Engineering Mechanics
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Module - 02
Dynamics
Lecture - 12
Kinetics I

Let us move on to our next chapter on plane Kinetics.

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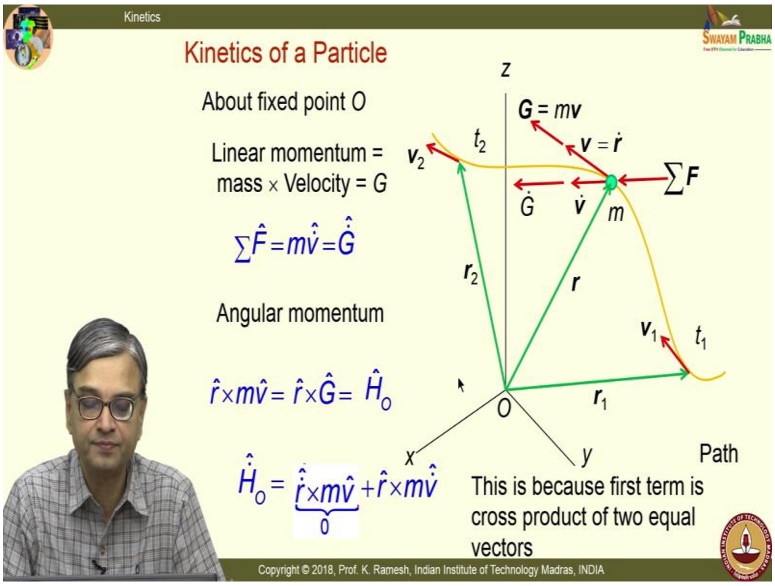

And we will start from kinetics of a particle, move on to system of particles then finally, rigid body and we would view this about a fixed point ok, I have a fixed frame of reference and I have an arbitrary path specified like this, I have a particle which is moving along this path and we could also label the positions like a position vector, r_1 at time t_1 and it has a velocity v_1.

It moves to some other point and we have this as a generic point, I have this as position vector r, I have the velocity is nothing but $\dot{\hat{r}}$ tangential to the path and it moves to the another point, at time t_2, I call this as r_2, I have this velocity as v_2 at time t_2. Many of these quantities you might have seen earlier in your courses, it is a more of a recapitulation and today is a day where you will be writing equations after equations, in a very comprehensive manner that will be good to have notes like this.

So, the particle has a mass m then, I can also define linear momentum, I have this as mass into velocity and I the standard symbol used for this G. So, do not confuse this with the center of gravity G. So, learn it from the context, interpret the symbol out of the context. So, I have $G = mv$, mass into velocity and I have a net force acting on the particle and I can write the acceleration that is $\dot{\hat{v}}$ in this direction and we also have the relation $\Sigma\hat{F}=m\dot{\hat{v}}=\dot{\hat{G}}$, that is rate of change of linear momentum.

So, I have this rate of change of linear momentum written down like this, one can also write the angular momentum and we have already said that we will do this about fixed point *O*, see in this chapter, when we move on to system of particles and rigid bodies, we would like to look at the mass point *G*, as well as an arbitrary point *V*. And, I have $\hat{r}\times m\hat{v}=\hat{r}\times\hat{G}=\hat{H}_O$, I get this as angular momentum, these all standard symbols, capital H_O. *O* depicts about which point, I have would this angular momentum and I can also look at the rate of change of angular momentum.

So, I can expand this and write it $\dot{\hat{r}}\times m\hat{v}+\hat{r}\times m\dot{\hat{v}}$ and you know very well, this goes to 0. So, I have $\dot{\hat{H}}_O$ = $\hat{r}\times m\dot{\hat{v}}$ and from simple mathematics you will know that the first term is cross product of 2 equal vectors that goes to 0, that is what is shown there.

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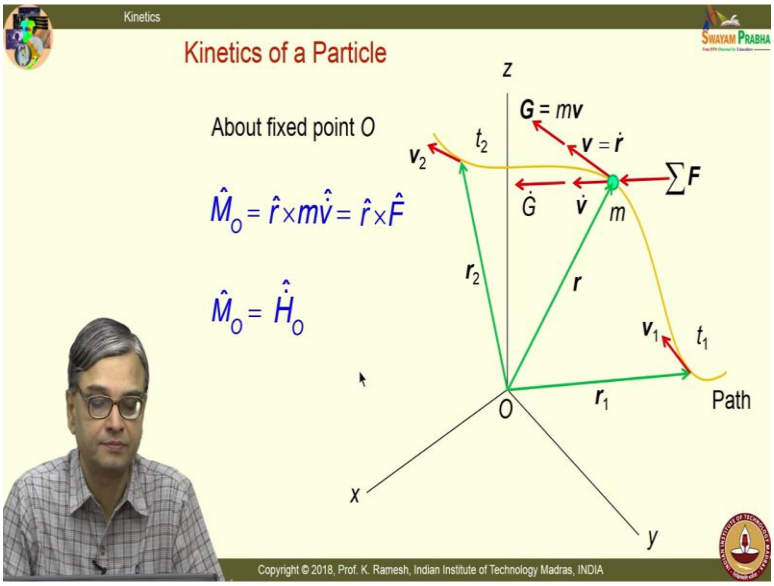

I can also label this as moment about *O* that as $\hat{M}_O=\hat{r}\times m\dot{\hat{v}}=\hat{r}\times\hat{F}$. So, you can go back to your statics and then look at meaning of these expressions and we get a relationship moment about *O* is nothing but rate of change of angular momentum about *O*.

