Engineering Mechanics
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Module - 01
Statics
Lecture - 04
Force Systems - II

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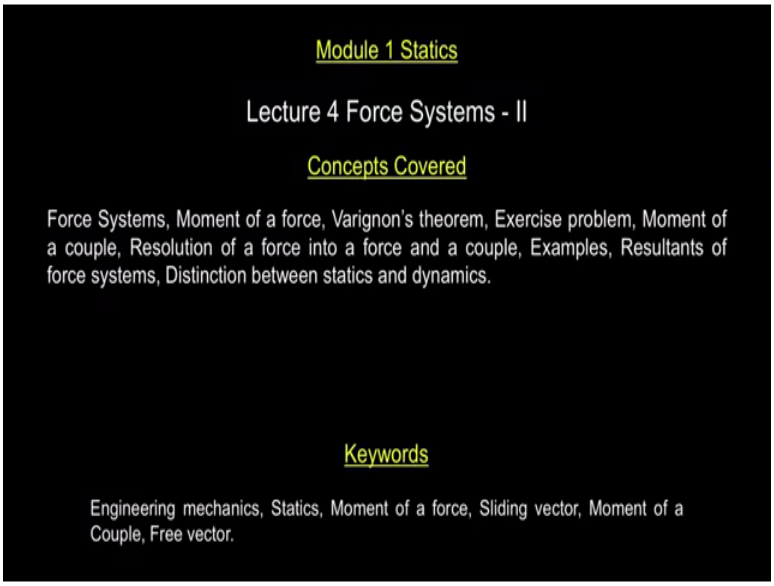

Let us continue our discussion on force systems.

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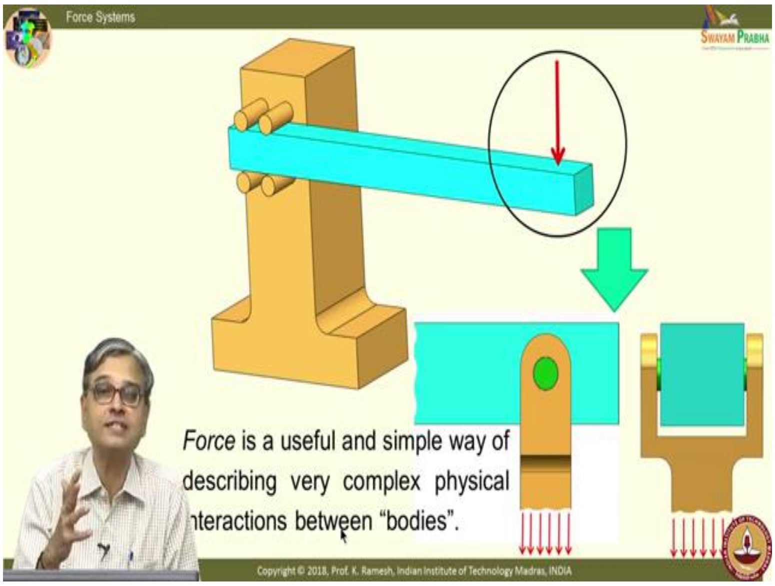
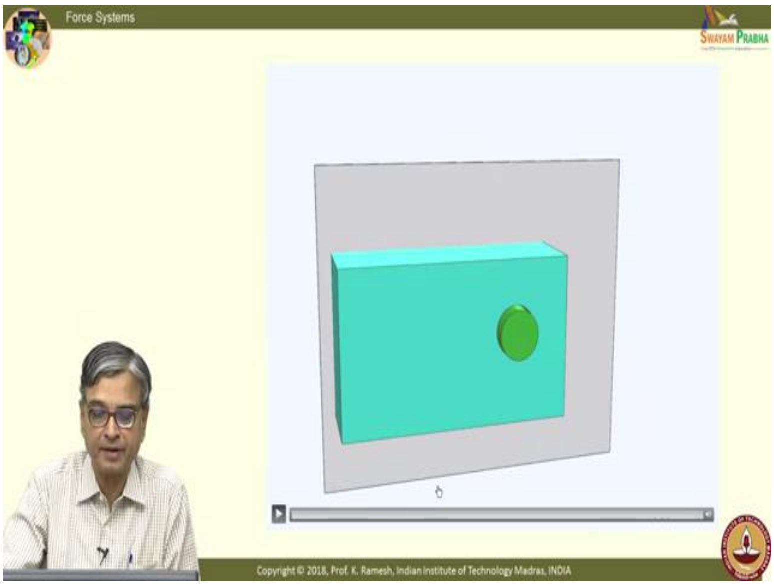


See, in the last class we had taken up a simple problem, where in I have shown a load acting at the tip of a cantilever beam and I said in order to apply this load one possibility is to have a pin joint with a fork; and when I view it on this side view, I get this or if I see there I can also called that a front view. And if I see the side view, I have the system like this. I have a pin going through the hole and I have a fork. A pan is attached to the fork and you put a weight on it. So, looking from another perspective force is a useful and simple way of describing very complex physical interactions between “bodies”. This is what we have seen in the previous class I will again go through it quickly for you to appreciate you should look at force as a concept.



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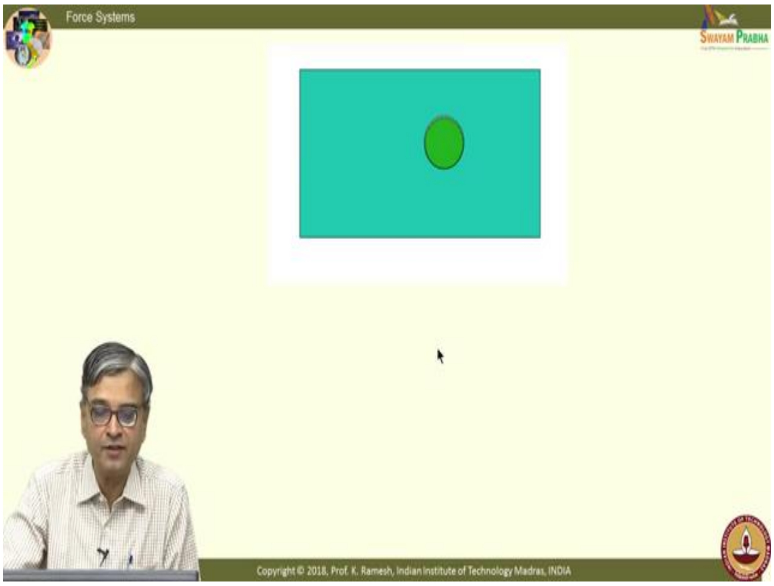
I pass an imaginary plane cutting the pin and the block assembly.



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And I would see the pin sitting on the hole like this.

