

Engineering Mechanics
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Module - 01
Statics
Lecture - 05
Equilibrium of Rigid Bodies - I

Module 1 Statics

Lecture 5 Equilibrium of Rigid Bodies - I

Concepts Covered

Necessary and sufficient conditions for equilibrium of – a particle, system of particles, single rigid body and interconnected rigid bodies. Equilibrium conditions for two force member, three force member and coplanar force system. Free-Body Diagram, Reactions at supports for a 2D structure - class I, class II and class III supports.

Keywords

Engineering mechanics, Statics, Equilibrium, Necessary and sufficient conditions, Rigid Body, Two force and Three force members, Free Body Diagram, Reactions at supports for 2D structures.

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Let us move on to the next chapter on equilibrium of rigid bodies, and you know Newton's first law provides the basis for equilibrium equations.

Equilibrium of Rigid Bodies

Introduction

- Newton's first law provides the basis for equilibrium equations.
- Rigid body: System of infinitely many particles with fixed distances between them
- A particle is said to be in equilibrium:
 - Resultant force acting on it is zero
 - Can moment act on a particle? $R = 0$
 - No!

This condition is both necessary and sufficient.

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And we have already said that we are going to deal with rigid bodies. When you say a rigid body, it is a system of infinitely many particles with fixed distances between them that is a key word there; Fixed distance.

I have a particle and I have a force R acting on it. And I would like to know can moment act on a particle, on a single particle. You cannot have a moment. The answer is no, and what would be the condition

for equilibrium? When do you say a particle is in a state of equilibrium? The force resultant acting on it is zero.

So, that is fine; see when you look at any condition you have to go to mathematics, in mathematics whenever you talk about conditions you talk of two things what is a necessary condition and what is a sufficient condition. Only when both these aspects are satisfied, you say the condition is achieved. For a particle when I say $\sum \vec{F} = 0$, it is

Equilibrium of Rigid Bodies

SHYAM PRABHA

- For a system of particles to be in equilibrium:

Necessary condition:

- Resultant force on the particles and resultant moment of these forces about any point should vanish.

$$\sum \vec{F} = 0; \sum \vec{M} = 0$$

Sufficient condition:

- The above conditions become sufficient if every conceivable sub-system satisfies them.

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necessary as well as sufficient. This case it is obvious, you would say that it is both necessary and sufficient; you do not need any other condition to verify whether the particle is in a state of equilibrium.

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Now, I move on to a system of particles, I have explicitly stated what are necessary conditions, and what are sufficient conditions. And I have a system of forces acting on it; please try to make a neat sketch of it, you will not find some of these concepts given in a book systematically, because people rush through the conditions of equilibrium.

In a system of particles, I can have both force as well as a moment. And when I have a system of particles there is no compulsion how they should behave under the action of forces that distances between them are free to vary not like a rigid body. So, what I have as $\sum \vec{F} = 0; \sum \vec{M} = 0$ is basically a necessary condition. When do you think that these conditions become sufficient so that you can say for sure that these systems of particle are in a state of equilibrium?

Then you will realize why we make in our first level of course idealize the bodies to be rigid. These conditions become sufficient only when they are applicable to every

conceivable sub-system which is shown as different circles here. When I have infinite particles, I could have infinite ways of forming the sub-system.

This is the reason when you go for deformable solids where the distance between the particle do not remain constant by the definition that you relax small deformations are possible, the equations of equilibrium appear as differential equations. And fortunately, in this course we make a very important idealization that we will confine our attention to rigid bodies.

Equilibrium of Rigid Bodies

- For rigid bodies

$$\sum \vec{F} = 0; \sum \vec{M} = 0$$

- Are both 'necessary' and 'sufficient'.
- This is because distance between particles is fixed.
- Only in a few real life situations one comes across a single rigid body for analysis
- Many practical situations are actually a set of interconnected rigid bodies.

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For a rigid body I have $\sum \vec{F} = 0; \sum \vec{M} = 0$ summation of these forces. From your earlier experience of what we have discussed, what can you say about necessity and sufficiency of the conditions. A rigid body by

definition can have both the resultant force and a moment.

Equilibrium of Rigid Bodies

$$\sum \vec{F} = 0; \sum \vec{M} = 0$$

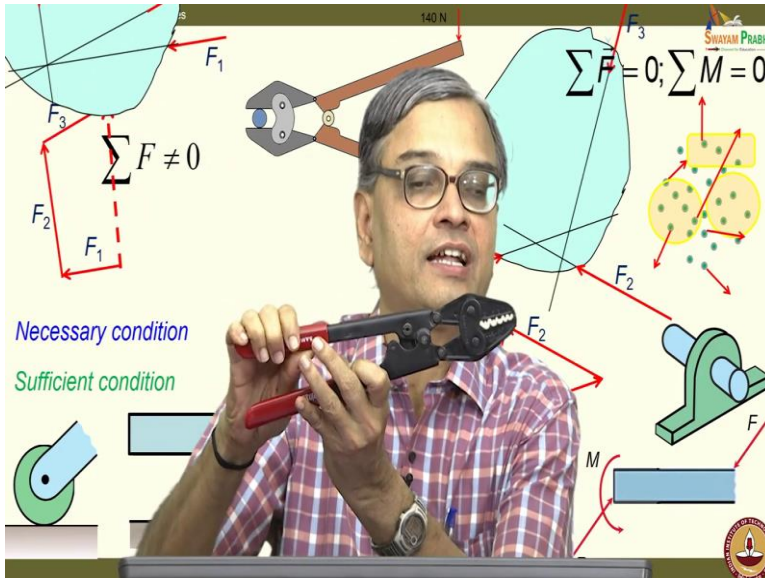
Necessary condition

Sufficient condition

If I am dealing with only one rigid body, these conditions are both necessary and sufficient. There is no two opinion about that. Mainly because the distance between particles is fixed, but if you look at only in very few cases you come across a single rigid body. (Refer

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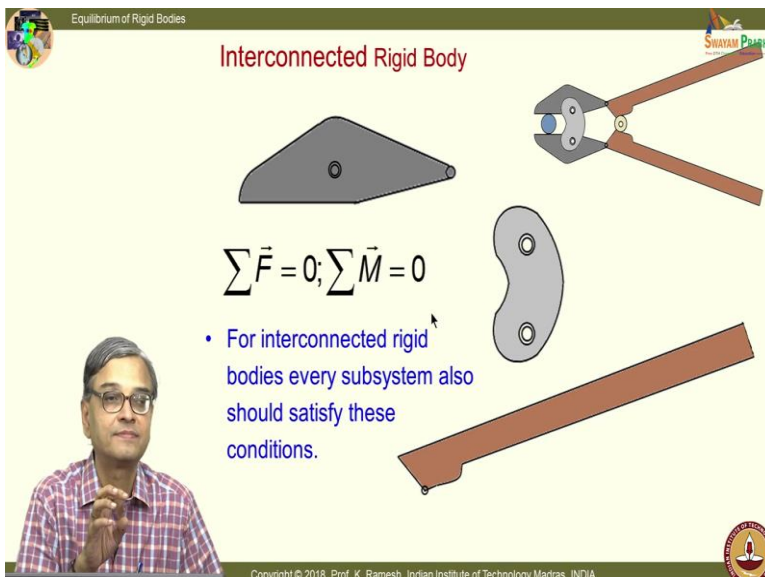
Suppose I take a simple cutting plier which everyone would have used it at some point in time. I need two elements, I have one element like this, I have another element like this; they need to be connected only then you can make a rigid body. So, what is practical is I would in general have interconnected rigid bodies.



