Engineering Mechanics
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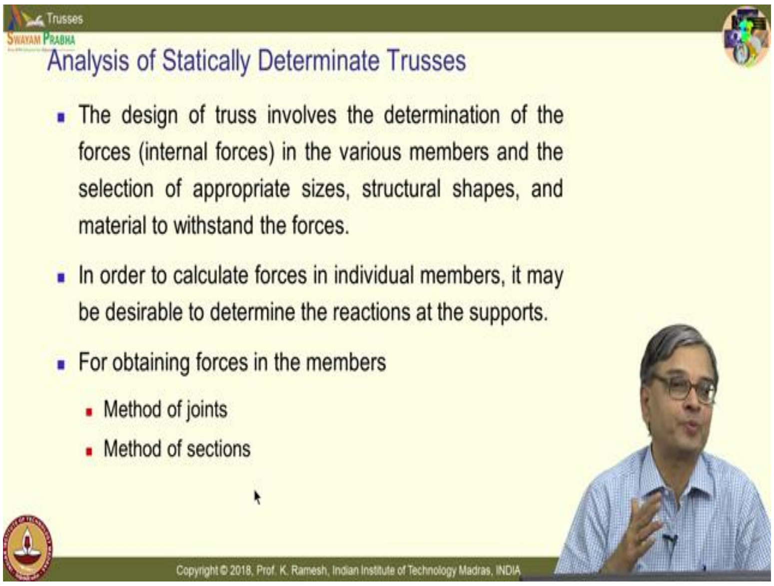
Module – 01
Statics
Lecture – 08
Trusses – II

Let us continue our discussion on Trusses. In the last class you have seen a variety of practical structures that use trusses for supporting the loads. And we have also seen what is the origin of the word truss. We saw that it has a French connection and the meaning was collection of things put together, now you can change it as collection of essentially two force members. And we have seen that no member is continuous through a joint whether it is a two-dimensional planar truss or a three-dimensional truss.



In a three dimensional truss you have a ball and socket joint and another aspect what we noticed was, if we have roof truss or if we have a bridge truss in all of this you have set of beams which are cleverly put and transfer the load only at the joints of the truss. So, what is important is in a truss no member is continuous through a joint and loads are applied only at the joints only then you can model the structure as a truss for analysis. What is the next stage?

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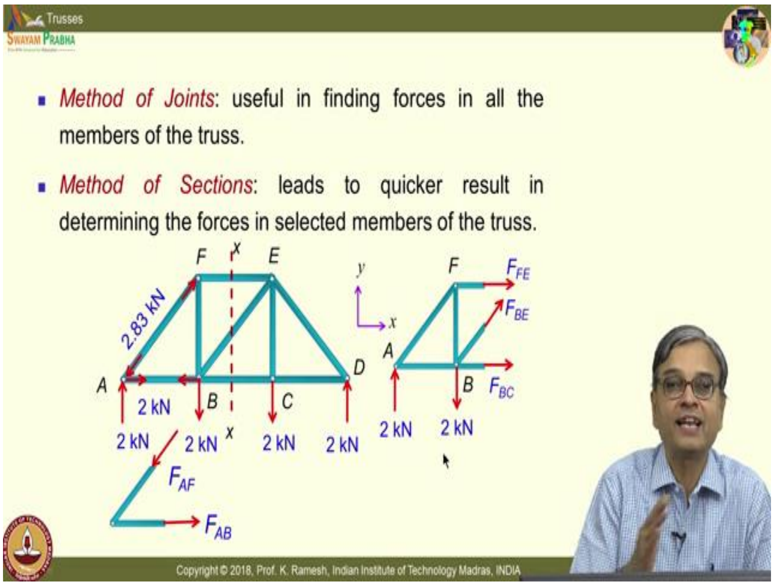
To find out the internal forces. Why do we want to find out that? To select appropriate size structural shape and material to withstand the forces. So, your analysis in engineering mechanics is only a first step in truss analysis, this information of forces will be used in the next level of course, to find out the deformation and also to decide on the structural shape.