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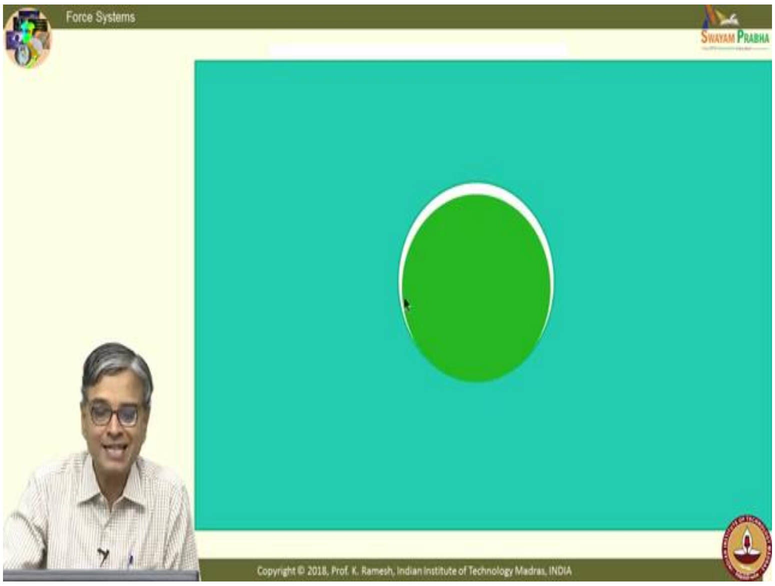
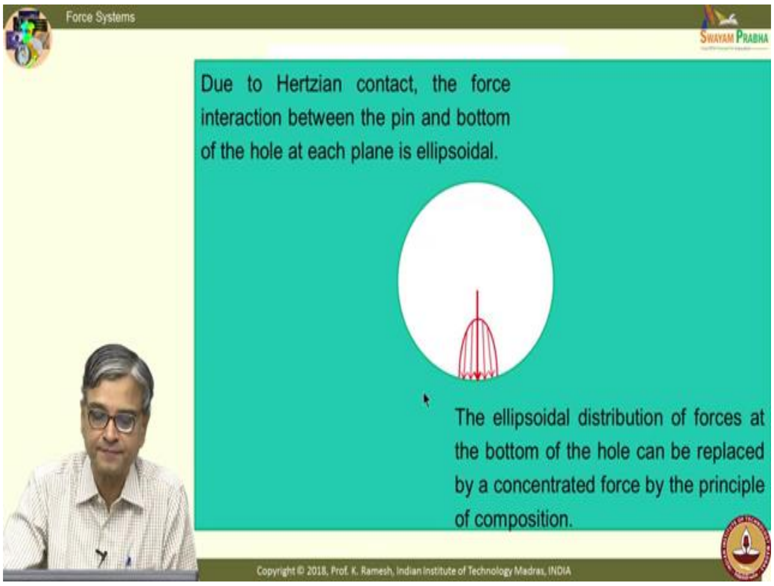


When I magnify it, I can see the clearance between the pin and the block and you have, contact over this length which is magnified several times.



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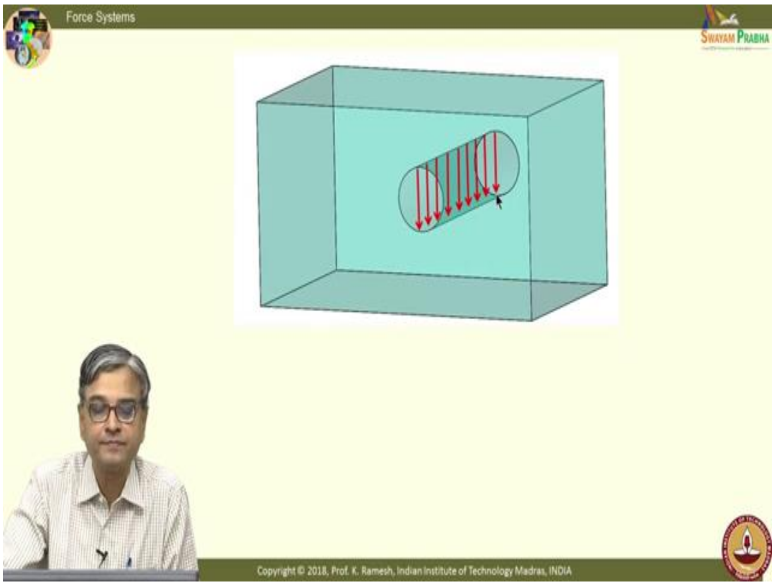
And by Hertzian contact principle you would have an ellipsoidal force distribution. In reality all bodies are deformable as an idealization we make it as rigid that makes our life simple. And by the principle of composition I can make this force distribution as a concentrated force.



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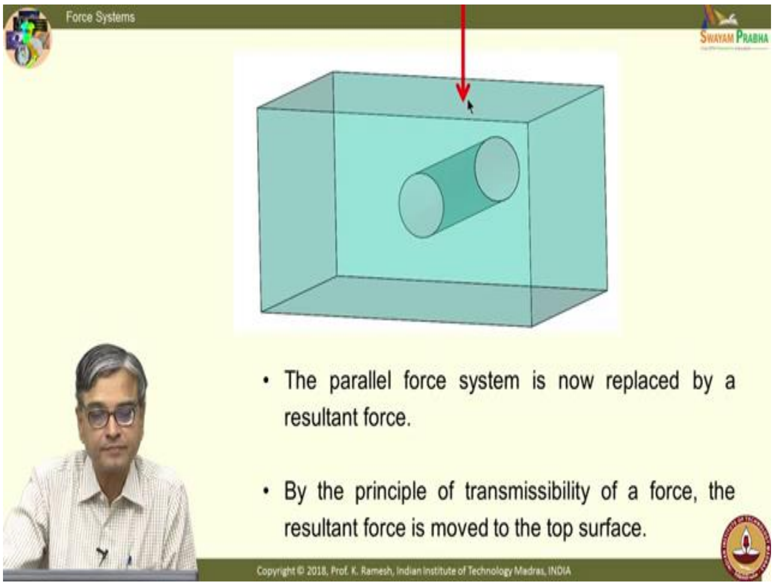
And when I see this for every other plane in the pin, I would have essentially a parallel force system.



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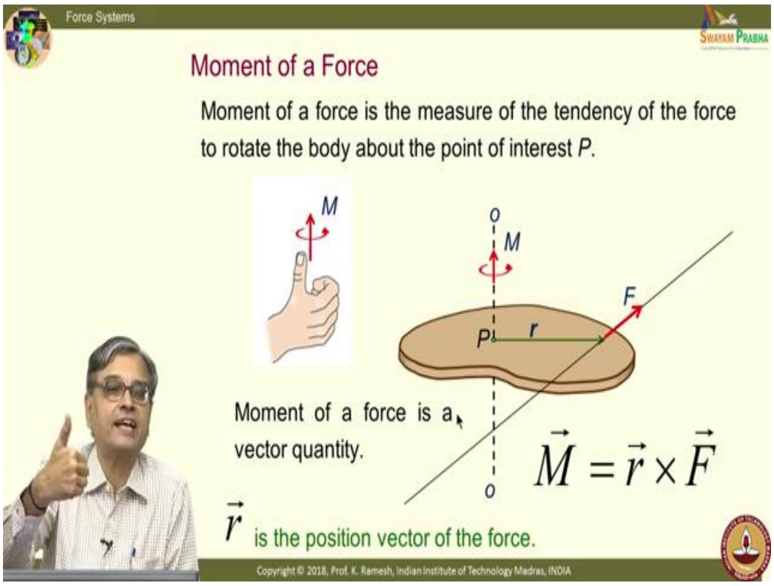
I can take a resultant of this which is simple to do, but the force is acting only at this point and we have learnt a very powerful principle, principle of transmissibility of a force. So, by that I move the force to the top end of the beam.



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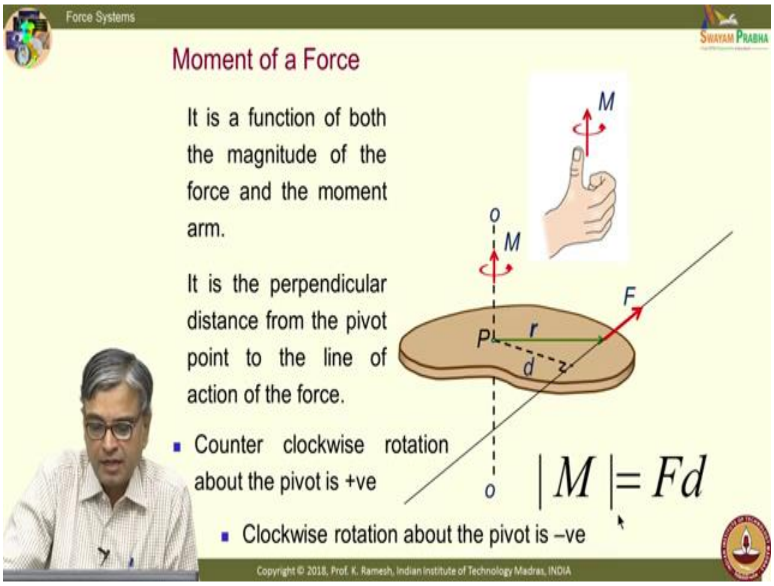
And I have already told you whenever I apply a force in general it produces a translation and a rotation; only translation or only rotation are exceptional cases. And I also said that we have to quantify these effects whatever the effect the force is causing, I should know what is the rotatory effect. That is what I get it from the concept of moment of a force. Make a neat sketch of this body and let me apply a force at an arbitrary point on the body. And I would like to find out what is the moment of this force acting at an arbitrary point P and what is shown now is I have told you earlier force can be treated as a sliding vector; as long as I am interested in the external effect of the force by using the principle of transmissibility I can treat the force as sliding vector.