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And I have a very nice tool this many of you may not have seen, this is called a crimping tool. This has many components you will see that and even for playing do not put your fingers here because it gives a very high magnification and you can

injure your fingers. And we would also try to solve this problem as part of maybe the next class.



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So, this is what is shown in this slide as well to emphasize the fact that it has many components for one arm you bring out these components. So, I have a component like this, I have a component like this, and I have another

component like this. This is replicated twice.

The conditions $\sum \vec{F} = 0; \sum \vec{M} = 0$ become sufficient only when subsystems of these rigid bodies also satisfy until then it is not. But what happens is in many problems we do not

have the patience to verify whether the condition satisfied for the entire system is also satisfied for subsystem. Because we live in a very fast world, but there could be occasions if you have not done the idealizations correctly you can have surprises.

In fact, I would recommend you to read one of the example problems given in the first chapter of Crandall and Dahl. The last problem he has taken the example of a universal joint, where he has shown if you have not verified $\sum \vec{F} = 0; \sum \vec{M} = 0$ for every subsystem.

The body as a whole is not in equilibrium, if you have wrongly idealized some of the supports. It sets a welcome relief because your mathematics also helps you to verify whether the solution, I get is acceptable or not. So, do not consider checking for sufficiency is a trivial exercise, it may become necessary in certain practical situations.

Equilibrium of a Rigid Body

$$\sum \vec{F} = 0; \sum \vec{M} = 0$$

- Writing in component form

$$\begin{aligned} \sum F_x = 0 & \quad \sum M_x = 0 \\ \sum F_y = 0 & \quad \text{and} \quad \sum M_y = 0 \\ \sum F_z = 0 & \quad \sum M_z = 0 \end{aligned}$$
- Unknown forces can be determined by applying the above conditions

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It sets a welcome relief because your mathematics also helps you to verify whether the solution, I get is acceptable or not. So, do not consider checking for sufficiency is a trivial exercise, it may become necessary in certain practical situations.

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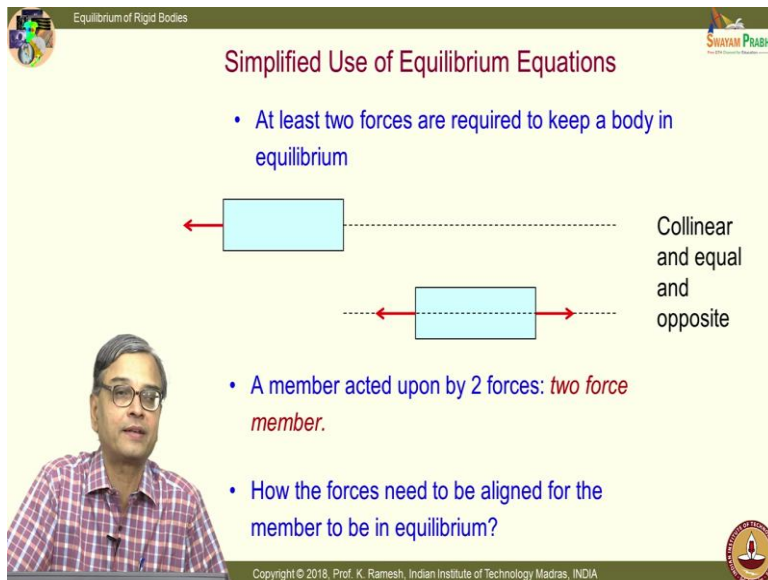
So, I have this $\sum \vec{F} = 0; \sum \vec{M} = 0$

In component form in three dimensions I could write it as

$$\begin{aligned} \sum F_x = 0 & \quad \sum M_x = 0 \\ \sum F_y = 0 & \quad \text{and} \quad \sum M_y = 0 \\ \sum F_z = 0 & \quad \sum M_z = 0 \end{aligned}$$

And why do you apply these conditions? The whole focus in statics analysis is you have to find out the unknown forces.

So, the focus of applying these conditions is to evaluate the unknown forces, and you would use these forces in your next level of subject and find out what are the internal reactions. It is not that you do engineering mechanics the problem is completely solved; it only provides you input for higher level courses.



The slide is titled "Simplified Use of Equilibrium Equations" and features a video inset of Prof. K. Ramesh. It contains the following text and diagrams:

- At least two forces are required to keep a body in equilibrium
- A member acted upon by 2 forces: *two force member*.
- How the forces need to be aligned for the member to be in equilibrium?

The diagrams show a light blue rectangular member. In the first diagram, a single red arrow points to the left from the center of the member, with a dashed line extending to the right. In the second diagram, two red arrows point in opposite directions (left and right) from the center of the member, with a dashed line extending through both.

Text to the right of the diagrams reads: "Collinear and equal and opposite".

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So, to idealize the body as rigid gives you enough room for you to solve it in a simplistic manner and the final result is also acceptable based on experimentation. If the final result is not acceptable then you have to re-idealize the problem.

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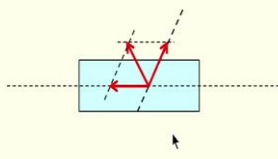
And let us ask the question how many forces are required to keep a body in equilibrium? You have already seen at the start of the slide the body with a force was moving. It acted like a smart board and said that I will I am not in equilibrium, so I have to go. So, the minimum what you require is at least two forces to keep it in equilibrium.

Do you agree there are two forces; the two-force assumption is valid under what conditions. I have said in aerospace and mechanical for a variety of problems you could get reasonably a good solution by neglecting the weight of the component. If you bring in the weight, I will always have three forces I cannot have two forces. So, you have to understand that we have neglected weight for a variety of problems and you could say a member acted by 2 forces as a two-force member.

There is a very great advantage; if you understand how a two-force member behaves, it would be easier for you to find out the unknown forces. When you solve a problem you will see its advantage, and how the forces should be? They should be collinear and equal and opposite.

Equilibrium of Rigid Bodies

Simplified Use of Equilibrium Equations



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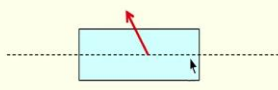
Now, let us take a situation suppose they are not collinear what would happen? I have taken one force here and I have put the second force here. And I can use the principle of transmissibility and move these forces to the point of

meeting. And when these two forces are concurrent what is the value of moment?

Moment is 0. When I have a concurrent force system moment is 0, but these two forces would definitely will have a resultant force.

Equilibrium of Rigid Bodies

Simplified Use of Equilibrium Equations



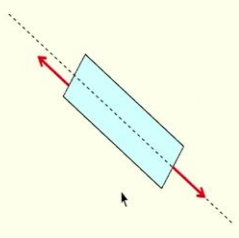
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And how does this resultant influence the body, if the screen becomes smart; body will not remain in equilibrium but it will translate like this.

