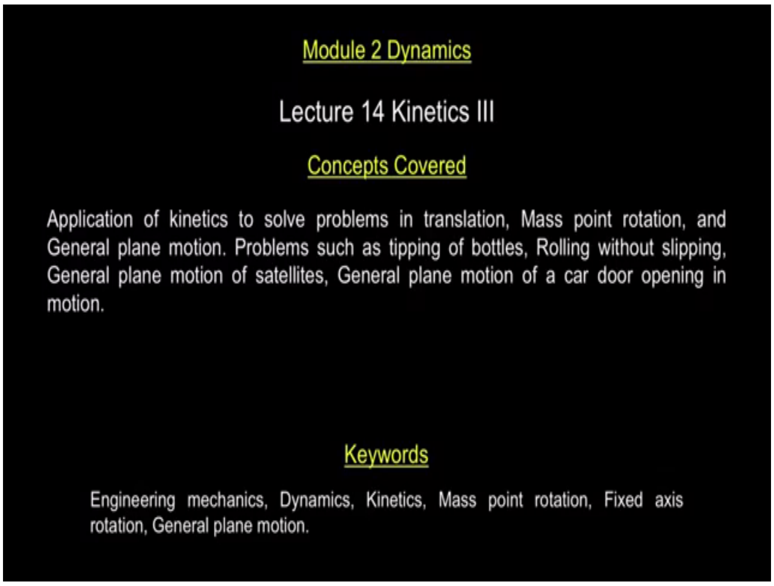
Engineering Mechanics
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module – 02
Dynamics
Lecture – 14
Kinetics III

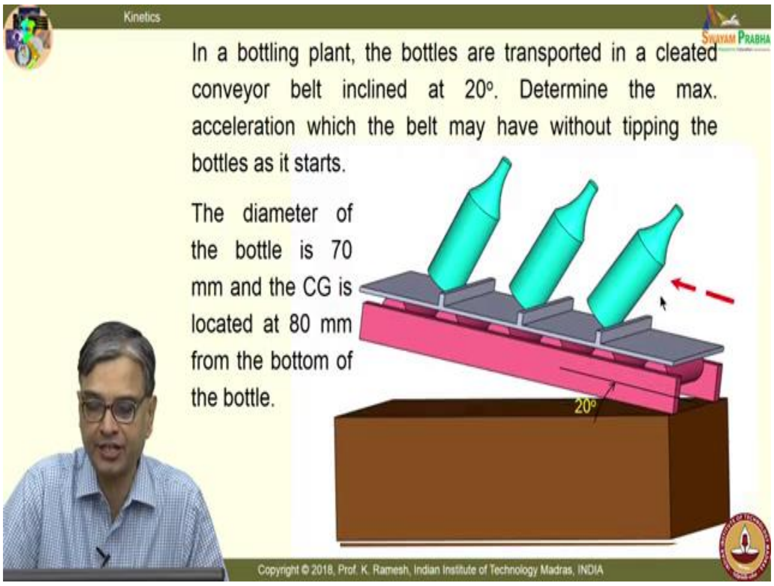


(Refer Slide Time: 00:30)

Let us continue with the discussion on Kinetics.

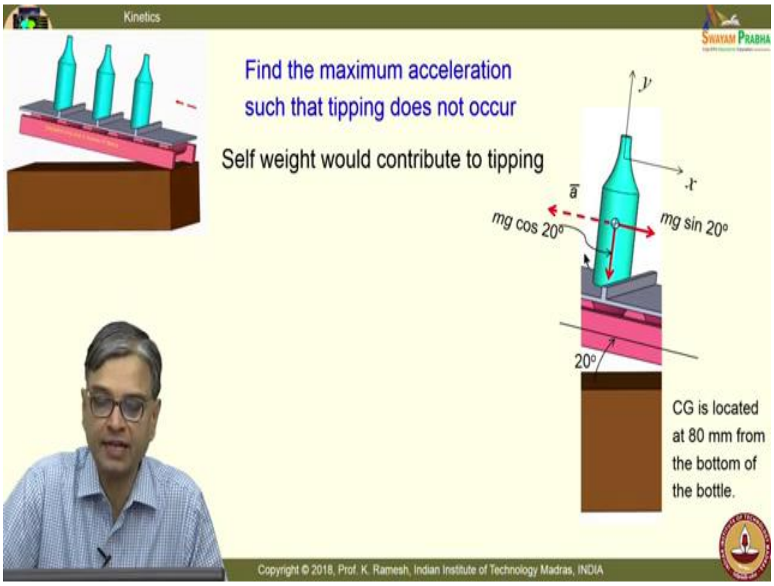
(Refer Slide Time: 00:45)

In the last classed, we have understood what is the requirement of this problem, we will have to find out, what is the maximum acceleration that can be tolerated by the system. So, that these bottles do not tip.



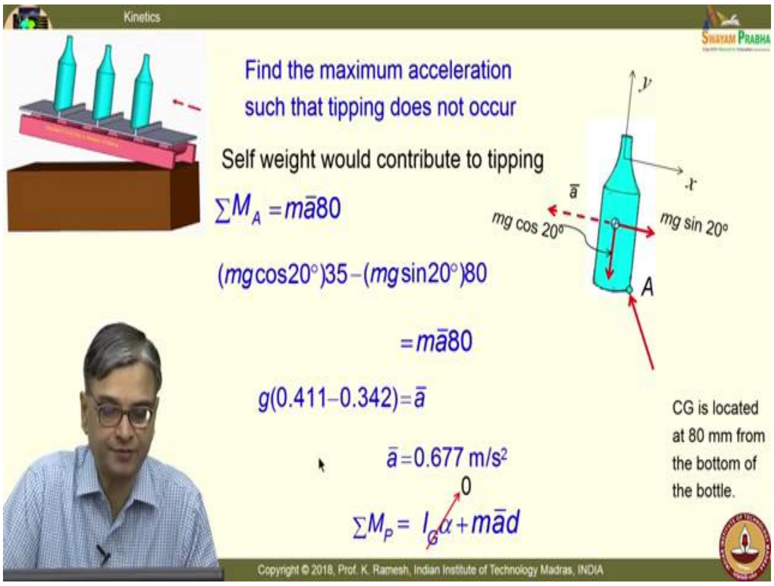
(Refer Slide Time: 01:03)

And, so the question is to find out the maximum acceleration and we need to put the free body diagram of the bottle and we have to investigate for tipping. And we have learnt in statics, how to investigate for tipping and it is easier to take the coordinate axis parallel to the conveyor belt and perpendicular to that. And, in this case it is a self weight of the bottle, that contributes to tipping and it is given in the problem, where the *CG* is located, it is about 80 mm from the bottom of the bottle. So, it is put approximately here and you have the weight acting on this, *mg*cos20^o and *mg*sin20^o we can put it like this and you also have the acceleration of the mass point.



