Engineering Mechanics
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Module – 02
Dynamics
Lecture – 15
3D Kinematics I

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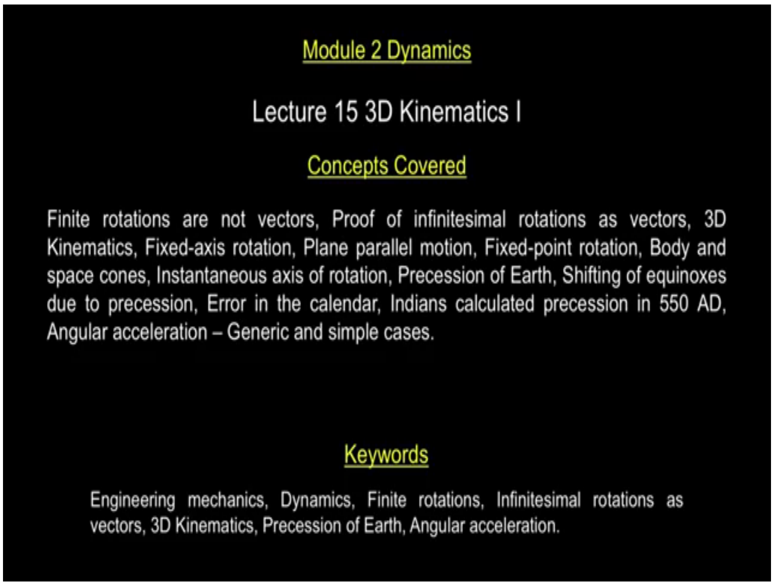

See you will have to appreciate everything is in three dimensions in the real world. So, we cannot stay away from solving 3D problems. So, we get certain glimpses of aspects related to 3D kinematics.

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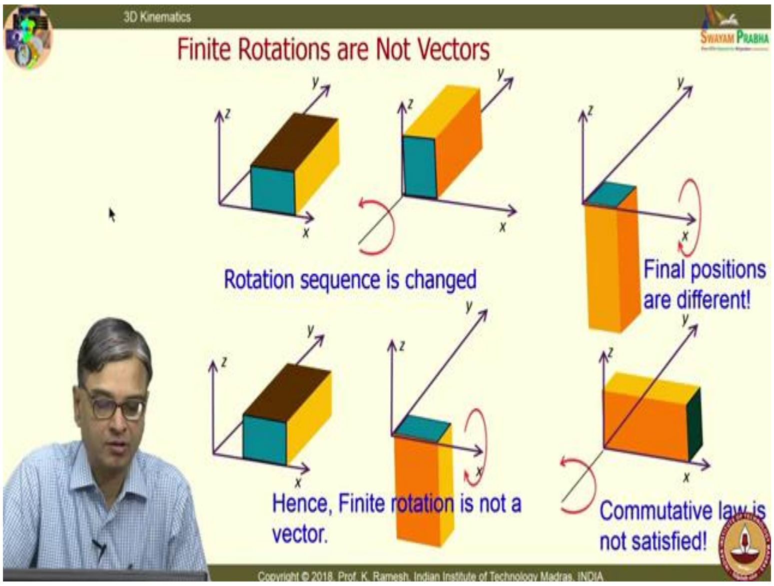
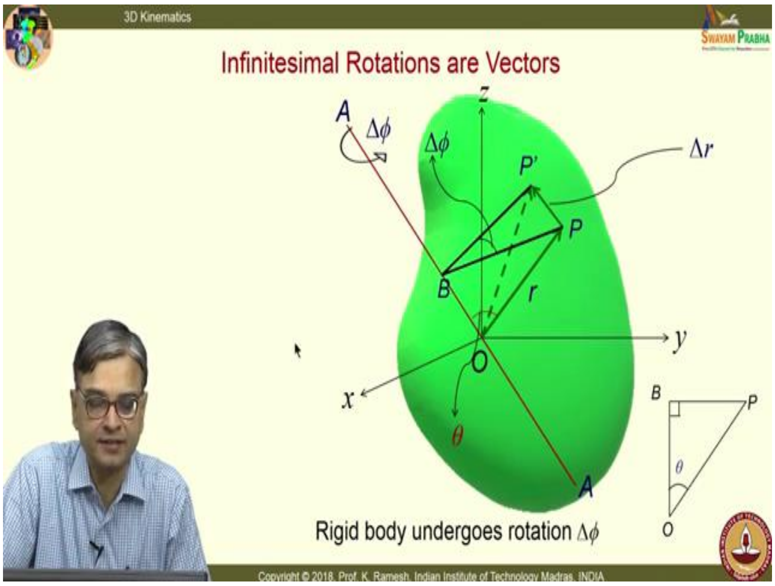
See one of the important concepts that you need to focus is to appreciate what happens to



finite rotations. Let us take the example of a block and let me rotate it in two different ways fine. I will give two sets of rotations. So, in the first case I rotated about *y* axis. So, when I rotated it takes a shape like this, to aid your visualization the sides are colored fine, then I gave another rotation about *x* axis. So, I first do the rotation about *y* axis, then I do the rotation about *x* axis.

So, whatever is the original object here finally, it takes a shape like this these are like finite rotations ok. The next step what I am going to do is let me reverse this rotation and see what happens. I have the same object here, now I rotate it first in *x* and what we need to get is, even though I change the rotation you may want the final answer to be identical let us see what happens.

So, I changed the rotation sequence, I find that this makes an influence which you can easily verify, you can take your book and then rotate it this way because it is rectangle. So, it is easy for you to see the sides and what you find here is when I have finite rotations they cannot be labeled as vectors, they do not qualify to become labeled as vectors. So, the final positions are different and commutative law is not satisfied. So, they are not vectors.



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And let us look at infinitesimal rotations I have a three-dimensional body and I have an axis *AA* and I try to give an infinitesimal rotation. See for the purpose of illustration even this infinitesimal rotation is shown sufficiently big for visualization. So, take it which a pinch of salt in visualizing this and make a neat sketch of this and this sketch will be repeated several times so, that you will have opportunity to fill in the gaps.

And what you need to look at is I can look at the triangle *BPO* this is at an angle θ which is what is the orientation of the axis *AA*. And, I have the line *BP* and the point *P* has shifted to a point *P*' located at a distance Δr and this angle is given as $\Delta\phi$ and we have just taken rigid body undergoes rotation $\Delta\phi$. So, you would see this being repeated at the subsequent slides. So, you can fill in the gaps.

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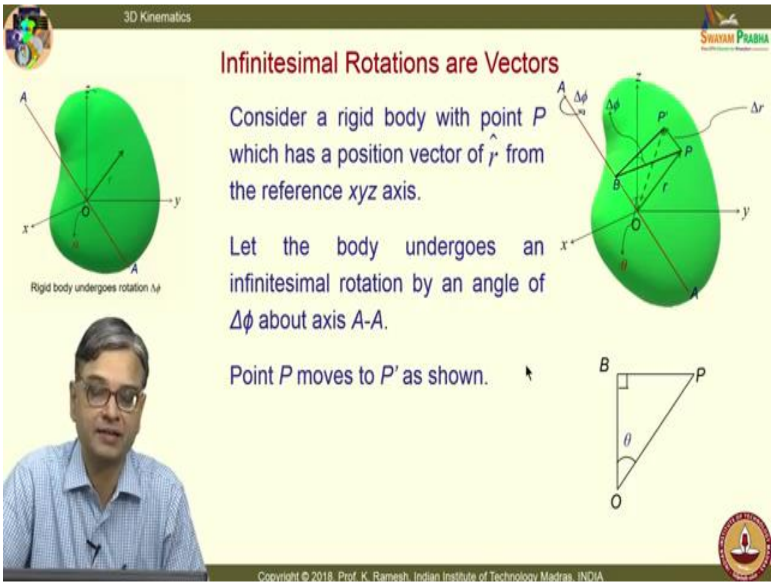
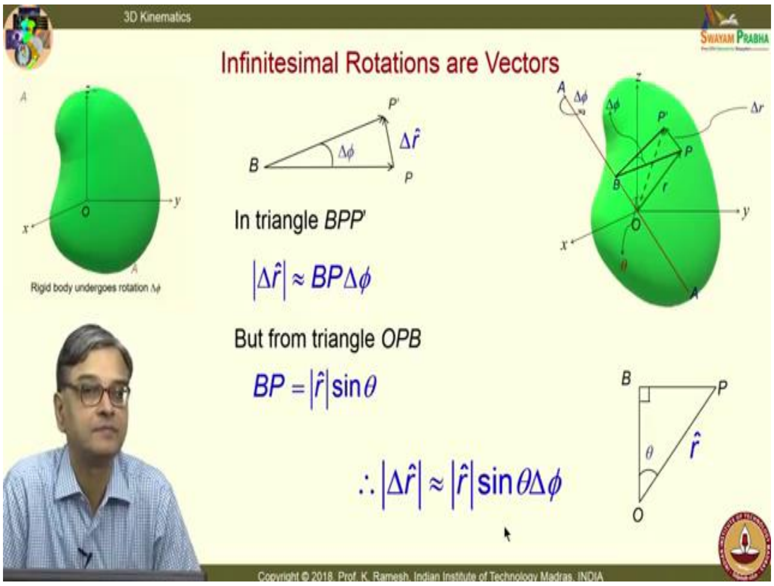


And, this is what is mentioned consider a rigid body with point P which as a position vector of r from the reference *x y z* axis have *x y z* axis have the position vector *r*. Let the body undergoes an infinitesimal rotation by an angle of $\Delta\phi$ about axis *AA* and you have the triangle *BPO* and the point *P* moves to *P*'. And, the arithmetic we are going to do is very simple, we all know how to find out the incremental change, express it in the mathematics and try to figure out whether we can argue that $\Delta\phi$ can be labeled as a vector.