Now, I take a point P which is arbitrary there is no special location to that, and if I want to know what is the position vector of this force from the point *P* I have this vector drawn. And I label this vector as r and you can easily find out what is the rotatory effect of this force about point *P* as

$$\vec{M} = \vec{r} \times \vec{F}$$

From a vectorial notation you can easily do that. And when I represent the rotatory effect, I see very clearly, I am having an anticlockwise motion, in this course I would urge you to visualize physically how the force tends to turn the object; do not just do the mathematics using vectorial algebra. You should always physically make an attempt to visualize the rotatory effect that will help you to appreciate the subject better.

And this is about the axis *O-O* as force is a sliding vector, moment is also a sliding vector. So, if I have multiple moments acting on the body; if they all meet at a point, I can use vectorial addition and get the resultant. And I use the right-hand rule and I find out what is the direction of the moment. Anticlockwise is taken as positive; clockwise is taken as negative. This is what is summarized in the next slide.



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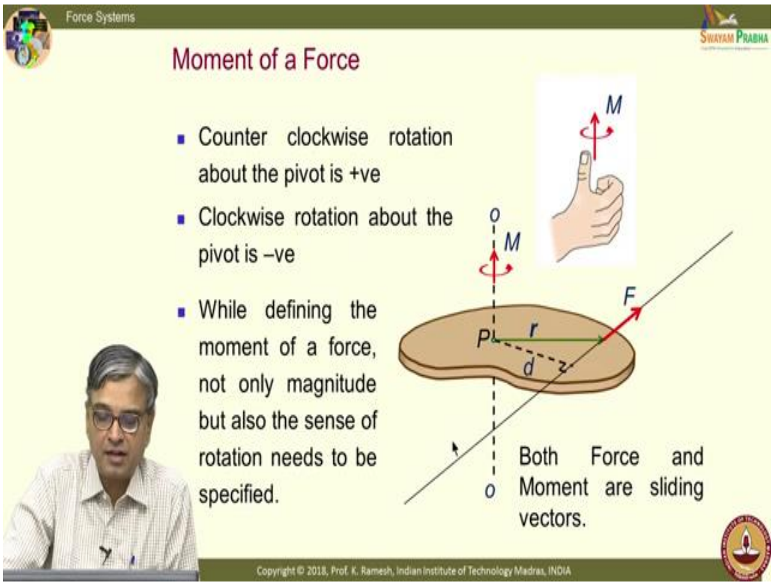
I can also find out moment of a force based on distances which I could calculate if I find out what is the perpendicular distance from point P to the line of action. I can simply write the moment as the magnitude as

$$|M| = Fd$$

and from the sense of the force acting on it I can perceive that this is acting anticlockwise and I label it as positive moment.

So, that is what is summarized here. So, I can do a vectorial calculation or I can physically look at what is the distance and also get the rotatory effect. And what would happen when I change the point P on the component, the position vector r will keep changing in a sense d also will keep changing. So, the rotatory effect changes from point to point; it is a very important concept. The rotatory effect of the force differs from which point I am trying to calculate. It again emphasizes that moment is a sliding vector.



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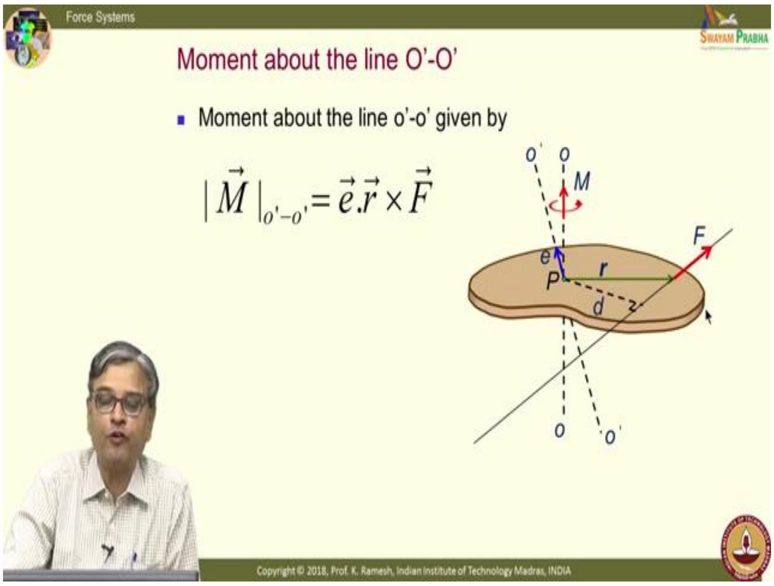
And clockwise rotation about the point is taken as negative. So, when you specify the moment not only the magnitude, but also the sense of rotation needs to be specified and also perceived by you while solving problems. Do not hang on to only vectorial algebra and say r x F; I get it I get all this components *ijk*. Then the subject becomes very dry; there is no life to the subject. You should always visualize how the force is trying to rotate the body and this again shows that it is a sliding vector, force is also a sliding vector; the concept is emphasized again and again so that you remember this as we go by in the course.



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Suppose I have an arbitrary line O prime O prime how do I find out the component. I have the line O prime O prime and it is again simple if you know vector algebra. I can easily do that; I have the unit vector as e. I can simply get that as

$$|\vec{M}|_{o'-o'} = \vec{e}.\vec{r}\times\vec{F}$$

It is very simple for you to do that there is no difficulty in getting it, but I can also appreciate it from the way how the force interacts with the line of interest.

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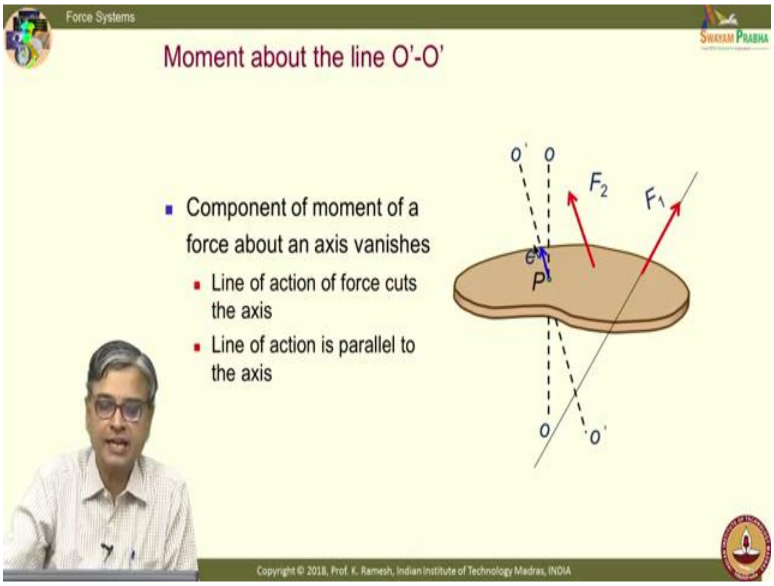

There are two cases where the component of moment vanishes about an axis, when the line of action of force cuts the axis like this or when the line of action is parallel to the line of interest. Suppose I have a force F_2 which is parallel to this component of moment along that axis will be 0; it would help you to simplify complex problems if you understand this property. It is again a useful property that you can employ whenever the need arises.