(Refer Slide Time: 02:19)

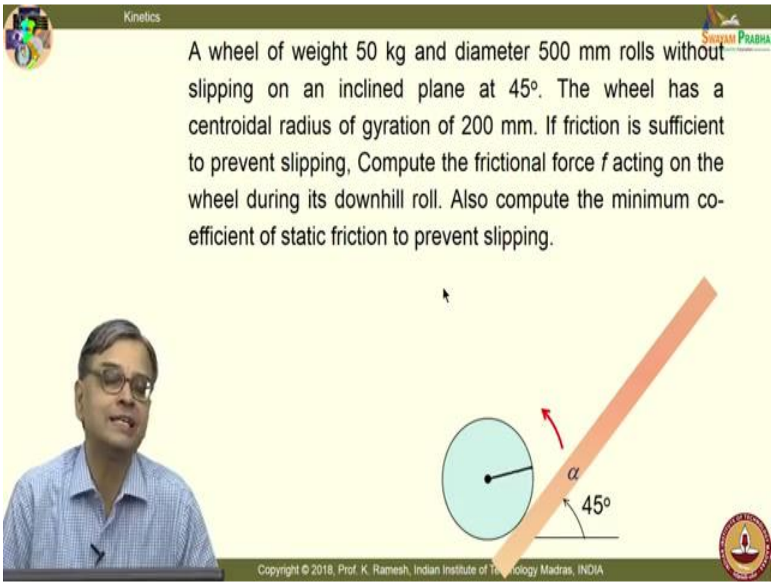
And this is where you have to investigate, for tipping to be prevented the resultant force should be within the bottle. So, the limiting case is you put it at the edge, we have already seen that it is going to tip like this, that is what we are anticipating it. Since, this diagram is complete you are in a position to write the appropriate equations and it is desirable that we do it at point A and this is nothing but, an arbitrary point b ok. So, you are going to invoke this equation, summation of $\sum M_P = I_G \overset{0}{\alpha} + m\bar{a}d$.



All the parameters that is required are given and this is the problem where the, you know there is no rotation. So, $I_G\alpha$ is 0. So, you write $\sum M_A = m\bar{a}80$ and write what is the value of M_A. So, you will have a contributing force from this and you will also have this force contributing to this moment.

So, that is all the problem is solved, I have $(mg\cos 20^\circ)35-(mg\sin 20^\circ)80 = m\bar{a}80$. So, that gives me the limiting value of acceleration, as 0.67 m/s^2. Say it is a very nice problem; it is a very practical problem where you have the packaging industry, when they want to fill some liquid. If they want to increase productivity, you have to apply mechanics and find out what is the maximum acceleration that the system can tolerate.

Otherwise, the functionality would be lost and it is a nice problem which brings in your understanding of how to analyze, whether tipping will take place or not and this is also an arbitrary point, in order to remove your mental block, I can have any point labeled with any symbol. In the discussion, when we develop the equations, we call the arbitrary point as *P*, the point *P* can be a in this context. So, you should be able to apply these equations comfortably, when the labels are also changed.



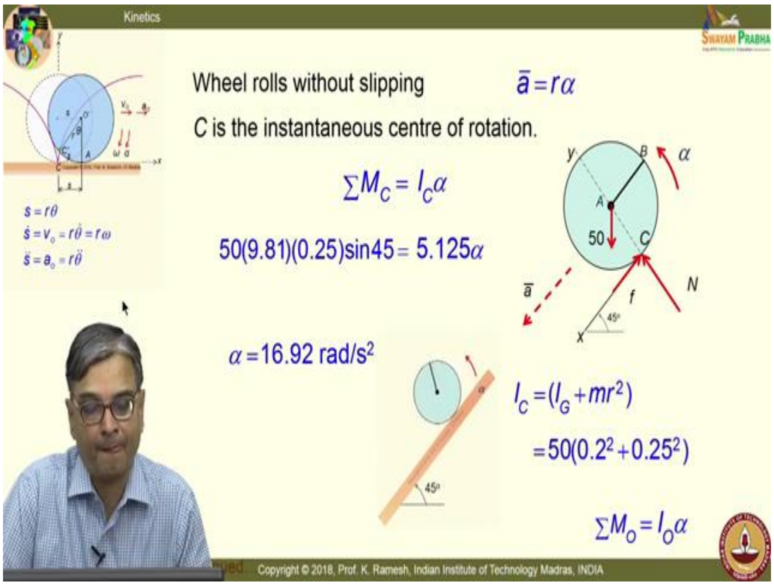
(Refer Slide Time: 05:04)

Then we move on to another simple problem, I have a wheel of weight 50 kg and diameter 500 mm rolls without slipping, that is a very important statement, on an inclined plane at 45°, the wheel has a centroidal radius of gyration of 200 mm.

See I have coin the problem such a way that, I do not wanted you to spend time on calculating the mass moment of inertia, I directly give you the radius of gyration; so, focus is on learning dynamics; not on calculating the mass moment of inertia and get stuck. If friction is sufficient to prevent slipping, compute the frictional force *f*, acting on the wheel during its downhill roll.

Also compute the minimum coefficient of static friction to prevent slipping and this is fairly straight forward problem and you know rolling is general plane motion fine.

