Engineering Mechanics
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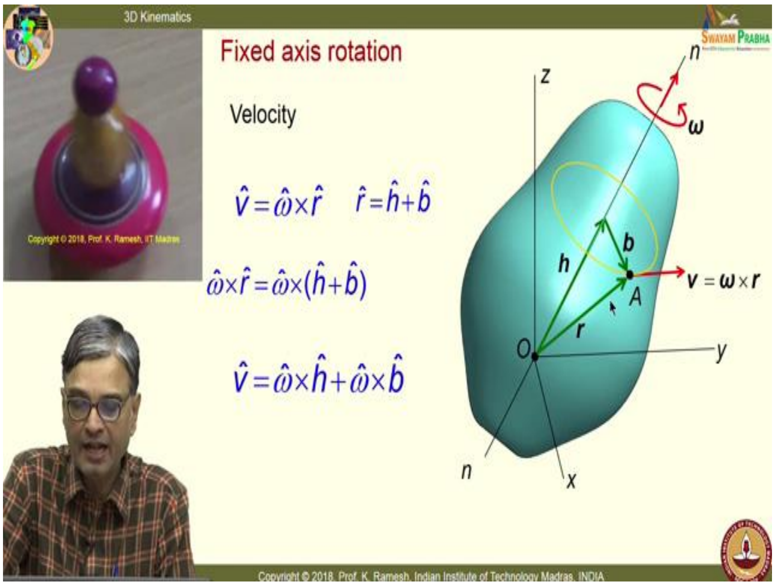
Module – 02
Dynamics
Lecture - 16
3D Kinematics II

See one of the important concepts that you learn in this chapter is actually fixed-point rotation, if you look at this top it starts with a fixed axis rotation then towards the end, it shifts to fixed point rotation. And fixed-point rotation is the new concept that you come across in 3D dynamics ok. So, it shifts to fixed point rotation towards the end.



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And if you look at fixed axis rotation, when the top starts rotating more like fixed axis rotation ok, some wobbling is there, but it is very small. And if you look at the axis, I have a generic axis put like this *nn*. So, it is rotating about this axis, it is having an angular velocity ω. And we have seen already, how do you define the position vector and how do you visualize the direction of velocity, what is the expression for velocity and how this can be rewritten in terms of the position vector *h* and *v*, all that we have looked at. But these are the equations for velocity that you have got is same as what you have done it for simple circular motion ok.

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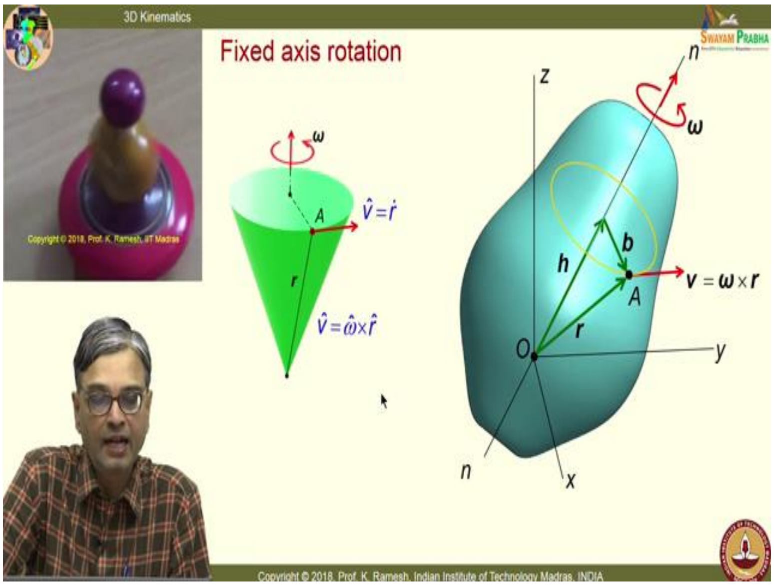
And what you need to notice is, when I have this is rotating, I take a particle we visualize that this is rotating and I visualized this as the cone. You know there was a difficulty when I use the analogy, where does the cone come from. You identify this as the position vector r and I have this angular velocity ω and we have the velocity as $\hat{V} = \dot{r}$, which is tangential to this circle. And, we have also got this expression $\hat{V} = \hat{\omega} \times \hat{r}$. And the same analogy is used to find out angular acceleration in a simple case.



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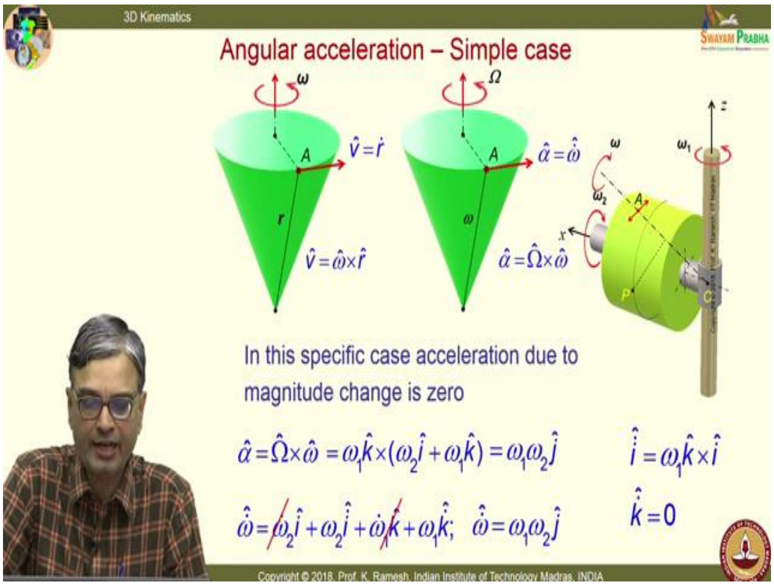

Here what we have discussed was imagine that, this is the transparent cylinder filled with black particles. It has a rotation about this axis ω_2 and there is a small precision about the vertical axis ω_2. And you would see, when you take a photograph, you will see a line seen in your photograph, a snapshot wherein the black particles will form this axis. This is the instantaneous axis of rotation; any other point will appear blur ok. And if you want to find out what is the acceleration for this special case where the precision is steady. It does not have an acceleration about this z-axis ok.

And you look at what you have done in the case of velocity earlier you use the analogy, and simply have the expression for α as $\Omega \times \omega$. And in this chapter, we would have a special symbol which will denote the rotation of the axis; rotation of the coordinate axis is put as Ω for simplicity, because when you have multiple omegas you do not know which ω you have to use. So, this comes from analogy this is something like a shortcut, when I have a steady precision, I can define what is the acceleration straight away from this.

And, we have also looked at in the previous class, evaluating α by this $\Omega \times \omega$ as well as when I have $\omega = \omega_1 + \omega_2$ differentiate, it systematically recognizes the unit vectors have i dot and j dot is valid. From that route also we have a plane ok. And while employing this you know books put this ω both are simply ω_2, because ω_2 is one which rotates with the precision of ω_1 and it is also written as the complete ω. Mathematically this is one and the same because $k \times k$ will go to 0 and in solving problems this is also exploited ok, and I get this as $\hat{\dot{\omega}} = \omega_1 \omega_2 \hat{j}$. This is a shortcut when I have a steady precision.

If you want to do it systematically from first principle, I can have this as $\hat{\dot{\omega}} = \cancel{\dot{\omega}_2}\hat{i} + \omega_2 \dot{\hat{i}} + \cancel{\dot{\omega}_1}\hat{k} + \omega_1 \dot{\hat{k}}$.

In this specific example, we have deliberately considered $\hat{\dot{\omega}}_2$ as 0, $\hat{\dot{\omega}}_1$ as 0 so, though those terms get knocked off and k dot remains same, and we have learnt a powerful way of writing $\dot{\hat{i}} = \omega_1 \hat{k} \times \hat{i}$.

So, when I do that also I get the same expression for acceleration. See you learn multiple recipes depending on the context you can use it. You can also use it for verification or do minimal mathematics, the choice is yours. And another way of looking at this is you know α essentially gives you what I have got as ω_1 ω_2 j as nothing but ω_2 i dot. So, this is the magnitude change and this is the direction change. So, you can also visualize this as an expression that gives you directional change. This is another insight into the problem fine. We will solve a variety of problem the ideas would become much clearer.