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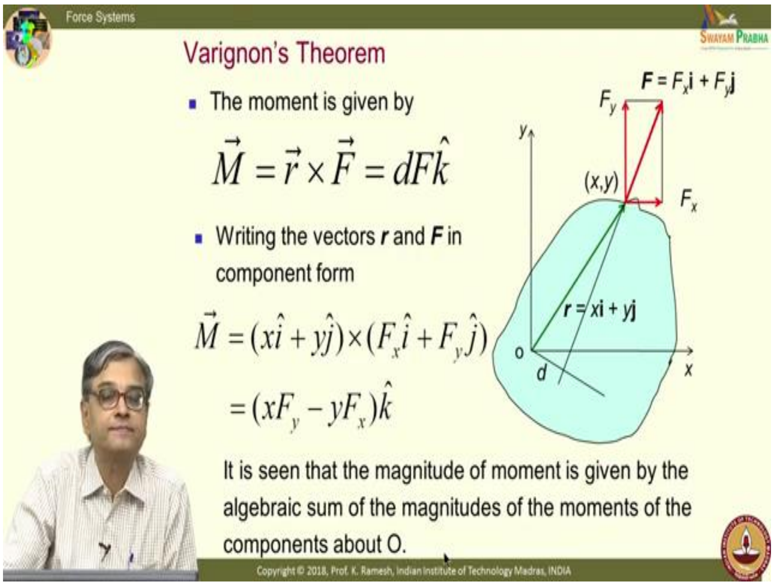

And there is also a very simple theorem goes by the name Varignon, I have a body. Make a neat sketch of it. I have an arbitrary force *F* acting on it at point (x, y) it is acting. And a force is depicted as

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

F_x and F_y are the magnitudes and i and j are the unit vectors in the *x* and *y* direction.

What is the moment given by this force? I have a position vector *r* and I can also find out what is the distance *d*. I draw a line through the force; from the origin I try to find out what is the distance which I can calculate let the distance be *d*; and this force tends to rotate the object about point o anticlockwise. So, I can easily write the value as $d \times F$ use the magnitude and it will perpendicular to the plane, so I get it on the *z* axis.

So, I put the unit vector as k. So, I can find out this. I can also find out the moment about point o writing vectors *r* and *F* like this.

$$\vec{M} = \vec{r} \times \vec{F} = dF\hat{k}$$

$$\vec{M} = (x\hat{i} + y\hat{j}) \times (F_x\hat{i} + F_y\hat{j})$$

And you have to go back to the product rule *i* × *i* is 0, *i* × *j* is k, *j* × *i* is -*k*, when you do all this, I can simplify this as

$$(xF_y - yF_x)\hat{k}$$

So, what does this show? Magnitude of moment is given by the algebraic sum of the magnitudes of the moments of the components about *O*; a very powerful theorem. So, I can find out moment by using the components, in many situations it may be easier to do the calculation that way; let us learn this by a simple example.

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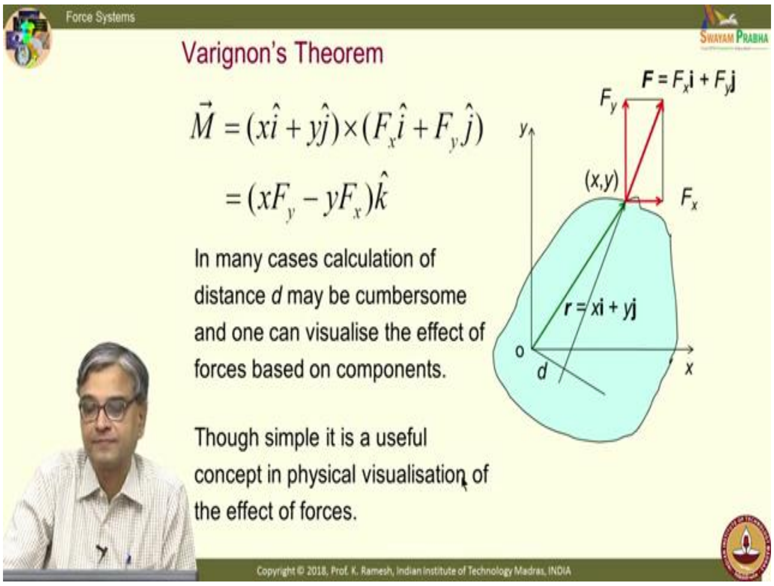

If the calculation of d is cumbersome, then you can go with the components. It is definitely simple, but a very useful concept in physical visualization of the effect of forces.

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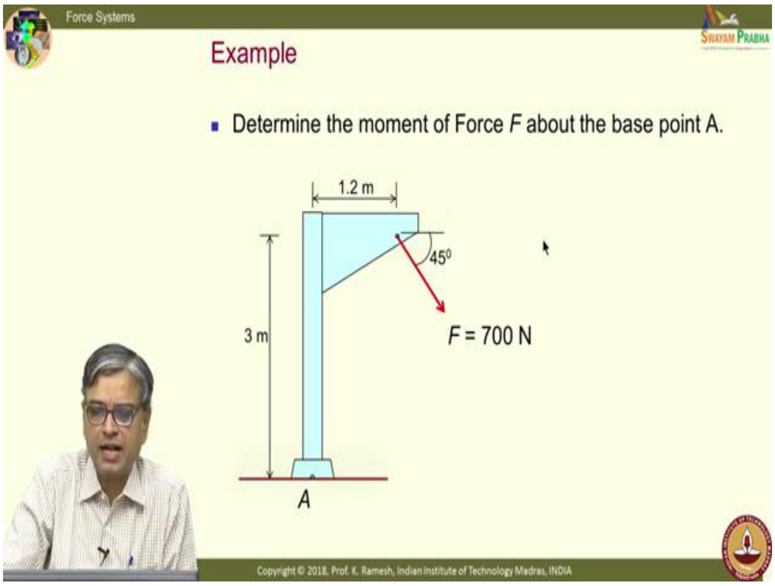
I take a simple problem; please make a sketch I have a mast. You know, before we even get into free body diagram, I am teaching you elements of free body diagram indirectly. We will take a formal appreciation of how to draw a free body diagram later. I have an object this is the ground. I have a point A this is marked on the ground and I have a force of 700 N acting at this point oriented at angle 45^o. And I have the distance of the point of application horizontal distance from point A is given as 1.2 m; and the vertical distance is given as 3 m. Now this is at the same level if you look at this point and this height depicted as 3 m are at the same level. It is very easy to do and we will solve it by three different ways.



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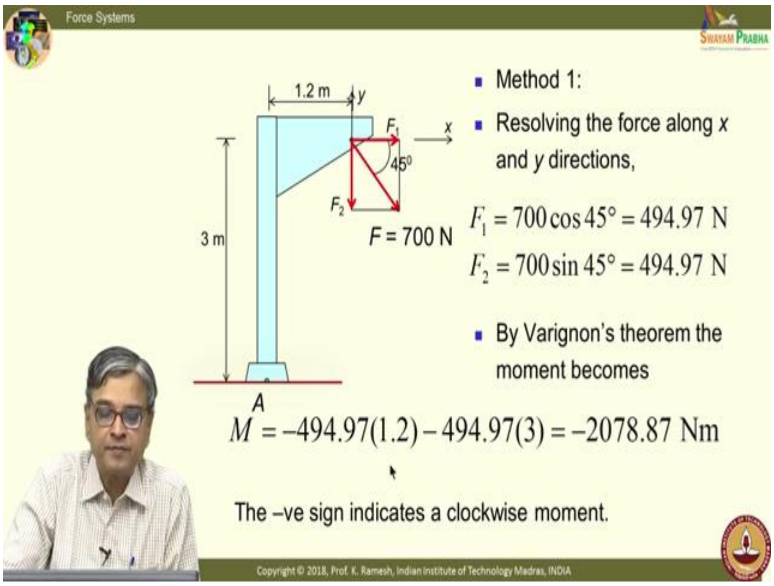
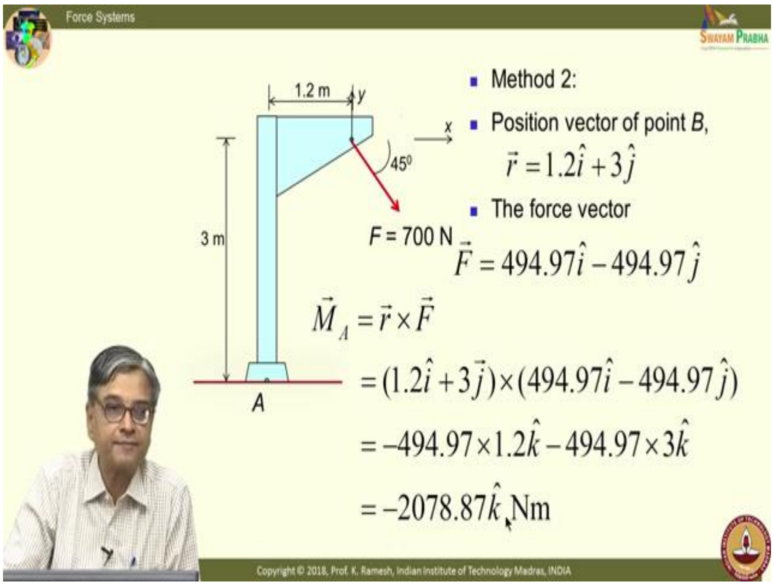


So, in the first method, resolve the force along x and y directions and you know it is

F cos 45 and *F* sin 45 and I can also get those values I get the values as 494.97 N. And use the Varignon's theorem; we have already got the distances horizontal distance *x* is 1.2 meters, vertical distance is 3 m. So, I can directly write the moment from this even before you do that can you visualize in which direction, you will have the moment; is it clockwise or anticlockwise. It is clockwise; you should visualize this right from the first problem.