Equilibrium of Rigid Bodies

Simplified Use of Equilibrium Equations



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Suppose I take another situation where I have these forces are not collinear, they are equal and opposite what would

happen? The body will start revolving.

Equilibrium of Rigid Bodies

Simplified Use of Equilibrium Equations

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Collinear and equal and opposite

- How the forces need to be aligned for the member to be in equilibrium?

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So, it is very clear for the body to remain in equilibrium in a two-force member the forces have to be collinear, equal and opposite. You would use this property when you solve a problem, when you understand this. So, the natural next step is from a

two-force member graduate to a three-force member.

Equilibrium of Rigid Bodies

Simplified Use of Equilibrium Equations

SHYAM PRABHA

- At least two forces are required to keep a body in equilibrium
- A member acted upon by 2 forces: *two force member*
- A member acted upon by 3 forces: *three force member*
- For a rigid body under the action of a co-planar system, the number of equilibrium conditions reduce from six to three

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

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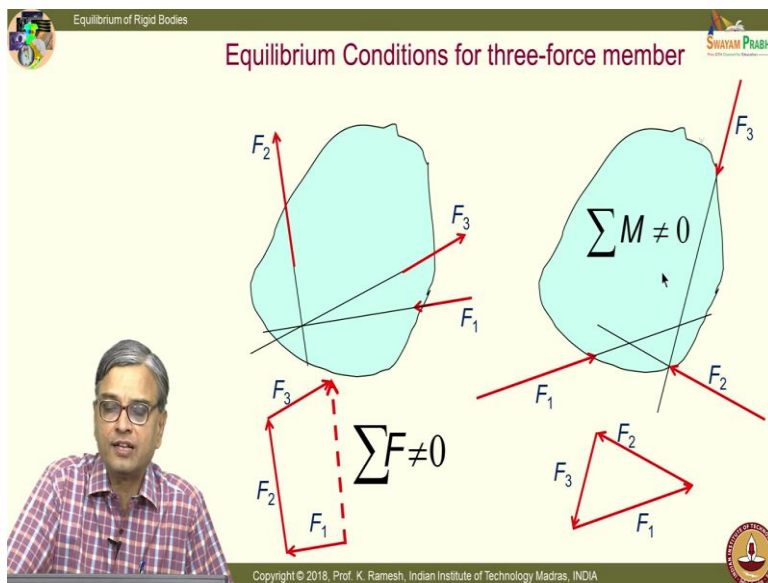
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So, we have discussed that at least two forces are required to keep a body in equilibrium, a member acted upon by two forces we call it as a two-force member. A member acted upon by three forces you call it as a three-force

member. And also, in order to make our life simple, we would initially start with coplanar force system.

If you understand the mathematics behind it, you can graduate to a three-dimensional analysis you will have one extra component from mathematics point of view. And what happens? From six conditions of equilibrium, it reduces to three for a co-planar system

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$



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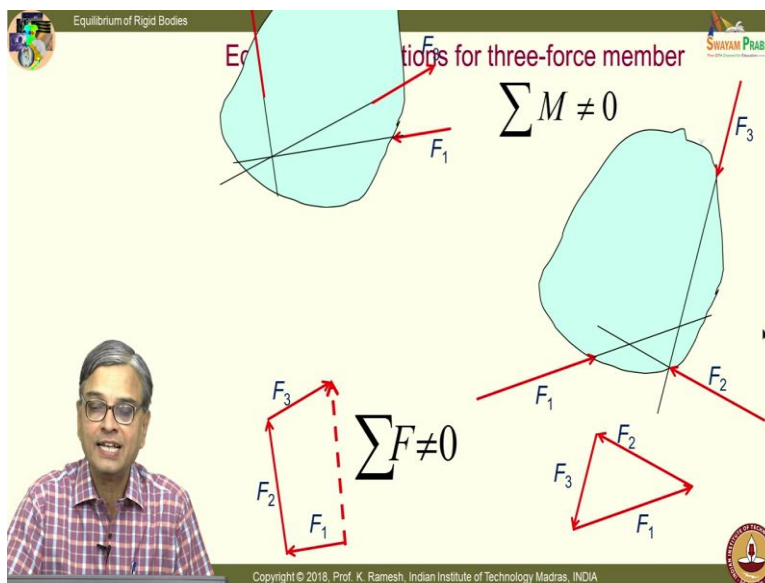
Please make a neat sketch of it I have a body; I have three forces acting on it. You know the animations are done slowly, so, that you have time to copy it down. Digest the concept and copy, you do not have

to verbatim copy the shape; shape is representative.

Let us see how these forces act, let me find out the line of actions. And you what do you find the line of actions meet at a point. What is information it gives you directly? The moment due to these forces F_1 , F_2 and F_3 is 0, because I have concurrent force system. Now, if I have to find out what is the resultant, I should draw the force polygon. Let me put these forces I have F_1 here, I have F_2 here and, I have F_3 here it does not close. So, I have a resultant acting on the body, so, I get the situation in this case $\sum F \neq 0$.

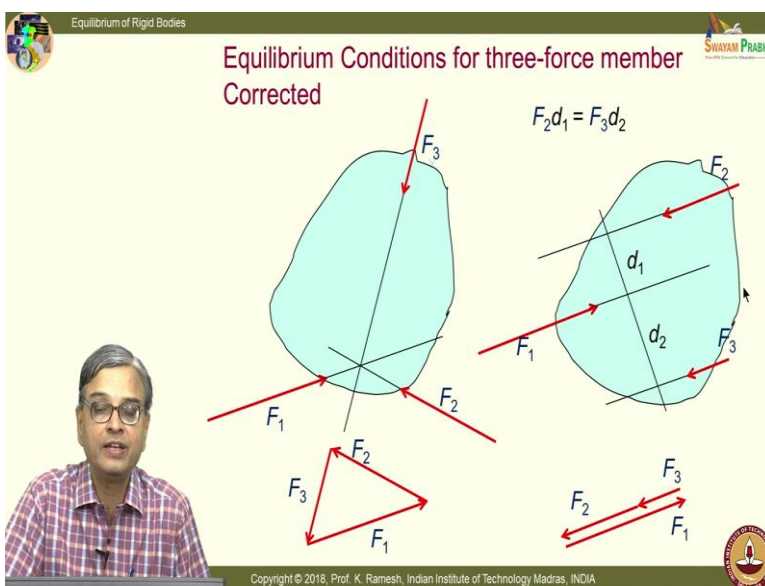
Now, let me take one more situation, I again take this body I have three different forces acting on it as before. I have force F_1 , F_2 and F_3 ; they are purely arbitrary, there is no special consideration in selecting these forces. Our interest is to find out what is the property of a three-force member, what kind of force system I can have so, that I can solve a problem intelligently when I identify this as a two force or a three-force member. I can take advantage of these properties.

Now let me see how these forces are the in magnitude? I have the force F_1 , F_2 and F_3 they form a closed triangle, so the net force is 0. Then what we will have to investigate, what is the moment? In order to do that let us see the line of action of these forces. So, I find it is not concurrent and I will have to find out that there will be a moment, let us say moment is not equal to 0 let us also give a direction as clockwise.