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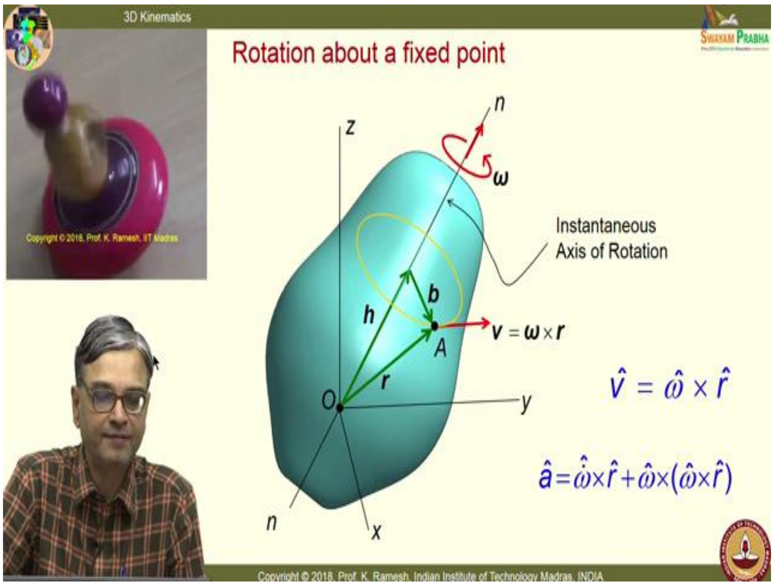
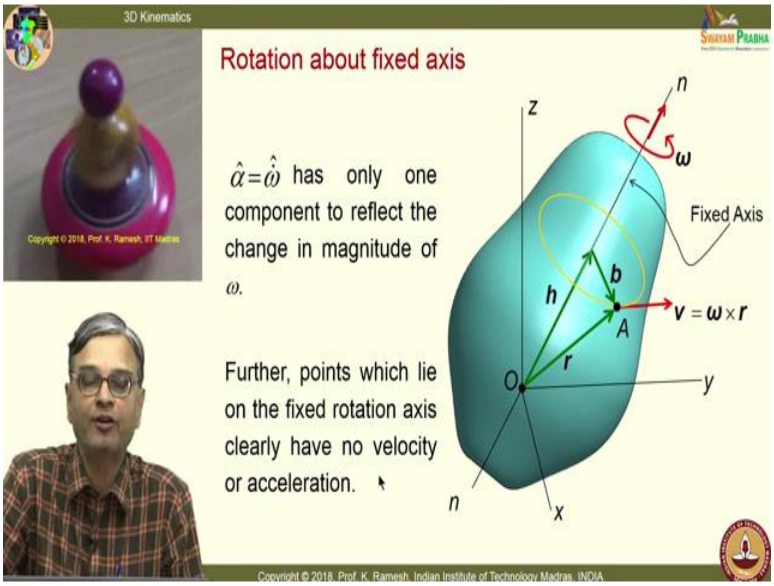
And let us compare fixed axis rotation and fixed-point rotation. I have object like this; this is rotating about the axis *nn* and this is the fixed axis and I have this angular velocity all this we have seen. We have a point a on the object, that moves in a circle like this and I have $V = \omega \times$ r, this expression is same for both fixed axis as well as fixed rotation. See in fixed axis rotation, we define $\hat{V} = \hat{\omega} \times \hat{r}$ and then we also get acceleration as $\hat{a} = \dot{\hat{\omega}} \times \hat{r} + \hat{\omega} \times (\hat{\omega} \times \hat{r})$.



If you go back to your two-dimensional analysis these expressions where identically same, we are moving from two-dimension to three-dimension, the vectors would have three components instead of two components. And another aspect is when I move from fixed axis rotation to fixed point rotation, if you look at the expressions for velocity and a, the expression still remains the same, but what you substitute becomes different fine. And, that is what I shown here, when I go to fixed point rotation you see that this also has a precision. And that precision is changing and finally, top collapses.



So, instead of this being the fixed axis rotation, this becomes the instantaneous axis of rotation, when I have a rotation about fixed point. So, that is the subtle difference fine. And what are all the other differences we will also have a look at it.

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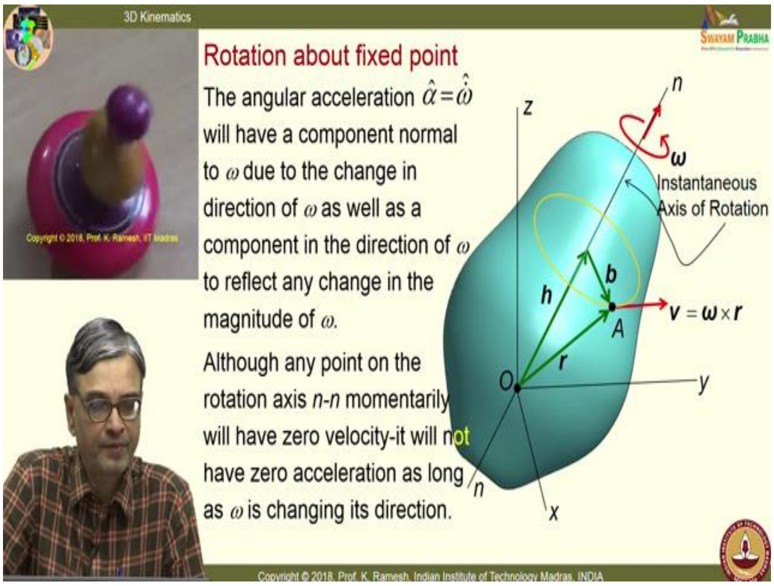
If I have a rotation about the fixed axis, α is $\dot{\hat{\omega}}$ has only one component to reflect the change in magnitude of ω. So, in a fixed axis rotation, when I say α that gives you only the magnitude change of ω, because it is rotating about a fixed axis; the axis is not changing and what else we see? The points which lie on the fixed axis of rotation clearly have no velocity or acceleration. That is what you see in a fixed axis rotation.



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Suppose I move on to fixed point rotation, what happens? You have seen the top transitioning from fixed axis to fixed point rotation here. The first observation is this axis becomes instantaneous axis of rotation that is only difference, the expression for velocity and ω, α expression wise they are similar. The angular acceleration α now also talks about the directional change of angular velocity that is a very important observation.

So, when I am looking at fixed point rotation. I was worry about both magnitude change of ω as well as directional change of ω. Although any point on the rotation axis n and momentarily will have zero velocity, it will not have zero acceleration as long as ω is changing its direction. It is a subtle point when I am coming to a fixed axis fixed point rotation, there is a directional change of ω, that is what you see as precision. So, these are the differences between fixed axis rotation and fixed-point rotation, but the expressions are very similar.

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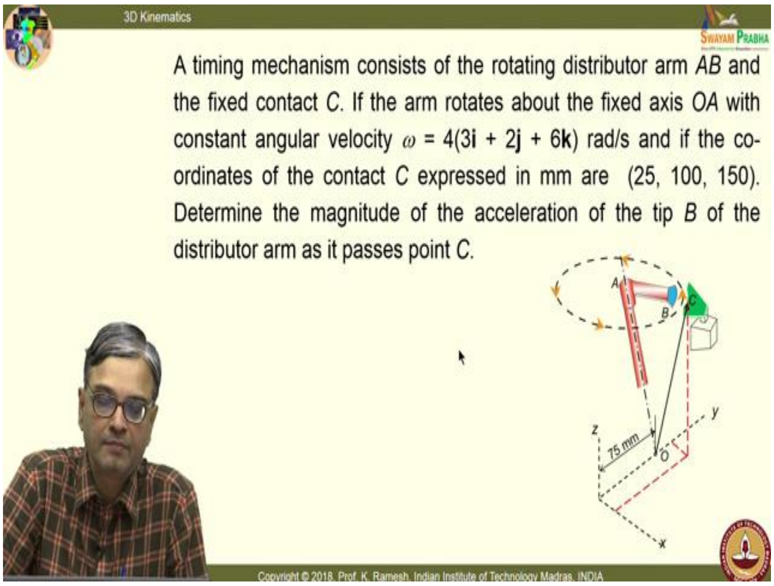
Let us solve simple problems. You make a neat sketch of it. This is just to give you a fixed axis rotation problem, wherein you have to do a determination of this vector product little more complicated; little more involved, you will have to go back to your determinants and then do it because I have three components. And this is a fairly straight forward problem, I have a rotating arm and this is the sensor which senses this. So, this has a fixed axis rotation in a three-dimensional sense, you have the axis *OA* about which this arm rotates ok.



