gives you that, fine. And what is the direction of this acceleration? This is actually along the coordinate axis. In this case it is also along the slot, but this is given by the component  $\omega \times (\omega \times r)$ . Now, we will also look at the term involving Coriolis acceleration. I have  $2\hat{\omega} \times \hat{v}_{rel} = 2 \times 7.5\hat{k} \times -1125\hat{i} = -16875\hat{j}$  mm/s<sup>2</sup>. Usually, this is the component people have failed to observe sitting on the rotating frame of reference. And you need to understand how this component is reflected in the mechanism, what is its direction ok. So, I have this the point A is like this. And if I interpret this and put the Coriolis acceleration, this is perpendicular to the slot and this is in the negative direction of  $j$ .

So, this is direction of Coriolis acceleration which is very clear from the mathematics, no difficulty at all. If you hang on to mathematics absolutely no difficulty in getting the Coriolis acceleration. See, one of the important learning that you have to understand is,

what is the direction of it, in relation to the slot how it is located. This is all become important when you analyze physical problems, fine. We will also see one interesting application. If you do not understand this, your design of space stations would really be jeopardized. We will see that, where there is a very nice, you have to recognize that this is perpendicular to this ok. That concept is very important.

Now, can you tell me what is the that you know on a rel. The problem itself says that you have to determine the relative acceleration value as well as direction. Can you comment at least on one of the two, magnitude or direction? See, I said when you are sitting on a rotating frame of reference, I would see the particle to move on along the slot, fine. So, what you will have to recognize here is,  $a_{rel}$  direction is along the slot and hence

$\hat{a}_{rel} = a_{rel} \hat{i}$ . It is a very important statement. You should not confuse with the absolute acceleration of point A. Then, you will not be able to write it. You have a long expression which contains several terms. You are only looking at a term what a rotating observer will observe. And you go back to your previous problem where we had done the quick return mechanism of shaper. There again, you had a slot which is straight. Visualizing velocity along the direction of the slot is ok, but saying acceleration will also be in the direction requires some imagination. So, you have to appreciate this,

$\hat{a}_{rel} = a_{rel} \hat{i}$ . Once you say this, your mathematics can help you to solve it because I would have grouping i terms and j terms and I would be able to calculate the value of quantities that are associated with i and j. So, a person sitting on the rotating frame of reference would observe the acceleration dictated by the slot. Because the slot is straight, it turns out to be a simple expression along the slot ok. If the slot is curved, then I have to go to n and t coordinates and find out what is acceleration. It will have a both the components, normal and tangential component.

So, I have unit vector as  $AO = -\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}$ . And I have the magnitude is  $|\hat{a}_A| = OA \times \omega_{OA}^2 = 150 \times 15^2 = 33750 \text{ mm/s}^2$ . And if I write it in vectorial form, it turns out to be like this. So, what I have done here is, I have my x y axis, fixed frame is like this, but my rotating frame is inclined. And I found, it is convenient to write all quantities with respect to the rotating frame of reference. So, you have to understand this has multiple terms, also their frame of reference is different for different terms. So, you have to put

the vectorial summation properly. If the rotating frame of reference and fixed frame of reference are not coincident or parallel or the any when any one of these conditions are violated. So, you have to handle this. So, it adds up to; little more of mathematics.


Rotating Frame of Reference

Substituting these terms in the Equation

$$\hat{a}_A = \hat{a}_C + \hat{\omega} \times \hat{r} + \hat{\omega} \times (\hat{\omega} \times \hat{r}) + 2\hat{\omega} \times \hat{v}_{rel} + \hat{a}_{rel}$$

$$-29228.4\hat{i} - 16875\hat{j} = 0 + 0 - 14614.2\hat{i} - 16875\hat{j} + a_{rel}\hat{i}$$

By grouping  $\hat{i}$  terms

$$\hat{a}_{rel} = -14614.2\hat{i} \text{ mm/s}^2$$


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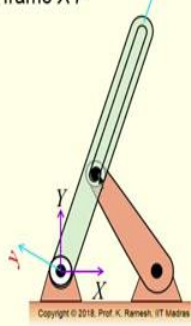
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So, substituting these terms in the equation, it is a child's play to solve it. I keep writing this because you know, you have to keep looking at this and you should remember it. See, by learnings a subject like this, certain things you have to remember. There is no choice. And if you keep writing it several times, you are in a position to remember that.