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It could be clockwise or anticlockwise because we are not getting into the numerical values to find out; just a representative. So, I have a situation in the first case, net force is not zero, in the second case net moment is not zero. So, both of these force systems are not going to give me a member under equilibrium. If the board become smart what would happen to the first case it will translate, in the second case the body would rotate.



equilibrium.

Now, the idea is if I have to find out an equilibrium condition, in the first case the forces have to be adjusted. So, that the resultant is 0; what we would now do is, is there any way I can make for the second case how to make this as in a state of

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Given a choice what would you do? I give you freedom to change the point of application of these forces, if I give you that freedom to you is there anything that you could do so that I can make the body in equilibrium; because we have already seen net force is 0. So, I am not going to modify the magnitude of these forces, I am also not going to modify the direction of these forces. So, what is the choice I have?

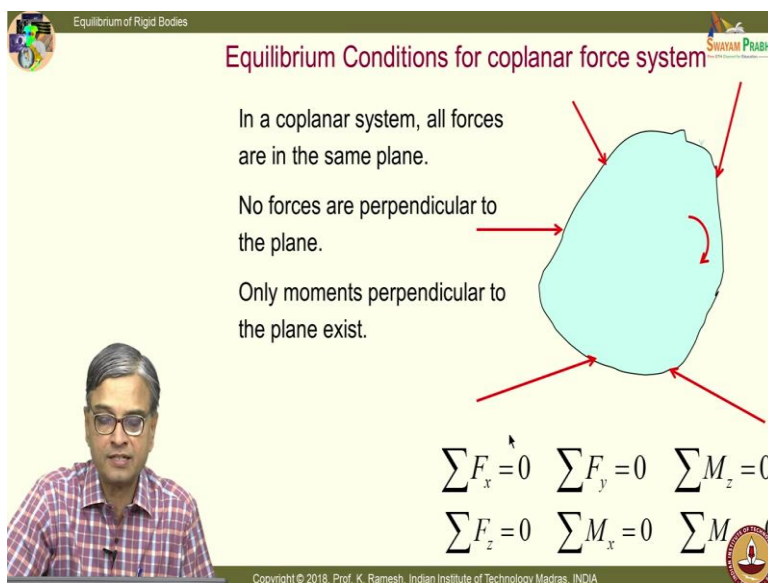
So, the idea what I have is the force F_3 has to be shifted, so, that it forms a concurrent system, so, it is a very important clue. So, if you identify a body is a three-force member, you can right away say the force system has to be concurrent, when I am looking at a coplanar system ok.

And when I say that they meet at a point what is the extreme of this? I could also have a parallel force system; they meet at infinity. If I have 3 forces acting that are parallel, then they have to satisfy the following conditions. The force magnitude should be balancing themselves and these are all the line of action, so, I can find out the distance between the forces.

Let me say that they are separated by distances d_1 and d_2 from moment equilibrium point of view I should have F_2d_1 which anticlockwise should be equal to F_3d_2 which is clockwise.

$$F_2d_1 = F_3d_2$$

I urge you to visualize the sense of moment physically. We would solve problems vectorially as well as direct competition you should get practice on both these approaches do not only stick on to vectorial approach. And the magnitude of these forces has to balance. I have $F_1 F_2 F_3$ unless these two conditions are satisfied you cannot have parallel forces giving a state of equilibrium to the body.



Equilibrium of Rigid Bodies

Equilibrium Conditions for coplanar force system

In a coplanar system, all forces are in the same plane.

No forces are perpendicular to the plane.

Only moments perpendicular to the plane exist.

$\sum F_x = 0$ $\sum F_y = 0$ $\sum M_z = 0$
 $\sum F_z = 0$ $\sum M_x = 0$ $\sum M_y = 0$

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And we will confine our discussion to start with on coplanar force system. I have shown a body; in a coplanar system all forces are in the same plane no forces are perpendicular to the plane. Only moments I have either a clockwise

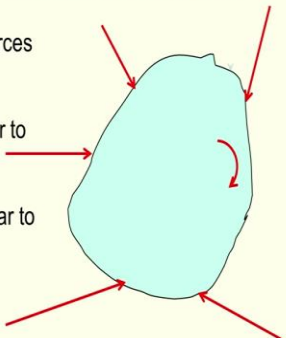
moment or anticlockwise moment which is perpendicular to it and you have general 6 conditions of equilibrium of this only 3 are required;

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

Equilibrium of Rigid Bodies

Equilibrium Conditions for coplanar force system

In a coplanar system, all forces are in the same plane.
No forces are perpendicular to the plane.
Only moments perpendicular to the plane exist.



$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

The subscript for M is often ignored. Instead the subscript denotes the point about which moment is taken

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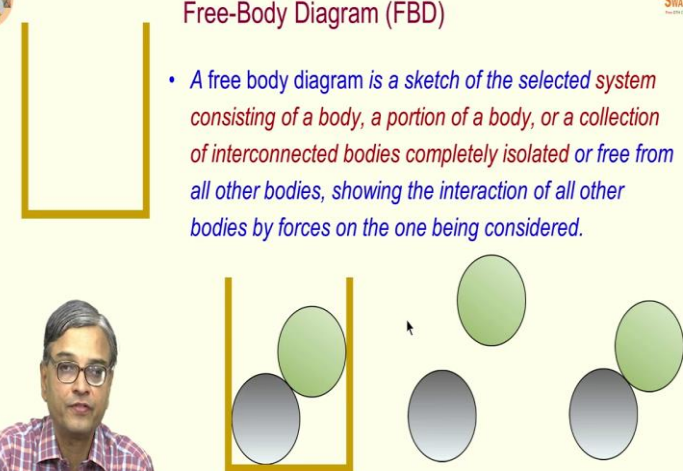
And what we would also do is we would not call it as $\sum M_z = 0$, usually we will not give the subscript or even if we put the subscript, we would put the point about which you take the moment. That is a general convention used

when you are solving problems.

Equilibrium of Rigid Bodies

Free-Body Diagram (FBD)

- A free body diagram is a sketch of the selected system consisting of a body, a portion of a body, or a collection of interconnected bodies completely isolated or free from all other bodies, showing the interaction of all other bodies by forces on the one being considered.



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Then we go to the most important aspect what is a free body diagram, you must understand the definition very clearly. And you know many students do not pay attention to free body, they all worry about answers from an engineering

perspective you should know how to draw your free body as clearly as possible. If you develop good habits it will stay with you, make your engineering education meaningful and enjoyable.

A free body diagram is a sketch of the selected system; the system can be a body, a portion of a body. It is very important; it took a very long time to scientists to come up with an imaginary cut of the object. People were not comfortable earlier; people were comfortable segregating the bodies.

And we would soon see in the case of trusses as well as a beam we would pass imaginary cuts physically it will be a continuous body. But we will cut one section and see what happens at that section, or it could be a collection of interconnected bodies what is important is completely isolated, it should not be connected to the surroundings. So, when I replace the surrounding, I should replace the interaction with the surrounding by appropriate force system.