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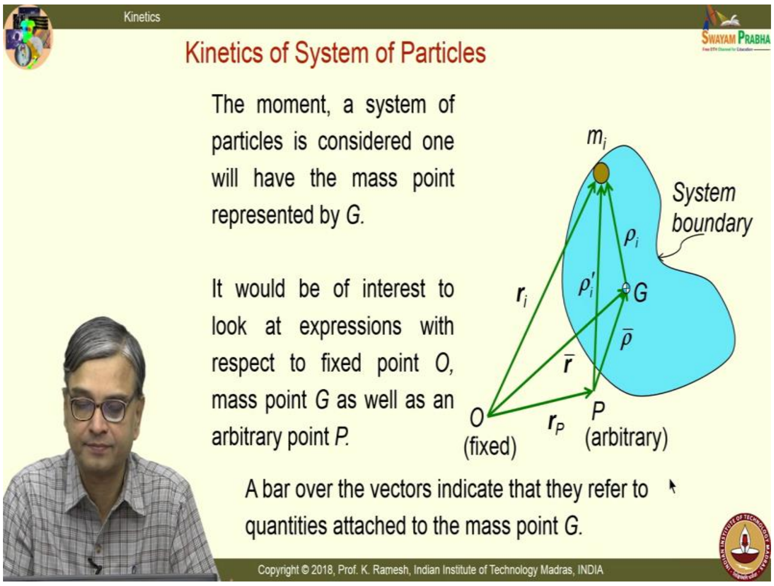
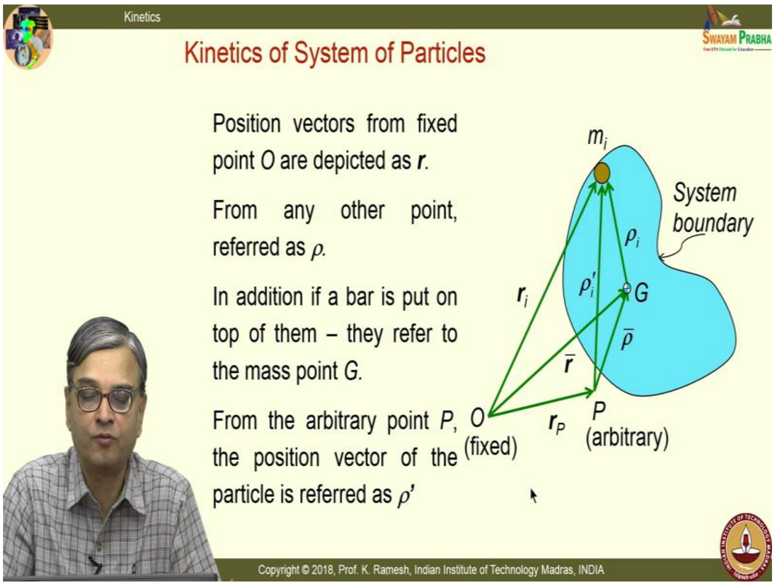
Now, we move on to system of particles, I have a system of particles like this, I enclose this by a system boundary, I have a mass point. See many natural phenomena you have system of particles, you see birds flying, when they go from one migratory location to another migratory location, they form a pattern. Similarly, fish, they also find within the sea, they form in a group and then go and look at Amazon forest, you have animals going in groups, something drives them to form into a group and then go. You can really see how the system of particles will move by looking at what happens at the mass point, provided no animal is killed by the predator fine because, we you are looking at a system, where the mass remains constant, we are not looking at the system where mass changes.



So, I have a representative point and we will also have some discipline in labeling them. So, we are going to develop expressions with respect to fixed point *O*, mass point *G* and an arbitrary point *P*. For clarity, I will put the arbitrary point outside the object here. So, that the diagram looks cleaner, when I say arbitrary point, it can be anywhere in, space that is convenient for us to do the calculations later and you should recognize when I have an arbitrary point, that point will have an acceleration, it is not like a fixed point *O*, which is fixed in space. So, that is the difference, I have an initial frame of reference attach to point *O*, such a restriction is not there for *G* as well as *P*. The points can have accelerations.

So, *O* is a fixed point and I have the position vector and you have a discipline, whenever I put a bar that indicates that I am dealing with the mass point. Quantities attached to the mass point *G*, we will have a discipline to put a bar over the quantity and any arbitrary point is located by position vector r_i, I can also view this point from *G*, I can also view this point from arbitrary point *P*. So, we will also have to label the position vectors appropriately, I am following the same symbolism use by Merriam. So, I have this position vector, represented by ρ, I have ρ_i, from the mass point G and I have an arbitrary point P, located at a position vector r_P, for clarity I am putting it outside, it can be anywhere that is convenient to solve a given problem.

So, when I want to find out what is the position vector, I label it with a prime, instead of simple ρ, I put this as ρ'. So, symbolism that is used is, I reserve the symbol r, from the fixed point. I use ρ from the mass point, I use ρ' from arbitrary point. I can also have a position vector connecting P and G. Now, I am going to attach this to a mass point. So, I will have a bar over it. So, I will call this as $\bar{\rho}$. So, I am consistent in my symbolism, the symbolism is consistent. So, once you understand the symbolism, you can easily write down these position vectors comfortably.



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So, this is reiterated here, position vectors from fixed point O are depicted as r, from any other point referred as ρ. In addition, if a bar is put on top of them, they refer to the mass point G, ρ or ρ' for the arbitrary point we put it as ρ'. From the arbitrary point P, the position vector of the particle is referred as ρ'. It better to have a convention so, that you do not get lost; when you have an array of equations, for you to interpret, we can recall from memory, what way we have labeled them.

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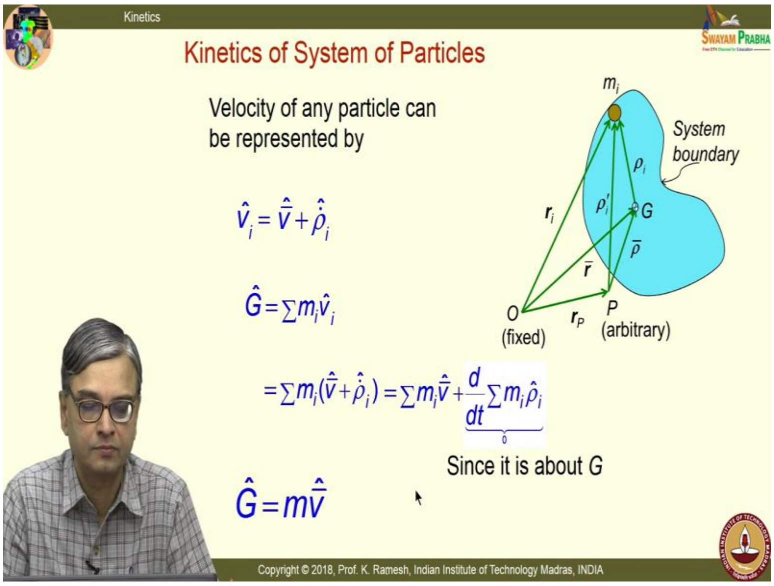

So, velocity of any particle can be represented by $\hat{v}_i = \hat{\bar{v}} + \hat{\dot{\rho}}_i$. So, we are going to get advantage of system of particles, where the mass is not changing. So, I have *G* as $\sum m_i(\hat{\bar{v}} + \hat{\dot{\rho}}_i)$, that reduces to $\sum m_i \hat{\bar{v}} + \frac{d}{dt}\sum m_i \hat{\rho}_i$, can you see something interesting here? You know we are going to do this kind of simplification, again and again. We recognize that we are looking at from a mass point.

So; obviously, this has to go to 0. Since it is about *G*, we would be doing the simplification again and again ok. Some of those things I would skip the steps also, for saving time and you must be able to figure out from the expressions correctly. So, I have a very interesting relation, $\hat{G} = m\hat{\bar{v}}$. Very well-known expression, it is not something new to you; we are only revisiting the expressions that you know already.