In order to calculate forces in individual members, it may be desirable to determine the reactions at the supports, I have deliberately put it this way. In some problems even without finding the reactions you can go hide and solving the problem, it depends on the problem situation; in several situations it may be desirable to start from the supports.



And how do we obtain the forces in the members? There are many approaches you have two of them method of joints and method of sections. The idea is to solve a problem, you need to have more than one method depending on the context select that is appropriate.

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I give a bird's eye view of what is method of joints in this slide, later on we will definitely spend time in learning the integrity details. Method of joints is useful in finding forces in all the members of the truss, that may be very rare, but if you want to do that, I can use method of joints. And you have method of sections where I separate a section of the truss and then try to find out the forces on selected members. The challenge would be what is the ideal section that I have to use? And if you look at the force system in the method of sections it is non concurrent.

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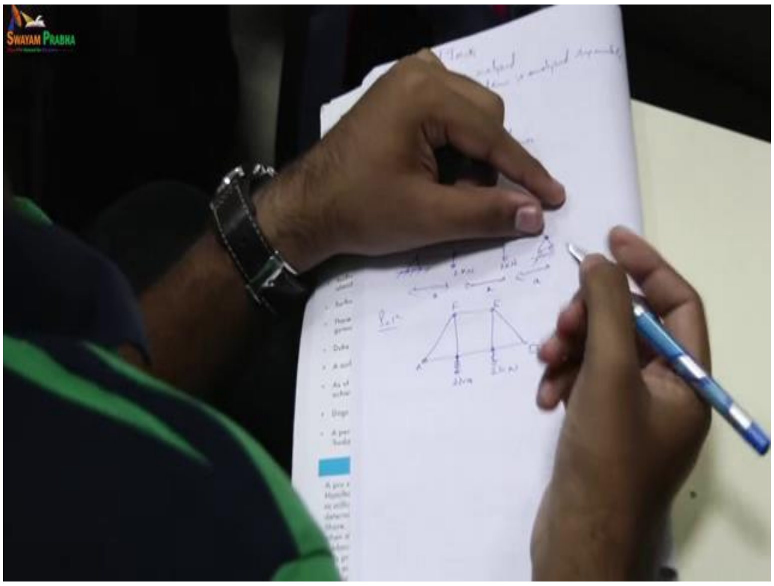

So, I can essentially use three equations of equilibrium to find out the unknowns. On the other hand, when I have an isolation of a joint, I definitely have a concurrent force system. So, I can use only $\sum F_x = 0$ and $\sum F_y = 0$ then we confine our attention to two-dimensional analysis.