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So, I am going to look at the other triangle *BPP*'. So, this is separated by angle $\Delta\phi$ and this is Δr; so, which can be written the magnitude as $|\Delta\hat{r}| \approx BP\Delta\phi$. So, $r\Delta\phi$ would be the distance of this and the distance is BP and I can get BP from the other triangle, I can get *BP* from other triangle we have the inter relationship.

So, this is BP is nothing, but $r\sin\theta$. So, this expression finally, reduces to this. I have $|\Delta\hat{r}| \approx |\hat{r}|\sin\theta\Delta\phi$. See from your knowledge of vector algebra can I rewrite this expression in the form that we are accustomed to and for that I need to make one small assumption to start with.

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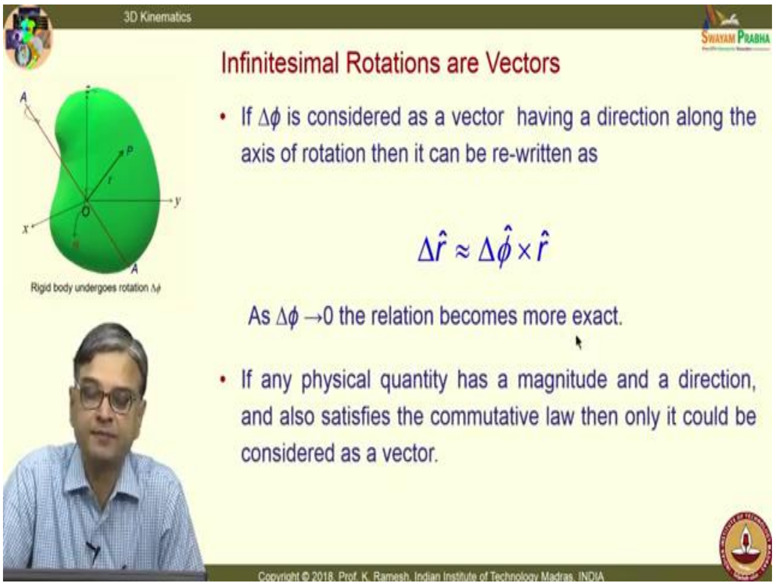
Let me consider $\Delta\hat{\phi}$ also as a vector. If $\Delta\hat{\phi}$ is considered as a vector having a direction along the axis of rotation, then the expression can be rewritten conveniently as $\Delta\hat{r} \approx \Delta\hat{\phi} \times \hat{r}$. So now what will have to do is we will have to disprove $\Delta\hat{\phi}$ cannot be assumed as a vector if I am not able to disprove then finally, I will say that this is a vector that is the way we are going to develop the line of argument.



As $\Delta\phi \to 0$, the relation becomes more exact and what you need to keep in mind is if you have a physical quantity that has a magnitude and direction; you cannot label that as a vector it should also satisfy the commutative law, only if satisfies the commutative law you can label that quantity as a vector. For me to do that I should not stop with one infinitesimal rotation I should also give another infinitesimal rotation and see what happens to the body.

So, that whichever way I do the rotation I must get one unique answer that is a meaning of your commutative law, but this animation is meant for one angle of rotation I am not simulated it for two angles.

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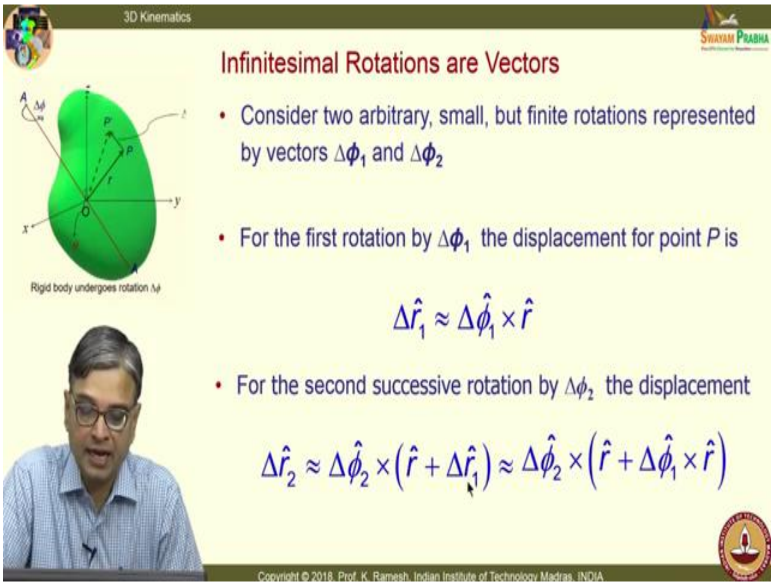
So, here I am going to give two arbitrary small, but finite rotations represented by vectors $\Delta\phi_1$ and $\Delta\phi_2$ you should still treat as an infinitesimal rotation only. For the first rotation by $\Delta\phi_1$ the displacement for point *P* is from the strength of what we discussed earlier you can write it as $\Delta\hat{r}_1 \approx \Delta\hat{\phi}_1 \times \hat{r}$.



Suppose I give a second successive rotation $\Delta\hat{r}_2 \approx \Delta\hat{\phi}_2 \times (\hat{r} + \Delta\hat{r}_1)$ and we already know how to write $\Delta\hat{r}_1 \approx \Delta\hat{\phi}_1 \times \hat{r}$ is related like this. So, $\Delta\hat{r}_2$ is now $\Delta\hat{r}_2 \approx \Delta\hat{\phi}_2 \times (\hat{r} + \Delta\hat{r}_1) \approx \Delta\hat{\phi}_2 \times (\hat{r} + \Delta\hat{\phi}_1 \times \hat{r})$. Now I do the cross product and the knock off term that are very small, then I get an expression what is that I have to do.

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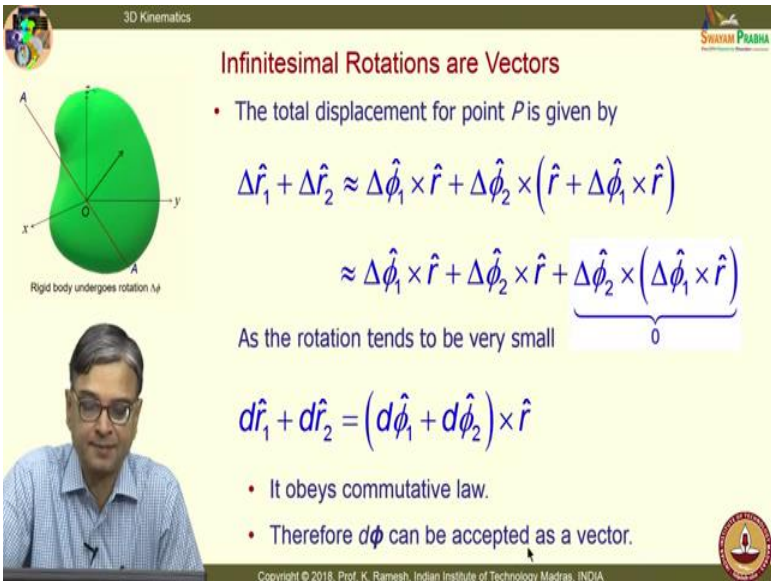
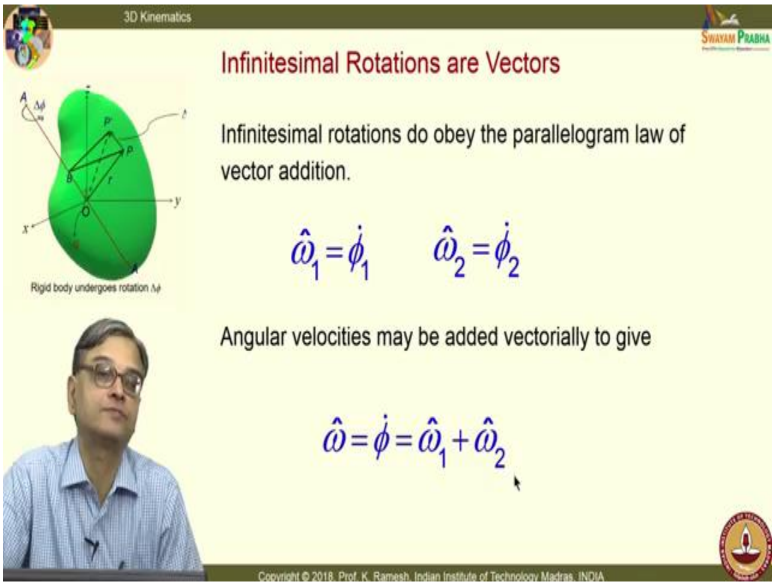