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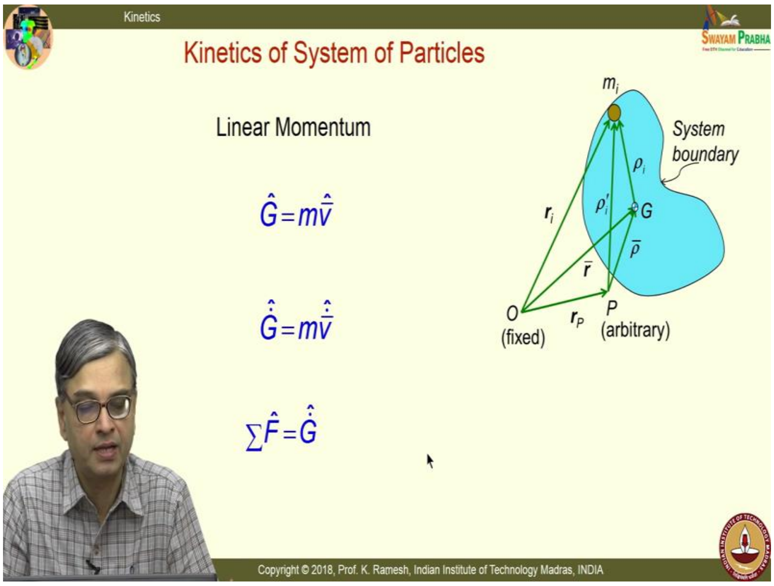

Then I go for linear momentum, $\hat{G} = m\hat{\bar{v}}$ and rate of change of linear momentum I get this as $\dot{\hat{G}} = m\dot{\hat{\bar{v}}}$ and $\sum \hat{F} = \dot{\hat{G}}$, then we can also move on to angular momentum about *O*.

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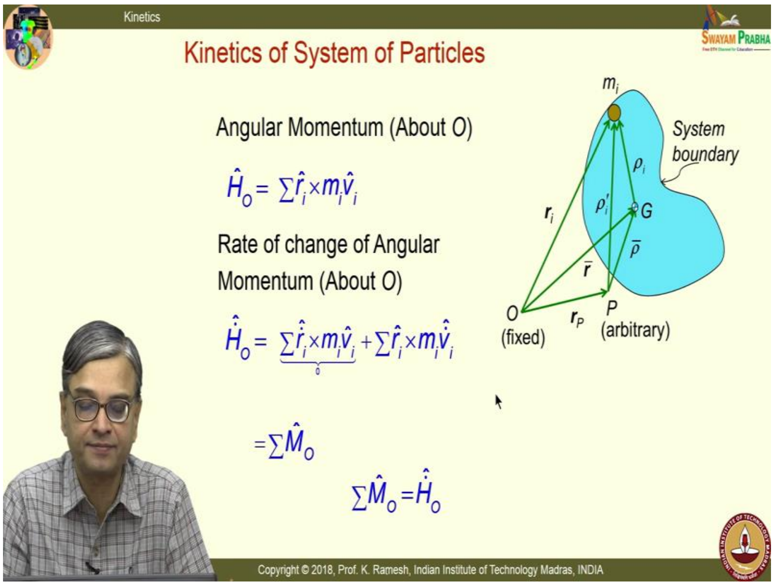
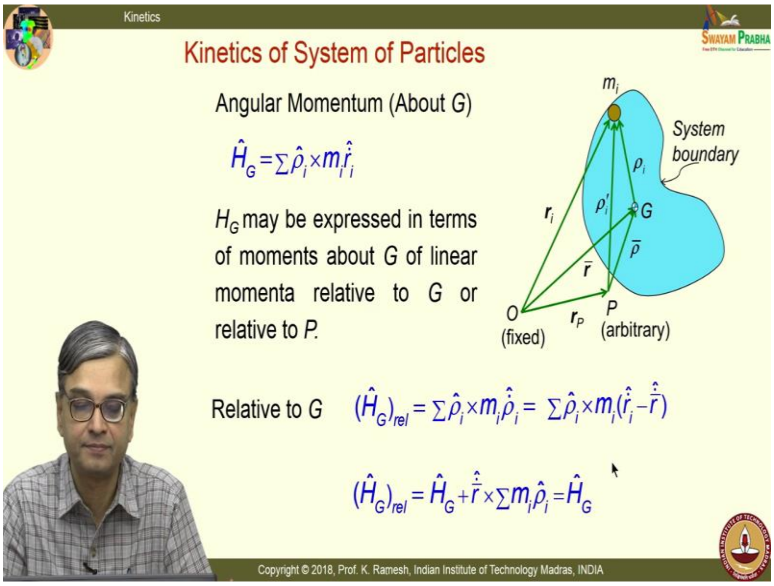
So, I have $\hat{H}_O = \sum \hat{r}_i \times m_i \hat{v}_i$. So, the position vector is r that is what you refer from *O*. So,



$\hat{r}_i \times m_i \hat{v}_i$ and when I look at rate of change of angular momentum, differentiate a correctly by applying the product rule, do not swallow up any terms. So, when I have $\dot{\hat{H}}_O = \underbrace{\sum \dot{\hat{r}}_i \times m_i \hat{v}_i}_{0} + \sum \hat{r}_i \times m_i \dot{\hat{v}}_i$, we would also simplify this. This goes to 0 and I have this as $\dot{\hat{H}}_O$ equal to moment about *O* and I have the identity summation of moment about O is nothing but rate of change of angular momentum about *O*. You know, once we come to a system of particles, we would also look at about point *G*, as well as point *P*, those are all the expression which are of interest for us to handle them later.



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How do I define angular momentum about *G*? So, look at here, this does not change, this is again *mV* but, the vector, which I use now is ρ_i, instead of r_i, it is ρ_i. How can I express this relationship? You have to understand this as a definition, we would also develop another symbolism, here I have set angular momentum about *G*, I can also have angular momentum $(H_G)_{rel}$, when I replace this r_i as ρ_i, I will call that as $(H_G)_{rel}$. So, H_G may be expressed in terms of moments about G, of linear momenta relative to G or relative to P

These become convenient, when we want to extend it for rigid body mechanics, when I derive the expressions, we will use these identities cleverly to minimize our computations but, the final expression which you are going to do for solving problems are very simple right now, you will have array of equations coming up one after the other, playing with symbols and so on and so forth, it is good that you have this as the summary in your notes.

So, relative to G, I define $(H_G)_{rel}$, I have already told you how I am going to define $(H_G)_{rel}$. I would call this as, $(\hat{H}_G)_{rel} = \Sigma \hat{\rho}_i \times m_i \dot{\hat{\rho}}_i = \Sigma \hat{\rho}_i \times m_i (\dot{\hat{r}}_i - \dot{\hat{\bar{r}}})$, when I have this as angular momentum about G, I use the appropriate position vector but when I put this as a relative quantity, I also play with the velocity, I am using a relative velocity. In some cases, it is easier to handle relative velocity computations and I would replace $\dot{\hat{\rho}}_i$ as $(\dot{\hat{r}}_i - \dot{\hat{\bar{r}}})$ because, ultimately you are going to simplify the expressions and you will get a very nice simplification here and I am skipping some of those steps, you know you have to expand this and bring in what are the knowledge that you have and also switch between the product instead of $\hat{\rho}_i \times \dot{\hat{\bar{r}}})$, I would switch it, when I switch it, the sign also will change.

So, all that is done and I have the expression like this, I could rewrite this expression in this fashion, I have initially written down the definition, replace this $\dot{\hat{\rho}}_i$ as $(\dot{\hat{r}}_i - \dot{\hat{\bar{r}}})$, when I rewrite this I could write it like this, taking on this identity, I can write this as H_G. Can you see something interesting here? What is the value of this term? I have a mass point. So, I have summation of $m_i\rho_i$ becomes what? That goes to 0. So, you have a very important relation, $(H_G)_{rel}$ is same as H_G that is identity you get, it is the very interesting identity, we have now done it with respect to relative to G; we will also do it relative to point P.