Rotating Frame of Reference

Rotating frame at C rotating with CB (two rigid bodies)

Fixed frame XY      Rotating frame xy



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And it is fairly easy to solve this and I get this

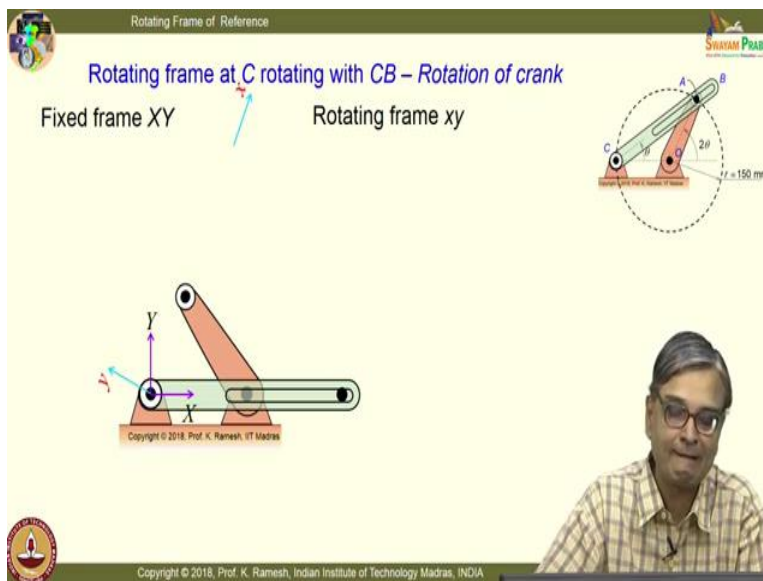
$$\hat{a}_{rel} = -14614.2\hat{i} \text{ mm/s}^2$$

We have used a very important understanding, that the  $a_{rel}$  would be along the slot. It is dictated by the slot. I will put it that way because if the slot is curved then it can have a normal component of acceleration as well as a tangential component of acceleration.

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Now, let us go and investigate the mechanism much more closely ok. This is supposed to help your visualization. You know, I have done all these simulations based on PowerPoint. So, I have been able to group them together, segregate them so that you get a better depth of understanding of what is happening, fine. Because, you know how eye gets taken away by the way that you think in a sequence ok. That is what I want you to look at it.

So, just keep your observational skills as high as possible. So, I have a rotating frame of reference attached to this. I am now going to look at as 2 rigid bodies. I have a rotating frame attached to the slotted member  $CB$ . And I have a frame like this, this is just moving. In my original animation I also had a dot here. Because, you know, the dot here is behind this member, it was not clear. And let me just see how I had animated this dot ok. This, a dot was animated like this. Do you see any motion of this dot? How does it move? It is going and stopping there fine.



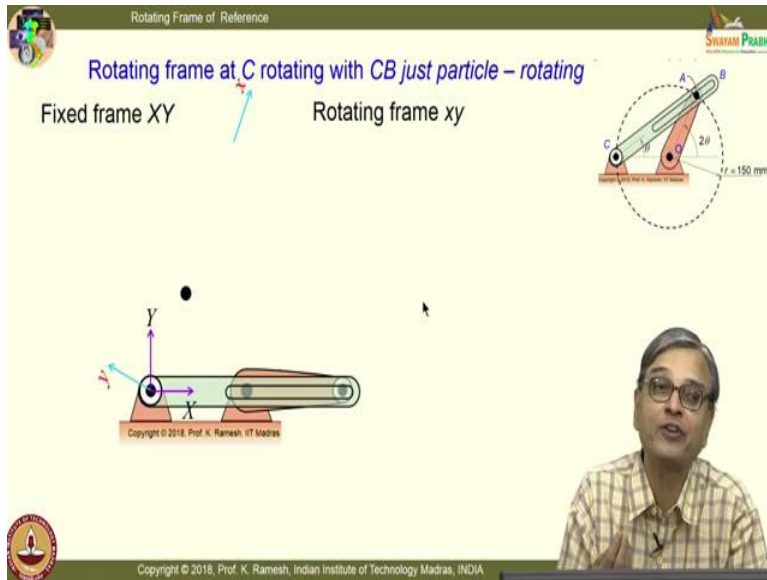
Now, let us look at the same animation from a different perspective.

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I am going to look at the rotation of a crank. I put the fixed frame. You know this is a luxury you enjoy because of multimedia type of development of the course. Otherwise, it is impossible to show any one of these variations. And now, I am having this rotating frame moving like this. And I am just having the crank rotating. I am not rotating any other member. You can see it again. You know, you find that there is no connectivity between that frame of reference and the crank fine.