Even though your mathematics will give you correctly what is the correct vectorial orientation. Physically you feel that; get that habit. Both forces F_2 and F_1 as well as the resultant F will produce only a clockwise bending; moment and clockwise by convention you have taken it as negative. So, I get this as a number 2078.87 Nm in the clockwise direction.

So, this is by method one straightforward I have used the Varignon's theorem, used the components and determined the value of the moment.



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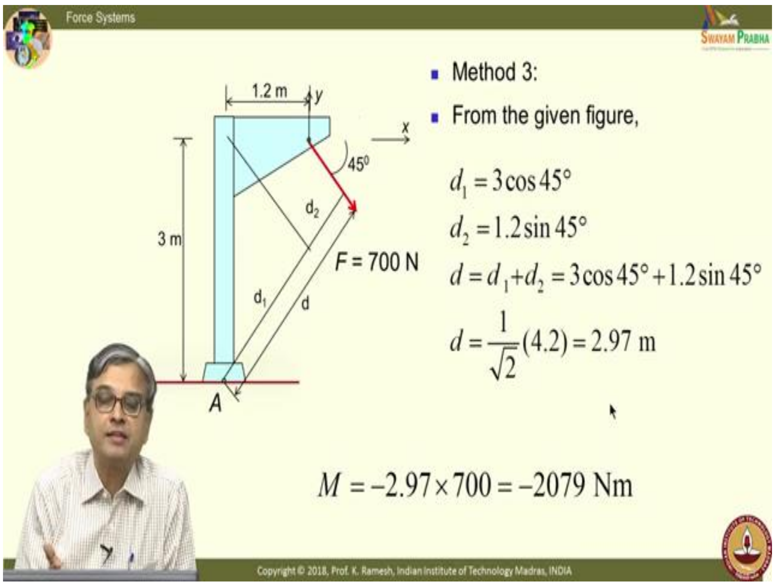
Then I can also do it by the vectorial calculation, I write the position vector

$$\vec{r} = 1.2\hat{i} + 3\hat{j}$$

because I know that this is at 1.2 meters and this is at 3 meters from the point of interest. The force vector I can write it like this, because I have taken the origin coinciding with the point of interest; I put the distances probably and I put this r cross F. So, when I do the r × F; I get the value again same as minus 2078.87 k. So. It is straightforward. The third method is I can also find out the distance d and do it all of them should give you the same answer.



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And how do I do this? I have the distance d from point A to the force and you know I can split this distance as d_1 and d_2, from this triangle I can find out d 1 and from this triangle I can find out d 2.

$$d_1 = 3\cos 45^\circ$$
$$d_2 = 1.2\sin 45^\circ$$

So, I get the value of d as 2.97 meters this is rounded off here. So, I will also get a rounded off answer minus 2097 Newton meter, I have just given the magnitude and because I know this is clockwise, I have associated the sign as negative.

So, a very simple problem you understand Varignon's theorem in all its totality, you know these are very simple concepts, but we will be repeatedly using it in the course; do not treat them as trivial. They appear trivial you have to have a clear concept then it is easier for you to do.

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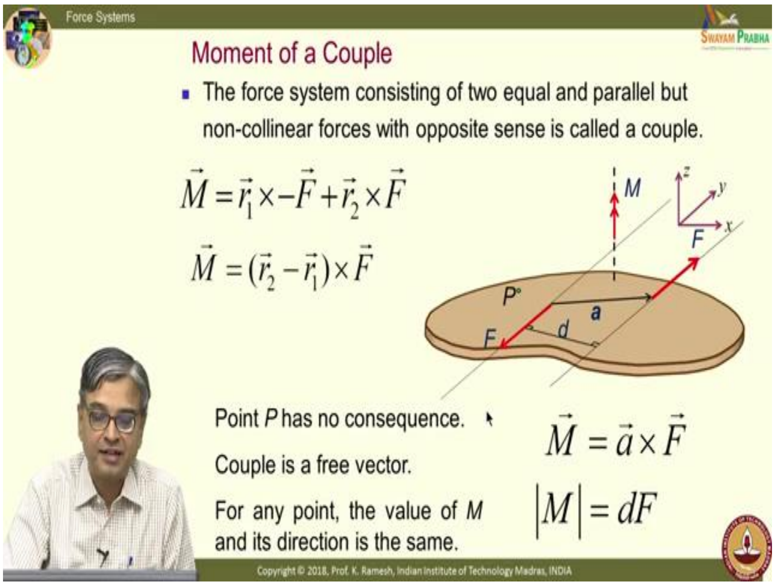


Now, we will go to a special system of force which will create a couple. The force system consisting of two equal and parallel, but non-collinear forces with opposite sense is called a couple. I have the same object on that I have a force *F* acting like this. I have another force acting in the opposite direction and these two are parallel. Now the question is what would be the net turning effect of this force system? So, let me take an arbitrary point P and before I do that, I also specify the axis orientation I have *x*, *y* and *z* like this so that I can write my math correctly.

So, I take an arbitrary point P. We have to find out the rotatory effect of each of these forces. I need to first find out the position vector. Let us put arbitrary the position vectors as r_1 and r_2. So, I have the position vector r 1 for this force and position vector r_2 for this force. We have already learnt how to find out the moment of these forces about point *P*. So, when I do that, I would essentially get a moment about this, we will have to find out the value and I have also used a different symbolism here. See in vectorial notation I can have a two headed arrow to depict the moment; either I can have one arrow and then show the sense of direction or put two heads which shows this in the positive direction or in the negative direction. These are two symbolisms used in depicting moment. I deliberately chosen a different symbolism here so that you will learn both of these.

And we have the reference axis as *x*, *y* and *z*, so when I write moment, I put

$$\vec{M} = \vec{r}_1 \times -\vec{F} + \vec{r}_2 \times \vec{F}$$

And I simplify this as

$$\vec{M} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$$

and when I do it vectorially, I get the difference as vector a. So, it is nothing but the effect of these two forces to produce a net moment is a cross F, this has a very interesting property that is why we look at this force system. And if I write it in long hand without the vectorial notation how will I replace *a*? I will replace *a* by the perpendicular distance *d*. Ok. So, when I have this what is it that strikes you immediately, no matter where I shift the point of interest; it will always be