One of the most challenging aspect in engineering is how to idealize a given problem situation, see fortunately or unfortunately in this course many times we coin a problem explicitly giving you the idealizations on the supports. A practical problem does not come to you like that; you will have to idealize what kind of support system I can model.

And you would also have lower bound and upper bound solution; if I assume the conditions like this, I may get the values in this range, I will have an upper bound solution so that I have the values between lower bound and upper bound for practical problems. So, idealizing the supports is a challenge in a given practical situation.

Suppose I represent the support by simple connections, which we would discuss how do

Equilibrium of Rigid Bodies

Free-Body Diagram (FBD)

- Three essential characteristics of an FBD:
 - (i) It is a diagram/sketch of the selected system
 - (ii) The system is shown completely isolated from all other bodies including foundations, supports etc.
 - (iii) The interaction on the system by each body removed in the isolating process is shown as a force(s) on the diagram
- Surface forces are applied through direct mechanical contact whereas, Body forces (gravitational, magnetic, electric) are applied through remote action.

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they behave it makes your life lot simpler. So, I should completely isolate the body free from all other bodies, showing the interaction of all other bodies by forces on the one being considered. And I have a system where I have 2 discs inside box, and when I say a system, I

could consider yes 1 disc is the system separately. I can consider the 2 discs separately or I can consider the system as both the discs and I can also consider the box.

So, one of the first steps in solving a problem is for a given problem what is the way you would identify the system for you to analyze. Here I have only separated the system; I have not shown the interaction of the system with the surroundings fine.

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And what are the essential characteristics of a free body diagram, first of all it is a diagram sketch of the selected system as engineers, you should be reasonably comfortable in drawing a neat sketch. It should be reasonably representative of the distances fine, unless you draw the free body clearly, it would cloud your thinking, if you draw the free body clearly, then your problem solving is also systematic.

The system is to be shown completely isolated from all other bodies including foundations, supports etc. Whatever the interaction of the system by each body that are removed in the isolation process is shown as a force or forces on the diagram.

And let us see some more points on how do we draw the free body diagram; I have already said that we classify the forces as surface forces and body forces. And surface forces are easy to identify there is very little chance for you to miss it, because it is

Equilibrium of Rigid Bodies

- Known and unknown forces to be indicated.
- Unknown external forces are usually constraining forces.
- Indicate both magnitude and direction of external forces.
- For unknown reaction forces, assume any direction initially.
- +ve answer means assumption was right; -ve answer means direction has to be reversed.
- For unknown frictional forces the actual direction must be indicated.
- Include only essential dimensions.

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applied by direct physical contact and body forces are by remote action. So, you must take care of both surface forces as well as body forces carefully in your free body diagram.

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You should mark both known and unknown

forces because the idea is to evaluate the unknown forces by applying the conditions of equilibrium. Then the question comes what are the unknown forces they are usually the

constraining forces, what forces that happen at the supports. And you have to indicate both magnitude and direction of external forces. See the thumb rule is, if you know from a physics of the problem you indicate it correctly, if you do not know the physics of the problem you can choose any direction to start with; your mathematics will help you.

So, you can assume any direction for reaction forces, there is, are they constraining forces. But if you know from a physics what direction the constraining forces you can put that directly. I would encourage you to physically visualize the problem each time. Take assistance from mathematics, but visualize the problem physically.

Whatever the direction you choose if my final answer is positive the assumption was right. If the final answer is negative direction has to be reversed. So, another practical way of looking at is, if you know the interaction correctly indicate it. Then also if your assumption is wrong or right your mathematics will help you. Usually if I do not know anything you take the unknown forces in the positive direction of the coordinate axis, this is another way of solving the problem systematically.

Suppose I have a problem where there is friction, I have said that why we have confined our attention to rigid bodies when we saw the necessary and sufficient conditions of equilibrium for a deformable body it becomes very difficult to handle. Similarly, in many problem instances even though in physical problem some level of friction exists, we idealize it to be frictionless.

In problems dealing with friction you have no advantage from your mathematical analysis to indicate whether your direction was correct or wrong. If you wrongly specify the frictional direction, you are solving all together a different problem please understand that. So, any problem dealing with friction you must make an effort to understand the problem statement clearly, visualize what is the physical interaction and try your level best to indicate the correct frictional direction. There is no luxury there.

And finally, in a problem you may have multiple dimensions include only essential

Reactions at Supports for a 2D Structure

- CLASS I: Support condition restrains translation in one direction
- Only one unknown: Magnitude of reaction force normal to the contact surface
- Examples: Roller support, frictionless surfaces, rockers, wall edges etc.

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dimensions. And other good practice is whenever you take a problem to draw the free body; indicate the coordinate system. You know we conventionally use right-handed coordinate system, please follow the convention do not say I have taken a left-handed coordinate system.

And it is extremely difficult to correct for hundreds of people. So, for the coordinate system use the right-handed coordinate system.

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And now let us understand what way can be idealize a support, I try to give it from an engineering perspective; see we have looked at a 3-dimensional situation if I have. I would have 3 forces and 3 moments. So, in general I can have 6 unknowns and if I translate the forces as the corresponding motion, I will have linear moments rectilinear motion.

Supports that restrain "Translation" in one direction

Support Type	Action on body to be isolated	No. of Unknowns
(a) Roller/Rocker/Knife edge		1
(b) Smooth Surfaces in Contact		1

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And if I have a moment, I will have a rotational effect. So, I would classify the simplest constraint as one that restrains translation in one direction. We would see the example and we will understand ourselves what they are in such a

situation only one unknown exists. We already know the direction only the magnitude of

reaction force needs to be determined. And what are the kind of supports; it is a roller support, frictionless surfaces, rockers, wall edges etc.

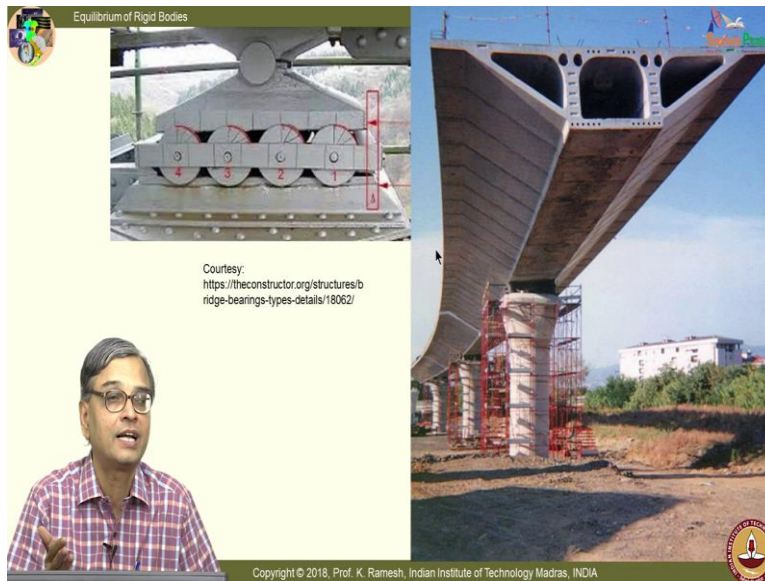
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I have a nice diagram please sketch the diagram neatly, I have a roller support. I have another form of roller with multiple rollers at the bottom, and what it shows is this is the body that is to be isolated. And this would essentially have a single unknown the direction is known; the value of this force is not known. If I look at this case, I would have 3 small forces by the principle of composition you can replace it by one single force. And essentially the number of unknowns in this case is just one.