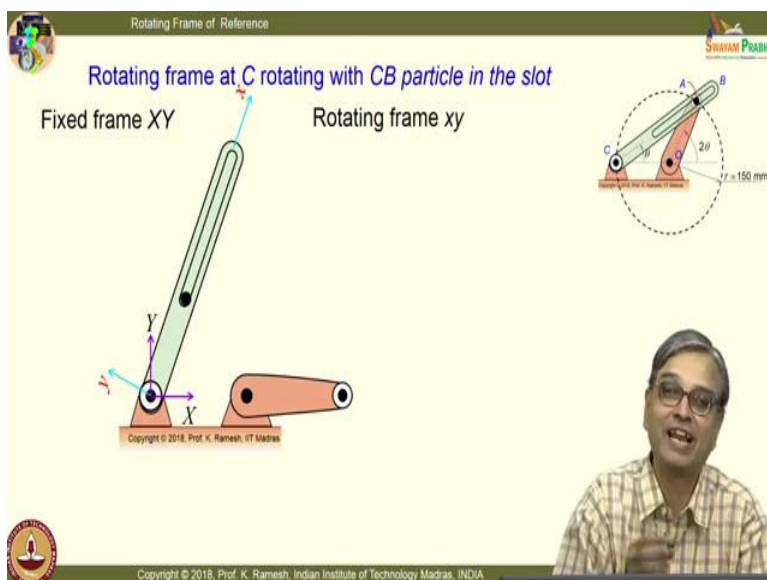
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Just the particle is rotating. Let us also look at just the particle is rotating. These all same animations where I play with the groupings; I just have the, I will have the rotating frame and also the particle. At least do you recognize that this particle is rotating as a circular motion? It is nothing, but the particle at the end of the crank. You do not see all of that.



Now, what I am going to do is, I am going to rotate this light green slot and this particle together. I am not going to rotate any other stuff. Let us see what is it that view because this is what we have been saying, if you are rotating with the frame of reference what would you observe.

See, you do not have an opportunity. There is a rotating frame, you cannot go and sit and watch. We have to only imagine, visualize. How best I can take you to that kind of imagination is the question.



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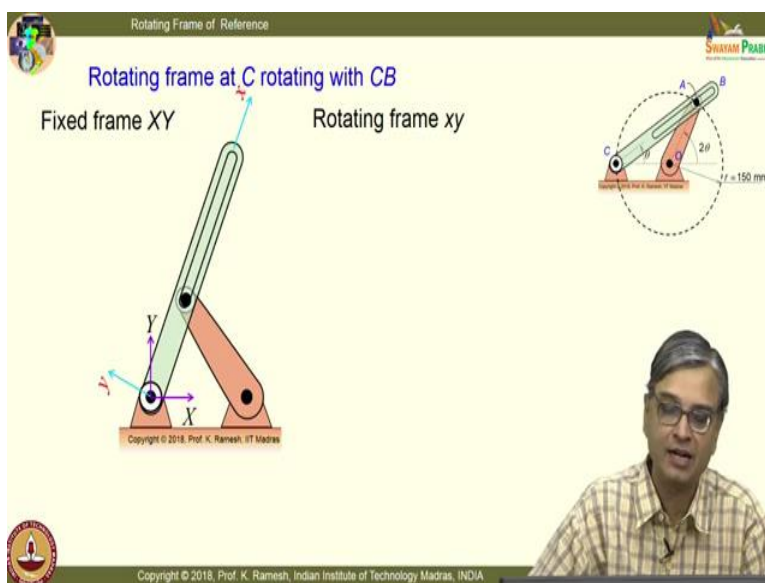
So, now I am going to rotate the green member and just the particle. I am not going to rotate the crank ok. And just look at the magical stuff. I see very clearly the particle is simply moving on the slot. When I individually look at the particle, it has a very

arbitrary motion. You saw it yourself. And what we have been saying is, when I say that I am sitting on the rotating frame of reference and observe, I would observe the particle to move along the slot. So, visualize it again. So, this is what we are trying to explain

you. Is idea clear? Again, the particle has a circular motion, but when you see these two animations together, you get a dramatic picture ok.