So, I have to now look at what is the summation of $\Delta\hat{r}_1 + \Delta\hat{r}_2$. We know Δr_1 independently will $\Delta\hat{r}_2$ we have written down in terms of $\Delta\hat{r}_1$. So, the expression turns out to be like this and this is fairly simple to simplify and I get this I write the products and this is product of very small quantities. So, I take this to 0. So, I get a very interesting answer when the rotations are very small in the limit, it goes to 0. So, I have this as $d\hat{r}_1 + d\hat{r}_2 = \left(d\hat{\phi}_1 + d\hat{\phi}_2\right) \times \hat{r}$.



Even from the appearance of the equation you would find that the order does not matter. So, it satisfies the commutative law. So, our original assumption $\Delta\phi$ can be treated as a vector stays now and you have an important understanding that infinitesimal rotations are vectors which is a very useful information when we handle 3D kinematics. So, $\Delta d\phi$ can be accepted as a vector.

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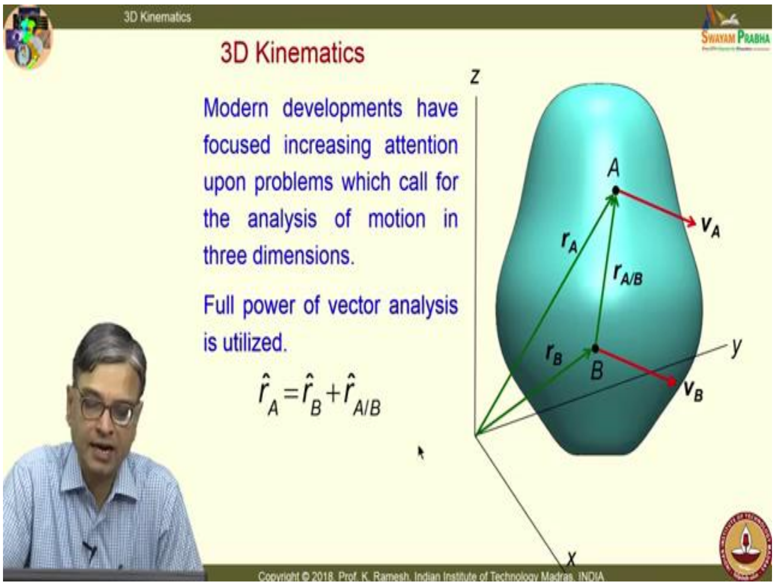

Infinitesimal rotations do obey the parallelogram law of vector addition. So, I can simply write if I have $\hat{\omega}_1 = \dot{\phi}_1$ and $\hat{\omega}_2 = \dot{\phi}_2$, I can add this angular velocity, $\hat{\omega} = \dot{\phi} = \hat{\omega}_1 + \hat{\omega}_2$; I can do the vectorial summation.

See in the earlier chapter we always had the rotation perpendicular to the plane of motion. We had α and ω both are parallel; in this chapter we will relax all of those requirements it can be in space; ω can change direction as well as magnitude.

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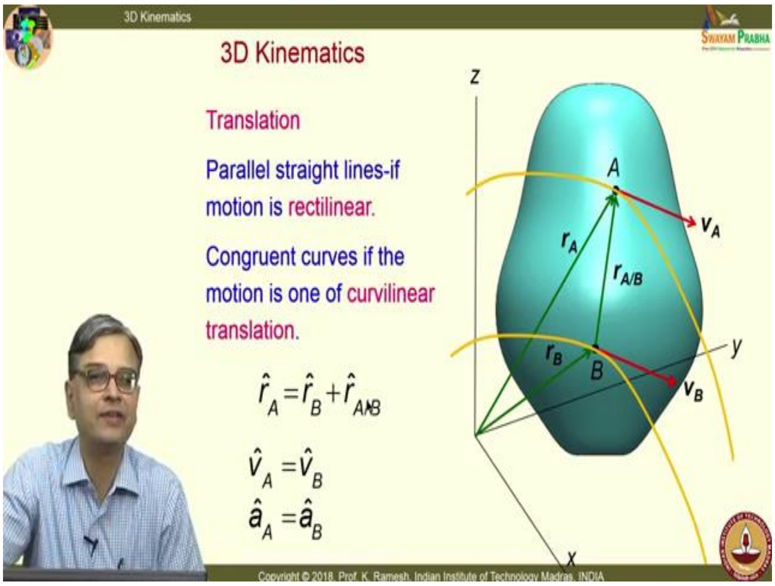
And many of the discussions on translation curvilinear translation is simply going from two dimension to three dimensions; it does not make much of a difference in those calculations. So, we have increasing attention upon problems will call for the analysis of motion in three dimensions, you cannot stay away from it and I have a frame of reference and I have some three-dimensional object. And I can also have a point *B* point *A* located like this. I can have the position vectors. And once you come to three dimensions; you know earlier have all base being saying try to visualize here you have to bank on your vectorial analysis.

So, what all the visualization training that you had for 2 dimensions, can assist you in some way ok, but you have to bank on the power of vector analysis because you are going to handle things in three dimensions. So, I have $\hat{r}_A = \hat{r}_B + \hat{r}_{A/B}$ it is all very simple and straight forward now each of this will have three components instead of two components, that is only difference and I have simple translation I have V_A and V_B.



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When I have translation parallel straight lines if motion if motion is rectilinear. So, I can have the situation that $\hat{v}_A = \hat{v}_B$ and $\hat{a}_A = \hat{a}_B$. So, then the motion is rectilinear translation.

We have looked at rectilinear translation; we have also looked at curvilinear translation in planar objects. So, you can also have a curvilinear translation for a three-dimensional object. So, I will have curved translation congruent curves if the motion is one of curvilinear translation and to an extent, I have drawn these lines it is all free hand sketch.

So, electronically I was not able to put it so, perfectly; so, visualize that it is congruent curves ok. The body as such does not rotate about itself; it has a curvilinear translation.

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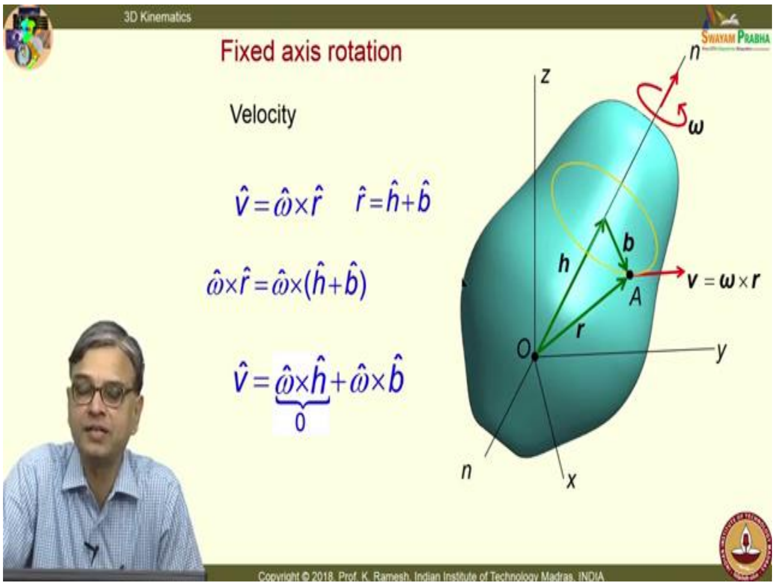
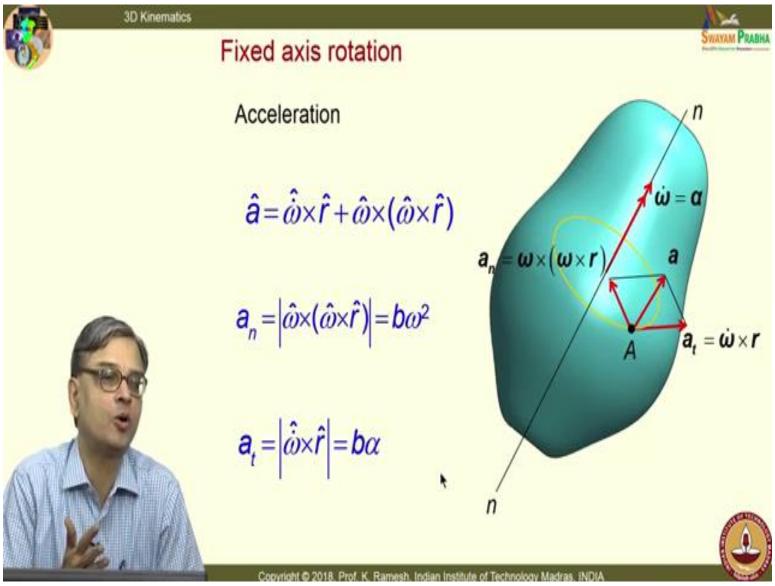


And right from your circular motion we were looking at fixed axis rotation. Fixed axis rotation is possible in three dimensional problems as well and look at what way we are getting it. I have an axis of rotation which is label like this, I can find out the velocity as well as acceleration and the body is rotating about this axis, the axis *nm*. So, I have this rotating about this axis and when I take a point A on the object, if you visualize what way this will have the motion, it can have a rotation in this the position vector is shown and I have always use the position vector in green color maintain that. So, this is how the point will have the motion.