(Refer Slide Time: 06:16)



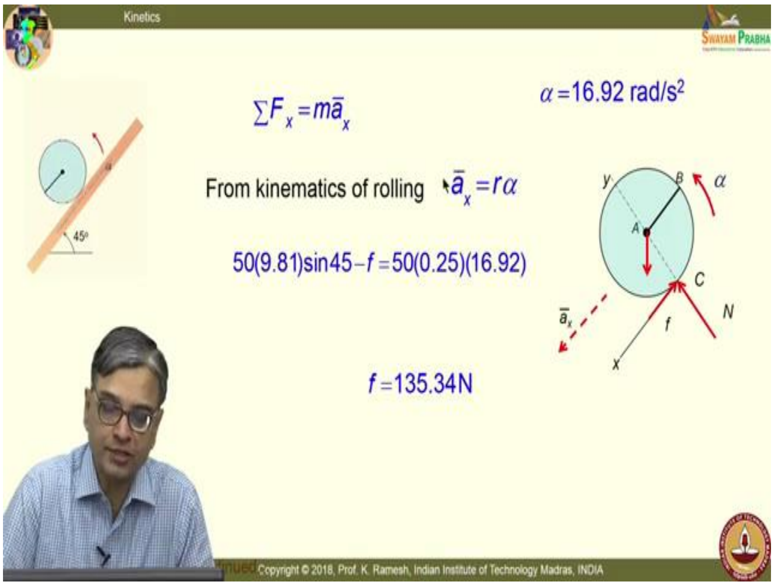
And the moment you look at rolling, you are expected to remember the kinematical conditions that you have learnt earlier. You know you need to remember certain quantities, it is unavoidable and rolling is such a common phenomena that you will come across in many applications, you are expected to remember the relationship between the mass point velocity and angular velocity, as well as mass point acceleration and angular acceleration, you are expected to remember these quantities and it says wheel rolls without slipping, when it rolls without slipping, these quantities we have learnt earlier, you are expected to remember and use it.

If you do not remember derive and use it and we have the important relation that $\bar{a} = r\alpha$. And when I have the axis as *x* and *y*, I could also label this as a_x and here you know from the physics of the problem, that *C* is the instantaneous centre of rotation.

So, I can apply centre of rotation as the point for me to do all the computations and make my life fairly simple. And you are given the weight of the disc and at *C* you will have an interaction of normal force and the frictional force will oppose the rolling motion. So, I have this as the frictional force, I label it has f. So, the diagram is complete including the rotation as well as linear translation acceleration components.

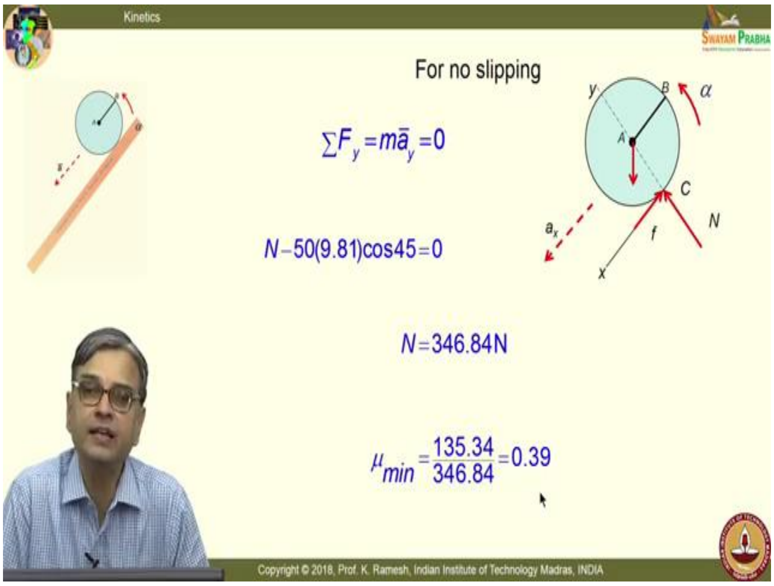
So, it is ready to be solved. So, identify the governing equation, we have $\sum M_O = I_O\alpha$, that is what we are going to use for this problem and this becomes point C here. So, I rewrite the expression as $\sum M_C = I_C\alpha$ and you have to use the parallel axis theorem to get the value of I_C, you are given I_G and then you can find out this ok.

So, I have this expression satisfied. So, I am in a position to get, what is the value of angular acceleration from this. So, the first step is I am in a position to get the angular acceleration and you have utilized the kinematical relationship, for a wheel that rolls without slipping. So, you are expected to remember these quantities, while solving problems.



(Refer Slide Time: 09:33)

And next equation I can write $\sum F_x = m\bar{a}_x$ and we have already seen $\bar{a}_x = r\alpha$ and $\alpha = 16.92$ rad/s^2. So, you have the basic equation returns very straight forward, please check the arithmetic and this gives me what is the frictional force, that is acting on the wheel so that it rolls without slipping. So, it makes your life lot simpler, if the wheel rolls without slipping, you have the kinematical conditions and you are in a position to invoke those interrelationships and solve for this problem.



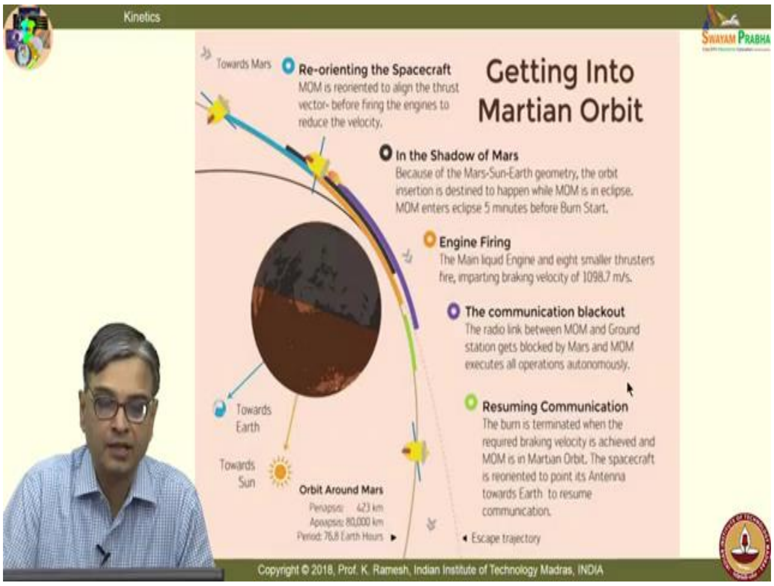
(Refer Slide Time: 10:34)

See the question also ask, what is the minimum value of coefficient of friction that you need to have, I can also write the other equation $\sum F_y = m\bar{a}_y = 0$. So, that gives me, what is the value of normal force and I know what is the value of frictional force that is required; so, it gives me, I should have a frictional coefficient of 0.39 is required; so that the wheel, rolls without slipping.