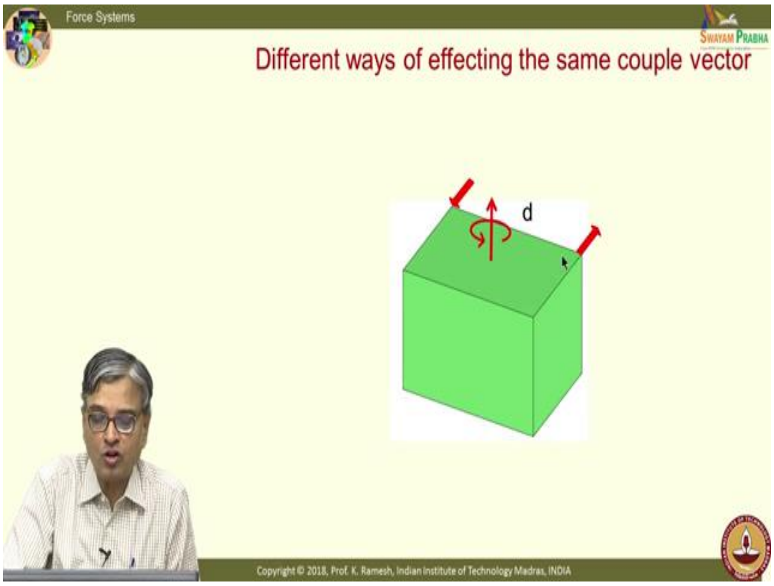
$$|M| = dF$$

I will always have the magnitude as

$$|M| = dF$$

So, the rotatory effect of this couple force system is independent of a point of interest, wherever I move the point of interest the value is going to remain same; it is very peculiar and people always get confused between a moment and a couple. Suppose I shift it to another point; there again the same value holds good, there again it is anticlockwise the point P has no consequence. So, wherever I move the point I have the same value. On the other hand moment of a force we found it is dependent on the point of interest; moment of a couple force system; I have a force system, I do not have one force, but I have two forces. So, it is very peculiar and you have to get the distinction between a moment and a couple very clearly. If you understand this rest of the chapters are child's play.

So, the point of application has no dependence on the value. So, couple is classified as a free vector. See we had looked at force and moment as sliding vectors from the principle of transmissibility; we are looking at the external effects. So, they can be translated along the line of action. The moment I go to couple I find the point of interest has no role to play and couple is a free vector.



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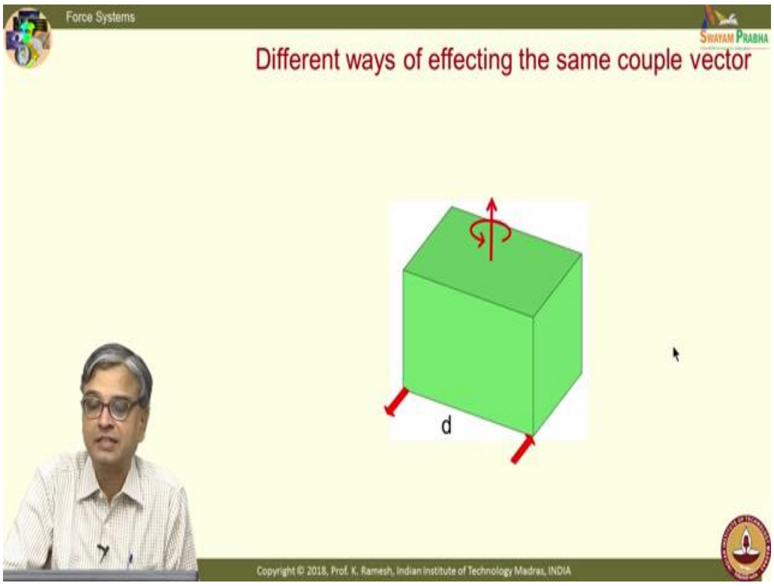

And let us look at the different ways of effecting the same couple vector. I take a block; I apply a force system like this; and suppose I have the distance between them as *d* this would give me the value of *Fd*. It is anticlockwise. Now, I apply the force not on this edge.

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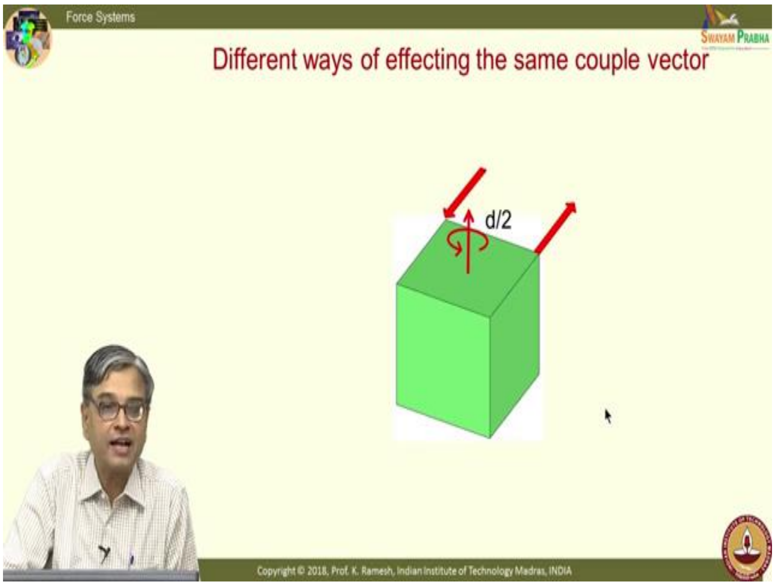
I apply it on these two edges, this force system also will result in the same couple; the rotatory effect for this system is identical.



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Then I take distance as half and I double the force application. I increase the force by two times and I cut this distance then again, the rotatory effect is identical.

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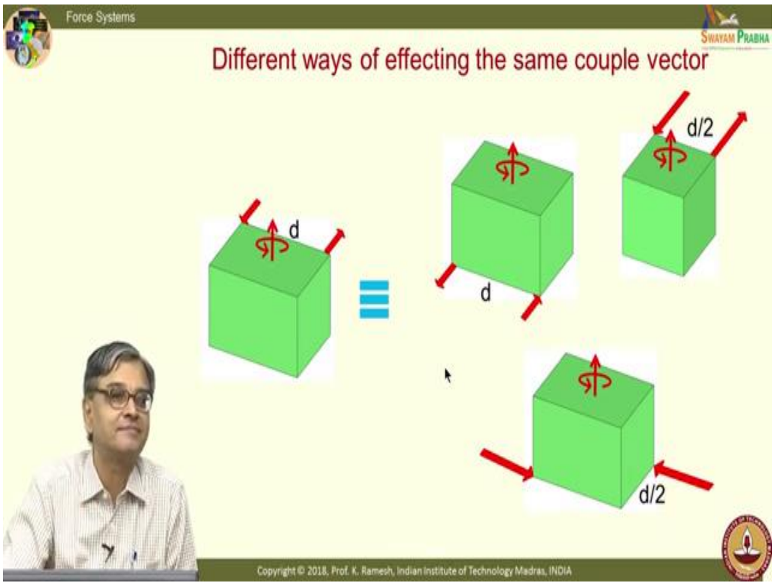

Whichever way I do it. So, in summary what you will have is, when I have a couple system like this; it is equivalent to doing it in multiple different ways. I can do it by doubling the force and shorten the distance by d by 2 or I can also apply it in different planes they are one and the same.

See you need to understand the distinction between a moment and a couple, while you learn it, they look simple but when you start solving problems you always get confused between a moment and a couple.

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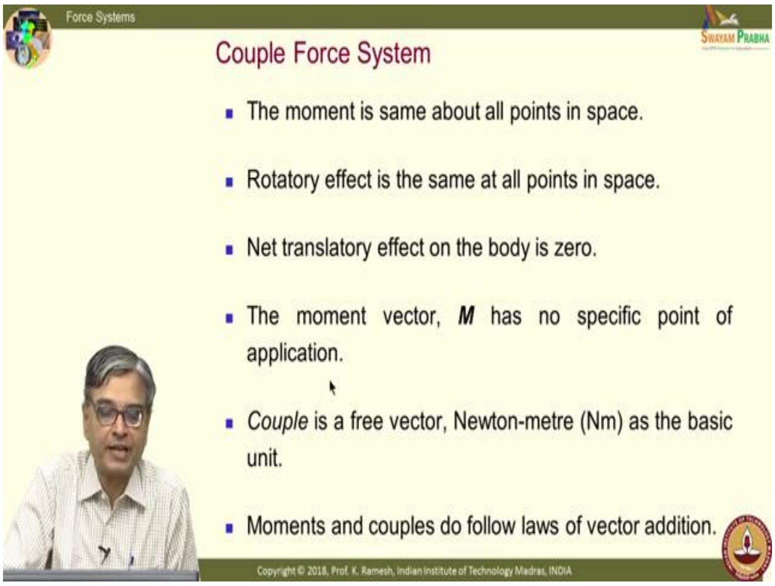

Let us summarize whatever that we have discussed on couple force system. The moment is same about all points in space. Rotatory effect is the same at all points in space. Net translatory effect on the body is zero; same idea put in different words ok. The moment vector *M* has no specific point of application. Its emphasized couple is a free vector, here again the unit is Newton-meter. Moments and couples do follow laws of vector addition ok. Let us look at a very nice example which illustrates what are the concepts that you have learned so far.