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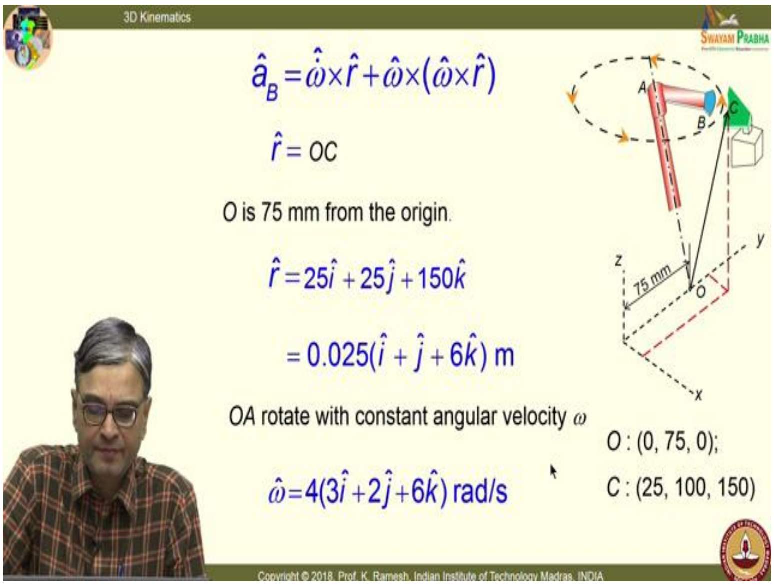
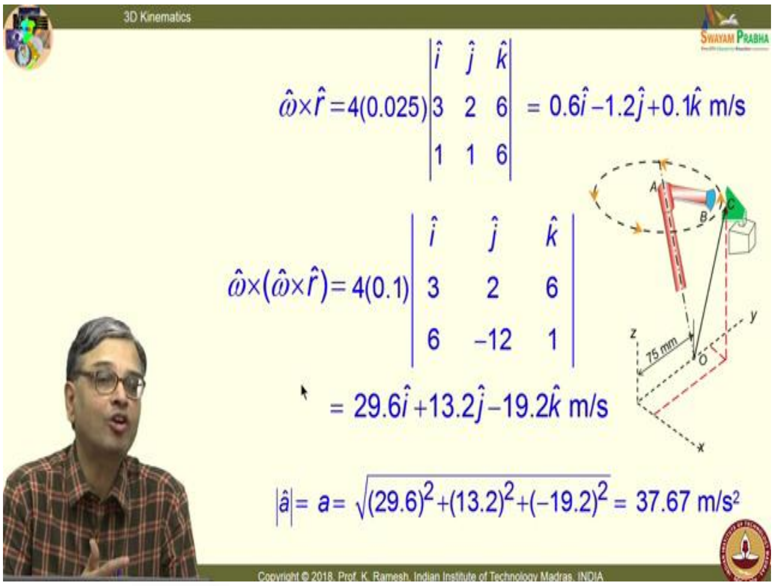


And if you look at the expression; expression is straight forward all the quantities have three components; you have x, y and z components ok. So, we have to find out the position the vector this is *OC* is the position vector of this. So, I have the coordinates for me to find out the vector, I have the coordinate *O*; $O:(0,75,0);$ and $C:(25,100,150)$.

So, you can easily find out what is the position vector, when the position vector *r* is given as $\hat{r} = 25\hat{i} + 25\hat{j} + 150\hat{k}$. And you will also have to know this axis *OA*. So, this is simplified to this *OA* rotate with constant angular velocity ω and that is given in your problem statement itself, what is the expression for the angular velocity. It is given in the problem statement. So, you go back to the problem statement you have this.



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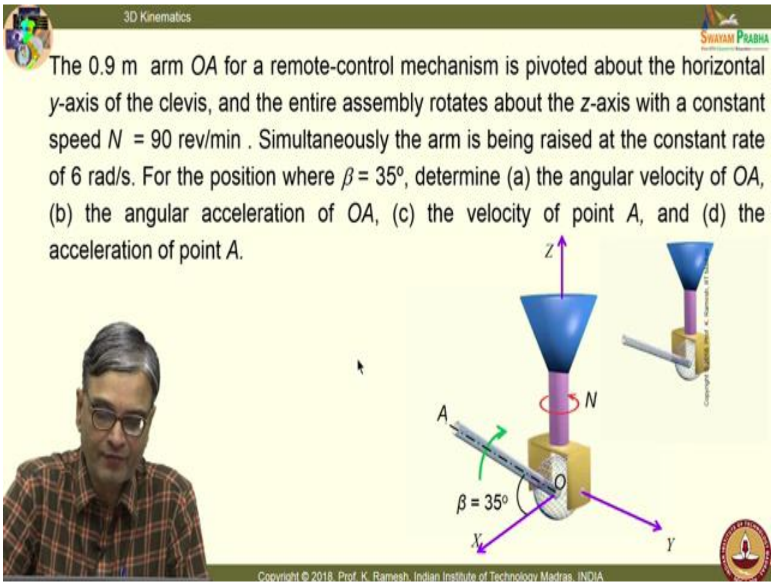
Now, I have to simply get this expression and $\hat{\omega}\times\hat{r}$, I have to do this and operate the determinant and then find out the value so, I get this value like this. And I get $\hat{\omega}\times(\hat{\omega}\times\hat{r})$. So, whatever the discussion that we had specially for two-dimensional situation you cannot extrapolate it here. You have to do the computation, just to alert you that you have to handle three components of each other vectors.



So, I have this finally, turning out to be this; this is just to alert you that go back and look at how to do the cross product of vectors when they have three components, otherwise the problem is simple and straightforward ok. There is nothing great thinking involved I have this as 37.67 m/s^2, it is a very straightforward problem just to alert you that you have to handle the vector products carefully.

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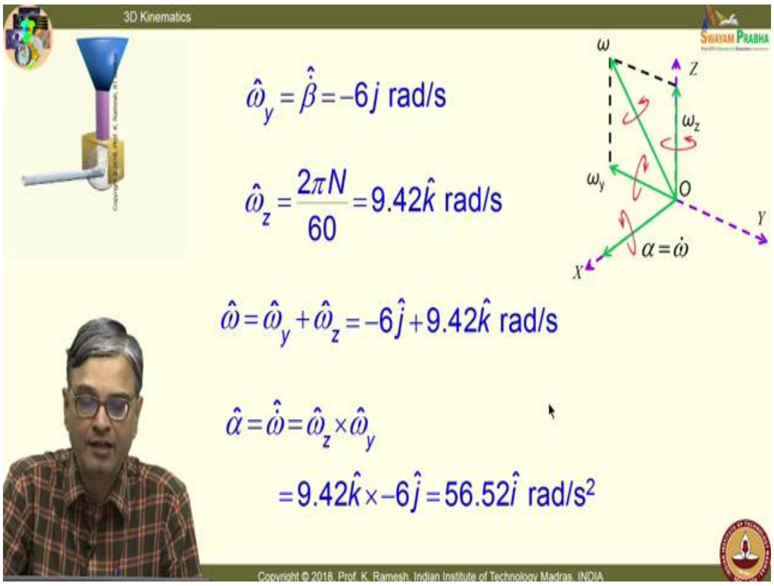
Let us move on to the next problem. This gives a very nice animation, what happens to this rod and only this is animated. See the animation software does not have all the facilities fine. You will also have to visualize in addition to this motion, the whole axis also have a rotation like this. So, can you recognise what is the kind of motion here? There is a rotation of this rod and that axis itself rotates, when I have a combination of this you could visualise this as a fixed-point rotation fine. So, that is what you will have to recognize and you are given this angle is 35°, and you are asked to find out the angular velocity of *OA*, angular acceleration of *OA*, the velocity of point *A* and the acceleration of point A. So, we are asked to find out these quantities.