So, here you say the position vector like this, when the origin of coordinate system you are looking at the position vector, but this also I can look at as summation of two other vectors. And like what you have learnt in your simple circular motion, here again $\hat{V}=\hat{\omega}\times\hat{r}$ and that will be tangential to the path v is $\boldsymbol{\omega}\times\boldsymbol{r}$, and I could identify the vector *r* as consisting of vector *h* along the axis of rotation and this radius *b*.

So, when I substitute on this what happens? $\hat{r}=\hat{h}+\hat{b}$, $\hat{\omega}\times\hat{r}=\hat{\omega}\times(\hat{h}+\hat{b})$, but it finally, reduces to simply $\hat{\omega}\times\hat{b}$ because the first term goes to 0 and you have to appreciate how we define the position vector *r*. I mean some of these simplifications can help you in solving problems; you are not getting any different result when I take $\hat{\omega}\times\hat{r}$ or $\hat{\omega}\times\hat{b}$ its one and the same, but you recognize that this is what happens to this. So, similar to velocity whatever we have learnt in circular motion for acceleration we could invoke and look at, what happens to the three-dimensional body.



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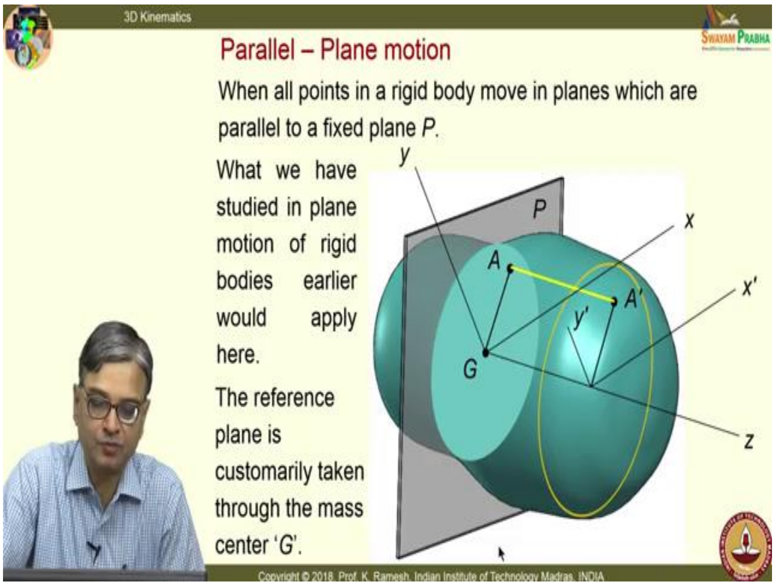
So, we move on to acceleration, and I have this as $\hat{\dot{\omega}}$ let us put as α and I have the point *A*, and you have the tangential acceleration and normal acceleration to that again borrowed from your simple circular motion. So, I have this component normal component as $\hat{\omega}\times(\hat{\omega}\times\hat{r})$, that we know from the earlier discussion on *r* and *b*, we can write this as simply as $b\omega^2$ and I have a tangential component that is $a_t=\left|\hat{\dot{\omega}}\times\hat{r}\right|=b\alpha$.



So, whatever you have learnt on circular motion, in various forms it shows up in the complete discussion of dynamics. So, you should be able to use them depending on the context comfortably that is very important. You are not learning any new equations only you understand how to use these appropriately in your problem solving.

And I have the resultant acceleration is like this, I have the normal and tangential components; so, I can visualize this. So, it is fairly straight forward what you have to recognize is each of these vectors will have three components instead of two components in 2 dimensions.

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And we have also looked at planar motion, you can have something similar to that in three dimensional problems as well. I have taken a very generic three-dimensional body, there is no symmetry along the *z* axis ok. So, it is very arbitrary. So, I could have some reference plane which is usually the mass point plane. So, I have the axis labeled as *XY* and *Z*. It is a simplified motion of the three-dimensional body in this what we are really looking at is whatever the motion happens on the central plane, a similar motion takes place on all the parallel planes ok.

So, you are taking baby steps, you are not bringing in all the complicities of three-dimensional motion. So, whatever you have learnt in kinematic relations for planar motion, you can invoke here provided you have parallel plane motion from kinematics point of view. But if you go to kinetics point of view because I do not have access is about the *z* axis it is not symmetric; the moment equations will be much more complex. It will have products of inertia, will have I_{yz} and I_{xz} components, kinetics will be little difficult, but kinematics will be very similar to what we have learnt in the previous chapter. So, whatever happens to this plane happens to the other plane also. This is the simplified visualization of motion; in some practical problems you could look at this idealization and solve your problem.

So, when all points in a rigid body move in planes which are parallel to a fixed plane *P*; the plane is defined by the like this; what we have studied in plane motion of rigid body is earlier would apply here. The reference plane is customarily taken through the mass point G. So, these are all handmade sketches. So, you have to visualize that there are difficulties in doing them, but gives we have perspective that we are dealing with a three-dimensional object which is generic in nature ok. And I have this as a mass point *G* that is what is located.

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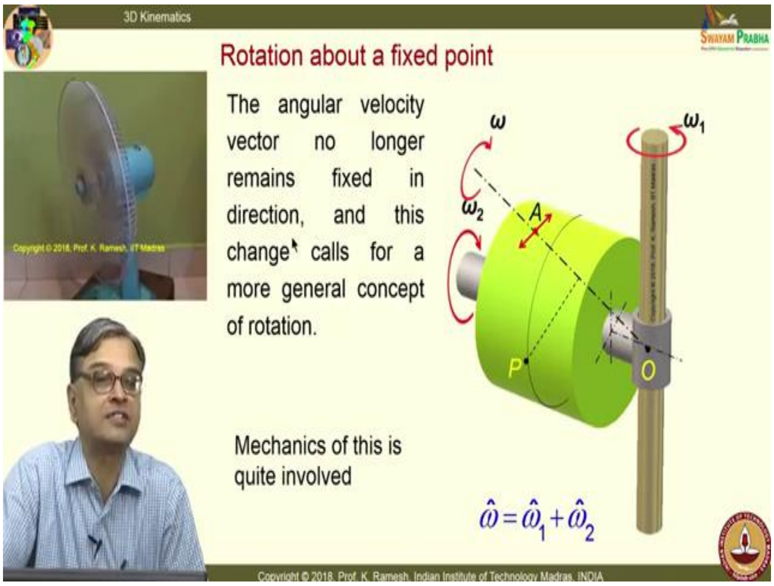
And I suppose when you are kids you had opportunity to play this single top ok. And please observe this motion very closely and particularly the face where you have a motion like this it is wobbling ok. And you have a very special kind of motion here you know you have a nail at the bottom of this top. So, it is rotating about a point and what you find here is the top is rotating about its axis not in this phase in the later phase, when it is about to collapse the axis also has a motion that is called a precision motion fine. But at on the same time you will have that the whole thing rotates about the point momentarily see it is a dynamic problem here and whenever we study the problem, we do not look at the transience, we look at the study state condition and look at it fine.



And, in this you have a very complicated motion to start with and the complete motion analysis we are not doing it, we are recognizing as a situation where it is rotating about its axis and it is also having a precision fine. And, you have this kind of motion you know is known as rotation about fixed point; this is what is different when you come to 3D kinematics, you have to recognize fixed point rotation what are its characteristics so, on and so forth.

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And you know you have a table fan, your table fan has this kind of a motion; I have the an blade which is rotating at a very high speed, and you can set the back portion appropriate, there is a knob that you can put it and appropriate position and it also has a slow motion in the other axis and this is called precision ok.