And you have this roller in many applications, you have wheeled chairs many applications you come across this roller. And if you look at bridges, we would soon see thermal expansion and contraction needs to be accommodated. So, you have bridges supported on these supports. And I can also have a simple roller supporting it or I can also have a knife edge; even in these cases the support can be replaced by a single force, the direction is known, magnitude is not known.

In a similar vein we also assume smooth surfaces in contact, because I said we want to make our life simple wherein we do not want to have friction. In reality you may have a very small value of friction which could be neglected for a given application, if it is appropriate you can do that. And when I have smooth surfaces in contact, I try to put a tangent, then I find out the normal to this.

So, in this case also the direction is known, only the magnitude is not known. And from a physical appreciation if we have a constraint like this, this restrains translation in one direction that is another way of understanding these supports from a physical perspective; it cannot move down, it can move horizontally fine. So, that moment perpendicular to this in the downward direction is restricted by these supports.



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Now, let us move on to class II supports, I think before we move on to that, I have a nice example of this. You have a bridge being constructed which is supported on this and it shows one possible support condition on this, mind you this is a very big

business. So, you have many numbers of bridges these supports need maintenance, replacement.

They are very essential when you consider in Northern India you have easily temperature changes of 10 to 12 degrees is possible, afternoon it may be 20 degrees, night it may be just 8 degrees. So, you have to accommodate for the thermal expansion and contraction of these systems. So, you need to have provision for expansion and contraction.

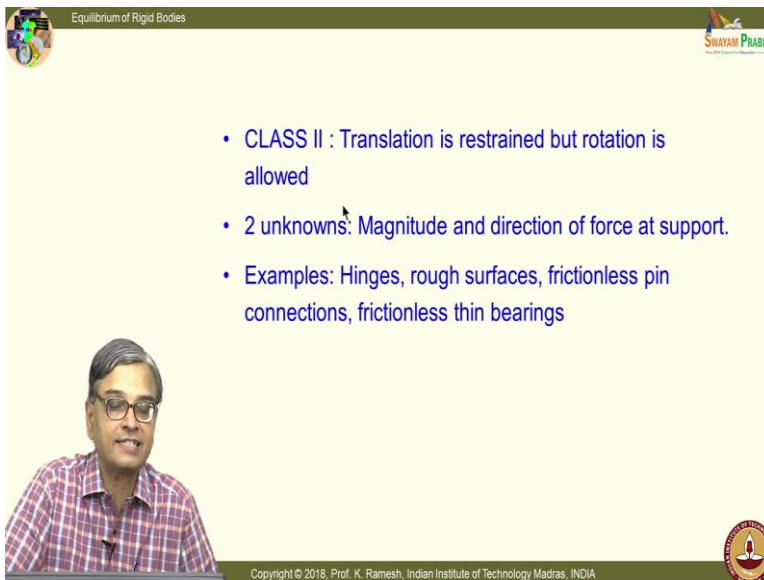


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And you also have another support I deliberately brought these slides to give you a flavor of engineering. I have always been saying that you should see physical systems around you and you have a huge truss bridge here and you could

see a lot of bumps on this they are actually rivets. That is, they are used for joining, they are very precision, mechanical, manufacturing operation.

And most of the structures that have that are very critical people join it by rivets, because it also has a inbuilt fracture resistance capabilities. If there is a crack developed the crack will get arrested. And you can see here I have a support which is very similar to what is the support we had seen in the previous slide there is three rollers, and you have a support like this for the bridge.



Equilibrium of Rigid Bodies

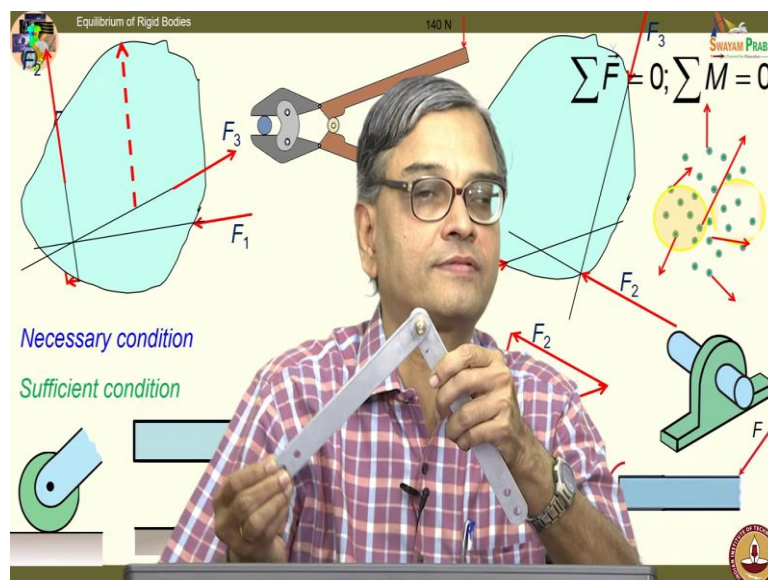
- CLASS II : Translation is restrained but rotation is allowed
- 2 unknowns: Magnitude and direction of force at support.
- Examples: Hinges, rough surfaces, frictionless pin connections, frictionless thin bearings

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Now, we move on to class II supports, translation is restrained, but rotation is allowed. And in this case, I have 2 unknowns: magnitude and direction of forces at support.