So, even before you start solving the problem you have a physical appreciation what happens to this only a part of it is animated you have to imagined that this axis is rotating about itself. So, the angular velocity axis is having a directional change. So, I have a fixed-point rotation.



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And I have ω_y that is given in the problem as $\hat{\omega}_y = \hat{\dot{\beta}} = -6j$ rad/s. And you have to recognize, I have the axis *XYZ* like this, when I am having a clockwise rotation, I put the sign appropriately. I am also given the rotation ω_z so, that gives me $\hat{\omega}_z = \dfrac{2\pi N}{60} = 9.42\hat{k}$ rad/s.

Now you are asked to find out the angular velocity. Now you get the angular velocity, because these are all infinity decimal rotations, you have already seen that you can apply the parallelogram law, they are all vectors. So, I could write this ω as $\omega_y + \omega_z$, we already have what is ω_y, and what is ω_z. And I get this as $\hat{\omega} = \hat{\omega}_y + \hat{\omega}_z = -6\hat{j} + 9.42\hat{k}$ rad/s. You are also asked to find out the angular acceleration, and you could see that this is a steady precision. I am not having $\dot{\hat{\omega}}_z$, I am also not having any acceleration of this rotation.

So, I could use a simple formula that α as $\Omega \times \omega$ here the Ω is nothing but your ω_z and then small ω, either I could substitute this or I can directly take ω_y, both will mathematically remain the same. So, depending on the problem context, you take which is simple for you to do that. So, I get this as 56.52 i rad/s^2 and the acceleration is like this.

So, this is something very interesting, see I have the angular velocity like this and I have angular acceleration at some other direction. This you have to recognize when you graduate from two-dimension to three-dimension, all these new aspects will come. You have to look at the mathematics, and look at what is the direction of α. In two-dimensional problems α and ω were parallel. Because it is all happening only in z direction here it is very generic ok

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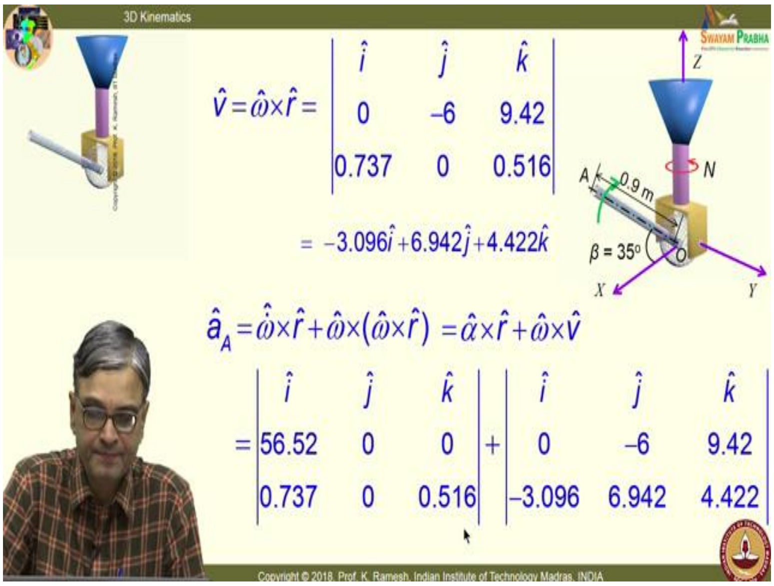

So, just to sensitize you I have taken a problem like this, and you are asked to find out for the point A. So, we have to look at what is the position vector r and then I put this $\omega \times$ r. So, you have to handle determinants for you to do the vectorial product, I get velocity like this. And I get acceleration has two terms, and we are given α is available, because you have directional change. You have to recognize, the angular acceleration has a value, because of directional change of ω.

Even though I do not have $\dot{\hat{\omega}}_z$ or $\dot{\hat{\omega}}_y$, I still have α that comes from directional change. And we have already got those expressions, and when you put them, I get this as two determinants for these two products. And you have to do the multiplication carefully do the mathematics. So, brush up these fundamentals, do some practice. So, that you are able to handle them quickly in the examinations ok.

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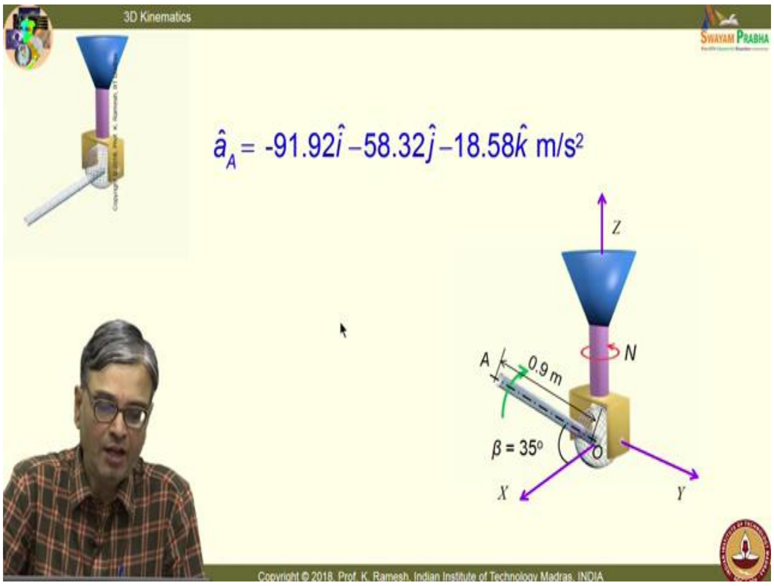
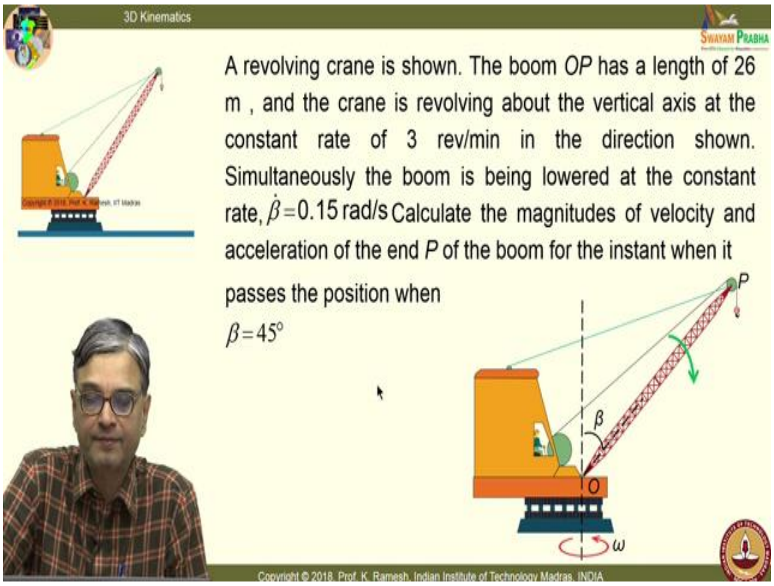


So, I am finally, going to get the expression for acceleration of point A, I get this as $\hat{a}_A = -91.92\hat{i} - 58.32\hat{j} - 18.58\hat{k}\ \text{m/s}^2$. So, this is the problem on fixed point rotation. We have seen earlier fixed axis rotation and fixed-point rotation.

Now, we will look at another interesting problem, because you know for all construction activities people use cranes. And here again the animation shows only one motion, it is the boom is moving in the vertical plane. In addition, the crane also has a rotation about this axis. The whole boom can rotate like this fine. The boom has rotation like this, boom moves up and down like this and whole boom can have a rotation like this. So, what is the kind of rotation that we can anticipate? This is again fixed-point rotation.



