

Engineering Mechanics
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Module - 02
Dynamics
Lecture - 13
Kinetics II

Let us continue our discussion on Kinetics

Kinetics of Rigid Bodies

The slide illustrates the transition from a physical rigid body to its kinetic representation. It shows three stages:

- Free Body Diagram:** A rigid body with centroid G and four forces F_1, F_2, F_3, F_4 acting on it.
- Equivalent Force-Couple Diagram:** The forces are replaced by a single resultant force $\sum F$ acting through G and a resultant couple $\sum M_G$.
- Kinetic Diagram:** The rigid body is shown with its mass m and acceleration $m\bar{a}$ acting through G , and a resultant couple $I_G\alpha$ acting about G .

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See we have learnt what is a kinetics diagram when I have a rigid body, I have forces acting on it and we call this as a free body diagram. Then I have summation of forces acting through the centroid and summation of moments acting through the

centroid. This is a equivalent force couple diagram which could be visualized as in terms of a Newton's law $F = ma$ and we have learnt at length the moment is nothing, but rate of change of angular

momentum.

Rigid Bodies in General Plane Motion

Taking moment about G

$$M_{Gi} = m_i \rho_i^2 \alpha + (m_i \bar{a} \sin \beta) x_i - (m_i \bar{a} \cos \beta) y_i$$

$$\sum M_G = \sum m_i \rho_i^2 \alpha + \bar{a} \sin \beta \sum m_i x_i - \bar{a} \cos \beta \sum m_i y_i$$

Origin of co-ordinates is taken as the mass centre.

$$\sum M_G = I_G \alpha$$

The diagram shows a rigid body with mass elements m_i at positions (x_i, y_i) relative to the centroid G . The distance from G to the mass element is ρ_i . The acceleration \bar{a} is shown at an angle β to the horizontal. The body is rotating with angular velocity ω and angular acceleration α .

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So, I could visualize this as translation and a combination of rotation and this is known as a kinetic diagram. See what you will have to keep in mind is your graduating from particle dynamics to

rigid body dynamics. Particles do not rotate whereas, rigid bodies rotate.

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Even though I say rigid body is now I am essentially considering a planar situation and we would also confine our attention to rigid bodies in general plane motion that is very important. We have still not taken a rigid body of a hot shape and trying to find out the kinetics of it. We are taking baby steps; we have looked at particles; particles do not rotate and now we have taken planar objects which are rigid bodies in addition to translation it will also have rotation that is the only difference.

And take a moment about G , where G is the centroidal axis take a mass point and then label it is at a distance x_i and y_i and find out what are all the forces acting on it have m into a . And, in all our discussion in this chapter we would have the angular velocity and angular acceleration perpendicular to the plane fine. Very simple problem graduating from particles to planar rigid bodies or you can make an idealization as a planar situation and we confine our attention to general plane motion not a three-dimensional complicated motion it is only general plane motion.

And, you can easily write in the case of rigid body, we already learnt if I sit on any point and view any other point on the rigid body it would appear as if it is rotating about it, we had long discussions on that when we discuss the kinematics. On the basis of that strength only I am able to write these components of forces and I write the moment equation about G and you invoke that we are working on the centroid.

So, these terms go to 0 and I get a very elegant expression $\sum M_G = I_G \alpha$, where I_G is the mass moment of inertia, which you will have to learn you must have learnt it already in your earlier courses. Please brush up there are several theorems also you have parallel axis theorem and all that you brush it up for this course again I am not going to discuss that.

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And we will look at simple motions, you can have a translation. So, I have a four system something typical like this; this would essentially make the object to translate, I have a kinetic diagram like this the object will only translate if I make the board as smart, the object will only translate is the simplest of the situations that you can think of. And what

Translation

Free Body Diagram

Kinetic Diagram

$\Sigma \hat{F} = m \hat{a}$

$\Sigma M_G = I_G \alpha = 0$

On the whole one gets three scalar equations.

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are a governing equation? I

have $\Sigma \hat{F} = m \hat{a}$ and $\Sigma M_G = I_G \alpha = 0$, I can all also have it as $I_G \alpha = 0$.

So, I essentially have how many equations? If I put this into components, I essentially have three scalar equations. And what is the difference between

statics and dynamics? In statics right hand side was always 0, in dynamics it is not 0 for translation, for translation the rotation the moment equals to 0. So, we would graduate to other simpler motions we have initially looked at translation. And what was the next

thing we learnt in the context of rigid bodies?