Let me also ask you one more thing since you are all wetted to your particle dynamics much more than in rigid body dynamics. It does not get into you. You resist. You solve any problem from particle dynamics. This is a very good problem in particle dynamics. I have simple polar coordinate system. I am looking at the  $\dot{r}$  and  $\dot{\theta}$  that is what the problem is. Or you can also have a problem, I have a slot like this and then I have an ant crawling from this end to that end. You have problem like that is not it? They are not mechanics problems. If I have to make a device, how do I have that particle to move on the slot? It has to be attached to a crank and then rotated. From a mechanical in perspective, I need to have a crank and rotate it. Here, I am able to disassemble all of them and then visualize it from different perspectives.

So, this is very interesting. This is what we have always been saying, when I say a coincident point. When I am sitting on the member and then observe it. I will observe the



particle to move on the slot which is very clearly brought out. Now, I will, let me see whether the next one is that or yeah.

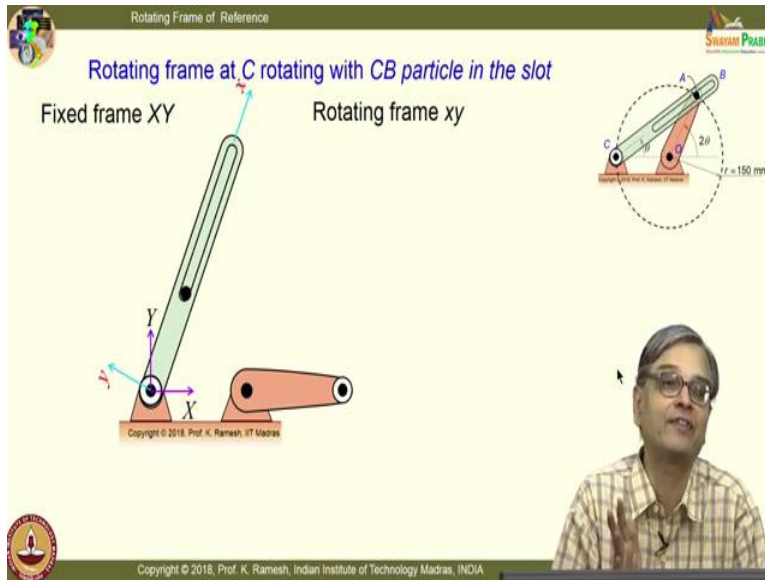
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Now, I go back to my original animation where I have this attached to this. I have all of them rotating. I have, the particle is also

rotating. So, that this particle looks brighter. So, to make this animation, it has so many layers. So, it is very complicated to do ok. My student had done this complicated one, gave it to me. I was able to disassemble it and then bring it before you a very important and subtle concepts.

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I hope that you have got the hang of what I am trying to explain ok. And this is to re-emphasize, we are looking at this. We will solve this problem using polar coordinates

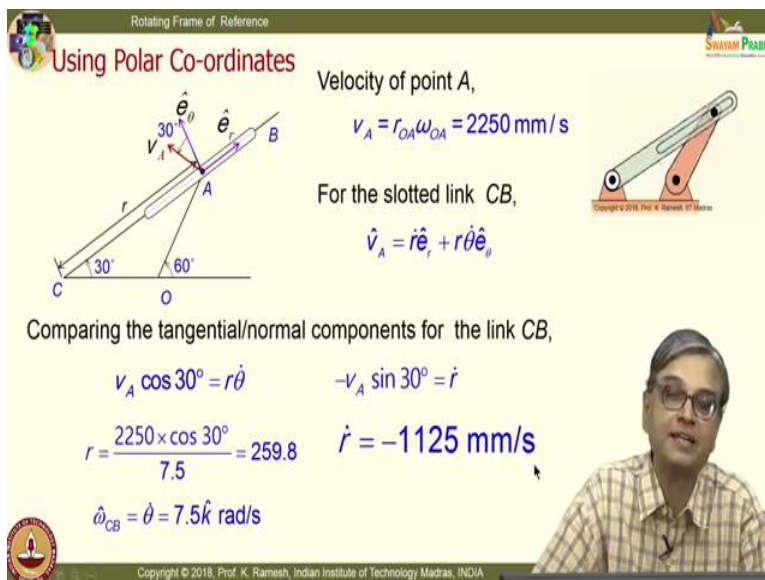


because you have a natural urgency to go and solve it by a polar coordinate. And then you are coming and asking, why not you give marks. I will give you 0 only because you need to understand how to tackle this problem from a rotating frame of reference because you are learning a

new methodology.