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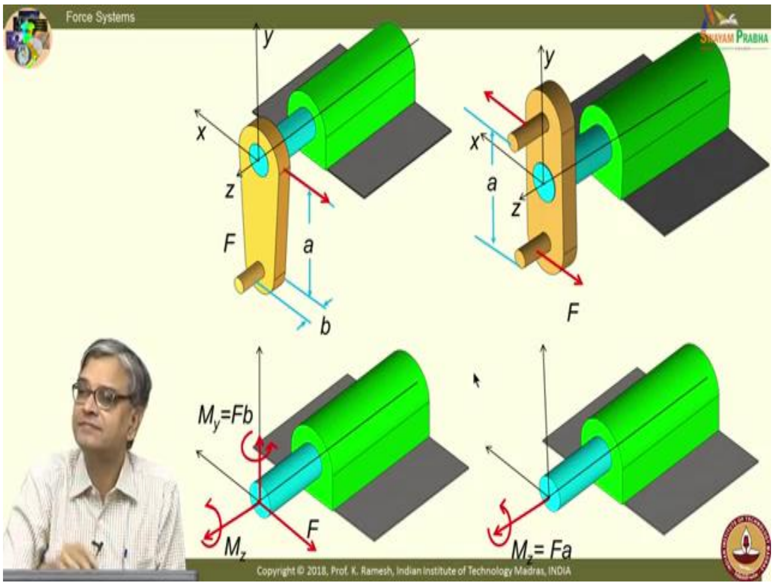


I have an object like this, I would like you to make a sketch, what I have is I have shaft here and this shaft needs to be rotated. For me to rotate this shaft, I have one type of handle here, I have another type of handle here. In this handle I can apply force only on this pin. So, I would apply a force to rotate it. On the other hand, this shaft has a handle which has two pins like what you have learned in the couple force system, I can apply an equal and opposite forces they are parallel on both these pins.

Now what I want you to visualize is in both cases shaft is going to rotate; are they same or different? What is your first take on it; before we do the analysis do you find that I rotate the handle with one hand or I put two forces on the pins and then, try to rotate it. Anyway, when I ask you the question there has to be some difference; fine? So, let us see what it is and we have you have enough background to do that; fine and what I have done here is, I have removed this handle, I have cut this; this is what you are going to do it in a free body.

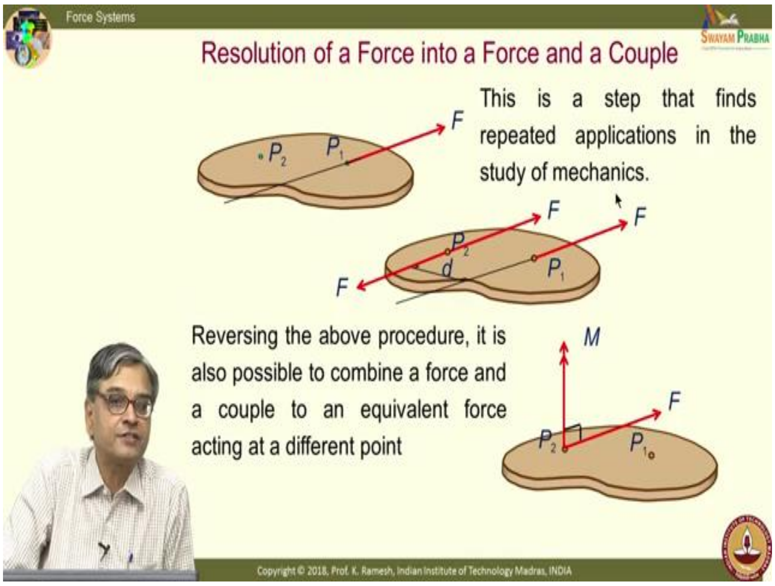
I will look at what are the forces that act at the end of this shaft; and I would do whatever we have done earlier; fine. Now I move this force to this place horizontally. I am also going to take this separately in the next slide, but we will see this in the context of couple force system. So, we will understand it better. I move this force this way horizontally, when I move this force this way what would happen?

If I have to move this force this way; it will also provide a rotation that will become clear in the next slide. And I will use principle of transmissibility to translate it like this, then I move this force to this place; this is what I am going to do. Moving the force from this position to this position would give me a couple here; value is F × b and, moving this force will give me a rotation this is what my ultimate aim, but I would have

$$M_y = Fb$$

M_z as well as force acting on the shaft; right now, it is not clear ok. I will take up the next slide and then explain you back.

From the context of rotating the shaft whether I rotate it with one handle or two pins a special handle with two pins where I can apply force at two places and mimic the couple force system; I will introduce only rotation there. I will not have any other additional forces; you will understand it more clearly by the next slide where I demonstrate what happens when I move a force from one line of action to another line of action; we will go there and then again come back.



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So, the idea is resolution of a force into a force and a couple. You may ask why do we need to do this. At the start of the class I had a simple force system as concurrent; when the force systems are concurrent by using the principle of transmissibility, I could move them to the point of meeting; and I can find the vector law for me to add. Suppose the lines of actions of forces are different; how am I going to find out the resultant. I must have a via media to do that and that you achieve by resolving a force as a force and a couple.

So, let me take point P_1. Let me have a force F acting on point P_1. Now my interest is I want to move the force from point P_1 to point P_2, while maintaining the same external effect as the original force F, I do not want to make any modifications on that. I cannot simply take the force and put it at point P_2. My problem is I have a force acting at point P_1; now I would like to see if I have to translate this force to point P_2; what would be the effect?

So, what I am going to do is I will introduce two forces at P_2, my interest is to see what happens when I translate this force to point P_2; my interest is I do not want to alter the external effects. If I move the force this to P_2; the external effects are changed. In order to cancel that what is it that I can do? I can put another force here, if I put force equal and opposite; have I applied a force at point P_2? I have not done anything. So, this is equivalent to what was original, but can I visualize this from a different perspective? We have learned what is a couple. Now I can quickly see I have a force; they are equal and opposite, they are parallel, so these two forces form a couple.

So, I can replace this by a couple vector; is the idea clear? So, now what I am going to do is; I have these two forces depicted as yellow, saying that that is what we are using them to form a couple force system. So, this will give me a moment about this point and I have a force. So, what you have to follow here is, if I have to translate the force from point P_1 to P_2; I cannot simply translate it. My external effects of this force would get modified. We have already seen when I apply a force; it produces a rotatory effect; it changes from point to point.

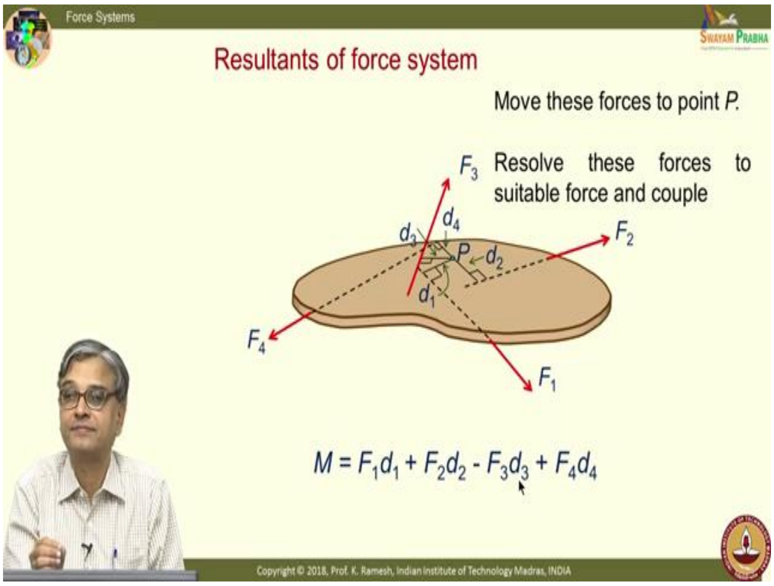
Now what I want to do is, at point P_2; I introduced two forces which are equal and opposite as good as no force at point P_2; when I put these two forces I get a clarity that if I have to translate this force to point P_2 without altering the external effect I have to consider force F, as well as a couple. And this is a very useful trick; later we are going to learn in the case of beams what are known as shear force diagram and bending moment diagram. You will repeatedly do this step and this also simplifies if I have a non-concurrent force system; how do I find out the resultant?