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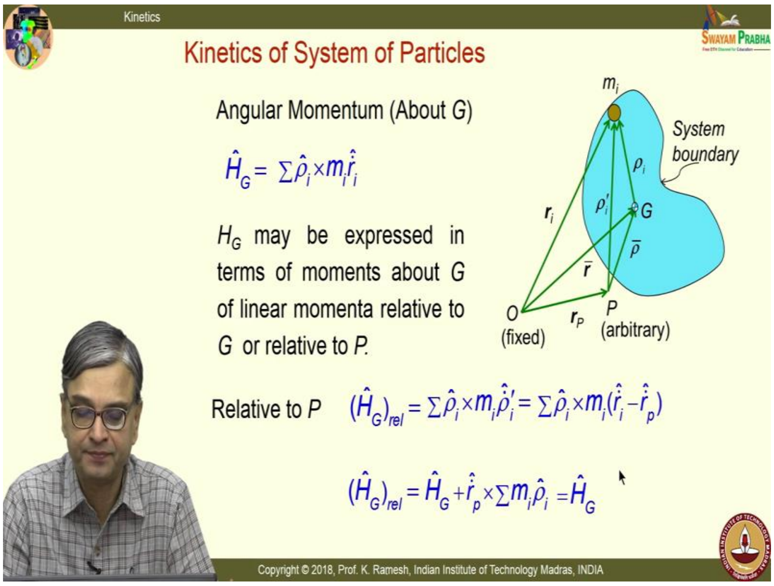
So, I have the same expressions here, I have written down what is H_G. Now, I do it relative to P. So, when I write it relative to P, I have this $(\hat{H}_G)_{rel} = \Sigma\hat{\rho}_i \times m_i\hat{\dot{\rho}}_i' = \Sigma\hat{\rho}_i \times m_i(\hat{\dot{r}}_i - \hat{\dot{r}}_p)$. So, I have $\hat{\dot{\rho}}_i'$, $\hat{\dot{\rho}}_i'$ is what I am going to do and this could be replaced in terms of the vectors and you put the product and group the terms appropriately, I can write it like this.



Here again, I get an identity, $(\hat{H}_G)_{rel} = \hat{H}_G + \hat{\dot{r}}_p \times \Sigma m_i\hat{\rho}_i = \hat{H}_G$, it is a very interesting identity, whenever we take advantage of it, we can compute it from relative quantities or the definition what is shown here. Please write this down and I hope it is free of typographical errors, if there are any errors please bring it to my attention.

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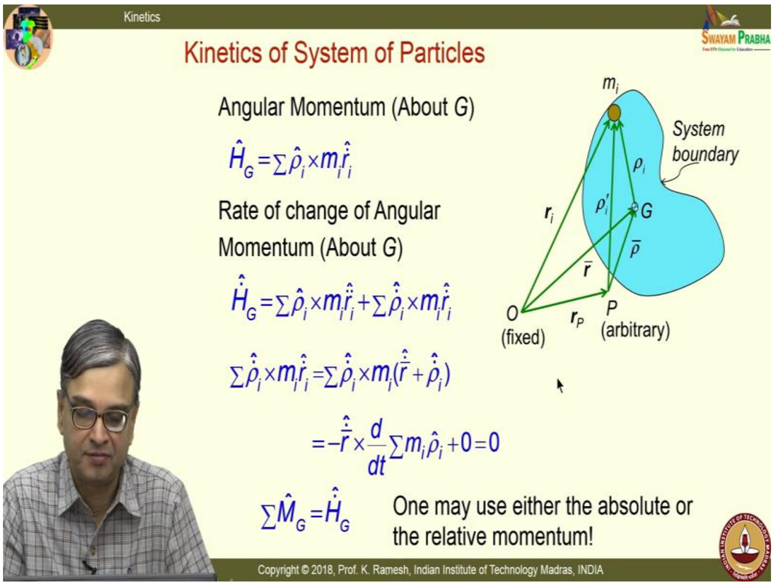
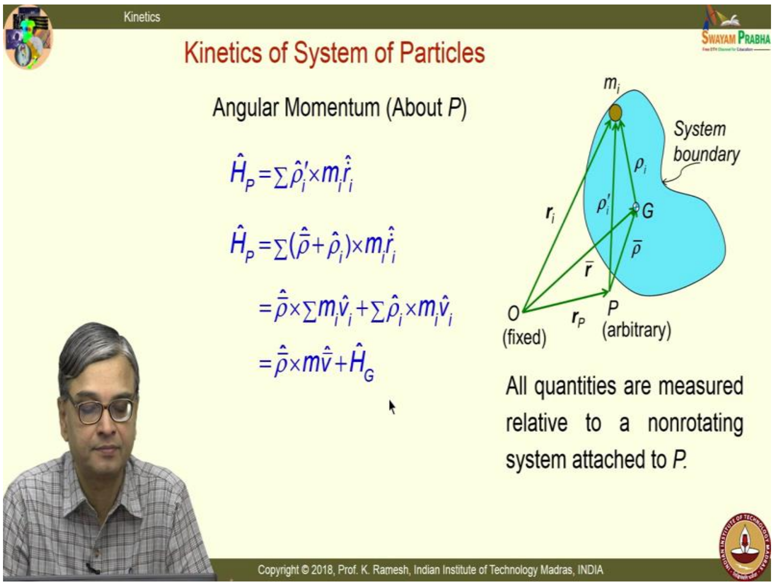


So, I have $\hat{H}_G = \Sigma\hat{\rho}_i \times m_i\hat{\dot{r}}_i$ and I have rate of change of angular momentum. So, I differentiate it by applying the product rule properly and when I look at this, this can be simplified further and you have a very interesting relationship here. So, $\hat{\dot{r}}_i$ can be replaced as $(\hat{\dot{\bar{r}}} + \hat{\dot{\rho}}_i)$ and finally, the whole thing reduces to 0.

So, this term goes to 0 that is what is shown here. Please take your time to write down, I have not done the simplifications step by step, intermediate steps I have skipped, fill in those steps in your notes later, not now in the class, you can go back and fill in the missing steps. So, I get $\sum \hat{M}_G = \dot{\hat{H}}_G$.