Curvilinear Translation

Free Body Diagram

Kinetic Diagram

$\Sigma \hat{F} = m \hat{a}$

$\Sigma M_G = I_G \alpha = 0$

$F = ma$ needs to be written for n and t directions.

On the whole one gets three scalar equations.

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I can have a four system like this and the rigid body also can have a motion like this; carefully observe how the rigid body has moved. It had taken a curvilinear path, but it has not rotated about itself, it is a very

settle point and you will have to appreciate that you can also have curvilinear translation. And, when I have curvilinear translation all my quantities are dictated by the path, so it is

easier for me to have a normal and tangential coordinate and express the forces like this ok. And what are the governing equations I have? I still have $\Sigma \hat{F} = m\hat{a}$ and I also have the situation that rotation is 0.

I can have a curvilinear motion, but it is a curvilinear translation we would solve problem related to that then you will appreciate this. Here again I have essentially three scalar equations and it is desirable that you write $\Sigma \hat{F} = m\hat{a}$ in n and t directions it is convenient for you to do the mathematics. In all the problems you will also find how to choose the axes of reference for you to solve the problem.

And in this chapter, we would say that by your kinematical analysis you are got the absolute acceleration or accelerations from an inertial frame of reference from a kinematical analysis. Once you have that quantity then I can find out the forces, that is the way you have to interpret it they are not relative accelerations, they are measured

Fixed-axis Rotation

$$\Sigma \hat{F} = m\hat{a}$$

Scalar form of these equations are

$$\Sigma F_n = m\bar{r}\alpha^2 \quad \Sigma F_t = m\bar{r}\alpha$$

$$\Sigma M_G = I_G\alpha$$

Caution: While taking moment about G account for the forces at O

Free Body Diagram Kinetic Diagram

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from an inertial frame of reference.

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And you have the fixed axis rotation, I have an axis fixed and this is rotating about this. So, I have

$\Sigma \hat{F} = m\hat{a}$ and if I remove this pin you have to

remember how do you ideal as a pin joint; pin joint can support forces in two directions ok. That means, you put the components in two directions I have an arbitrary force you have to recognize that you should not forget how do you ideal as a pin joint. So, I will have an arbitrary force acting at the pin joint, this becomes the free body diagram I remove the support and a kinetic diagram will be like this is the kinetics diagrams, so it has a rotation and I have $\Sigma \hat{F} = m\hat{a}$ because I know the axis of rotation I can find out the tangential and normal components.

Scalar form of these equations are I can also replace the acceleration into $\bar{r}\omega^2$, you know in this book anything referred to the centroid we are putting a bar, \bar{r} means it is referring the centroid. So, $\Sigma F_t = m\bar{r}\alpha$ that is dictated by have axis of rotation like this, so I can write these quantities and I also have $\Sigma M_G = I_G\alpha$.

And there is a word of caution while taking moment about G when I take the moment about G when I write the LHS of it RHS you will not have difficulty you will write $I_G\alpha$, but when you want to write the LHS you must also account for the forces at O . Now to be very systematic, so the essential difference what you find here is compared to statics right hand side is non-zero in general.

We are taking a baby step moving from a particle to a planar rigid body, it is not a three-dimensional rigid body having a general motion we also say it has general plane motion or translation or the curvilinear translation or fixed axis rotation. We simplify the

problem to the extent possible.

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You can also write the expression from different perspective. So, it is easier to do it from a rotation axis O , I can also write this as $\Sigma M_O = I_O\alpha$. So, we can easily reduce it from the

Fixed-axis Rotation

It is generally useful to apply the moment equation about the rotation axis O

$$\Sigma M_O = I_O\alpha$$

The above equation can be easily deduced from the kinetic diagram

$$\Sigma M_O = I_G\alpha + m\bar{r}^2\alpha$$

$$= I_G\alpha + m\bar{r}^2\alpha$$

$$\Sigma M_O \triangleq I_O\alpha$$

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kinetic diagram. So, I have you have this parallel axis theorem $\Sigma M_O = I_G\alpha + m\bar{r}^2\alpha$. So, I have this $I_G\alpha + m\bar{r}^2\alpha$, so I can out this as $\Sigma M_O \triangleq I_O\alpha$.