With this knowledge can we go back and then see the previous slide? So now I have a force F acting on this. So, I have to move this force from this line of action to this line of action. So, you add and subtract forces there, then you can recognize two forces will form a couple ok. So, when I move this force to horizontally like this, I can recognize I would have a couple acting on this. It is a free vector and I am putting it as the force acting on it.

Now, I translate this force acting at this, see I could also have the force on the side just for aesthetics I have put it on the side. I can easily translate it from this to this point. So, when I translate this to this point what will happen; add a force and subtract a force here; they form a couple that in turns tries to rotate the shaft; is the idea clear? So, that is what is shown in this diagram; I have a force, so I have a twisting moment.

So, when I apply a force only on one handle; then net effect on the shaft is like this, I not only have rotation, but I also have a couple F_b, as well as force F whereas, if I apply an equal and opposite force properly I would just have a rotatory effect of the shaft; is the idea clear? So, I would like to emphasize that this is the step that finds repeated applications in the study of mechanics; it appears very simple, if you understand it will remain simple for you; if you do not understand it, it will become the bottleneck for this course ok, that is why I wanted to spend sufficient time on this concept.

I can also reverse the above procedure; when I reverse above procedure it is possible to combine a force and a couple to an equivalent force acting at a different point. So, these are different tricks; you will choose these tricks appropriately when you need them, but you definitely master these tricks comprehend it; see you have automobiles and some of them you know they try to drive it with one hand, some of them will try to drive it with two hands. In earlier days when people are not having power steering; they need two hands rotate the steering wheel. So, what is the force that exerts on the bearing is different when I do it with two hands and when I do it with one hand. They are not same from a mechanics point of view, though physically you have the steering wheel is rotated; from mechanics point of view the bearing that is available in the car will experience different force system.



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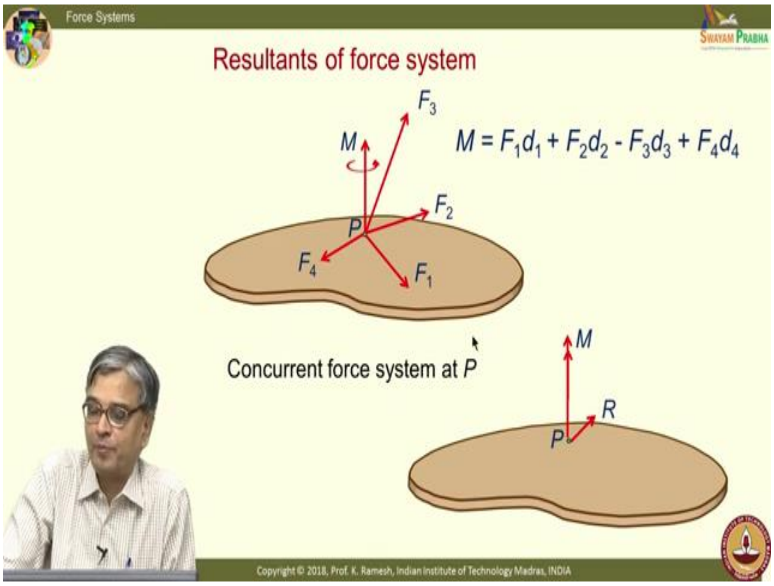
So, now, I have a trick, I have a body with forces acting at different directions. They are not having a common line of action; they do not meet at a point. I have multiple forces. I have F_1, F_2, F_3, F_4 like this. How do I find out the resultant of these forces because both in statics and dynamics you have to find out what is the resultant force acting on the system? Now you have the trick; what I have to do? I have to translate this force to point *P*. I have taken a simple situation where all the forces are lying in the same plane. So, I have to simply translate it to that. So, this is equivalent to finding out the perpendicular distance and give me a moment I will have a moment for this force F_1, I will have a moment for the F_2, I will have a moment for F_3, I will also have a moment for F_4.

I put these distances as d_1, d_2, d_3, d_4 and I get the net moment like this because you can find out that forces F_1, F_2 and F_4 introduce an anticlockwise moment whereas force F_3 introduces as a clockwise moment. So, that is negative. I would like you to visualize sense of rotation physically that would greatly help you to understand when you solve problems. Then with these moments I can move these forces to this point. I cannot simply take a force and move to that point; then it is a different problem. If I have to maintain the same external effects of the original force, I have to recognize when I translate, I should put in the appropriate couple components fine.



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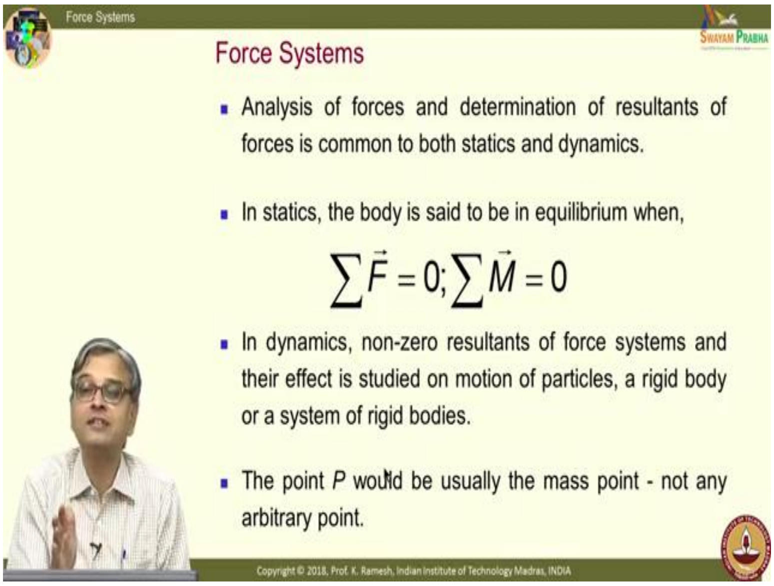

So, I have the moment acting like this, then I have a force system whatever we saw as F_1, F_2, F_3, F_4. They are all now meeting at one point; now it is child's play for you to find out by vector addition what is the value of the resultant; is the idea clear? So, I can replace this by a resultant and this also shows a general force system introduces a translation and rotation of the body; this is what we started with and that is what you see here, after learning the force system after how to do composition and resolution and also employ that trick of translating a force from one point to another point; you find any force system can be finally reduced to a resultant force and a resultant moment.

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So, in summary what you find here is analysis of forces and determination of resultants of forces is common to both statics and dynamics. In statics, the body is set to be in equilibrium; you must have been told probably you had exposure that I am not quite sure; when

$$\sum \vec{F} = 0; \sum \vec{M} = 0$$

you say the body is in equilibrium. On the other hand, in dynamics, you have non-zero resultants of force systems and their effect is studied on motion of particles, a rigid body or a system of rigid bodies. In such a case what would be the point *P*? You look at the mass point as the important point in dynamics. Whatever we have learned in this class is equally valid for statics as well as dynamics; there is no difference at all. So, in this class you know we have looked at what are force systems, we have looked at collinear force system, we have looked at parallel force system and we have also learned tricks like how to move a force from one point to another point. We have learned the concept of moment and concept of couple force system and you should make distinction between a moment and a couple; try to make an understanding right today, do some reading extra because if you understand a final distinction between moment and couple you will find the course very simple.

Thank you.