And suppose you have a different arrangement where in I have this as a transparent object with lot of particles inside which are easily floating, when I rotate this with ω_1 and when I rotate this with ω_2; you would see that this particles get aligned they do not move; it is a visual representation that the axis of rotation is different ok. Make a neat sketch of this diagram also will again and again that is why I am also little slow in explanation.

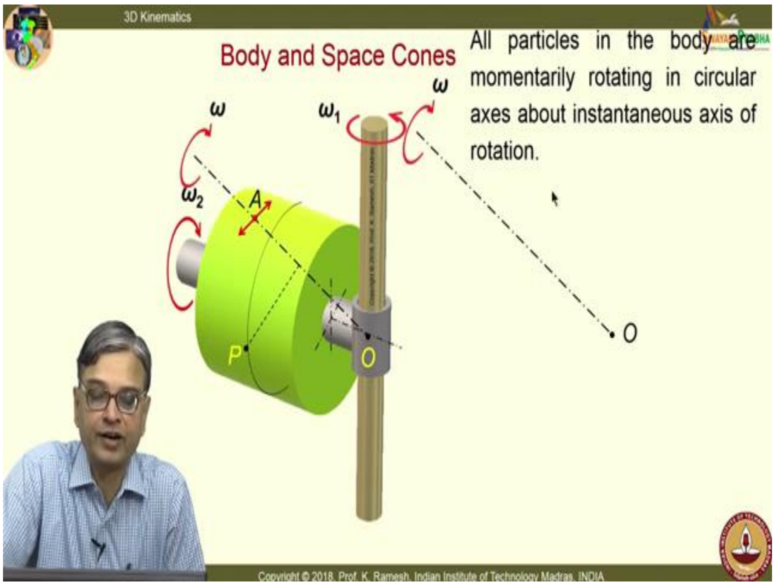
So, I have a shaft for you to visualize can have these two motions I just put the table fan. You know when you see a table fan you never recognize that these motions it has, you take it for granted, it should blow air to your face you should feel comfortable you do not think anything beyond that ok.

This is not exact reproduction of this; this is another way of looking at the whole thing to appreciate what happens to the rotation axis physically. We have already learnt when I have ω_1 and ω_2; how to find out the ω of the system? I can have a vector addition simply write this as $\hat{\omega} = \hat{\omega}_1 + \hat{\omega}_2$. I can have this as $\hat{\omega} = \hat{\omega}_1 + \hat{\omega}_2$ and that would appear as a line like this with angular velocity ω.

So, what the discussion is if you take any point on this axis, it will have 0 velocity. It will have one velocity component from ω_1, another velocity of component from ω_2 they cancel each other. So, along this axis momentarily the velocity is 0. So, this adds like instantaneous axis of rotation and this rotates about the fixed-point *O* ok. And if you take any other arbitrary point suppose I have this cylinder filled with particles which are in some color maybe I have put this as a black line I can consider all of these points as a black particle. And if suppose this cylinder is at transparent when I take a snapshot of a dynamic event, I would find the particles will be focused along these lines indicating momentarily they are not moving.

For any other general point because the point is moving; you will not be able to focus it properly. So, you will have smear of that. So, this is an indication indeed you can observe an axis of instantaneous axis of rotation like this when I have motions like this have ω_2 on this and ω_1 on this. You know in all other problems in 2 dimensions we have never had a situation the axis of rotation itself changes in three dimension the angular velocity vector no longer remains fixed in direction and this change calls for a more general concept of rotation. This is what makes three-dimensional motion little more complicated, but if you understand the integrity details it is not that difficult to handle, it is quite simple if you follow a systematic procedure.

So, what you find here is, you can expect an instantaneous axis of rotation now the question is suppose visualize what happens to this instantaneous axis of rotation when it has a complete motion.



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And that is what is depicted in the next slide; you have a concept of body and space cones. So, I have this axis like this. So, this is the axis of instantaneous axis of rotation, all particles in the body are momentarily rotating in circular axis about instantaneous axis of rotation fine. So, this is all particles are rotating about this axis.

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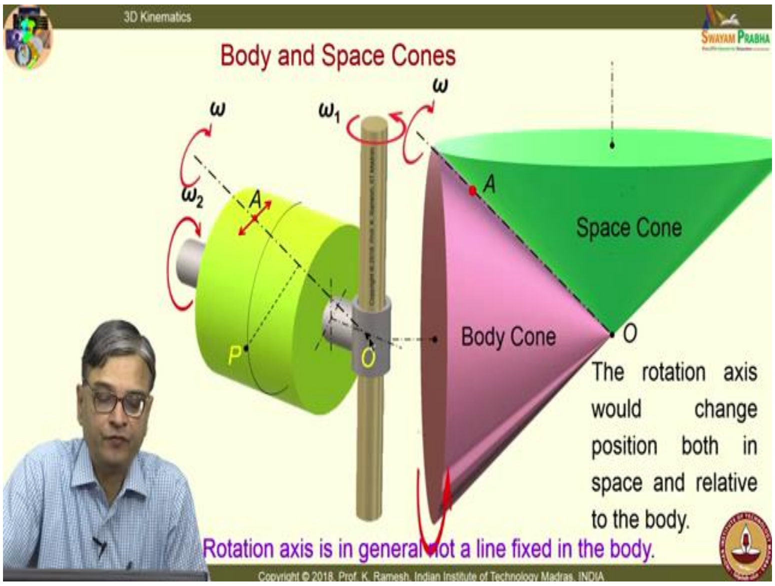
And if you look at this the rotation axis would change position both in space and relative to the body. So, you have what is known as a space cone and what is known as a body cone for this. And we have taken a situation where the angular velocity remains constant fine, I have taken a situation.

See in the top what you saw? The angular velocity was changing it is a much more complex dynamics which you saw the top analysis is not simple ok. And now you can visualize that this axis on the body it will move like this and this body cone will roll on this space cone. And in this case, they are simple cones like this rotation axis in general not a line fixed in the body. So, this is the fixed point of rotation.



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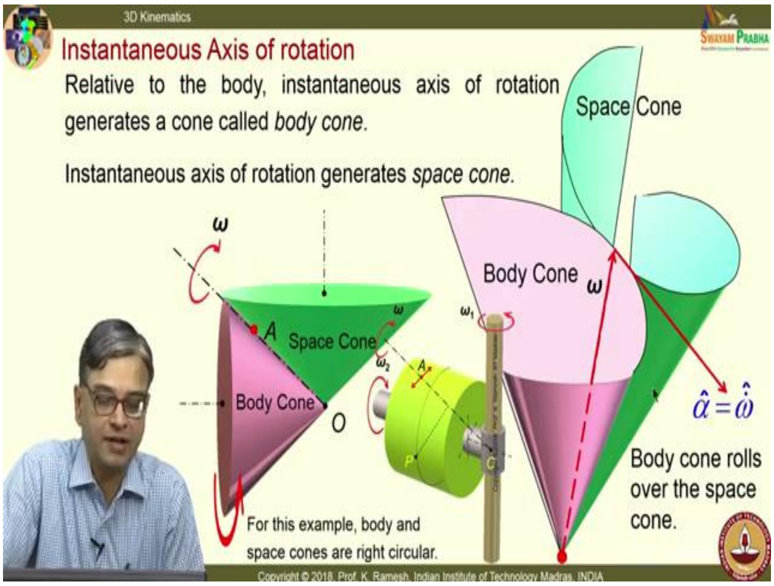
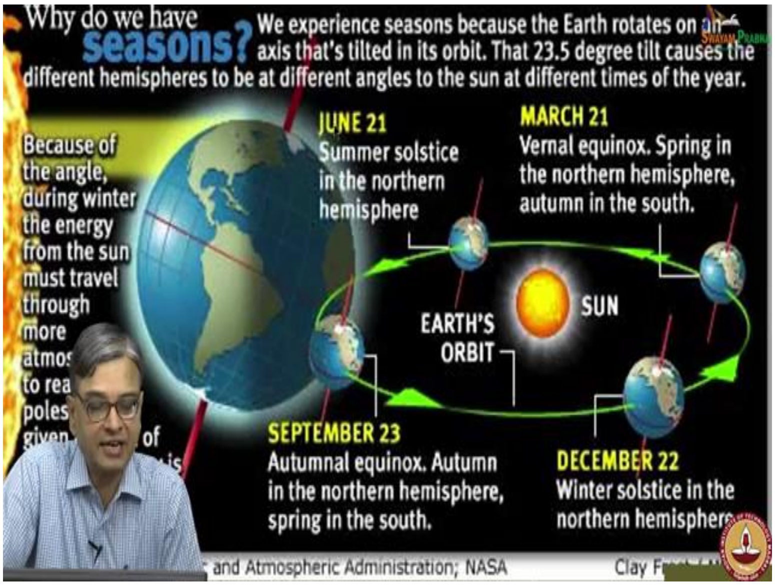
So, the relative to the body instantaneous axis of rotation generates a cone called body cone. So, I have a body cone, now I am looking of a situation where the angular velocity does not remain constant, we had a situation angular velocity remained constant. Suppose I allow angular velocity to change its magnitude as well as direction fine. I would in general have an arbitrary cone like this and I would have instantaneous axis of rotation generates a space cone that would be another arbitrary cone like this; the only requirement is that this will roll over on that ok.