In many problems this may be easier to do you can find out I naught. So, you should know how to get from I_G to I_O parallel axis theorem all these fundamentals please brush it

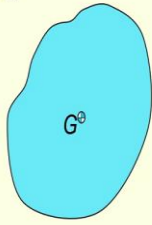
up. You already know for area moment of inertia; you must also look at for mass moment of inertia those details I would not be explaining it again fine.

Fixed-axis Rotation

Suppose the fixed-axis rotation is about G

$$\sum M_G = I_G \alpha$$

Further

$$\hat{a} = 0 \quad \text{and hence} \quad \sum \hat{F} = 0$$


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And, if I have a fixed axis rotation about G that is also possible, then this reduces to $\sum M_G = I_G \alpha$ and then further I have this $\hat{a} = 0$ and hence $\sum \hat{F} = 0$ ok.

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Then you also have a very interesting concept called centre of percussion, in statics we have already learnt this see in statics what did you learn if I have to move the force to a

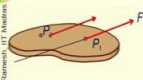
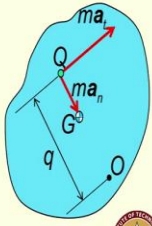
Centre of Percussion

In statics a force and a couple can always be represented by only a force at a suitable point.

One can identify the centre of percussion (point Q) in a dynamic problem such that the resultant of all the forces pass through it and the moment about Q is zero.

From fixed-axis of rotation, let point Q is located at a distance q,

$$m \bar{r} \alpha = I_o \alpha$$

$$q = \frac{I_o}{m \bar{r}} = \frac{k_o^2 m}{m \bar{r}} = \frac{k_o^2}{\bar{r}}$$



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point P_2 ; I need to replace it by a couple as well as a force or in other words if I have a couple and a force I can also find out a suitable location it is repaid only as a force. You have an equivalence of it in dynamics where you have the concept of centre of percussion and will see

what it is.

So, I have point Q, one can identify the centre of percussion point Q in a dynamic problem such that the resultant of all the forces pass through it and the moment about Q is 0. In fact, when people play golf whatever that bat and that they use for golf has centre of percussion is used you need to have at knowledge of it.

And I have this distance as Q ; how do you find out the Q ? The fixed axis of rotation let point Q is located at a distance q , this is very similar to the concept that you looked in statics fine, reverse of it there we replaced the force as a force and a couple here I have a rotation as well as this I make it only as a force there is no rotation.

So, I have $m\bar{r}\alpha q = I_o\alpha$ write the moment equation. So, I get $q = \frac{I_o}{m\bar{r}}$ if I express naught as radius of gyration square multiplied by mass then I have this and I have expression for q as $\frac{k_o^2 m}{m\bar{r}} = \frac{k_o^2}{\bar{r}}$. So, you can find out the centre of percussion, it is useful in a class of day to day applications. It is nothing, but reflection of the concept that you have already learnt in statics ok.

Useful Expressions

$$\Sigma \vec{F} = m\vec{a}$$

$$\Sigma F_x = m\bar{a}_x; \quad \Sigma F_n = m\bar{a}_n;$$

$$\Sigma F_y = m\bar{a}_y; \quad \Sigma F_t = m\bar{a}_t;$$

About P (nonrotating axes attached to this point)

$$\Sigma M_p = I_G\alpha + m\bar{a}d$$

Diagrams show a rigid body with center of mass G and pivot O . The distance between G and O is \bar{r} . The distance between G and point P is d . The acceleration of G is \vec{a} , with tangential component a_t and normal component a_n . The angular acceleration is α . The moment of inertia about G is I_G , and about O is I_O .

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Then we move on to solving problems. We also have a summary of useful expressions.

$$\Sigma \vec{F} = m\vec{a}$$

$$\Sigma F_x = m\bar{a}_x; \quad \Sigma F_n = m\bar{a}_n;$$

$$\Sigma F_y = m\bar{a}_y; \quad \Sigma F_t = m\bar{a}_t;$$

I have F equals to ma bar which could be expressed

in terms of Cartesian coordinates, it could be expressed in terms of n and t coordinates also it could be expressed in terms of $r \theta$ coordinates, but most of the problems we will confine to Cartesian and n and t .

So, when I have this as a vectorial expression in component form it could be written in a convenient fashion to solve your problem at hand and we also looked at you know in the case of statics what you learnt I can take moment about any point on the object or outside the object.