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Equilibrium of Rigid Bodies

$\sum \vec{F} = 0; \sum M = 0$

Necessary condition

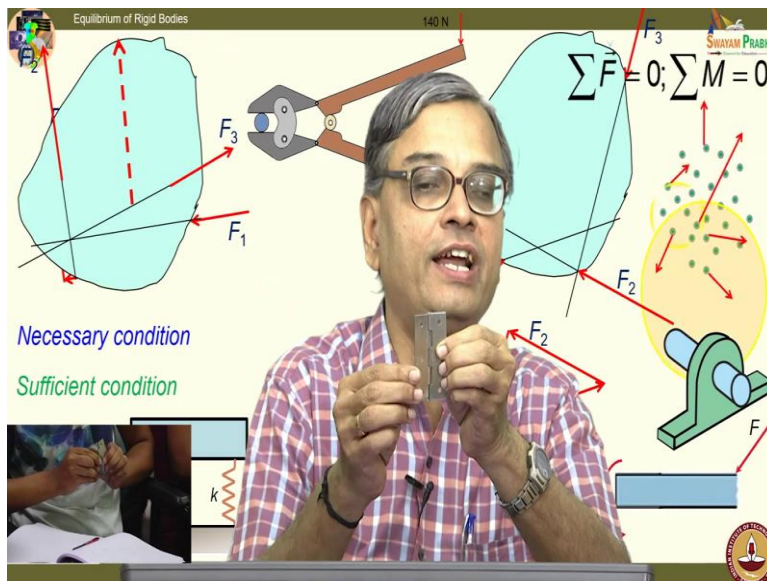
Sufficient condition

140 N

F_1 , F_2 , F_3 , F

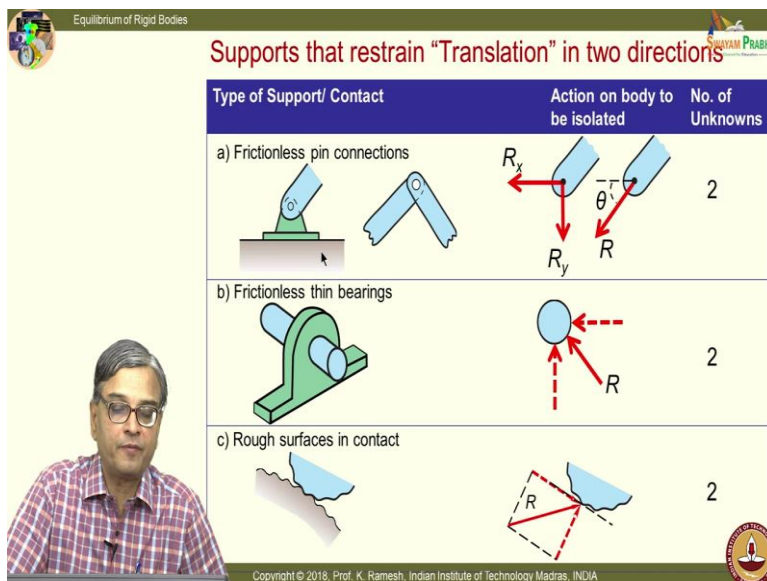
And what I have here is two links and then I join it by a pin, I have a pin here and then I put it here, and I can freely rotate this. So, the rotation is allowed can I translate; I cannot translate it; translation is arrested whereas, rotation is completely allowed this is seen in hinges also.

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You know the common door hinges you have I have also given this to the class please circulate. And you have a look at it I have these hinges, whatever the doors that you have that allow you rotation and you want rotation. You want the door to open and close and you do not want

somebody to take away the door. So, it is restrained from removing.



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And this is what is shown in this as nice picture, so, I have one representation like this, you should also know these representations. In future classes when we give a problem you should be able to interpret these representations, see how a

roller support was indicated, how a hinged joint is indicated this is also called a pin joint. And this is what I have demonstrated to you physically, I have two links and I put a pin. And in this case, I do not know the direction of the force, so, I represent the unknown force as a horizontal component and a vertical component.

Force is a vector I neither know the magnitude, nor the direction. So, I have 2 unknowns in this case. And in many mechanical engineering applications, you have a bearing to

support a shaft there could be thick bearings as well as thin bearings. Suppose I consider the bearing to be thin and frictionless, here again I could replace the support by an unknown force. So, I have 2 unknowns to be determined the horizontal component vertical component or what is the orientation θ and the magnitude whichever way you look at it.

And naturally when I have a rough surface which is very highly exaggerated here, it would have both a frictional force as well as a normal force and I would have a resultant, so, I would have 2 unknowns. So, in problem solving if a joint is represented like this, you should figure it out that, this is a pin joint. And then replace the support by an appropriate force system by an appropriate force system you should do it yes.

Conversion:

Student: (Refer Time: 42:51).

Which one?

Student: (Refer Time: 42:55).

It does not matter I can assume it in any way I like it fine.

Student: (Refer Time: 43:02).

Which one?

Student: (Refer Time: 43:04).

See this is only a representation fine, and when I draw a free body diagram what you say in a sense, I agree with you fine. Here it is shown correctly when I have a bearing, it will; the bearing support will push it like this here I cannot say how it is I am putting a very generic directions ok.

And my mathematics is going to tell me whether my assume direction is correct or not as long as I am not dealing with friction, I have no issue. But when I am having an interconnected body, if I assume for one body by Newton's third law, I should only put the opposite in the connected body. There I cannot bring in a choice of my own and then do it when you solve a problem these concepts will become clear fine.



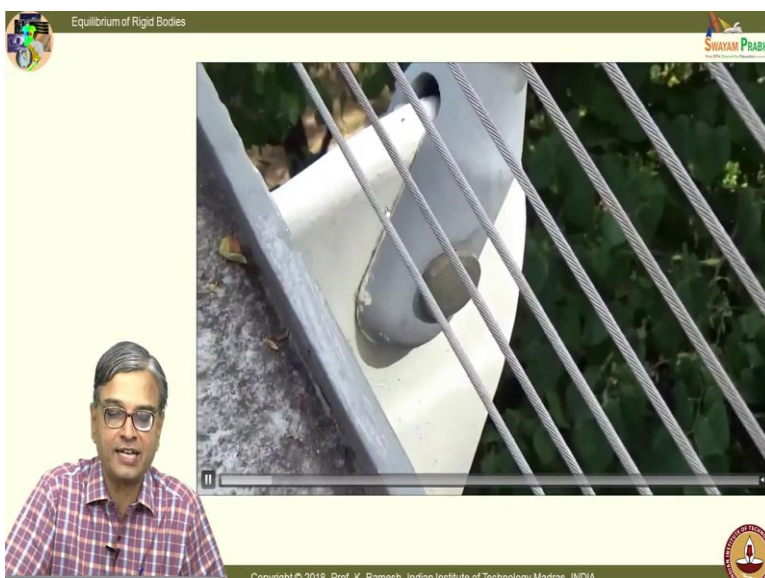
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And you know I have been telling that you should see structures around you and I should also live by example. And you find a big mast I recently visited the Greenville South Carolina there is a very nice hanging bridge. And you could see here very clearly a pin joint, and if you go to an airport all modern airports in the country, you have huge pin joints pin joints is so, well utilized in several structures.



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You can see that and we will also see what the bridge is this is a bridge of about 300 meters in length. This is fully supported by cables and by the big mast which you had seen, and it is about 30 feet above the ground; there is no support from the bottom.



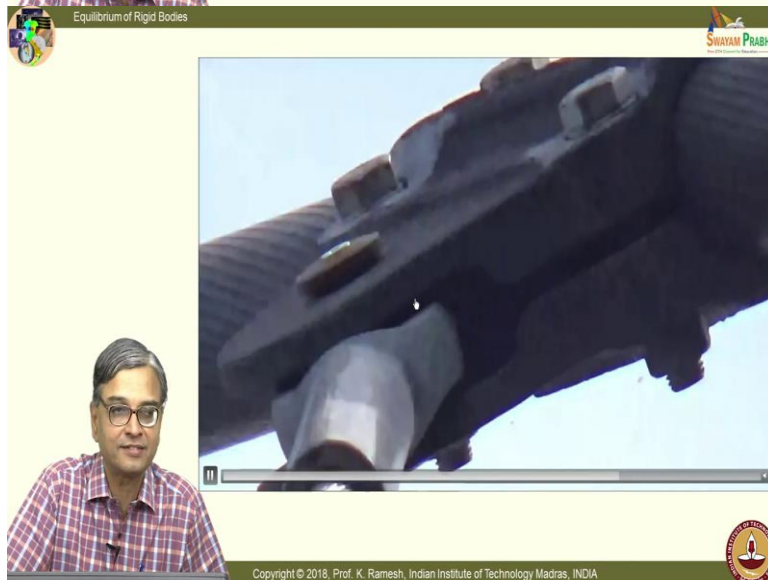
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And this shows how the pin joint is there in one end supporting the bridge.