So, friction aids friction is needed in many applications, when friction is significant and its role cannot be ignored as engineers we bring in, until then we solve problems without friction, that is how engineers make the problem solvable ok, you get the first hand solution may be apply a factor of safety and then design criteria to do you a design rather than complicate your problems with all aspects of the physical system ok.

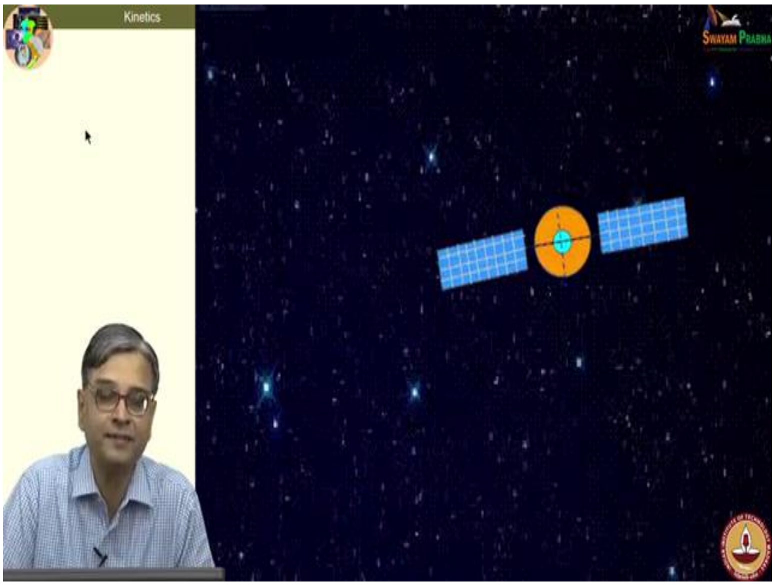
(Refer Slide Time: 11:57)

Then we move on to a very interesting problem, see I want to give you the feeling whatever the knowledge that you learn in this course is of use somewhere that we have come across recently and it is very interesting to see our Mars mission was a success



because, we had being able to do this in our first attempt, very first attempt and. In fact, the NASA probe that was a revolving around Mars welcomed this Mangalyan, they sent a message.

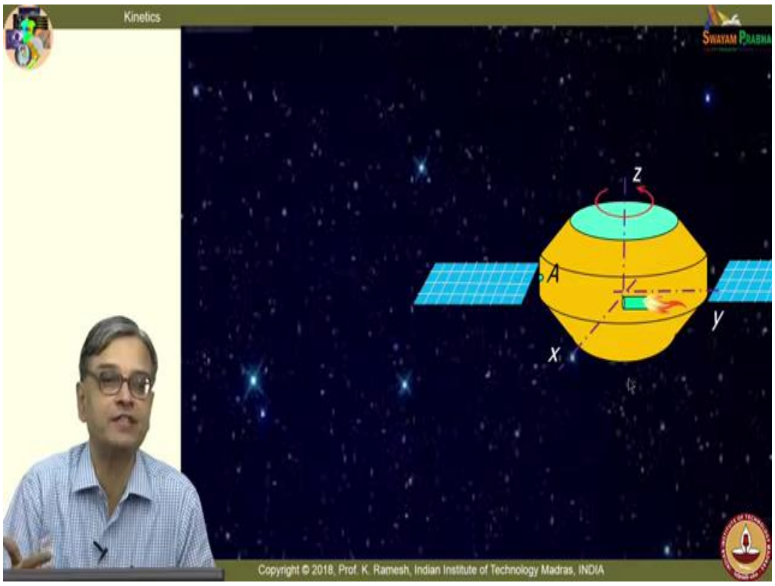
So, that is the proof that is somebody watching there, that you have reached Mars ok. So, the problem here is see the satellite is an orbit like this and if it has to go to Mars it has to take another orbit and what is to be done is you will be surprised that when you reduce the velocity, it shifts to the other orbit and there are various steps you know you have the blue line, you have the black line, you have the orange line, you have the green line, you have a slightly this kind of a indigo color ok.



So, you have definitions for all these stages of the craft. So, they have to align the satellite and then fire it, when communication is not possible from earth that is what is shown as a black strip and once you reduce the velocity, it gets locked into the Martian Orbit.

(Refer Slide Time: 13:47)

So, it all involves quite a bit of mechanics calculations fine and so this is the spice to the problem. So, I have a satellite which is spinning.



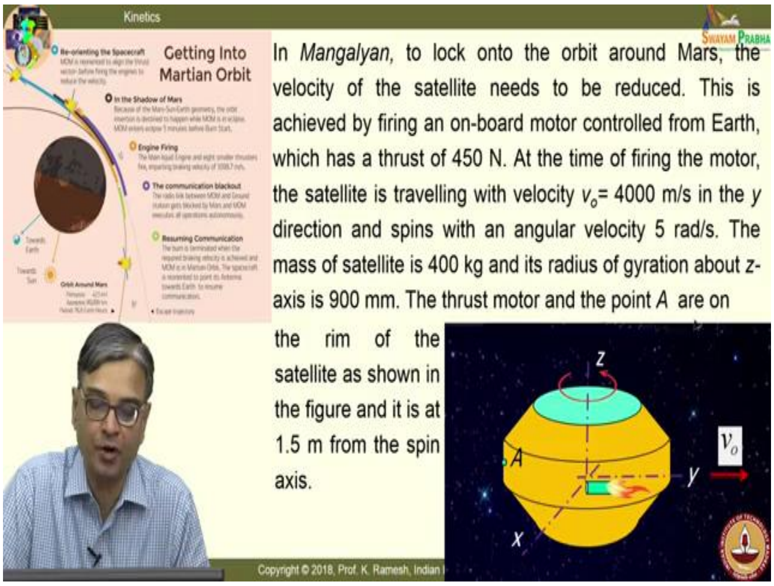
(Refer Slide Time: 13:52)

And we are going to look at, when there is an onboard thruster, which reduces it is velocity because, it is moving in this direction, when I fire it, I am applying a force this way. So, I try to reduce the velocity of the satellite then, it is in a position to shift the orbit. So, that is the physics behind it.

(Refer Slide Time: 14:15)

So, the problem statement is like this. So, let us also feel we will solve a simple problem, we also take credit that we have also done something to do with Mangalyan. So, to lock into the orbit around mars, the velocity of the satellite needs to be reduced, this is achieved by firing an on-board motor controlled from earth, which has a thrust of 450 N.