So, you also need to have a via media we have actually looked at from the angular momentum root and also developed about any point P also we have developed the

equation. If I go back to that expression have a point P on the object now and you must recognize that point P is not static it is having an acceleration, but that acceleration you should know as absolute acceleration and handle this equation. So, whatever the conveniences you had in statics you also have the same convenience in dynamics we have got those expressions for you to use.

So, I have this M_P , if I want to take moment about point P I have the expression

$\Sigma M_P = I_G \alpha + m \bar{a} d$ and if you look at these are all measured from inertial frame of reference. And there are simplified cases for this, I have a situation where there is rotation, so this reduces to $\Sigma M_G = I_G \alpha$, I have a fixed axis rotation reduces to $\Sigma M_O = I_O \alpha$.

So, in the subsequent problems you will just play with equations, please understand we have looked at three points; one is mass center other is arbitrary point both of them can have acceleration understand this only the point O which we have labeled as a fixed point when you attach an axis to that that becomes inertial frame of reference.

So, when we develop the equation, we have made our lives simple, so that I could write moment about any point of my interest. So, I would choose like in statics I can choose

A loaded grain truck of weight 2400 kg is shown in the figure. It is lifted up by a mass M . Determine the max. mass M such that the loaded cart will not overturn about the rear wheels B . Neglect the mass of all pulleys and wheels. For simplicity neglect frictional effects.

All dimensions are in mm

All dimensions are in mm

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any point I have shown an arbitrary position P , I can choose an appropriate position so that my mathematics is reduced and remember that point P on the rigid body can have acceleration. But you should know the absolute acceleration; in other words, the acceleration measured from an inertial

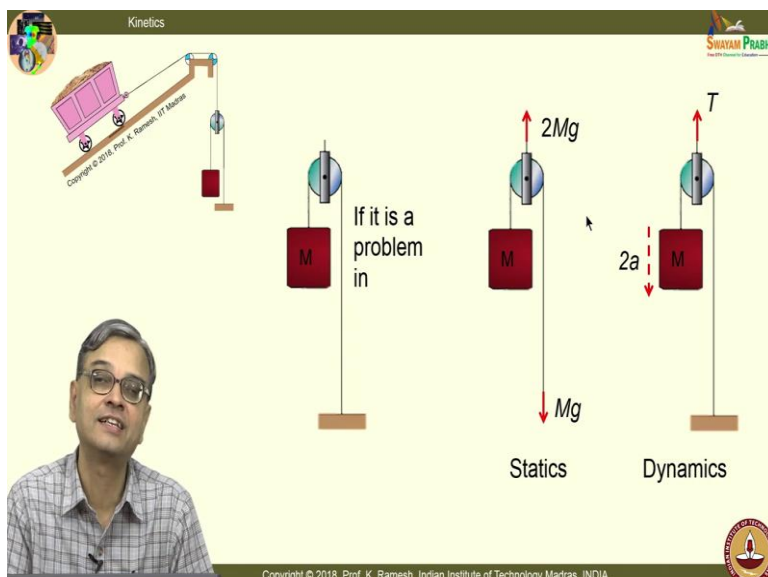
frame of reference for you to use these equations fine.

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You take up this problem it is a very interesting problem and you also have the animation to aid your thinking, see consciously throughout this course wherever possible when we discuss a problem have made an effort to bring in what is it happening in the problem which you should understand, so that you can translate it for solution and you have to understand every statement of the problem.

So, what it says is I have loaded grain truck of weight 2400 kg it is lifted up by a mass M which is an animation here so that label is not there, but you have a label put in this sketch. And you are asked to determine the maximum mass M such that the loaded cart will not overturn about the rear wheels, here the labeling is done the front wheels are put as A and rear wheels are put as B . So, you have to interpret this step; step and appropriately to solve the problem.

Neglect the mass of all pulleys and wheels for simplicity, neglect frictional effects, so we make our life simple, so do not bring in any friction here although there is friction present in this problem it is very obvious and we neglected for the purpose of simplicity it is a very interesting problem.



So, what is the kind of motion this cart is experiencing? Plane motion is a very general term among the plane motion what is it that is experiencing because the simplest problem you have taken it up it is essentially having simple translation nothing else ok. So, you

are moving away from a particle dynamic to rigid body dynamics starting with a simplest aspect of translation fine ok.