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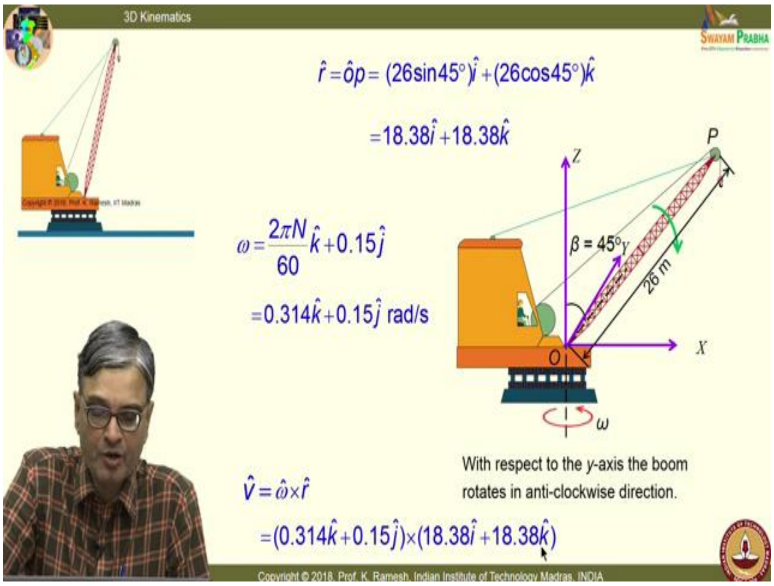
See fixed point rotation is a new concept you learn in this chapter. So, I thought it is better that, I solve a variety of problems to give you a practical flavour. And also, you recognize quickly that this is fixed point rotation. So, you are given the length of the boom as 26 meters and you are also given about the vertical axis, it is rotating at a constant rate of 3 revolutions per minute.

And you are also given see this is having a clockwise motion. It is coming down from vertical position, the boom is coming down and I want to caution you the animation is showing only one motion. You know the, it is easier to do this if you have to do show the other animation, it become very complicated we have not been able to get that opportunity to do that. And you are given this $\dot{\beta}$ as 0.15 rad/s and you are asked to calculate the magnitude of velocity, and acceleration of the end *P* of the boom.

For the instance when it passes the position when $\beta = 45^\circ$ so, it is lowering down and you have at $\beta = 45^\circ$ what is the value, it is straightforward problem again fine. And here I have to choose my reference axis, let us see what is the reference axis; I am choosing, and what is its Advantage? I can choose reference axis in anyway, I have taken a right-handed coordinate system, I take this as *X*, this as *Y* and this as *Z*.



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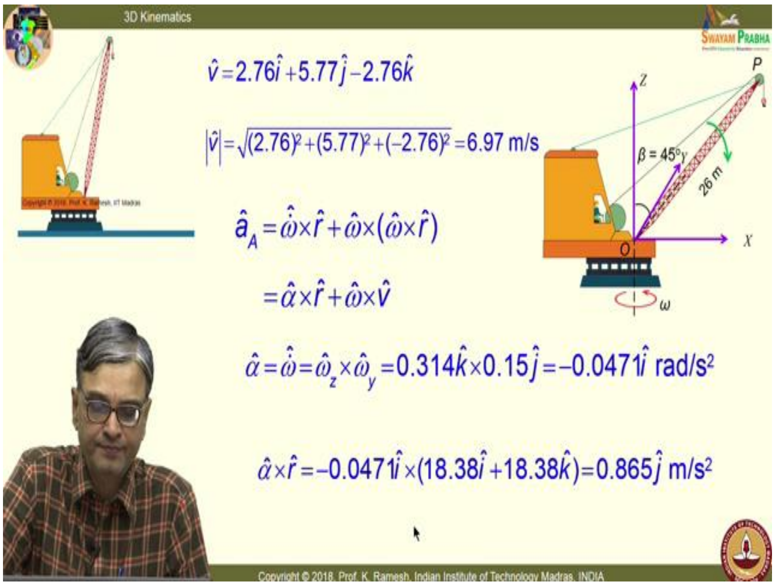
And, when you look at the motion of the boom, the boom is coming down ok. From our point of view this is clockwise, but from the way I have put the *Y* axis what is the rotation? About the *Y* axis this is anti clockwise, you have to recognize that ok. These are not trivial things, do not say instead of putting plus *j* I have put minus *j* that is only a small mistake do not argue like that, you have to solve the problem that is given to you in all its totality.

So, with respect to the *Y* axis the boom rotates in anticlockwise direction you have to recognize that, and rest of it is simple. So, this is just to give you practice and so, I have this position vector r that is given as from the problem statement we can write this so, it is in the plane *X* to *Z*. So, you put we are asked to find out at $\beta = 45^\circ$ so, you write the vector properly. And I have this as $18.38\hat{i} + 18.38\hat{k}$. So, I have the position vector, I have the ω I have directly written; I have the vertical axis rotation plus this is given as 0.15j that is given in the problem, and I this is what we have discussed whether I should put plus *j* or minus *j*.

From the diagram for us we are finding that it is moving from vertical to down, but when I look from the axis it is still positive. So, I have the angular velocity written like this and I also have the expression for *V* it is nothing but $\hat{V} = \hat{\omega} \times \hat{r}$. I have ω I have r just do the multiplication, and I get the answer probably, it is put in next slide. So, this mathematics you should develops speed. So, please develop speed as you solve many problems. So, I have this as $(0.314\hat{k} + 0.15\hat{j}) \times (18.38\hat{i} + 18.38\hat{k})$.



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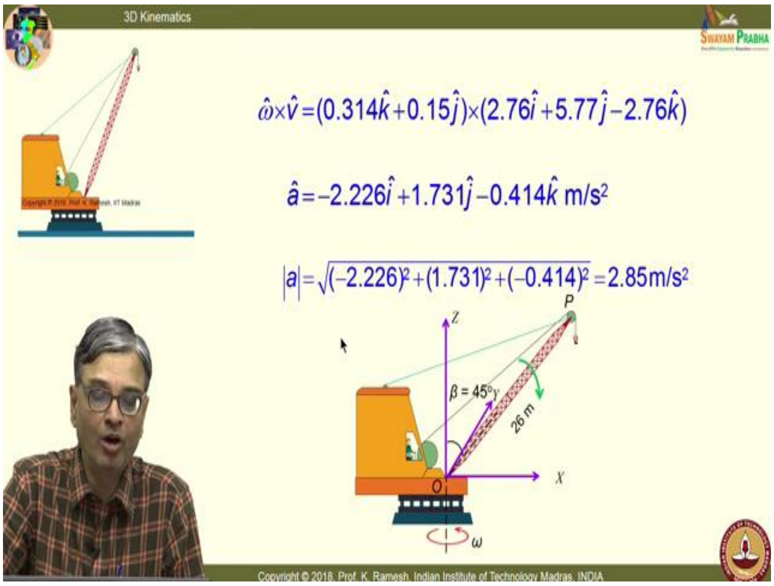
So, when I do this, I get this value of V as $\hat{v} = 2.76\hat{i} + 5.77\hat{j} - 2.76\hat{k}$. And you are also asked to find out, what is the acceleration, when I get the magnitude; the magnitude turns out to be 6.97 m/s. And how do I write what is the kind of precision I have in this problem. See the precision is always slow, that is what we have seen in problems also. And we have also seen in the case of earth. Its precision takes 26,000 years approximately. If you do the actual calculation it is 25,920 or something like that, but when you are talking about such a large number no harm in saying that has 26,000 years.

So, here I have steady precision. So, I can also write a simpler expression for angular acceleration. The acceleration is given like this and I need to find $\dot{\hat{\omega}}$ and $\dot{\hat{\omega}}$ is written as α. And α I could easily write it as $\hat{\omega}_z \times \hat{\omega}_y$. So, this is the precision cross product of precision, and the angular velocity $\dot{\hat{\omega}}$. The axis has a precision like this ok.

So, I am in a position to get the value of α. So, this is purely a directional change and because I do not have $\dot{\hat{\omega}}_z$, and $\dot{\hat{\omega}}_y$ I can directly take this as α; the shortcut gives you directly the value of α. So, once I know this rest of it is simple algebra, which you can comfortably do. So, I have $\hat{\alpha} \times \hat{r} = -0.0471\hat{i} \times (18.38\hat{i} + 18.38\hat{k}) = 0.865\hat{j}$ m/s^2. So, this product is fairly easier to do. And I can also do this $\omega \times V$. So, I have $\omega \times V$ like this. And when I substitute the quantities, I get acceleration as $\hat{a} = -2.226\hat{i} + 1.731\hat{j} - 0.414\hat{k}$ m/s^2.



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Again, fairly a straightforward problem, but this gives a very practical example, you find cranes are used extensively for construction in India also. So, you need to find out the practical flavour for it and I have also got the magnitude; the magnitude is given as 2.85 m/s^2. So, you should recognize that, α can come from directional change of ω. You should not only say only the magnitude change, because we are accustomed to only magnitude change in two dimensional problems. In three dimensional problems directional change does come into play ok.