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So, using polar coordinates I have this like this. And I am also going to put it on this, the



animation. And then show how they are beautifully illustrated. So, what I will do is, I will put this member here. I will rotate only the green member and this. And I have this beautifully rotated like this. And I can have a radial vector like this. And the  $e_\theta$  like this. And this is

oriented at angle  $30^\circ$ . So, from your polar coordinate system, I can write this

$\hat{v}_A = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ . Because, you have used polar coordinate system for circular motion where you had  $r$  as constant, you are not solved problems where  $r$  is varying. You can



also solve problems where  $r$  is varying,  $\dot{r}$  I have,  $r$  double dot, all these possibilities exist. And whatever the problem that you have, can also be visualized from the perspective of a particle analysis which misses out a need for a crank whereas, a mechanical engineer needs to have a crank to have that particle moving. So, like I said, in statics you have to visualize how the loads are applied. In dynamics, visualize how the motion is affected. So, so comparing the tangential and normal components of the link CB see, I should also get identical answers. Method does not influence my final result fine. So, I have  $v_A \cos 30^\circ = r\dot{\theta}$ . And I can also find out what is the value of  $r$ . And I can find out what is  $\dot{\theta}$ . Then, I have  $-v_A \sin 30^\circ = \dot{r}$ . So,  $\dot{r} = -1125 \text{ mm/s}$  which is your  $V_{\text{rel}}$  in our earlier symbolism. So, I can also solve this problem using polar coordinates

Rotating Frame of Reference

Acceleration of point A,

$$a_r = -r_{OA} \omega_{OA}^2 = -33750 \text{ mm/s}^2$$

$$a_\theta = r_{OA} \dot{\omega}_{OA} = 0$$

$$a_A = -33750 \text{ mm/s}^2$$

$$\hat{a}_A = -29228.4 \hat{e}_r - 16875 \hat{e}_\theta$$

For the slotted link CB,

$$\hat{a}_{CB} = (\ddot{r} - r\dot{\theta}^2) \hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{e}_\theta$$

$$= -14614 \hat{e}_r - 16875 \hat{e}_\theta$$

$$\hat{a}_{A/CB} = -14614 \hat{e}_r, \text{ mm/s}^2$$

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because you had a very nice illustration that the particle is coming down. It is not just coming down it is also rotating.

So, you have  $\dot{r}$  as well as  $\dot{\theta}$  existing. So, you have done this. Now, I can also go ahead and write the expression for the

accelerations.

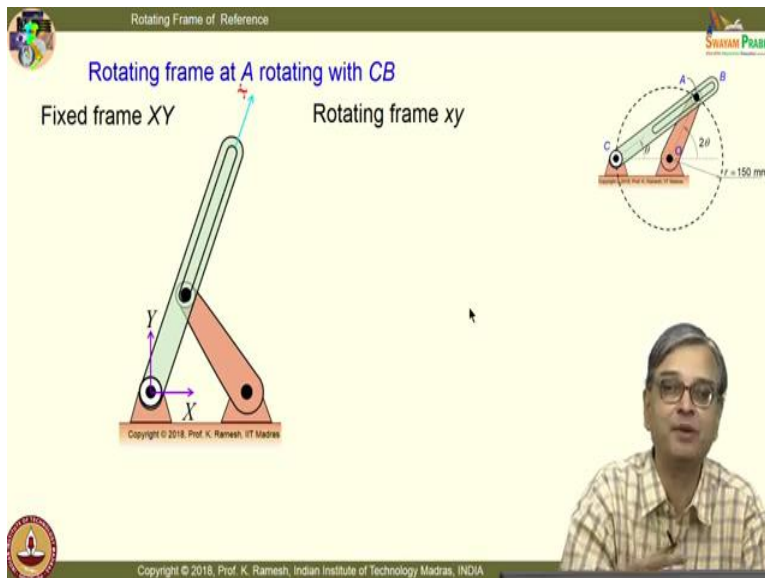
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So, acceleration of point A, I can write it like this. I have a radial component and also a tangential component, normal and tangential component. So, I can write this as in terms of  $e_r$  and  $e_\theta$ . So, velocity of A is completely known, which is expressed in terms of  $e_r$  and  $e_\theta$ .

So, for the slotted link CB, I have  $\hat{a}_{CB} = (\ddot{r} - r\dot{\theta}^2) \hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{e}_\theta$ . So, here again you see, I have very similar to your Coriolis component. See, the grudge was, a person sitting at the

rotating frame does not observe, but mathematically that term was always existing, no problem. If you have your mathematics right, the expressions are flawless.