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Now, let us look at the other side of it this is very interesting ok. So, I would like you to visualize how would you handle it in the context of dynamics. How will you handle it in the context of statics and how will you handle it in the concept of dynamics. You have an advantage see, if it is in a problem of statics you will simply say that I have a force acting like this because it has been balanced by this force and I can also find out what is the tension this is nothing but two times the Mg nothing is moving it is in a state of statics when I say statics this motion is not there.

And in the case of dynamics I have this motion, you have to visualize what way I would model what all nitty gritty details that you have to look at, that is why the animation is put. You have to recognize you know this pulley is moving down with some acceleration and what is the acceleration with which the mass is moving, is it one and the same? Is the question clear you have to visualize all that you have to visualize when you analyze the problem it is the problem in dynamics?

So, I have my right hand side non-zero I have to put the appropriate mass into acceleration only then the equation of motion is complete, so need to know that acceleration and what I have I can only say there is tension in the cable I cannot quantify it as $2Mg$ like what I could do in statics that is number one.

And from the physics of the problem you know this cart is moving at the same string is coming down. So, I can say that this has some acceleration, but what happens to this mass you look at here how what happens to this mass what is the acceleration of the mass.

So, it is $2a$ you have to recognize that see the problem is simple have taken a simple problem and thread bare let us analyze it, so that you do not miss out any details. See my anticipation is you do not memorize the solution understand the solution, so that you can attack any problem logically. You can analyze what is happening to the problem physically visualize what it is, then use your equations appropriately fine so that is the idea behind it.

So, in a problem in dynamics I will have to handle this kind of a diagram, I have to recognize things are moving, things are not static that you have to recognize and from a kinematical understanding of the problem should be in a position to find out this, the problem is so simple. So, I can directly write what is the quantity I do not have to do a kinematical analysis to get this quantity.

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Then when I analyze the truck what is the access that is easier for me it is better that take it along the inclined plane itself x and y ok. So, here I write the basic equations, so have the mass acting on the centroid of cart and I can find out what is the

component $mg \sin 30$ because this angle of orientation is 30° . So, I can find out mass will be acting downwards, so I can have the component here. So, that turns out to be 11.77 kN.

Then I have the other component $mg \cos 30$ that is acting like this. So, let us go and complete what are all the forces that is acting on it and also complete what are all the accelerations that is acting on it only then the problem is complete for you to write the equation of motion. So, we have done for the centroid. Now when I go to this I will have a tension acting on the string and I know the geometry it is not absolutely parallel, there is a small inclinations of the order of 4° is there and we will also invoke as engineers I will take the \cos component $\sin 0$ is 0, so I will neglect the \sin component.

So, I have component $T \cos \delta$ as the force acting there and I also have $T \sin \delta$, but I would say $\sin 4^\circ$ is so small, so I can make this as 0 for all my practical analysis and in addition you also know from the physics of the problem the cart is moving up. So, the centroid

has an acceleration, since I have put this as x direction put this as \bar{a}_x and I put a bar to indicate the convention that we are dealing with centroidal quantities. Now, what is the statement in the problem? Cart will not over turn about the rear wheels. How do you translate this into your expression?

This is the statement that you need to understand and translate it into your mathematics; we have to look at the limiting condition. What should be the force interaction at B and what should be the force interaction at A ? You do not want the cart to tilt like this. So; that means, what? In the limiting condition the mass should be identified such that, the force interaction here is what that is 0 that is the limiting case you have to recognize this; this is the meaning of the statement, the cart will not over turn about the rear wheels B .

So, if you interpret this then you have your diagram complete, then it is simple mathematics no difficulty at all. The challenge in this problem comes at two places; one is how do you handle the mass here, how do you find out the acceleration and how do you put the force interaction between the ramp and the wheels. If you understand these two this there is nothing more in this problem, it is fairly straight forward, is the idea clear?

So, what we are writing is you know you have to recognize that $\Sigma F_x = m\bar{a}_x$ this is what is

the difference between statics and dynamics. I can write down the x components I have this; this; this all written here that is equal to mass is 2.4, you have taken the 2.4 that factor you have taken and then I have put this as \bar{a}_x . So, where T is in kN and \bar{a}_x in m/s^2 ok.