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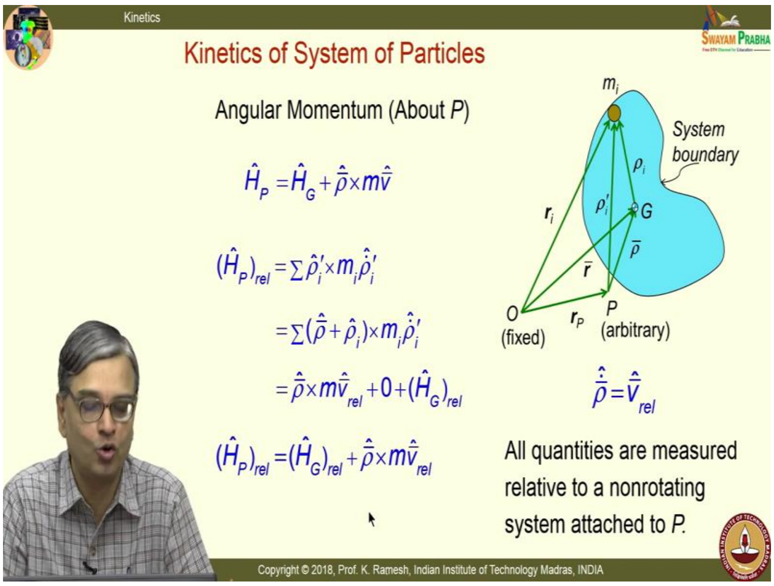
So, we will write angular momentum about P and when I say point P, you can imagine that there is a non rotating axis attached to P. When I say a non rotating axis attached to P, that axis can have acceleration, you do not lose sight of it, the point P what I take it is not a static point in space, it can have acceleration.

So, I have $\hat{H}_P = \sum \hat{\rho}_i' \times m_i \dot{\hat{r}}_i$, similar simplifications we have done for the earlier case, we will continue to do it here. So, I will replace $\hat{\rho}_i'$ as $\bar{\hat{\rho}} + \hat{\rho}_i$, I have $\bar{\hat{\rho}} + \hat{\rho}_i$, I get this $\hat{\rho}_i'$. I get this expression, then simplify this and then knock of terms that go to 0. So, I can replace from the earlier definition, this as simply H_G. So, for an arbitrary point, I can write this angular momentum, consisting of two terms, one from the mass point G, plus $\bar{\hat{\rho}} \times m \bar{\hat{v}}$.



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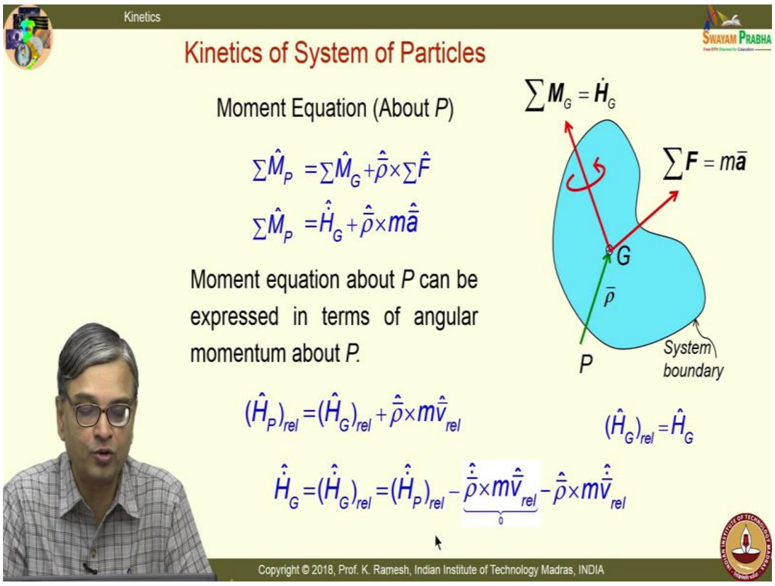
So, that is what is rewritten here in an elegant fashion, about a point *P*, I can write it as $\hat{H}_P = \hat{H}_G + \hat{\bar{\rho}} \times m\hat{\bar{v}}$ and I can also define what is $(H_P)_{rel}$. See we have been consistent in defining these quantities, when I say put relative, I replace by the relative velocity. So, here it is $\hat{\dot{\rho}}_i'$ is the velocity now and this also can be simplified; I can replace this as $\Sigma(\hat{\bar{\rho}} + \hat{\rho}_i) \times m_i \hat{\dot{\rho}}_i'$.

So, when I also bring in another interpretation, you know there is positive of symbols. When I say $\hat{\dot{\bar{\rho}}} = \hat{\bar{v}}_{rel}$, do not confuse it with your V_{rel} what you have seen in a rotating frame of reference. So, these are two different contexts interpret it in the current context, we are just defining $\hat{\dot{\bar{\rho}}} = \hat{\bar{v}}_{rel}$ ok, the definition is given right here, definition of this terminologies given right here.

So, I could rewrite this as do the simplification yourself later. So, I get this as $\hat{\bar{\rho}} \times m\hat{\bar{v}}_{rel} + 0 + (\hat{H}_G)_{rel}$. So, I have $(\hat{H}_P)_{rel} = (\hat{H}_G)_{rel} + \hat{\bar{\rho}} \times m\hat{\bar{v}}_{rel}$ that is a final identity I get. These are multiple equations; it will look as if it is a bit confusing but, all these are easily derivable very systematic, it is better to have all these equations in your notes.



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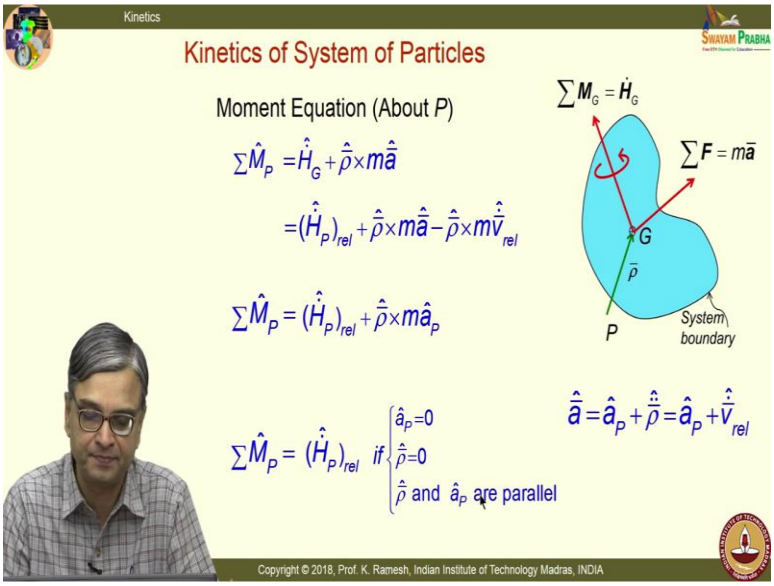
So, we will look at the moment equation about *P*. So, I have put the net force acting on the system of particles as $\Sigma F = m\bar{a}$ and any quantity I am writing it with respect to the mass point, I always put a bar. The same convention is used, the position vector is put with bar, the acceleration is also put with a bar; velocity is also put with a bar. Whenever I have a bar over it, it indicates that I am talking about mass point quantities; I also have a moment M_G, that is equal to rate of change of angular momentum about G.