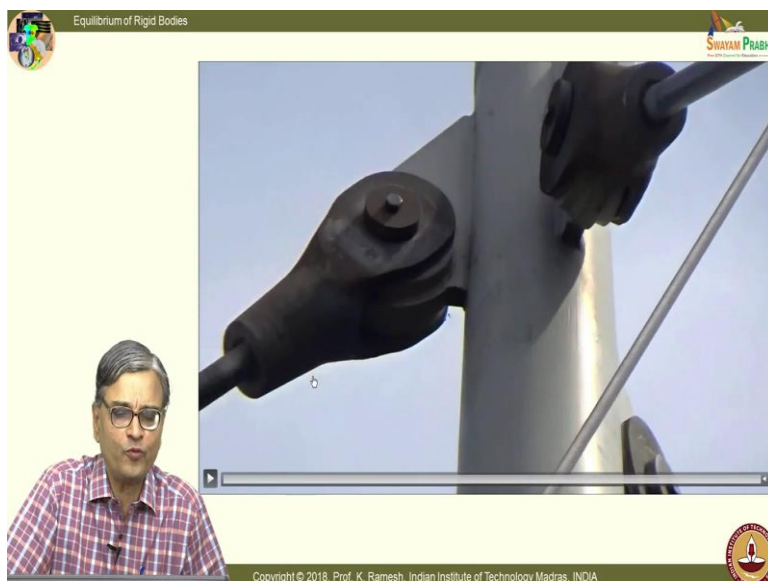
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And you have the cable that goes to the cable coming from the mast.



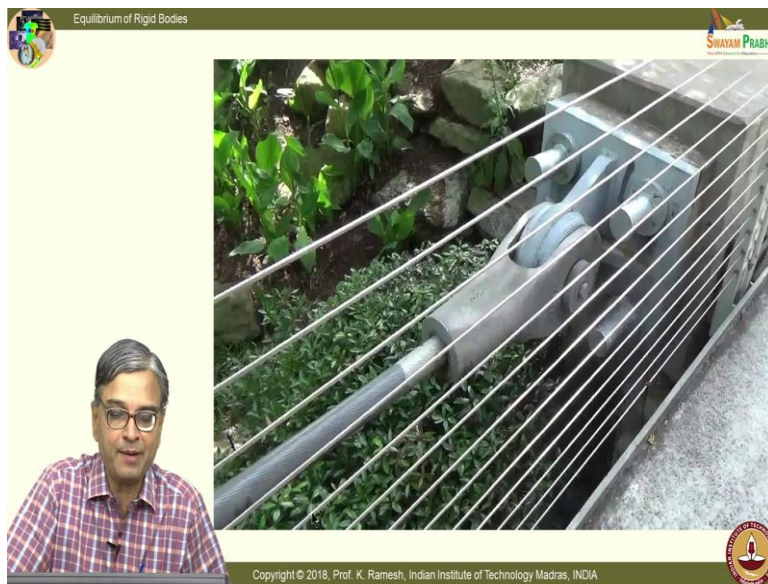
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Here again you have a pin joint fine. So, thinking of this course I recorded all of this in my camera.



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And this goes to the mast, here again you have a pin joint, so well utilized.



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And you could see the main cable is connected to the structure at the bottom; a huge pin joint. And another interesting thing is you see the barricade, the barricade is not made of any thick railings they have just used strands of wire.



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And you also learn little more engineering, this is the close-up view of it; another view shows very clearly you have threads on this. These are actually sophisticated turn buckles what they do is they turn it, and then tighten the cable.

Cable by definition cannot sustain compression, it can sustain only tension. So, the structure as a whole becomes very light and they are in a position to support.

Equilibrium of Rigid Bodies

- CLASS III: Translation and rotation are totally restrained
- Unknowns: Reduced to a force and a couple
- Examples: Fixed supports, welded ends, 2 closely spaced pin joint

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And you could see play of pin joints everywhere; pin joints are very nicely used in several engineering structures.

And we will also look at class III supports in which translation and rotation are totally restrained. You

have an example right here in this classroom can you identify where it is restrained.

You have a beam which is right here and beam is supported on both ends completely fixed. When I have a support like this, the support can be replaced by a force and a couple in this category you have fixed supports, welded ends; even 2 closely spaced pin joint can behave like a fixed support, because I already told you in engineering, if you come across a practical problem reduce that into a problem for solving is a challenge.

Unfortunately, in this course we do not give you separate training on that, when you solve some problems many problems are specified with supports. Some problem we also specify only the physical structure and it is for you to figure out what way you would

Equilibrium of Rigid Bodies

Supports that restrain both Translation and rotation

Type of Support/ Contact	Action on body to be isolated	No. of Unknowns
Fixed Support		2
Weld		2
		1

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model the support. Under such circumstances you will realize suppose I have two pin joints you can idealize; I will also show it by a simple practical demonstration of it.

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So, I have a fixed support

represented like this also look at the representations, it could be even welded like this or while making this support, they would have inserted it inside they would might have made a cut and then provided appropriate glue. And I have said finally, even simple two pin joints space together closely can behave like a fixed support.

And what did I say? It would arrest both translation and rotation. And this is how you represent the force interaction. I have a force horizontal force as well as the vertical force because I do not know the direction of this force. So, I have two unknowns, in addition I also do not know the moment acting at this point.

See let me take the same problem of the pin joint. I have taken it like this, I have put only one pin I am able to rotate it freely fine. Now, I said if I have two pin joints it is as good as a fixed support, you can model. When I say fixed support what I said rotation is restrained. I am not in a position to rotate ok, so, the rotation is restrained. So, when you come across physical situations you have to apply your engineering judgment and ingenuity to idealize the support conditions. It is very important.

So, in this class what we have looked at is what are the conditions for equilibrium of a particle, system of particles, single rigid body, and interconnected rigid bodies. And I have emphasized that you should not confine your attention to only necessary condition, you should also verify for sufficient conditions. Only then you can be confident that your idealizations have been correct and the body is indeed in a state of equilibrium.

And why do we apply equilibrium conditions; our main focus is to find out unknown forces by using these conditions I am in a position to estimate the value of the unknown forces. And in order to apply the equilibrium conditions, we have looked at properties of a two-force member, properties of a three-force member which you could take you to advantage in simplifying a practical situation.

Then we moved on to learn how to write a free body diagram, it is not trivial, it is very important for engineers. So, you start with the coordinate system, depict it in your picture, learn neat sketching of the body, put the essential dimensions and show the unknown and known forces properly. Then finally, we discussed about few types of supports that restrained translation in one direction, that restrained translation both directions and finally it restrains translation as well as rotation. Thank you.