At the time of firing the motor, the satellite is travelling with velocity of 4000 m/s, in the Y direction and spins with an angular velocity 5 radians per second, the mass of satellite is 400 kg and it is radius of gyration about Z axis is 900 mm, the axis are shown already in the problem, I have axis X Y Z attached to the satellite and you are also shown a point a, which needs to be located the thrust motor and the point A are on the rim of the satellite as shown in the figure and it is at 1.5 meter from the spin axis.

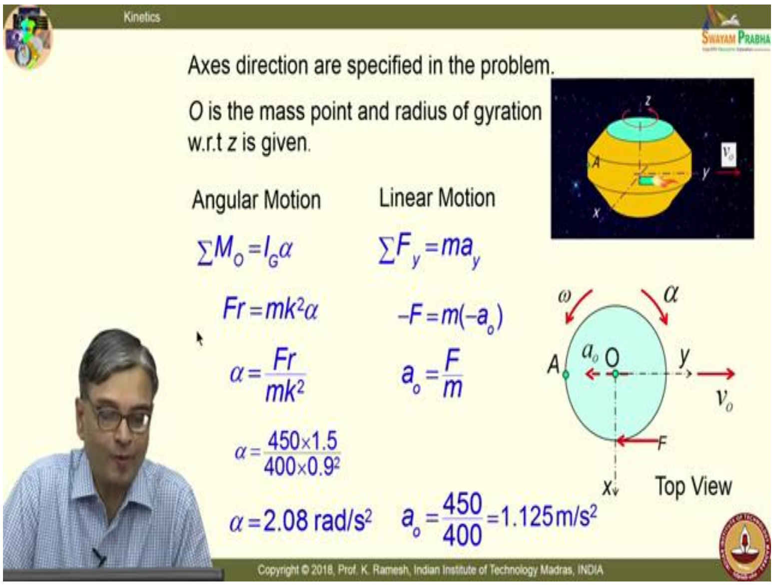


So, you are given all the particulars, you know this is very nice problem it brings in your basic understanding of how do you write the velocity equation relative velocity or relative acceleration equation, from one point on a rigid body to the other point on a rigid body. If you are attaching a non-rotating translating axis, you will see the other point, as if it goes in a circular motion. So, all those memories we have to bring in back and then handle this problem. So, the parameters are given which are very clear and simple.

(Refer Slide Time: 16:01)

And you have to recognize that the satellite is spinning, as well as having a linear motion and originally it is travelling with a velocity V_0 in the y direction. So, it is spinning counter clockwise and this is the angular velocity and you have the thrust motor attached here. So, when it is started, it is going to create a thrust and that needs to be put and it is labeled as O that is nothing but, the mass point. See we had specific annotation while we developed the theory; we have always referred the mass point as g, fixed point as O and arbitrary point as *P*. So, you must come out of that symbolism and learn to apply the correct equation to the symbols given in the problem, it may appear trivial here but, people get confused when they write the examination.

So, I have this and I have the onboard thrust motor introduces a thrust, that values also given in the problem. So, it has an angular motion, as well as a linear motion. Let us look at each of these. So, when I have this; obviously, this is going to create a deceleration like this ok, it will also reduce the velocity, it will also reduce the spin speed and you are given what is the acceleration a_0 ok, angular motion we look at, I have $\sum M_O = I_G\alpha$, this is the mass point of a satellite and you are given the radius of gyration, I can easily write what is M_G, only this force introduces that moment and I have $\alpha = \frac{Fr}{mk^2}$, I have all the parameters given.

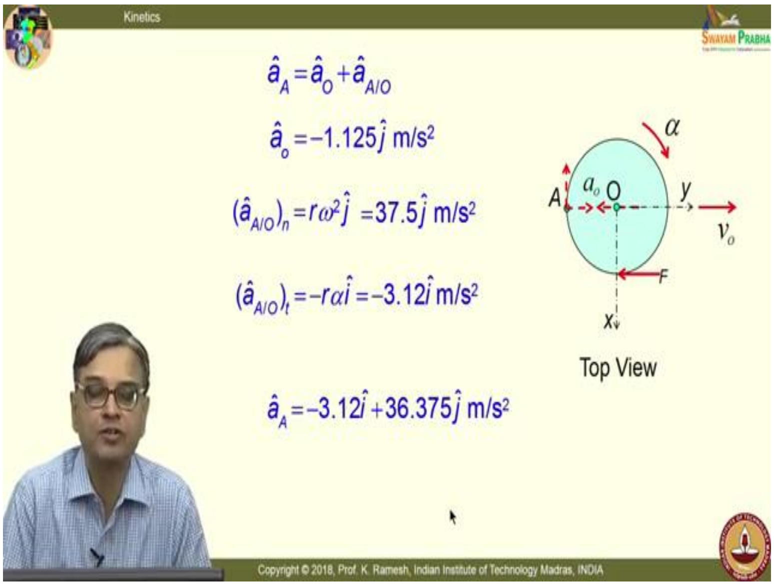


So, when I substitute, I am in a position to get the angular acceleration or deceleration whichever way you want to call it, this is 2.08 rad/s^2. Now, let us look at the linear motion, I have $\sum F_y = ma_y$ and we are given the coordinate axis with respect to the coordinate axis, I can also put the sign of the quantities and all these quantities are given *F* and *m* are known. So, I can find out a_0 and a_0 as 1.125 m/s^2. The idea is what is the values for the point a, that is what is asked. So, here this is the rigid body, which has a general plane motion, I sit on this body, I know this absolute acceleration, I need to find out what is the acceleration of point a.

(Refer Slide Time: 19:42)

So, go back to your kinematical relationships and write down the relative acceleration and you should be in a position to write what is the acceleration of *A* with respect to *O*, relative acceleration equation and this is nothing but, it is rotating and we know the direction of ω and α.

So, it is easy for you to write this component, from the previous calculation we have got this a_0 and you also assign the sign appropriately. So, we know a_0, what is acceleration of *A* with respect to *O*, I have the tangential component dictated by the direction of α and I have this normal component. So, I have this normal component is $(\hat{a}_{A/O})_n = r\omega^2\hat{j} = 37.5\hat{j}\ \mathrm{m/s^2}$ and tangential component is $(\hat{a}_{A/O})_t = -r\alpha\hat{i} = -3.12\hat{i}\ \mathrm{m/s^2}$. So, I have these quantities; so, it is easy for you to calculate, what is acceleration a_A.