So, I have $\sum \hat{M}_P = \sum \hat{M}_G + \hat{\bar{\rho}} \times \sum \hat{F}$. So, this is nothing but $\sum \hat{M}_P = \dot{\hat{H}}_G + \hat{\bar{\rho}} \times m\hat{\bar{a}}$ and we would also rewrite it in multiple different ways, which can be used later for simplification. So, the moment equation about P, can be expressed in terms of angular momentum about P. We have already looked at $(\hat{H}_P)_{rel} = (\hat{H}_G)_{rel} + \hat{\bar{\rho}} \times m\hat{\bar{v}}_{rel}$ and we also know the identity $(\hat{H}_G)_{rel} = \hat{H}_G$ and we have also seen what is the definition of this velocity $\dot{\hat{\bar{v}}}_{rel}$, you should not confuse with the rotating frame of reference, in this context it has a particular interpretation.

So, I have $\dot{\hat{H}}_G = (\dot{\hat{H}}_G)_{rel} = (\dot{\hat{H}}_P)_{rel}$ and take it to the other side, you do the differentiation properly and write all the terms, then bring in the simplification. So, I get this as $\underbrace{\dot{\hat{\bar{\rho}}} \times m\hat{\bar{v}}_{rel}}_{0} - \hat{\bar{\rho}} \times m\dot{\hat{\bar{v}}}_{rel}$, very simple it straightforward I have just differentiated this and then written it here and this goes to 0. So, I have expression for H_G. So, this could be substituted in this equation fine. So, that is what I am going to do.



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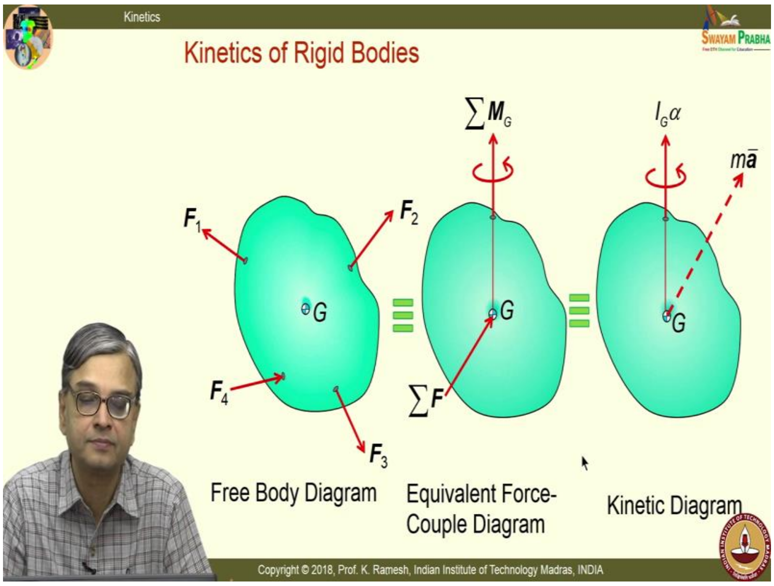
I am going to write, we have seen $\sum \hat{M}_P = \dot{\hat{H}}_G + \hat{\bar{\rho}} \times m\hat{\bar{a}}$, replace $\dot{\hat{H}}_G$, as $(\dot{\hat{H}}_P)_{rel} + \hat{\bar{\rho}} \times m\hat{\bar{a}} - \hat{\bar{\rho}} \times m\dot{\hat{\bar{v}}}_{rel}$. This is just reproduced the equations that I have written down in the previous slide and you also have another identity, see when I am looking at a point *P*, if I have the acceleration *a*, that is $\hat{\bar{a}} = \hat{a}_P + \ddot{\hat{\rho}}$. So, I can write this as $\hat{\bar{a}} = \hat{a}_P + \ddot{\hat{\rho}} = \hat{a}_P + \dot{\hat{v}}_{rel}$.

So, I would replace use this and then rewrite the expression in a different fashion. So, I can write $\sum \hat{M}_P = (\dot{\hat{H}}_P)_{rel} + \hat{\rho} \times m\hat{a}_P$. So, the idea is, the point *P* has an acceleration a_P. So, I can write the moment about point P, in terms of the angular momentum rate of change of angular momentum about *P*, plus $\hat{\rho} \times m\hat{a}_P$. This is the most general expression. So, I have M_P becomes $(\dot{\hat{H}}_P)_{rel}$, if any of these 3 conditions are satisfied.

$$\sum \hat{M}_P = (\dot{\hat{H}}_P)_{rel} \quad if \begin{cases} \hat{a}_P = 0 \\ \hat{\rho} = 0 \\ \hat{\rho} \text{ and } \hat{a}_P \text{ are parallel} \end{cases}$$

So, the second term goes to 0, when any one of these conditions are satisfied, then I have $\sum \hat{M}_P = (\dot{\hat{H}}_P)_{rel}$. See this is nothing but, playing with mathematics, you get multitude of expressions, we have looked at identities in between them, we will use them judiciously later.



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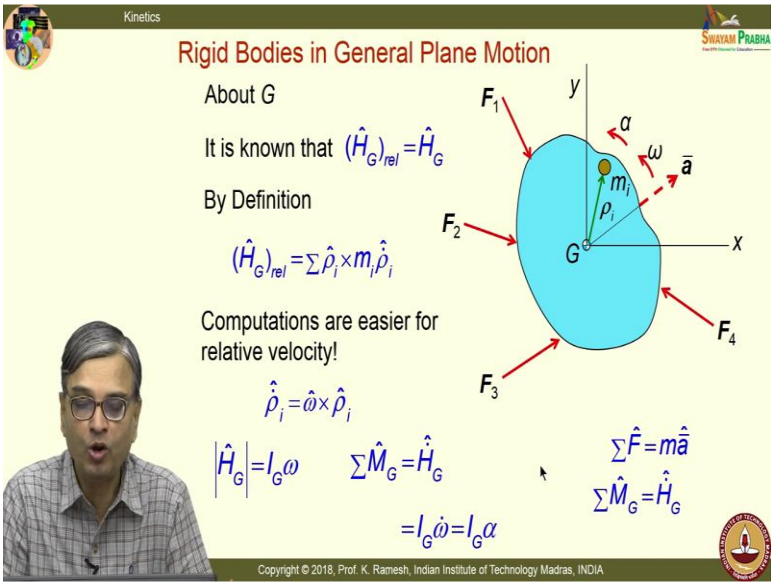
Now, let us move on to kinetics of rigid bodies. So, imagine that I have a rigid body like this, acted upon by arbitrary system of forces. Under the action of these forces, we have already seen, that the body will have a translation as well as a rotation and we call that as a general plane motion, if the body has translation as well as rotation put together, we call that as general plane motion.

So, this is a free body diagram and that is equivalent to saying that, this as a summation of forces, as well as a summation of moments, this has a translation, as well as a rotation and you have this as equivalent force couple diagram and you also bring in another terminology here, you call something as kinetic diagram and I have this as a kinetic diagram. This has and translation like this and a rotation dictated by $I_G\alpha$ and this is known as a kinetic diagram so, in many of the problems when we do it in kinetics.

We would like to have the kinetic diagram properly represented. So, that you visualize what happens to the rigid body, use those quantities appropriately in doing the computations. So, I have a basic free body diagram, which could be reduce to a simpler representation of equivalent force couple diagram, which can also be viewed as a translation and a rotation, all respect to the mass point *G*.