Kinetics

Neglecting the $T \sin \delta$ component,

$$\sum M_B \text{ gives}$$

$$T \cos 4^\circ (360) + 20.39(720) - 11.77(1080) = 2.4(\bar{a}_x)1080$$

$$T \cos 4^\circ - 11.77 = 2.4 \bar{a}_x$$

On solving

$$\bar{a}_x = 3.59 \text{ m/s}^2$$

$$T = 20.44 \text{ kN}$$

All dimensions are in mm

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And neglecting the $T\sin\delta$ component I can also write moment about point B and moment about point B is what? It is an arbitrary point. We know the recipe how to write the moment about this equation is not it, you have this as like this because I have this quantity, I can find out the moment about all of this. So, $m a d$ so this is the distance d and because this is the problem in translation you do not have the $I_G\alpha$ component, that is 0 it is only a translation, so this reduces to mad .

So, I get this is the previous equation I have got, I have two simultaneous equations. I can solve for the acceleration as well as the tension, I can solve for these two quantities because I have a simultaneous set of equations $\bar{a}_x = 3.59 \text{ m/s}^2$ and your tension in the cable is 20.44 kN. The problem is not complete because we have to find out what is the mass M . I have taken one reference axis for me to solve this, whatever you call it as car

or cart one and the same.

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And have to analyze this portion of the problem, I already know what is the acceleration here, I already know what is the value of tension here. From a kinematics know what is the mass of this what is the

mass of this is also shown.

So, we know that this is $2a$ and $\Sigma F_y = ma_y$ I write this; I write this as I have taken y as positive downwards. So, I have this equation goes like this, we already know the acceleration as 3.59 from the previous calculation, so this gives me the value of the mass. Fairly straight forward it is a very simple problem, nevertheless you should go in stages and understand what is the motion, how do you translate the conditions given in the problem as mathematical entities and solve the problem.

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The homogeneous rectangular plate of mass 30 kg is supported in the vertical plane by the light parallel links shown. If a couple $M = 150 \text{ Nm}$ is applied to the end of link AB with the system initially at rest, calculate the force supported by the pin at C as the plate lifts off its support with $\theta = 30^\circ$

900
600
 mg
600
 $\theta = 30^\circ$
600
 $\theta = 30^\circ$
All Dimensions are in mm

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Let us move on to the next problem, I would like you to observe in fact, you know in one of the earlier classes have shown you an animation of this. The problems are similar, here I have applied a moment instead I have just put a load here that was easier for me to do it experimentally, but from

the motion point of view it is one and the same. And you should recognize what happens to this object, that is the reason why have given, if I give you a problem like this you are left your imagination what happens to the object?

Now, I am giving you lot of aids, not only this I also giving going to give you some more animations to aid you what is happening to this object and write down this problem statement you have a homogenous rectangular plate of mass 30 kg, supported in the vertical plane by the light parallel links shown. And you are given the value of the couple that is applied and you are asked to find out the force supported by the pin at C as a plate lifts off its support with $\theta=30^\circ$, this $\theta=30^\circ$ this is what is I mentioned.

So, the problem statement is fairly straight forward. So, in dynamics you will have to recognize what is the kind of motion the object has. I have given enough clue in the animation; the animation gives you a very clear indication. What happens to this body, does this body rotate? There is no rotation about the body fine and let me also give you another way of looking at the whole problem ok.

I have this object moving look at this animation, the problem statement remains the same. To aid your thinking you know I have put initial state is like this in green and finally, it has gone to the position pink. You can see very well that this object has moved

like this, see my idea is that you appreciate the motion because of the way the object is constructed.

Can you guess what is the kind of motion this plate is undergoing? Can you guess that it is the first of all it is translation fine, is it a simple translation or is it a curvilinear translation? See you will have to recognize if you want have given you the animation in an examination, I do not think I can give you an animated question paper.

So, you will have to visualize based on your past experience and looking at problems, read the problem statement and interpret what way the object has a motion. To aid your thinking I have also done one more requirement, what I have done is see this is pin joint I have a pin joint at C , D , B and A .

The moment it is pin joint you can replace it by if I have a Cartesian coordinate system I can replace it by two forces because I do not know the direction of force I can replace it by two forces or if I take tangential and normal coordinates I can replace it by two component, that you should be able to understand. I have taken this problem specially because it provides a bridge between statics and dynamics, the concepts that you learned in statics you have to revive it fine.

So, any pin joint I can replace it like this and it also allows you rotation. So, what happens to this point see these are parallel links, the links are parallel of same links, when I have parallel links the clue is it has curvilinear translation. Suppose you find out the instantaneous center of rotation for this, it will be at in ingenuity; that means what? The object is having a translation.