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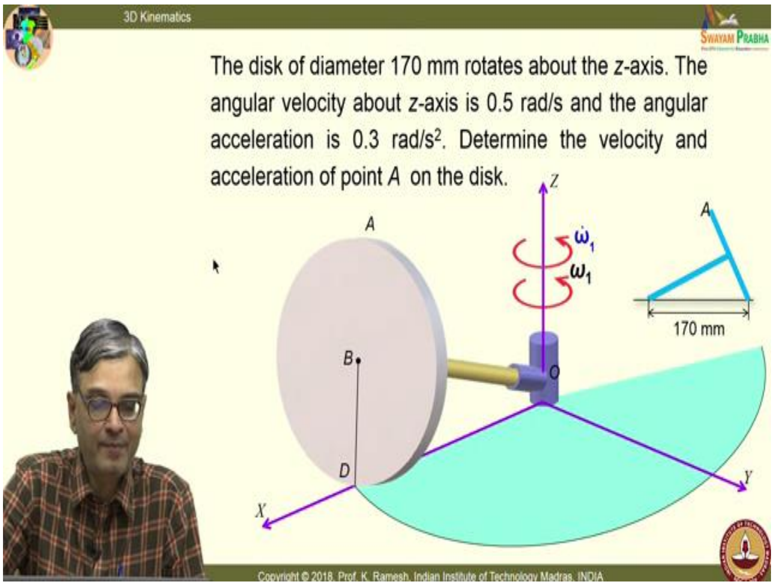
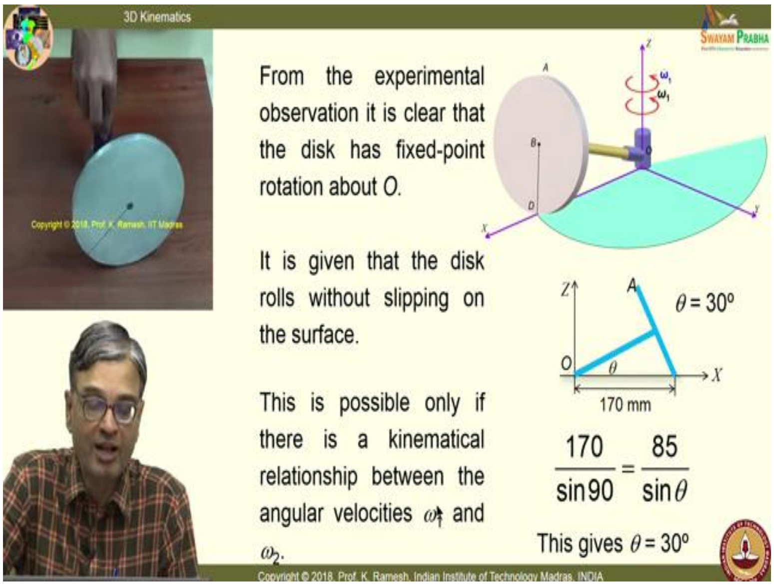
Then we move on to another interesting problem. So, I have a problem like this, I have a disk, this is having a rod firmly connected to this and there is holder, which is kept here. And you have this is the brass rod, this is put in a steel sleeve and there is clearance. So, this rod can freely rotate above this and the problem statement is like this. I have this is I



am rotating this about this axis ω_1 having an angular velocity of ω_1 and in the problem statement you are given, a disk of diameter 170 mm rotates about the *z*-axis. The angular velocity about *z*-axis is 0.5 rad/s.

In addition, it also has an angular acceleration of 0.3 rad/s^2. So, I also have $\dot{\omega}_1$ hence I have ω_1 as well as $\dot{\omega}_1$. And on this disk, it is actually the point diametrically opposite to *D*. I have a point labelled A and if I take the side view and look at other dimensions are also given. I have this as 170 mm and this also as 170



mm. And you have to find out, the velocity and acceleration of point *A*.

Say the problem statement is only like this, but if I have to solve the problem, I should understand, when I rotate about this axis what would happen to this disk. That is also very very important, is not it; see, unless we have a physical appreciation of the problem statement, how will you go about and solve it and it is not mentioned in this statement, I will mention the statement little while later.

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So, what I have done was, we have created this fabricated in the workshop and I have the gadget like this. So, this is what is happening to this; you have this being rotated, you get the idea? I have a line attached to this disk and you could see that, the disk is also rotating.

So, what you will have to recognize is, when I rotate this, the disk also rotates and the problem statement is still not complete, we also make one simple convenient assumption which is also seen in the actual display of the experiment, this wheel rolls without slipping on the platform; that is a very important clue only with this will be in a position to solve this problem or with this we will solve the problem either way you can take it ok.

And when, I am rotating with hand, I cannot give a constant angular velocity so, I will also have some angular acceleration. So, the problem is embedded with that I have ω_1 as well as $\hat{\dot{\omega}}_1$. And when, I have ω_1 and $\dot{\omega}_1$, when this roll; this will also have a magnitudinal change of angular velocity. Directional change is already evident from the way that you are rotating it the angular at a velocity has a directional change, it also will have a magnitudinal change.

And another thing, what you will have to visualize is, you have to bring in the kinematical relationship. There is connectivity between, what is the rotation about the z-axis and what is the rotation about the axis *OB*, fine. So, that is what I am going to summarize it and before that you know, see this is not exactly same as this; the dimensions are different; I have taken the dimension of the diameter as 170 mm, that is there in this disk and this length was slightly different, I have tweaked the values. So, that I get nice value of angle; the angle turns out to be 30^o; just to get this, I have tweaked these values. So, there is some disconnect between, this experimental observation and what you have it here ok, but the concept of relative rotations is neatly perceived in the experiment.

So, from the experimental observation, it is clear that the disk has a fixed-point rotation about *O*. We have been discussing fixed point rotation and you are also getting a physical appreciation of the problem, because the problem is being demonstrated to you. It is given that, the disk rolls without slipping on the surface. So, you take this also as part of the problem statement that makes our life simple to analyze this problem.

So, if I say that it rolls without slipping, I must bring in the kinematical relationship between ω_1 and ω_2. I have to establish, I have to go back to the fundamentals, what way we analysed rolling in two dimensions, bring in that knowledge and then establish first an interrelationship between, ω_1 and ω_2, is the idea clear, ok.

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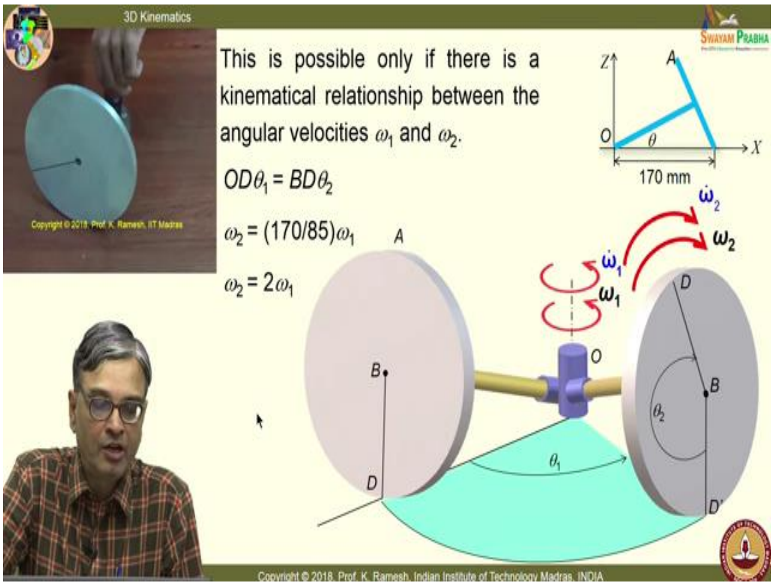
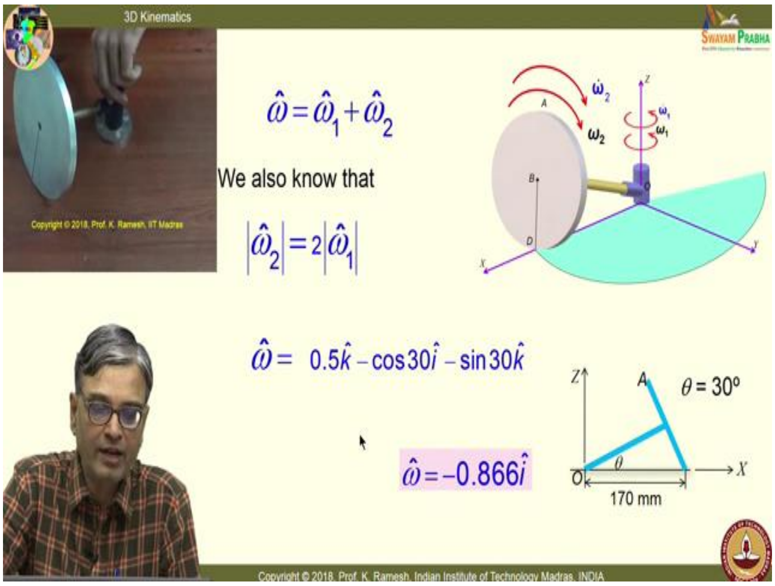