I have the value as $\hat{a}_A = -3.12\hat{i} + 36.375\hat{j}\ \mathrm{m/s^2}$.

Second a very interesting problem, you try to use, whatever you have learnt in kinematics and you have learnt to apply it and you have got the values that is asked in the problem.

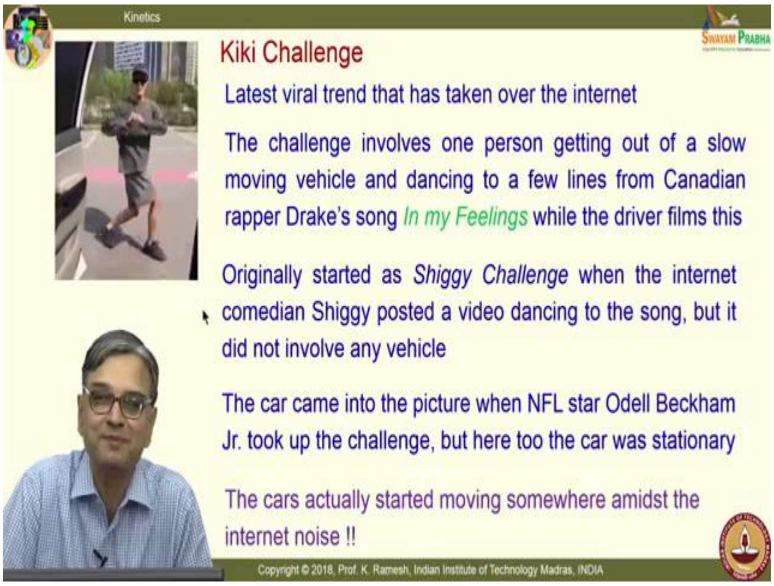
(Refer Slide Time: 21:24)

Now, there is a rage everybody is looking at the internet and then whatever that comes in the internet, whether it is right or wrong, you jump on to it and then do it and I thank my students, who have collected these details and you know it is very dangerous, you cannot simply open your car door and then dance and somebody films it on a public road and it tells you the history of development, it appears the cars started moving somewhere amidst the internet noise.

Initially, the proponent has not asked the car to move and what is the mechanics part of it? We are interested only in the mechanics part of it, what happens to this poor door ok, that is what we need to analyze fine.

(Refer Slide Time: 22:18)

And before me move on let us also see, how Indians have modified it and it became a real challenge to the Indian police. So, the advice is please do not do any of these challenges, fine it is all going to hurt you and my focus is on what happens to this door from mechanics point of view. So, you have warning from Bengaluru city police, Jaipur city police and so on and so forth ok. The idea is not to do this but, look at what is the mechanics behind the problem.

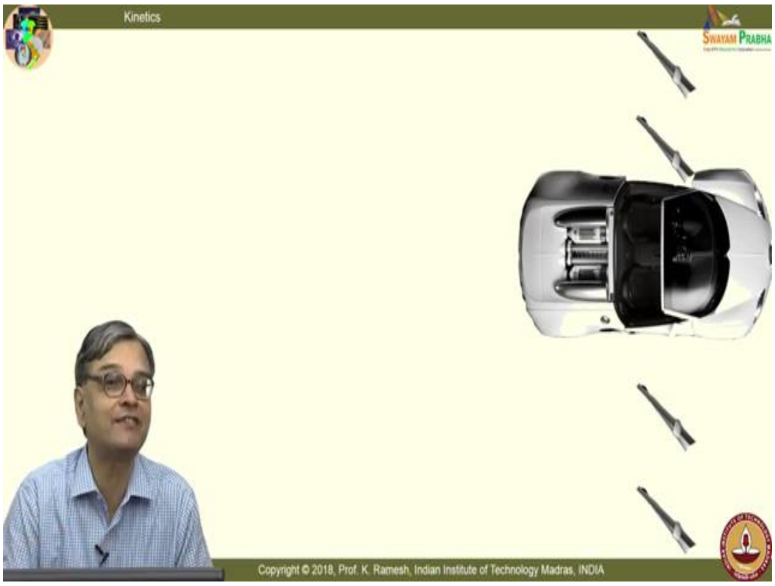


(Refer Slide Time: 22:52)

So, this is what you have, it is not exactly the same problem, please write down the problem in advertently the door is kept open and it is decelerating and what happens to that door, it is a very nice problem, you take any book on mechanics, they would have a car with a door opening. So, in the current context I thought that I can spice it up by bringing a Kiki challenge ok. Do not do any of those challenges publicly circulated in internet, many of them are injurious to you ok.

So, you are given the weight of the car door and you have very nice animation which gives you complete visualization of what is happening so that you can understand the problem statement better and look at the strategy to solve the problem. And we have to compute the reaction forces at the hinge of the 36 kg car door, when θ=45^o, the car decelerates at a constant rate of 3 m/s^2.

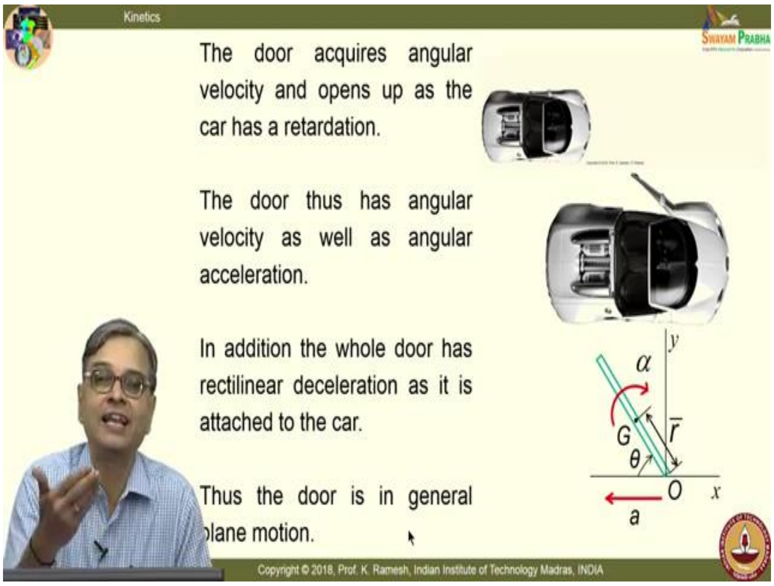
The radius of gyration k_0, at the vertical axis of rotation, you are given only at the axis of rotation, this is the axis of rotation, you are given only k_0 is given as 500 mm and the mass center $\bar{r}$ is 400 mm from the vertical axis of rotation. You are given all the particulars of it and let us go and look at how do we handle this problem.