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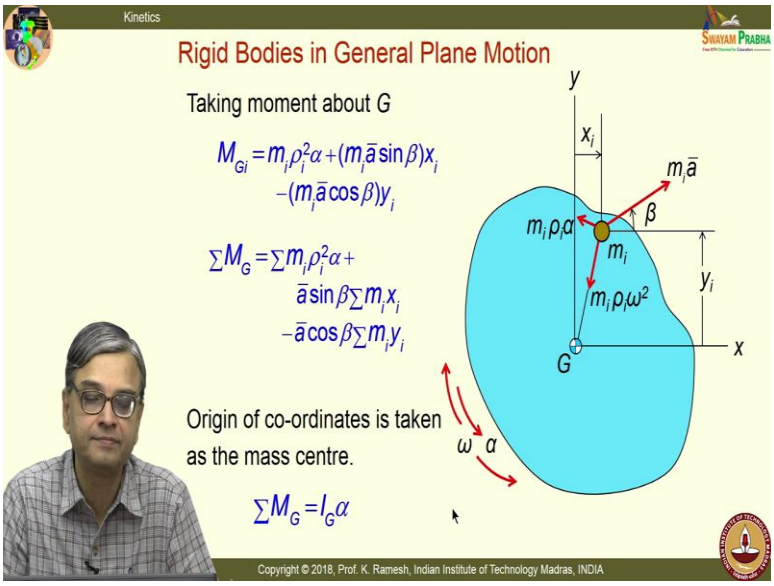
So, we will look at rigid bodies in general plane motion and we will look at about G. So, we have these identities, $\Sigma\hat{F} = m\hat{\bar{a}}$ and $\Sigma\hat{M}_G = \dot{\hat{H}}_G$. So, I have a system of forces acting on the rigid body and I have reference axis *X* and *Y*, let me take a let me also represent the accelerations and rotation of this and take an arbitrary point, we have the identity $(\hat{H}_G)_{rel} = \hat{H}_G$. I am going to use this, I take an arbitrary point on the rigid body, this mass point has mass of m_i and the distance is $\dot{\hat{\rho}}_i$ and I have already said that, we know $(\hat{H}_G)_{rel} = \hat{H}_G$.

So, instead of writing H_G from the first principle, I will write HG rel that is easier for me to write and use the expression by definition, I get $(\hat{H}_G)_{rel} = \Sigma \hat{\rho}_i \times m_i \dot{\hat{\rho}}_i$. So, this is where we use the identity, we have proved earlier, $(\hat{H}_G)_{rel} = \hat{H}_G$. Now, my interest is H_G but, I will just estimate H G rel from the definition basic definition and simplify it. Computations are easier for relative velocity, that is the advantage, I have and I can write from our understanding of rigid bodies and general plane motion, $\dot{\hat{\rho}}_i = \hat{\omega} \times \hat{\rho}_i$.

So, when I substitute here, I can simplify it as H_G the magnitude, turns out to be $|\hat{H}_G| = I_G\omega$ and when I take the differentiation of that, I have $\dot{\hat{H}}_G$, I get this as $I_G\dot{\omega}$, $\dot{\omega}$ is nothing but, $I_G\alpha$. This is one way of deriving the expression; we have taken the advantage of our earlier identity, $(\hat{H}_G)_{rel} = \hat{H}_G$. I can also do it from another fundamental prospective, we will derive this expression from both the ways ok, here I have use this identity and I could directly come and say that $\sum \hat{M}_G = I_G\alpha$ ok, that is a final expression that will be of used to us repeatedly, we will also look at from a different prospective.



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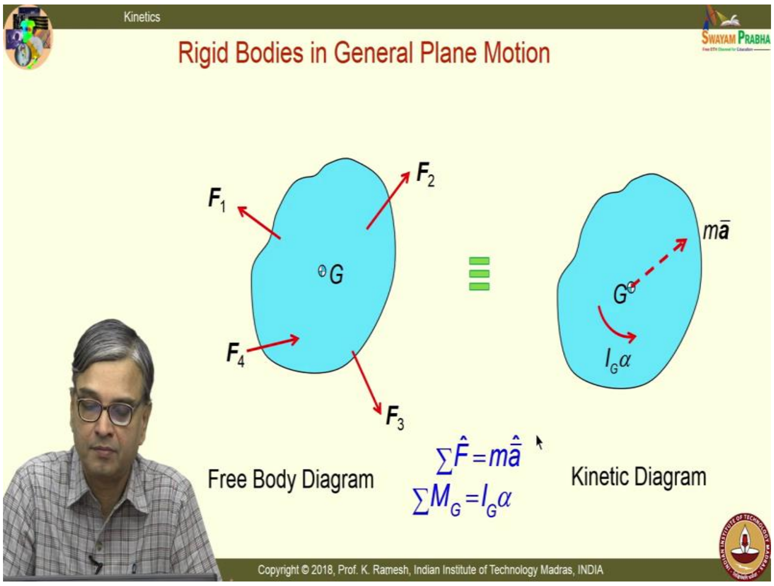
For clarity I have shown it little bigger ok. So, make a neat sketch of it, I have the representative point and which is located at a distance of Y i and X i like this and the mass point is put as m i, saying that it is a generic point and I have a acceleration of this as $m_i\bar{a}$ and I have the force, which is put as $m_i\bar{a}$ and this body has a angular velocity, as well as the angular acceleration and I have this as α.

So, these are all the other components of forces which you get and I can write M_{Gi}, if I want to find out moment about the point G,

$$\sum M_G = \sum m_i \rho_i^2 \alpha + \bar{a}\sin\beta \sum m_i x_i - \bar{a}\cos\beta \sum m_i y_i$$

you have done the Varignon's theorem, it is nothing but, writing it from the Varignon's theorem and we have a very interesting when I sum them up, the terms go to 0, that is what we are going to look at. So, you know very well, that we are writing this with respect to the mass point G.



So, by definition, we go to 0 and I simply get $\sum M_G = I_G\alpha$. So, the whole idea is, moment about G, in a rigid body reduces to your moment of inertia I_G into α, α is your angular acceleration of the body. We have derived it in one way earlier, we have re derived it from first principles in this slide, the final result is this, it is a very useful result.

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And when I have a rigid body in general plane motion, I always recognize $\sum \hat{F} = m\hat{\bar{a}}$ and $\sum M_G = I_G\alpha$. We will now look at, what are all the simple motions that we know off, we have looked at translation, we have looked at curvilinear translation, we also have general plane motion, we also have fixed axis rotation. So, what we are going to do is, simplify these expressions, for the given context, any one of these simplified motions.