So, I start with the disk like this and disk has been rotated to some other position for clarity, I have taken a larger angle for us to discuss, I will have some position like this. What is the *BD*? *BD* would have rotated to some other position like this and I will also show you the experiment. I have the experiment starts with this and then, this has come to some angle like this; see, this is neatly visualized from the experiment also. So, now, I can find out, what is the distance that, I have travelled and then, bring in the kinematical relationship.

So, I have this is yeah; this is also very important. I am rotating anti clockwise; I forgot to alert you, which way this disk is rotating? This disk is rotating clockwise. See, these are all very subtle things, you cannot understand it, you just problem statement is given; if you have a prior exposure how to solve the problem you will close your eyes and solve it in 2 minutes. Because you write it from memory, you are not understanding the problem statement and doing it.

So, in this problem, the subtleties are, you have to find out that I have to establish a kinematical relationship that is very important. And you should also recognize while, doing it that this is having a clockwise rotation, fine; it is having a clockwise rotation. So, I have; I will label those angles, I have this has theta 1 and from this angle, I will put it as 2 so, I can find out, what is the distance travelled? Because, I have $OD\theta_1 = BD\theta_2$, I can write the kinematical relationship and I have tweaked some of these dimensions. So, that I get an very elegant relationship so, I have $\omega_2 = (170/85)\omega_1$ because, we have seen that, this distance is 170 and this, diameters also 170. So, when I look at the radius, it is just half of it. So, I get a kinematical relationship $\omega_2 = 2\omega_1$.

See, in this experimentation it is not exactly 2, it is slightly less than 2, it is around 2. So, when you look at this rotation 3 or 4 times, it will not exactly match with this kinematical relationship because, I have the line, this is what I alerted you earlier, fine. By from mathematical handling for us to solve, if I have round number, it is easier to do the calculations. So, that is the spirit with which, the problem has been coined.



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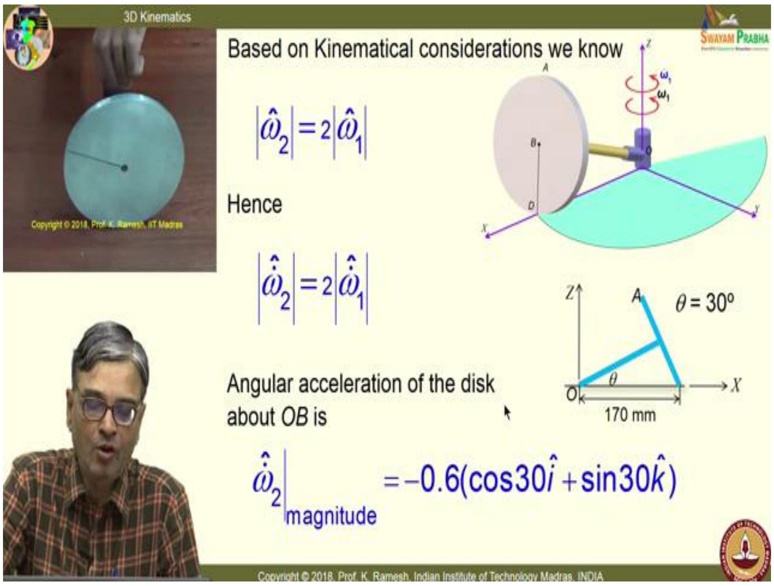
So, I have this, I have to recognize that I have ω_1 and $\dot{\hat{\omega}}_1$ and $\omega_2$and $\dot{\hat{\omega}}_2$. So, when I have all of these, I can directly write, $\hat{\omega}=\hat{\omega}_1+\hat{\omega}_2$ and α as $\dot{\hat{\omega}}_1$ plus ω_2; as $\dot{\hat{\omega}}_1$ + $\dot{\hat{\omega}}_2$. And $\dot{\hat{\omega}}_1$ if I want to do it, I will write it as the individual vectors and also look at the here in this case, I have *k* dot as well as I will have *i* dot. Instead I am going to do it from a physical appreciation, ok. We have already said that, α comes from two aspects a magnitudinal change and directional change. You can take either way of solving the problem; choice is yours.

So, this is straight forward, $\hat{\omega}=\hat{\omega}_1+\hat{\omega}_2$ is straightforward, we have already learnt this. And, we also have the interrelationship $|\hat{\omega}_2| = 2|\hat{\omega}_1|$ and you should also write this unit vector properly and we are also given this ω_1 as 0.5 rad/s. So, ω_2 becomes 1, ok these all done to simplify the mathematics.

So, I have ω is now written as $\hat{\omega} = 0.5\hat{k} - \cos 30\hat{i} - \sin 30\hat{k}$. Because, I have this as clockwise, you should recognize the sense correctly; sense of this disk is rotating in clockwise, ok. So, that is why minus comes and since I have ω_2 as 1, if the mathematics looks very simple and straightforward, ok. So, I get $\hat{\omega} = -0.866\hat{i}$ so, this is what I said; if I have to find out $\Omega \times \omega$ for me to get the directional change of acceleration. In this problem I can directly use this because, I have the number it is one and the same, ok; mathematically it is one and this same.



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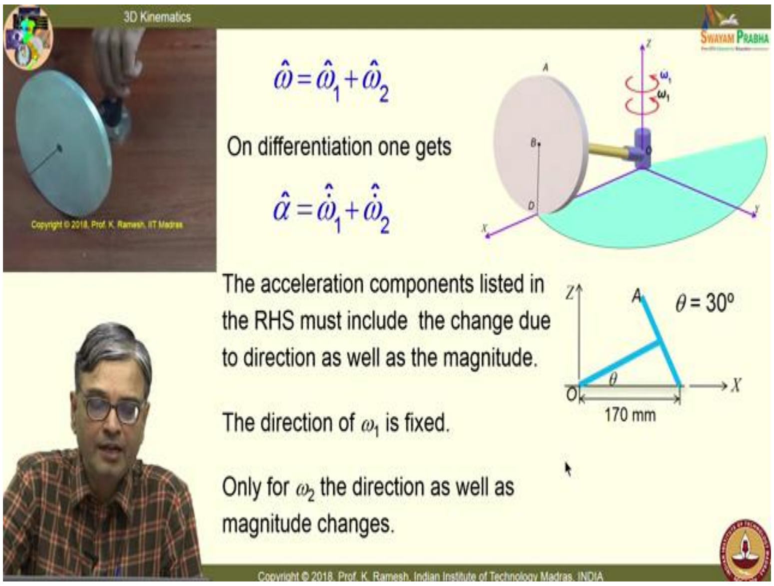
Now, let us look at the acceleration. When I have $|\dot{\hat{\omega}}_2| = 2|\dot{\hat{\omega}}_1|$; when, I have $\dot{\hat{\omega}}_1$ then, $\dot{\hat{\omega}}_2$ is also fixed, $\dot{\hat{\omega}}_2$ gives you only the magnitudinal change; it has magnitudinal change as well as directional change. You have given $\dot{\hat{\omega}}_1$ so, I can find out, $\dot{\hat{\omega}}_2$ no issue. And, I can write the angular acceleration of the disk as, I can put the magnitude as this, I am putting the appropriate unit vector and then, putting this, that is what, I have done and I am also attaching, what I am writing here as a magnitudinal change.