(Refer Slide Time: 24:44)

Let us understand the motion little systematically, I could think of this as linear motion, then followed by a rotation or I can think of rotation first and then the linear motion and reality both of it happen together, that is what happens I have rotation as well as linear motion takes place, that is what we call it as general plane motion. So, you have a reasonable appreciation of what is happening to the door and you should also recognize to start with how the door was, what was it is angular velocity.

All that is hidden in the problem fine, that is what you have to look at, that is why I have put this animation, you have a very important clue, it starts from rest. So, the ω is 0, it is starts from 0 angular velocity and it acquires a finite value of ω, when it reaches 45^o, you should recognize all that. Only when you recognize all that, you will be able to use the appropriate equations to handle it.



(Refer Slide Time: 26:02)

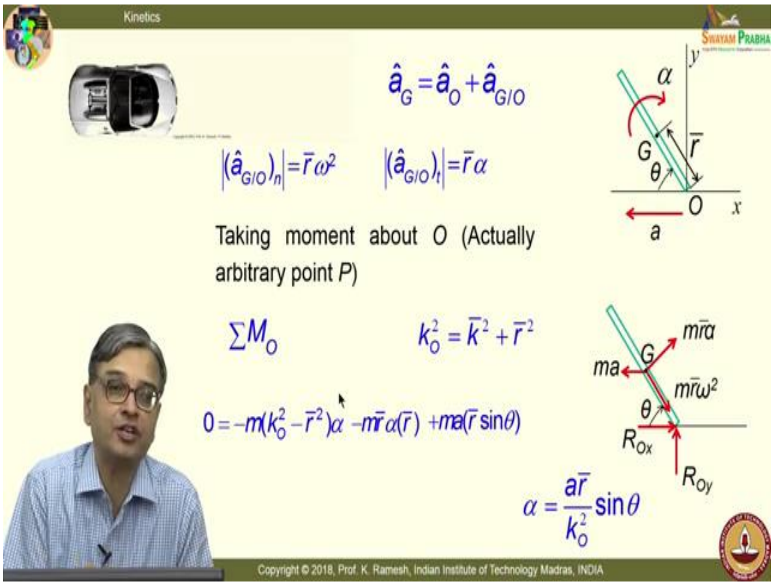
So, the door acquires angular velocity and opens up as the car has a retardation. So, that is the first observation, that we make out of it, door thus has angular velocity as well as angular acceleration and this is what is shown, you know I have a door which is very nicely shaped fine. But, as the engineers when you want to do an analysis, you simply take it as a rectangular unit and I have a radius of gyration, if once I have the radius of gyration do not need any of it is shape fine.

How we model the practical problem into a solvable problem. So, I have this as the rotation about which it takes place and you have to investigate what is this point, I have labeled this as *O*. We solve point *O* as a fixed point, you have to ask yourself is it behaving like a fixed point or something else, fine just because the problem statement says it is *O*, do not conclude anything out of it, you must investigate what is the motion.

So, I have this door, I have this angular acceleration, I have the location of the mass point, the distance is also given. So, you have to recognize the whole door has rectilinear deceleration. So, the point *O* is not fixed, it is having an acceleration. In this case it is deceleration because, we know the direction of the acceleration, it is coming to a stop.

So, you have to treat this as point p in our calculations and what I have. So, I have the acceleration shown here, which is a deceleration from this problem context, the door is in general plane motion, that is what you have to recognize you have to recognize it is in general plane motion. And once you draw the neat sketch and put the quantities arithmetic is fairly simple, there is no difficulty at all in arithmetic.

(Refer Slide Time: 28:27)

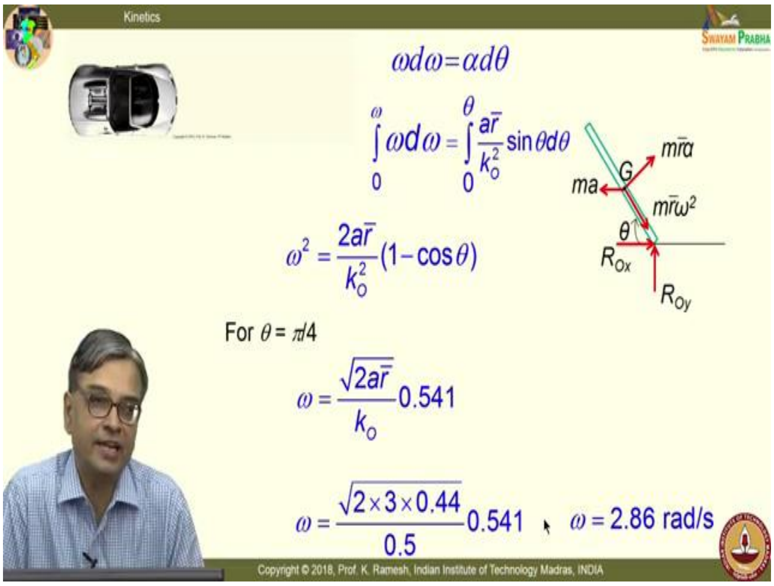


So, I have hinge. So, when I have a hinge, what is the way I will do it? I will assume two components because, I do not know the direction of the force represent it as component at the *x* direction and component at the y direction and an unknown force can be taken in any way, the general recommendation is; take it along the positive coordinate axis.

My mathematics will come and tell me whether this is right or wrong, all those principles you learnt in statics are equally valid in dynamics, there is no alteration here. And we have already looked at how to write this relative acceleration equation for a rigid body. And I have what are all the components of force that act at mass point, I have *ma* like this and then I have a normal component, $m\bar{r}\omega^2$ and a tangential component, $m\bar{r}\alpha$ like this. So, my diagram is complete. So, it is easy for me to write all the quantities that I require.