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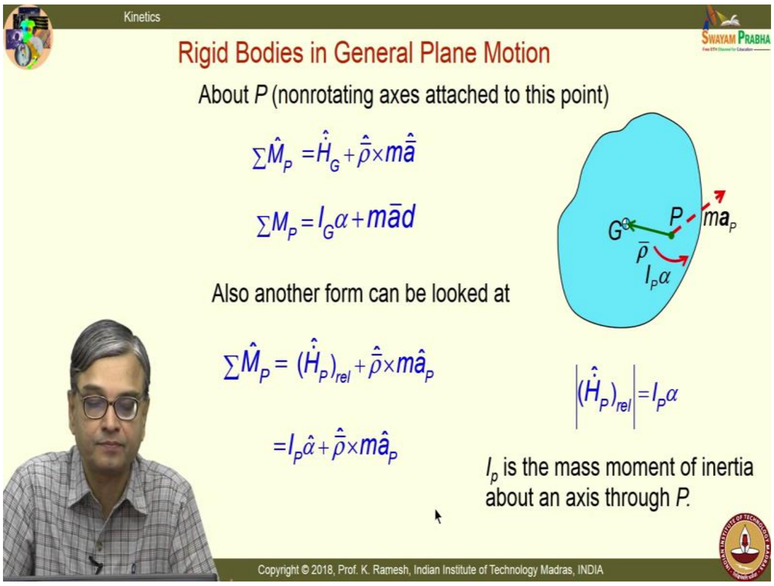

And about point P, just emphasize that we have attached a non rotating axes attached to this point, I can also write the expressions and for generality, you know the point P can be on the body, can be outside. So, I have taken it between the body here and I represent any quantity related to the mass point with a bar.

So, I have $I_G\alpha$, $m\hat{\bar{a}}$ and $\sum\hat{M}_P = \dot{\hat{H}}_G + \hat{\bar{\rho}} \times m\hat{\bar{a}}$ and I have $\sum M_P = I_G\alpha + m\bar{a}d$, *d* is the perpendicular distance. So, I can write this, simply as $\sum M_P = I_G\alpha + m\bar{a}d$ it is a very important expression you will repeatedly use it later on in solving problems, also another form can be looked at. Now, we would like to get ourselves armed with different forms of these expressions, depending on the context of the problem, we would invoke one of this and you know I can also recognize that the point *P*, has an acceleration a *P* then, I can put the differently, I can also have $I_P\alpha$.

So, $\sum\hat{M}_P = (\dot{\hat{H}}_P)_{rel} + \hat{\bar{\rho}} \times m\hat{a}_P$, go back and then see which we have derived it earlier, I have invoke that and $(\dot{\hat{H}}_P)_{rel}$ is nothing but, $I_P\alpha$, the magnitude of this is nothing but, $I_P\alpha$. So, I I P is the mass moment of inertia about an axis through *P* and I can write this M_P as $I_P\hat{\alpha} + \hat{\bar{\rho}} \times m\hat{a}_P$, where a_P is the acceleration of the point *P*. So, these are again another representation, either I could use this or I could use this depending on, which way I would like to solve the problem.

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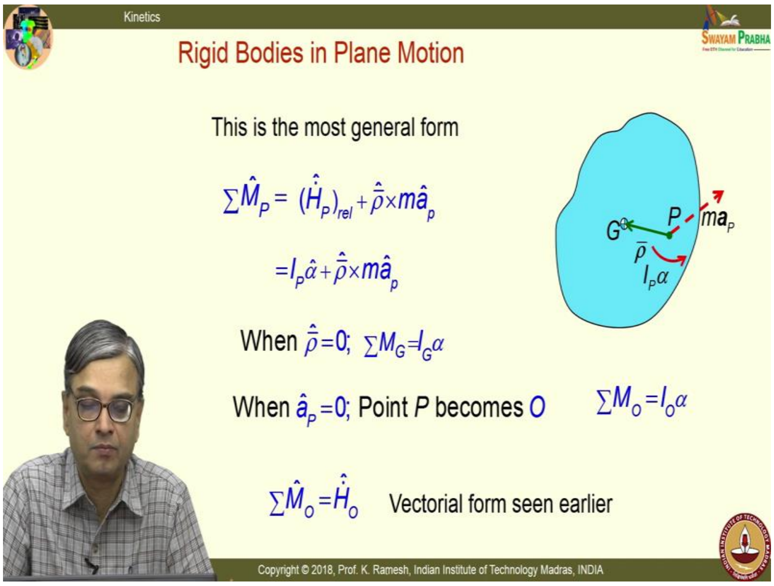
And the most general form is this, $\sum \hat{M}_P = (\dot{\hat{H}}_P)_{rel} + \hat{\bar{\rho}} \times m\hat{a}_p$ and I can put this as $I_P\hat{\alpha} + \hat{\bar{\rho}} \times m\hat{a}_p$ and I could look at specific conditions, when $\hat{\bar{\rho}} = 0$;, P merges with G. So, I have this as $\sum M_G = I_G\alpha$, we have also seen it earlier, $\sum M_G = I_G\alpha$. When a P = 0, point P becomes O, the definition of fixed point is you have an inertial frame attached to this, the point does not move.



So, a_P becomes, when a P become 0; point P becomes O and I have $\sum M_O = I_O\alpha$ or and I have $\sum \hat{M}_O = \dot{\hat{H}}_O$. So, this is the most general form of representation.

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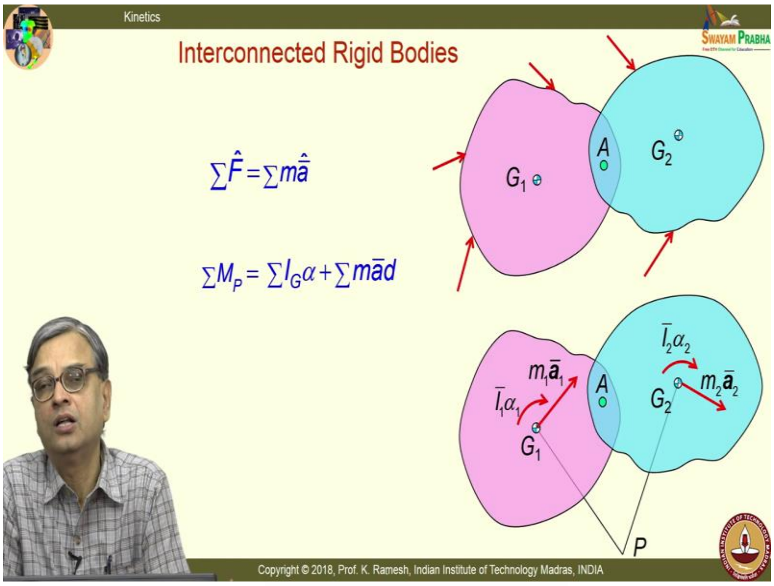

And you can also extend it for interconnected rigid bodies, you can also put the kinetic diagram for the interconnected rigid bodies. So, I have $\sum M_P = \sum I_G\alpha + \sum m\bar{a}d$. So, I have a summation here and I have $m_1\bar{a}_1$, $m_2\bar{a}_2$ and I have $\bar{I}_1\alpha_1$ and $\bar{I}_2\alpha_2$, I also have an arbitrary point P located vectors.

So, in this class, you know we have looked at systematically kinetics of a particle followed by system of particles and then finally, to rigid body. We have got the most general expressions and we would reduce them for specify rigid body motions for translation, curvilinear translation and general plane motion in the subsequent class.

Thank you.