Suppose I do not have an animated picture to aid your thinking what way would you reason it out? You should first recognize that it is translation and then qualify what is the kind of translation it undergoes. So, when I have a link like this it can only have a circular motion like this, for this I will have a circular motion like this I will go in an arc and if I have point D it will also go in an arc and you know very well we have discussed if the velocity of two points are known in a rigid body, the complete story of rigid body is known these two bodies have a curvilinear motion. So, the mass point also has a curvilinear translation ok.

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And we will do the FBD of the rectangular plate and we have already seen when I have a curvilinear translation it is better that you look at the normal and tangential coordinates is

The slide contains the following content:

- Top Left:** A photograph of a white mechanical linkage mechanism.
- Top Center:** "FBD of rectangular plate" showing a pink rectangle with dimensions 900 (width) and 600 (height). Forces F_{Bt} and F_{Bn} are shown at the top right corner, and F_{Dn} at the bottom right corner. Acceleration components ma_t (tangential) and $mg\cos 30^\circ$, $mg\sin 30^\circ$ (normal) are shown at the center.
- Top Right:** "FBD of link AB" showing a blue link of length 600 at an angle. Forces F_{Bn} and F_{Bt} are at point B, and F_{An} and F_{At} are at point A. A moment M is shown at point A.
- Bottom Center:** "FBD of link CD" showing a blue link of length 600. Forces F_{Dn} and F_{Cn} are shown at points D and C respectively.
- Text on the slide:**
 - Link CD is a two force member and considered massless
 - Initially, plate is at rest i.e., $\omega = 0$
 - Hence, normal acceleration is zero
- Bottom Left:** A portrait of Prof. K. Ramesh.
- Bottom Right:** IIT Madras logo.
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not it you have already looked at that, we have taken generic situation and I said I have the curve put the normal and tangential coordinates. How do you locate the normal and tangential coordinate here? The clue is that I have the value of θ that decides me how do I put that is at 30° .

So, I can have this to aid your thinking this is the curvilinear motion that you have.

So, it is tangential to that I have this; this is the most important step of the problem if you understand that the mechanism that I have shown introduces curvilinear translation and what is the axis you chose half of the problem is solved. So, you should not rush at this stage, that is the most important step in this problem and I have this mass into acceleration. Then rest all is very; very simple and you have to go back to your statics and then see you have learnt about two-force member; you must invoke all of those that is why the problem is so interesting and important.

So, how do I put the FBD of the link I have a pin joint, so you know this is at 30° , so am taking the axis along and perpendicular to that. So, replace this pin joint by two forces normal and tangential forces and similarly you have this and I have followed the principle in statics I have already assumed this. Once I have assumed this is fixed ok. Since I assume this what happens in the other link is also fixed and here at the end of it I also apply a couple for me to lift, the only difference between the two is I have a force, but this is the animation I have, this is good enough to understand what way the plate will have a motion ok.

So, FBD of link CD is like this and you must be able to quickly use your earlier knowledge. Do you see what I am arriving at, because we have simplified problems involving pin joints by recognizing them as assembly of two and three force members ok. In fact, I have also cleverly shown that only one force here that comes from the string that this is the two-force member, the force has to be along the member because it is a very linear member. So, this is the only way you can have a force for this, this can remain in equilibrium.

Because this is not, this is attached at point C and attached at point D ok. So, I invoke this then my problem becomes much simpler and its considered mass less there is all the assumptions that we have made to make our life simple. Initially plate is at rest that is also you have to look at, when the plate is at rest ω is 0 and normal acceleration is 0,

so one of the components go to 0.

For link AB , $\sum M_A = 0$ gives
 $0.6F_{Bt} - 150 = 0$
 $F_{Bt} = 250 \text{ N}$

For rectangular plate, $\sum F_t = ma_t$ gives
 $250 - 30(9.81)\cos 30^\circ = 30a_t$
 $a_t = 4.00 \text{ m/s}^2$

$\sum M_B$ gives
 $30(9.81) \times 0.45 - F_{Dn} \cos 30^\circ \times 0.6 = 20(4)(\sin 30^\circ \times 0.3 - \cos 30^\circ \times 0.45)$
 $F_{Dn} = 207 \text{ N}$

$F_{Dn} = F_{Cn}$
 $F_{Cn} = 207 \text{ N}$

The slide also includes a small image of a hand holding a plate, a free-body diagram of link AB with forces F_{Bt} , F_{Bn} , F_{At} , F_{An} and a moment M , and a free-body diagram of the rectangular plate with forces F_{Dn} , F_{Dn} , F_{Cn} , F_{Cn} , $mg \cos 30^\circ$, $mg \sin 30^\circ$, and acceleration ma_t .