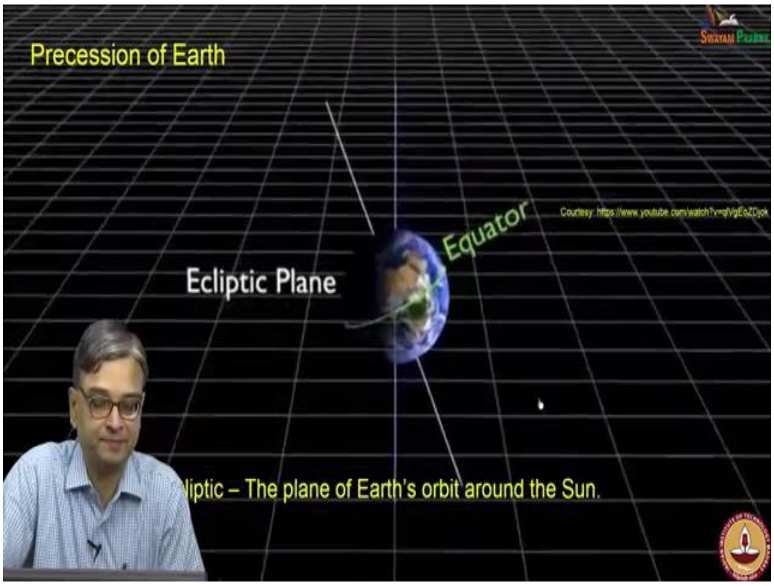
So, if I look at this as defining the angular velocity, you can see that the magnitude and directions are changing ok. And, you will have the acceleration tangential to this, and we will have to see how to get this $\hat{\alpha} = \dot{\hat{\omega}}$ how to get this? We would look at a simple case where there is no angular acceleration; we would also look at a case where there is angular acceleration. So, you have a small simple recipe when the situation is simple and how to handle it in a generic sense.

In all the cases whether I have a simple motion or complicated motion, body cone rolls over the space cone this is how it happens. And I have a very simple situation here where the angular velocity remains constant, then I have the cones that we saw earlier ok. The



body cone and space cones are simple, they are space cones are right circular this happens because of simplicity in the motion, in a generic sense it can have a complicated situation like this ok.

So, it is also very interesting to note you know in schools you learn you have the earth which is tilted and it is rotating, and this tilt only causes seasonal change you do not give a dam to it so, it all school portion you have to afford to forget it ok.



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And you also have a plane about which this moves around the sun and what you have to look at is, you have a equinoxes March 21st and September 23rd, they do not remain same what you will have to look at is, like we have looked at a precision on the top; the earth also has a precision.

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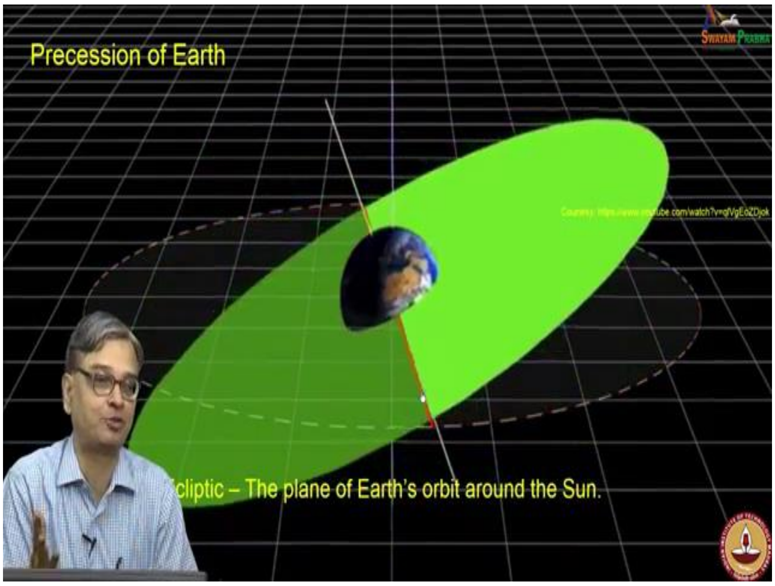
See these are all animations artificially made.



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So, certain aspects are brought out clearly, certain aspects are not brought out you have to understand that is not that as if the earth rotation is fully captured.

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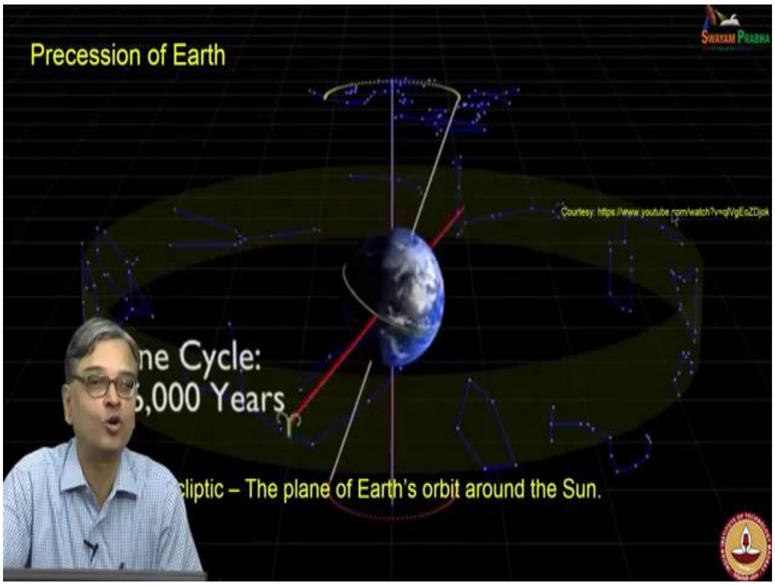
And you can go to this site and then get more details on the comments ok.



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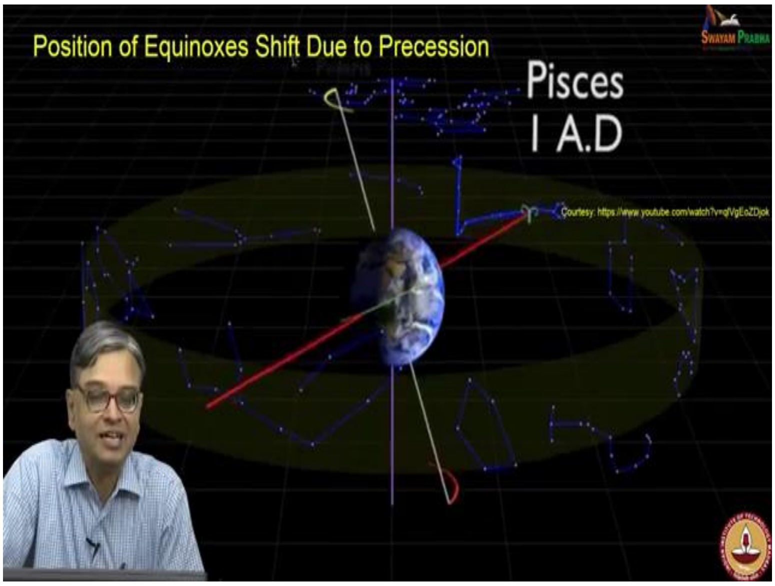
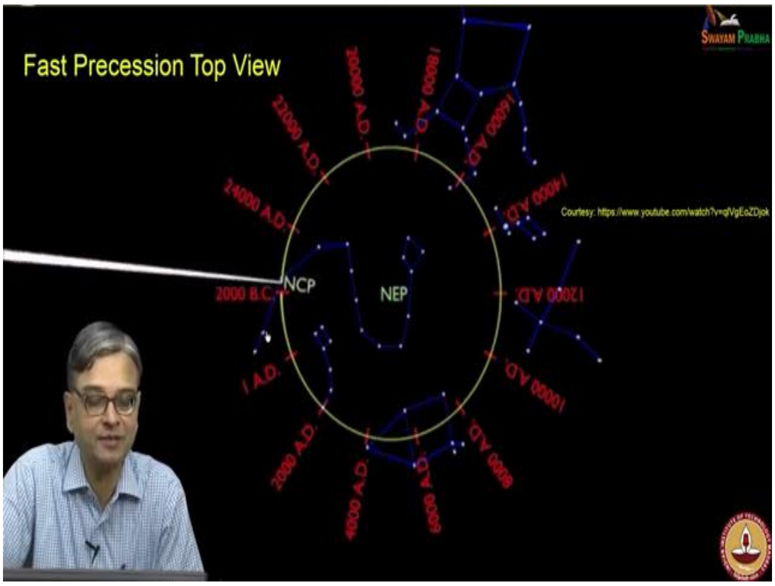
Nevertheless, this is a very interesting information, it provides you certain basic explanation of the complicated motion. Here what is shown is this axis has a precision.