Because. in the problem, magnitudinal change is specifically given and I thought I would discuss as a magnitudinal change as well as the directional change. You can also write it as, $\dot{\hat{\omega}}_1 + \dot{\hat{\omega}}_2$, in terms of vectors where, I will have $\dot{\hat{\omega}}_1$ plus ω_1 i dot that, kind of expression also you can do it. I mean, here I have put this ω_1 about z. So, I will have k dot there and for this will have i dot, that is the only difference.



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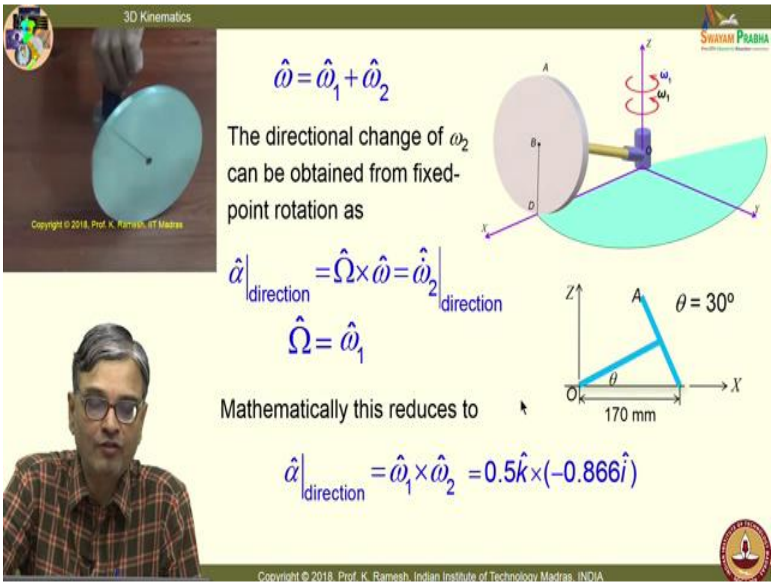
So now, what I will have is, I will also look at the this is what I said; when I have just put the vector, I have not put the appropriate vectorial directions here; when, I put the vectorial directions, I will have to handle the differentiation properly. So, instead, I am arguing it from a magnitude and directional change, the RHS must include the change, due to direction as well as magnitude. And in this case, directional ω_1 is fixed so, there will not be any directional change of ω_1, only the magnitude change of $\dot{\hat{\omega}}_1$ is there, ok.



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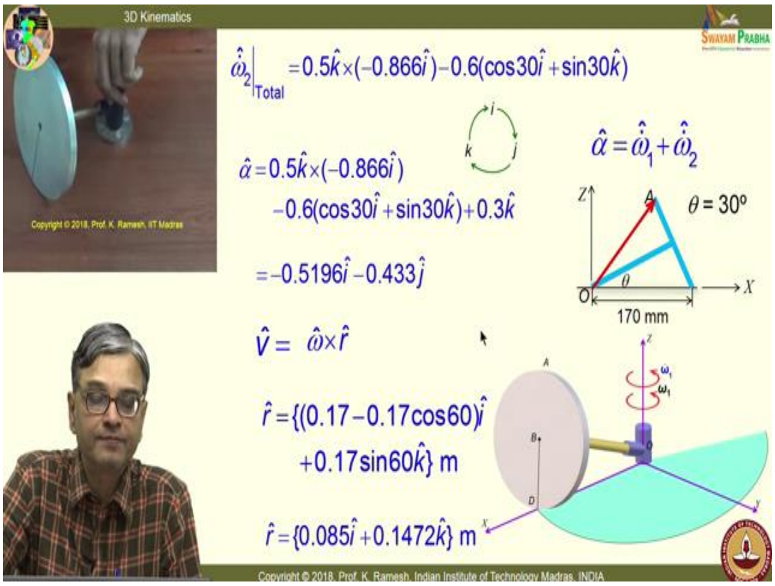
So, only for ω_2 which this disk is rotating, I have to find out the acceleration due to directional change, fine. That, we already have a recipe; we already have a recipe and you do not write it as α, but you write it as α specifically of direction, this is the one change which I am asking you to recognize I am use the same expression $\Omega \times \omega$ and I recognize this as, directional change which, we have already discussed.

There are two ways of looking at the same thing and I have chosen to discuss it like this and we have ω is nothing but ω_1 and then we already have the expression for ω. So, I can directly write that ω, it is $\hat{\omega}_1 \times \hat{\omega}_2$ so, that reduces to this, ok. Mathematically this, but when I substituted the ω itself directly, if I have to write it mathematically, I will also put the k component. And the k component, $k \times k$ will go to 0 while writing this I have used directly the value of ω ok.



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So, I recognize that, what I get from this expression is the directional change of ω_2 contribute in to the acceleration term. Now, you have all the quantities finding out α is not difficult. So, ω_2 total, I have the; I think this is the directional change and this is the magnitude change. So, I have this and then, we have $\dot{\hat{\omega}}_1$, what it is? So, the α reduces to this $\dot{\hat{\omega}}_1$ is this, this is $\dot{\hat{\omega}}_2$; now, I know α; rest of it is straightforward.

I have the expression for finding out the acceleration straightforward expression for you to do that. I have this α reduces to $-0.5196\hat{i} - 0.433\hat{j}$ and I have $\hat{V} = \hat{\omega} \times \hat{r}$ and we also have to write the position vector properly; this is the position vector. So, write the position vector based on the mathematics fine and so, I have this like this. So, you multiply $\hat{\omega} \times \hat{r}$, I have the value of r is given like $\hat{r} = \{0.085\hat{i} + 0.1472\hat{k}\}$ m so, getting V is straightforward.

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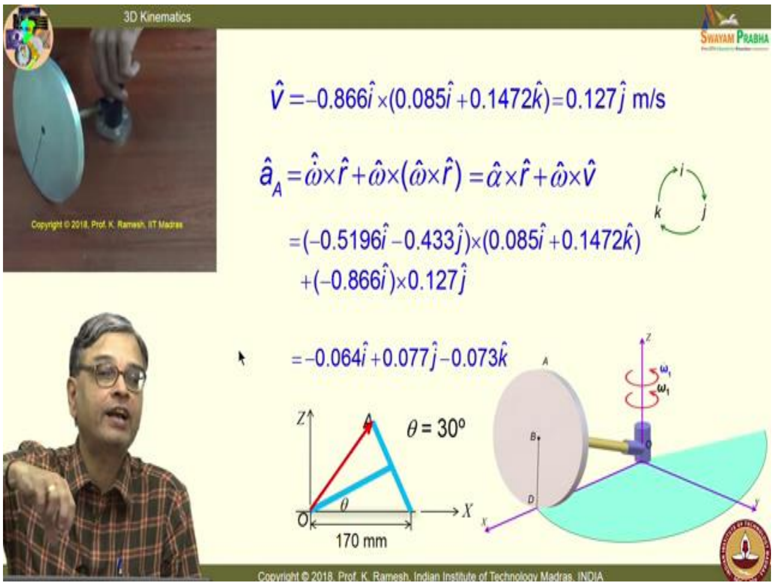
So, I get v as this is the multiplication, this simplifies to 0.127 j m/s and you also have an expression for acceleration. See, this expression is very similar to what you have learnt in two dimensions, what you have learnt in a fixed axis rotation, what you substitute for

$\dot{\hat{\omega}}$ and ω are different from fixed axis and fixed point, that is the only difference. So, you are not learning any new equations altogether, we are all one and the same equation which was seen it umpteen number of times.

So, now I substitute for these quantities, do the mathematics. So, I get this as $-0.064\hat{i}+0.077\hat{j}-0.073\hat{k}$. So, it is a fairly straightforward problem provided you understand the motion ok, to aid that you have this experiment is conducted and you are shown, what is happening to the disk, when I rotate it about the *z*-axis?



So, in this class we have looked at, what is the distinction between fixed axis rotation and fixed-point rotation and I have also emphasized in *3D* kinematics what you come across as new is fixed point rotation, and we have looked at how to identify the acceleration. The acceleration will have two components; one because of magnitudinal change of ω, another because of directional change of ω. And we have also solved variety of problems and bulk of them was on fixed point rotation so, that you get a flavour how you have to handle the fixed-point rotation properly.

Thank you.