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And I have this for link AB
 $\sum M_A = 0$ gives I or in a position to find out what is the force $F_{Bt} = 250 \text{ N}$.

Please check the arithmetic, if there are any differences bring it to my

attention and I have the sketch of the plate shown here for easy visualization. For rectangular plate $F_t = ma_t$, that is what you have to write because this has a translation, I have this and when you substitute the terms, I am in a position to get what is the acceleration component a_t .

And $\sum M_B = 0$ gives a long expression like this, so I have $F_{Dn} = 207 \text{ N}$. So, I have

$F_{Dn} = F_{Cn}$ from the two-force member. So, we have been able to find out the force in the point C we have been able to find out. So, the challenge in this problem was first

recognize that the plate has curvilinear translation, one easy way of looking at this is if you have parallel links connected in variably it will be a curvilinear translation. Your mini drafter is an example a mini drafter is an example where you have parallel links.

But you have to look at the problem visualize the motion and also chose appropriate coordinate system that is very important, appropriate coordinate system is very important and when I say this is a curvilinear translation the links at that position itself forms the reference for you to write the coordinate system. These are all the learning that you have from this interesting problem and this animation has really helped and beautifully you

are able to see that this plate has no rotation about itself.

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There is another way interesting problem see packaging industry needs lot of mechanics and so you have a bottling plant, the bottles are transported

Kinetics

In a bottling plant, the bottles are transported in a cleated conveyor belt inclined at 20° . Determine the max. acceleration which the belt may have without tipping the bottles as it starts.

The diameter of the bottle is 70 mm and the CG is located at 80 mm from the bottom of the bottle.

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in a cleated conveyor; that means, you have steps like this on the conveyor and then you have these bottles are going. Determine the maximum acceleration which the belt may have without tipping the bottles as it starts.

You have some geometric details of the bottles it is 70 mm and the CG is located at 80 mm from the bottom of the bottle and this is oriented at angle 20° and if increase the acceleration. You will see in the animation that this acceleration vector becomes thicker and bigger; that means, the acceleration is more.

So, when you are having this, see from production point of view what they want they want to accelerate everything, they want to move it fast as possible. If you increase the acceleration you have a danger you cannot increase beyond a certain limit that is the essence of this problem. You cannot have any acceleration of your choice just to increase

your productivity you cannot simply increase the speed; functionality will be lost. So, you should know what is the maximum acceleration.

Look at this; this is what the animation shows when the whole thing acceleration increases the bottles do not remain straight, they tip off. You may want to fill something in the bottle, so now bottles will not be ready to hold the fluid. So, it just to increase the productivity do not blindly go and increase the speed, increase the acceleration that is the essence of a, first you have to understand what is the essence of the problem.

You know in a book it does not show you that the bottles are tipped like this I have the advantage to show you and visualize to aid your thinking when I increase the acceleration. It is natural to expect that this will tip because the problem can be solved in few steps if you understand it correctly and you have to go back to your tipping analysis. In a problem dealing with friction we have looked at tipping, you have looked at the resultant force is at one of the edges, that is the limiting case.

So, it is a very nice problem which is linking your statics and dynamics, you cannot afford to forget the concepts learnt in statics once you step on to dynamics that is a message ok. I think what I would suggest is why do not you think about solving this problem will continue with the problem tomorrow ok.

So, in this lecture we have looked at a basic equation of kinetics of rigid bodies in plane motion, that is the key point here you are actually graduating from particle analysis to rigid bodies. And we have the simplest of motions the motion of the rigid body is in a plane not only that you idealize situations where the bodies also can be considered as planar.

So, you learned this from simple translation to rotation possibility and we have also looked at like in statics, we have developed the equations in dynamics also where I can find out the moment from any convenient point, so you have the complete freedom to select the point at which you will take the moment. And you know how to write that expression, whatever the quantity that go into the expression you should have got from your kinematical analysis.

From the kinematical analysis you are expected to provide the absolute acceleration in those equations please remember that; the accelerations measured from an inertial frame of reference.

Thank you.