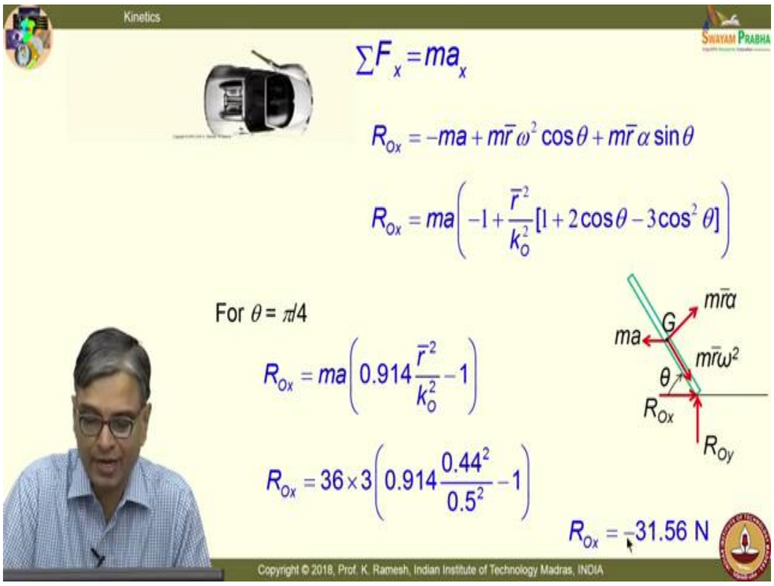
So, taking moment about *O*, you have already seen, that is actually arbitrary point *P* in our theoretical development, you should not lose track of it, that is very important, you should recognize that the point is not fixed in space, even though labeled as *O*, it is having a acceleration or deceleration whichever way you look at it. So, I have; you are given $k_O^2 = \bar{k}^2 + \bar{r}^2$.

So, this is a hinge. So, it has to go to *0*. So, I have $0 = -m(k_O^2 - \bar{r}^2)\alpha - m\bar{r}\alpha(\bar{r}) + ma(\bar{r}\sin\theta)$, you should recognize that the point *O* is in reality point *P*, it is not *O* in our theoretical development. So, labels can be misleading in a problem and you cannot put restriction on a person who is coining the problem to use your symbols to label it.



(Refer Slide Time: 31:27)

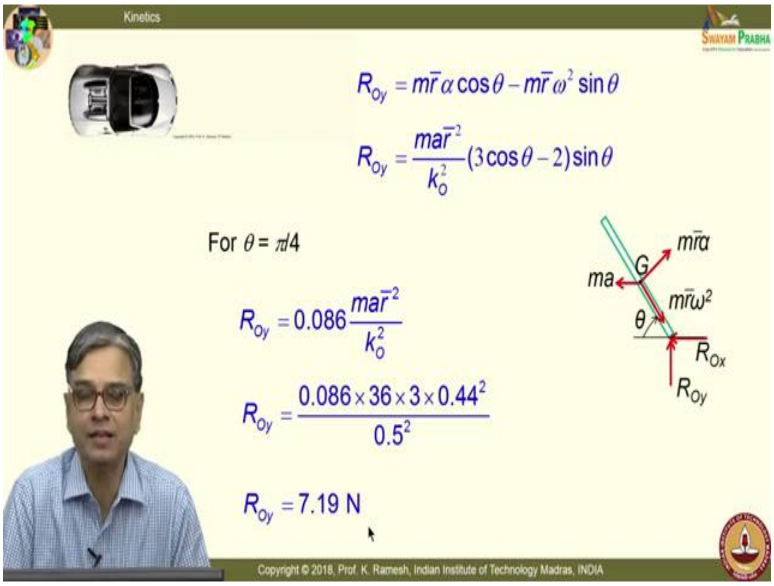
And we have already discussed, the doors start from rest. So, it has a *ω* is *0* to start with, it acquires angular velocity because, of the motion and this expression you have learnt long back in your particle dynamics. So, you are using that here. So, $\int_0^\omega \omega d\omega$ = $\int_0^\theta \alpha d\theta$, *α* we have got the expression from the previous step. And then when you do this, I get $\omega^2 = \frac{2a\bar{r}}{k_O^2}(1-\cos\theta)$ and $\theta = \pi/4$, you have to get the calculation.



So, I have this and finally, I get the value of *ω* as 2.86 rad/s, is it idea clear? It is a very interesting problem and it is quite dangerous; you should not have any door open in a car while it is moving. In fact, the modern cars have sensors, if any one of the doors is not locked, it gives you a beep or it gives you some visual indicator. So, you must attend to it, it can cause accidents ok, you should not drive the car with lose doors.

(Refer Slide Time: 33:10)

Then I go to $\sum F_x = ma_x$ and you can easily write this expression, if the diagram is complete, writing this is straight forward, it is not difficult all the quantities are put, you have the angle θ. So, I put the appropriate cos and sin θ components and you substitute the relevant quantities and for $\theta = \pi/4$, I get R_{Ox} is like this and I get the value of R_{Ox} as negative, that is perfectly alright, I have assumed it in a particular direction, the mathematics can come back and tell me whether my assumed direction is right or wrong.



So, you can start your problem with this and bank on your mathematics to assist absolutely no problem. So, I have determined the value of R_{Ox}.

(Refer Slide Time: 34:24)

Then I need to get R_{Oy}, R_{Oy} is given like this and substitute the relevant quantities for $\theta = \pi/4$, I get R_{Oy} = 7.19 N. So, it is a very interesting problem, you take any book on engineering mechanics, they would have a problem with car door opening, whichever way moves in the left or right or it is a very interesting problem. And you have to recognize that the door is in general plane motion, apply the appropriate equations, it brings in your knowledge of statics, it brings in your knowledge of kinematics, which we have learnt all that have to be used systematically to arrive at the final answer. And when you are designing the door hinges, you need to take care of these forces so that it last long.

So, in this class you know we have solved a variety of problems in kinetics. In fact, if you look at the problems that we have solved, we have solved problems with translation, curvilinear translation, fixed axis rotation, mass point rotation, as well as general plane motion. So, you have a variety and the focus was on how to visualize the problem situation and how to use the equations you have learnt appropriately.

The training was also given we have learnt it with certain symbolism, the problem may be given with some other symbolism use your symbols appropriately and do not do it in a rush because, your symbol and the problem symbol are identical, the problem symbol denotes what you have used in theoretical development, it need not be.

Thank you.