See in the top you saw that it is revolving very fast; do you know how long that this takes to make a one full circle? A mind-boggling figure it is a very slow phenomena it takes about 26000 years the precision takes 26000 years to do and this results in shifting of equinoxes.

And this is shown as the zodiacal sign that also I had I think have another slide which explains this is how you will perceive your cosmos around you. So, the equinoxes shift it is a very subtle phenomena



it takes about 26000 years; that means, how many generations should have observed it to calculate this fine. So, it is a very mind-boggling observation.

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And you also have this, this is 2000 BC and we are in 2000 AD ok. So, it is like this if you look at the top view it is like this. So, it will keep changing all this is very complicated fine this takes about 26000 years. Let us go back to history what way people have struggled to understand this.

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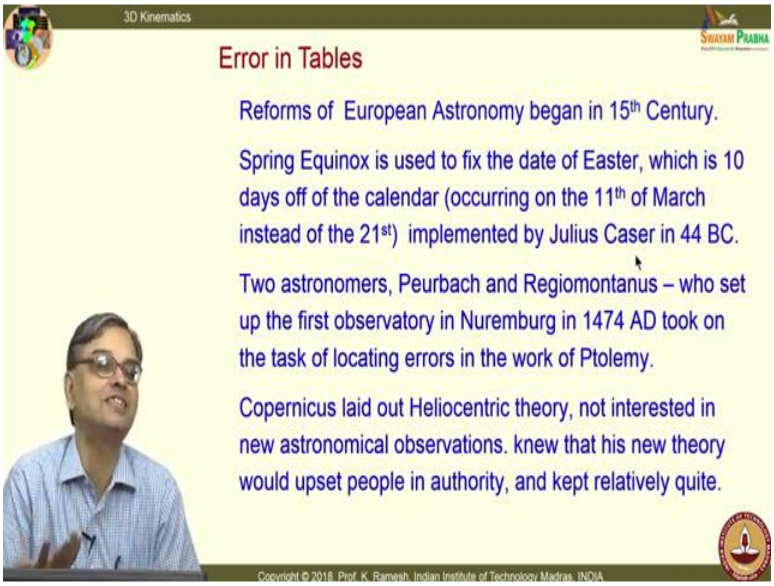
You know reforms of European astronomy began in 15th century, making a calendar is



not simple. You saw our academic calendar it has gone through a lot of revisions in this semester and making an actual calendar it is not that simple task, we take them for granted we normally take them for granted. And Spring Equinox is used to fix the date of Easter which is 10 days off of the calendar occurring on the 11th of March instead of 21st implemented by Julius Caser in 44 BC.

So, whatever the calendar that was generated by Julius Caser you cannot use it; you have to keep changing it fine only that is correct fine. So, there was a shift of 10 days this is what I discussed as a shifting of equinox. And you know there were lot of discussions on how to locate the errors, they had only the calculation by Ptolemy; I said Ptolemy believed in geocentric theory and he gave a complicated explanation of retrograde motion of planets and you know human mind wants complex explanation not simple ones ok. And Copernicus had the idea of heliocentric theory not interested in new astronomical observations knew that his new theory would upset people in authority and kept relatively quiet.

This was the kind of environment in the western world and we have already seen that shifting of equinoxes happens, naturally people looked at cosmology and then only got inspiration and he took 26000 years for one full rotation.

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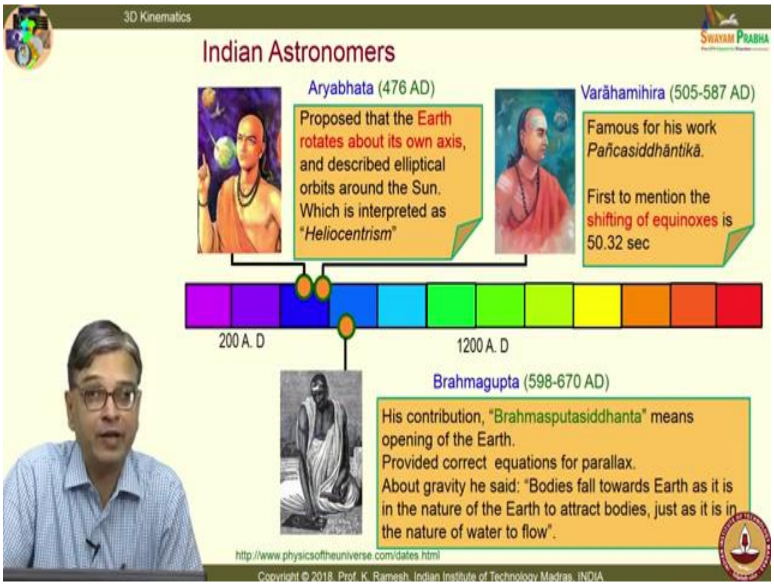
And if we go back to our history, you will be surprised much before Newton came in 1700 and then explained many of these concepts, you had record of Shifting of Equinoxes by Vrahamihira way back in 500 AD. How they have done it? Only god knows you have to give definitely give credit to the Indian aspect of science, we do not know the methodology that they have followed, but they have definitely made very significant contributions in understanding.



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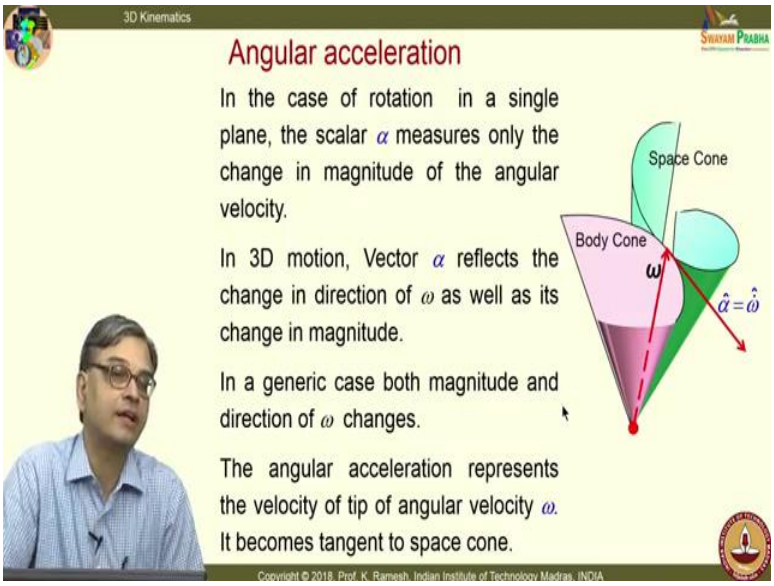
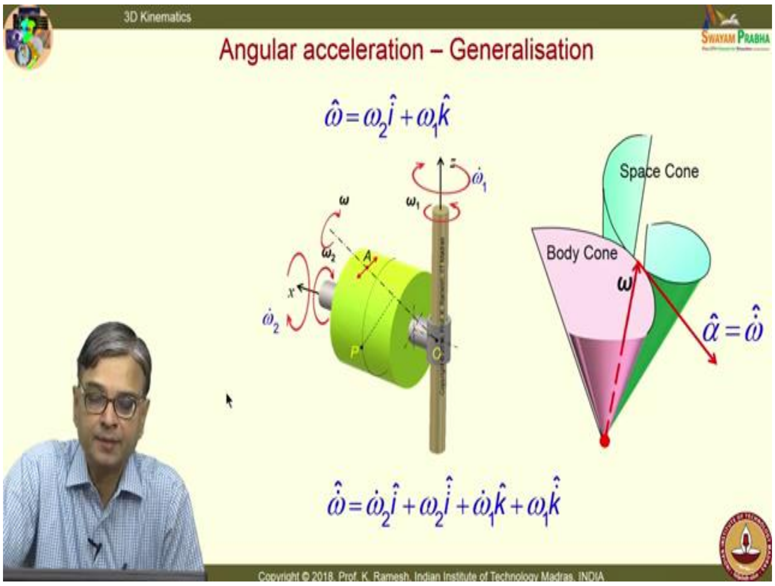


So, let us come back to this angular acceleration. So, in the case of rotation in a single plane the scalar α measures only the change in magnitude of the angular velocity this is what you had learnt in your 2-dimensional kinematics. We have never looked at that ω changes its direction; we have seen example of fixed-point rotation where ω changes its direction ok. In 3D motion vector α reflects the change in direction of ω as well as its change in magnitude.