So, I have  $a_{CB}$  as  $-14614\hat{e}_r - 16875\hat{e}_\theta$ . So, I have acceleration of A with respect to CB as  $\hat{a}_{ACB} = -14614\hat{e}_r$ , mm/s<sup>2</sup>. So, this is again along the slot, which again reinforces the discussion, that when I sit on the rotating frame of reference, the slot dictates the



direction of velocity as well as the acceleration direction. Slot, if it is straight, it turns out to be along the slot. If it is curved, it is dictated by the normal and tangential components.

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We will also have a peep into the other way of solving the problem. You know, this is like touching the nose the other way. I can touch the nose easily like this. I can also touch the nose putting like this and then touch it. Do an acrobat and say you are doing a great yoga. Add some spice to it, everybody will start doing it.

Because, this is physically not easy to visualize because some students have solved it like this. When we correct the paper, we will also have to solve it like this and see how much we can give marks. You understand? Because we do not simply cut your answers straight away as bad. So, whenever I have a slotted member where the member is moving, take the axis on the slotted member is fine. If there is a choice between putting it along with the fixed frame of reference or putting at the point, put it with the fixed frame of reference. Do not put it on the point because now, I have reversed the problem completely. I have to view the point, fixed point like this.

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Let me do some initial calculations and then you just see what is the way, circus I have to do to get the similar numbers because it is not physically visualizable. A physical

**Given**  
 $OC = OA = 150 \text{ mm}$   
 $\omega_{OA} = 15 \text{ rad/s} = 15\hat{k} \text{ rad/s}$

**From previous solution**  
 $v_A = 2250 \text{ mm/s}$   
 $v_P = 1125\sqrt{3} \text{ mm/s}$   
 $v_{A/P} = 1125 \text{ mm/s}$   
 $\hat{v}_{A/P} = -1125\hat{i} \text{ mm/s}$

$\hat{v}_{rel} = \hat{v}_{P/A} = -\hat{v}_{A/P} = +1125\hat{i} \text{ mm/s}$

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visualization is when there is a point is moving in a slot. View the point. Do not sit on the point. You get the difference? This is what I am trying to say. You may ultimately get the numbers because you know, many times you do not have time to visualize. You know this is what it is

and then you simply put some direction and then put some mathematics and then if the answer is ask you to prove, you will always know how to prove. So, it is like working

**Attaching a rotating frame  $xy$  to arm  $CB$  at  $A$  as shown.**

**From previous solution**  
 $\hat{\omega}_{CB} = 7.5\hat{k} \text{ rad/s}$

**From Rotating Frame Approach**  
 $\hat{a}_C = \hat{a}_A + \hat{\omega} \times \hat{r} + \hat{\omega} \times (\hat{\omega} \times \hat{r}) + 2\hat{\omega} \times \hat{v}_{rel} + \hat{a}_{rel}$

**Note the important change on the LHS**

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backwards. So, it is a really physically not comfortable to do. From previous solution I have all this. I will just write the expression. So, I have a  $\hat{v}_{A/P} = -1125\hat{i} \text{ mm/s}$  and then I have to have  $\hat{v}_{rel} = \hat{v}_{P/A} = -\hat{v}_{A/P} = +1125\hat{i} \text{ mm/s}$ , all this circus I have to do.

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And then, when I want to write the acceleration expression because we have solved the problem. Acceleration expression I have to write from  $a_C$  to  $a_A$ . So, note the important change in the LHS. So, you have to do this, then you will be able to get  $a_{rel}$  correctly fine. But you will get  $a_{rel}$  correctly, but if you have to interpret the Coriolis acceleration

you will again have to do a circus. So, the advice is, please do not sit on the point which is moving on the slot. Sitting on the point means what? You are attaching the coordinate of the rotating frame of reference. The observer sits on the coordinate origin and then looks at what happens. So, do not sit on the point. That is also a learning. When I have a slotted member, you take the rotating frame of reference on the slotted member. On the slotted member at an appropriate point, so that I can view the point moving along the slot. Do not sit on the point. That would be the recommendation that I would say. These are all thumb rules. See, these thumb rules you must verify for a given application whether the, this is applicable.

So, in this class we have looked at very simple problem and we have dissected the animations. So, that you are able to appreciate what we actually mean when I am hamming a rotating frame of reference. What is the advantage? I must have an advantage by selecting a rotating frame of reference. And we have learnt a thumb rule if I have a slotted member attach a rotating frame of reference that slotted member. And you have also learnt; do not attach the coordinate of rotating frame of reference to the point of interest. You will not get any benefit out of it. So, what not to do is also you should learn.

Thank you.