So, it is better to visualize the body cone and space cone. So, this illustrates the change in direction of ω as well as its magnitude, and the tangent to that gives you the value of α. In a generic case, both magnitude and direction of ω changes, I have given an example for that. The angular acceleration represents the velocity tip of angular velocity ω, it becomes tangent to space cone ok.

So, make sketch or complete the sketch that you have started drawing it earlier, and we know the angular velocity. Once you know the angular velocity you can find out the angular acceleration by differentiation fine, we will just see that.



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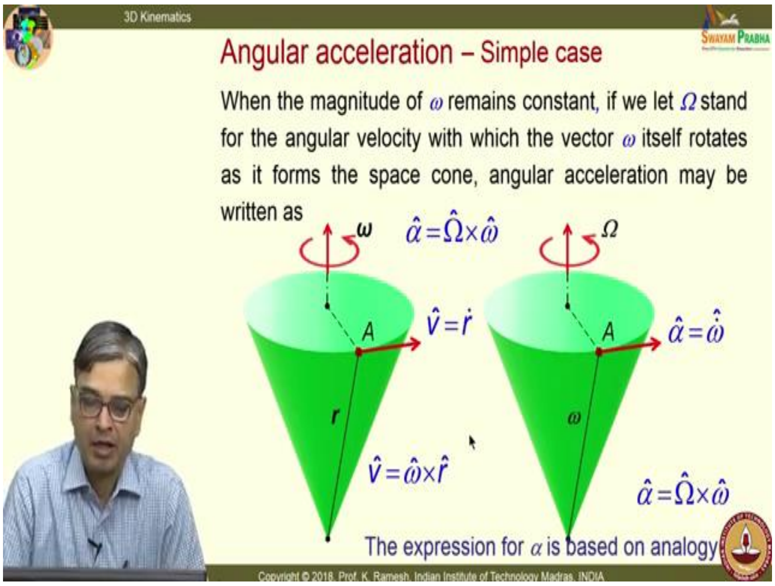
And I have a generalization when I say is generalization, I would also bring in angular acceleration to this as well as angular acceleration to this, then did it becomes a generic case and angular velocity you know *ω* equal to I have put this as *x* and *z*. So, on this axis I have ω_2. So, I put this as $\hat{\omega}=\omega_2\hat{i}+\omega_1\hat{k}$. So, if I have to get *ω* dot naturally, I will have to do this differentiation and do it systematically $\dot{\hat{\omega}}=\dot{\omega}_2\hat{i}+\omega_2\dot{\hat{i}}$. And we have already learnt for convenience we use this as *i* and *j*, but we recognize these axes rotate. So, I will have dot i dot is not 0, j dot is not 0, k dot is not 0 I have to calculate those quantities.



So, the acceleration will take an expression like this, I have $\dot{\hat{\omega}}=\dot{\omega}_2\hat{i}+\omega_2\dot{\hat{i}}+\dot{\omega}_1\hat{k}+\omega_1\dot{\hat{k}}$. And here I can do a very simple case to start with; so, when I look at this as a generic case, I have the body cone and space cone like this, which is very generic which has a very complicated shape.

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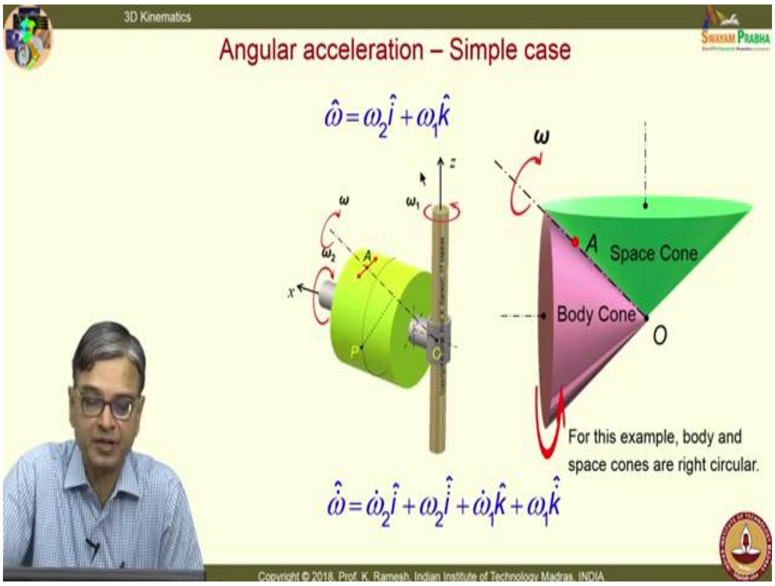
Suppose I have a simpler situation where I have no angular accelerations ω_1 dot and mega 2 dot 0, people also invoke analogy I have $\hat{V} = \dot{r}$. And similarly, I can put $\hat{\alpha} = \dot{\hat{\omega}}$ and we have set $\hat{V} = \hat{\omega} \times \hat{r}$ based on analogy you also call this as $\hat{\alpha} = \hat{\Omega} \times \hat{\omega}$. Then the magnitude of ω remains constant if you let $\hat{\Omega}$ you know in three dimension you will bring in this another symbol just for convenience if you let $\hat{\Omega}$ stand for the angular velocity, with which the vector ω itself rotates; as it forms a space cone angular acceleration may be written as $\hat{\alpha} = \hat{\Omega} \times \hat{\omega}$ this is by analogy.

The expression for α is based on analogy in some cases just to solve the problem quickly you can invoke this. But the most generic way of doing this is differentiate it and put *i* dot and *j* dot or *k* dot whatever that comes into it appropriately you can handle it.



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So, in a very simple case I have this as $\hat{\omega} = \omega_2\hat{i} + \omega_1\hat{k}$ and then I have this has $\dot{\hat{\omega}}$ written down like this. And in this case, I will have a simple right circular cone as defining this, and you will have $\dot{\omega}_2\hat{i}$ and $\dot{\omega}_1$ or 0 because that is how we have listed the case.

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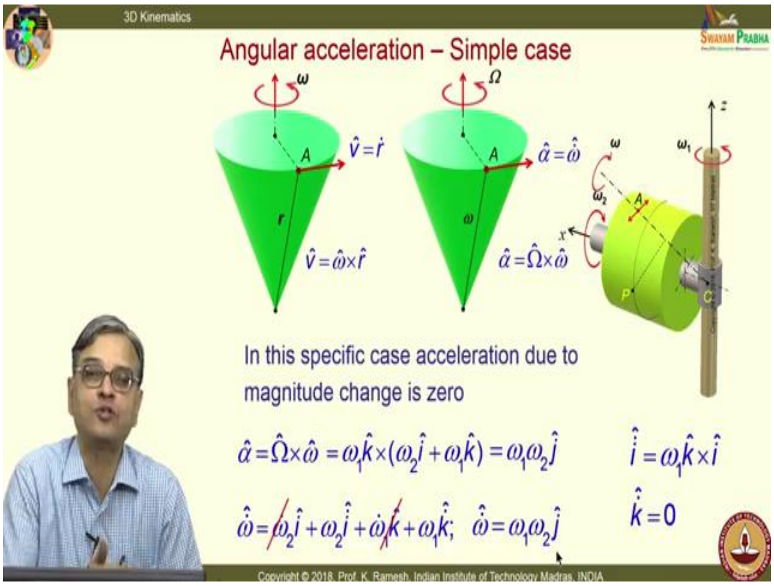
So, which is what is continued here I have this, we have learnt by analogy I can write it as $\hat{\alpha} = \hat{\Omega} \times \hat{\omega}$. So, when I do this, I have $\omega_1\hat{k} \times (\omega_2\hat{i} + \omega_1\hat{k})$. So, this reduces to $\omega_1\omega_2\hat{j}$ and if I have a generic expression, we know that $\dot{\omega}_2\hat{i}$ and $\dot{\omega}_1$ or 0 and we have to look at what is *i* dot. And we have already learnt i dot as can be written as $\dot{\hat{i}} = \omega_1 \hat{k} \times \hat{i}$ you go back to your rotating frame of reference; we have expressed it in this fashion also.

So, when I invoke this $\dot{\hat{k}} = 0$ because this does not rotate in space only this rotates by this root also you get $\dot{\hat{\omega}} = \omega_1 \omega_2 \hat{j}$. So, you can choose the generic expression, my recommendation is; use the generic expression because you are not making any compromise on your basic understanding, but books also discuss if the case is simple rather than going through a circuitous process, simply put α based on analogy. So, in this lecture we have looked at infinitesimal rotations can be treated as vectors.



Then we looked at fixed point rotation a very special case, in the case of a three-dimensional analysis we saw the simple top. We saw that it was precising, we also looked at how the earth precision has been a point of discussion in astronomy and what was the Indian contribution to that. And we have also learnt how to find out the acceleration in a generic sense for fixed point rotation.

Thank you.