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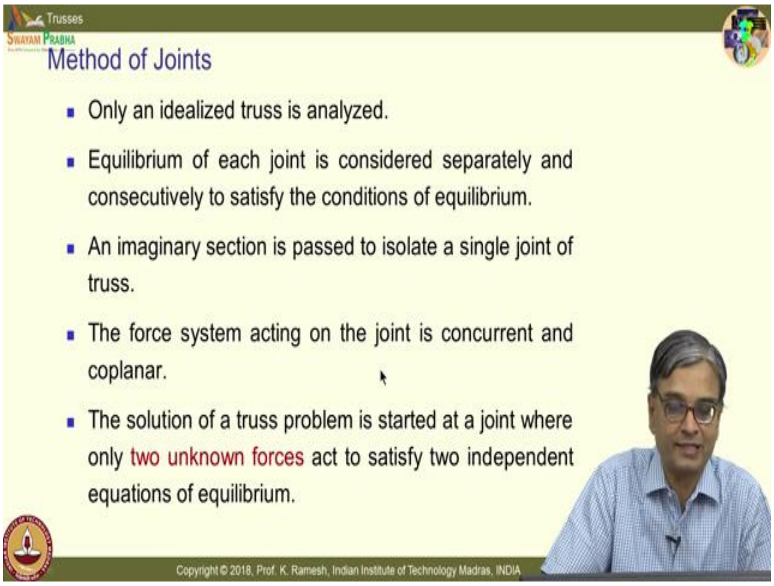
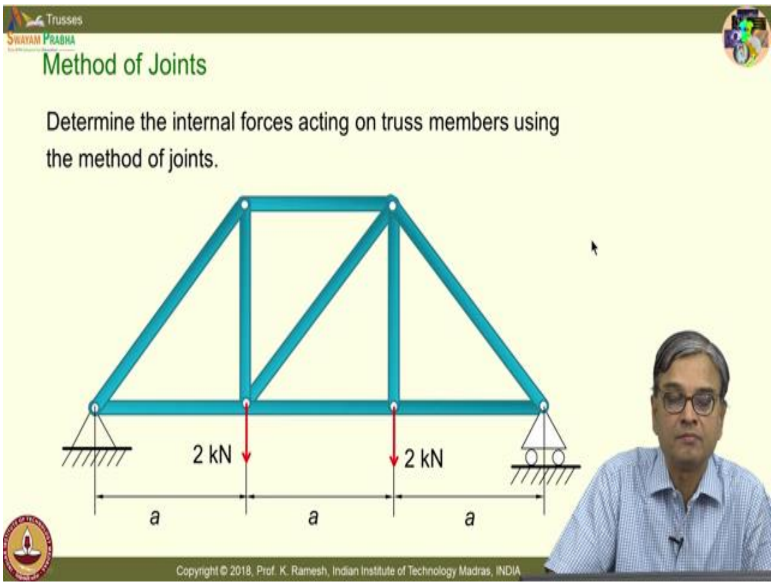


In method of joints you have to be very clear only an idealized truss analysed we have seen the elaborately a joint has a gusset plate several members join, you may have riveted connection, you may have bolted connection or you may have welded connection. But we idealize the joint to behave like a pin joint and what you do? Equilibrium of each joint is considered separately and consecutively to satisfy the conditions of equilibrium.

So, the idea is where to start and how do you navigate through the trusses? And you pass an imaginary section to isolate a single joint of truss and I have already mentioned a force system acting on the joint is concurrent and co planar. And we have already seen I can write only two independent equation from statics. So, you have to select a joint where you have at a time only two unknowns, if the more unknown sources there are also tricks that you can employee that we will postpone it for the time being.



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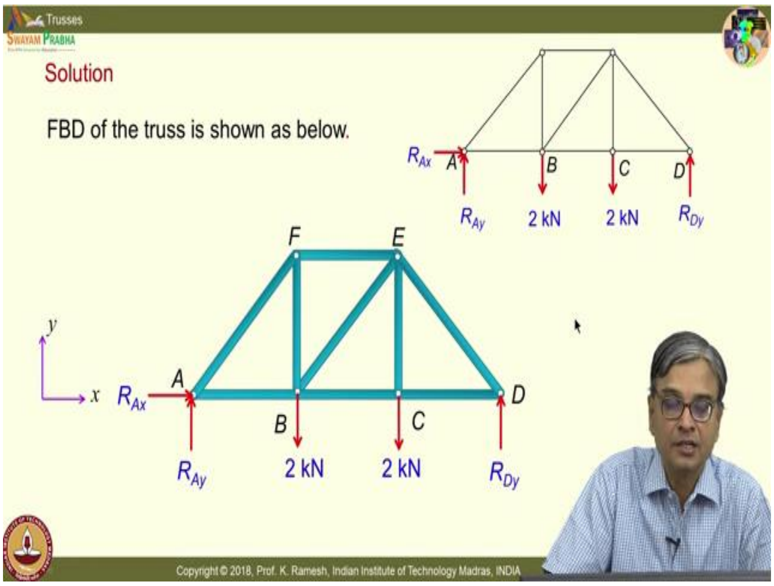
You make a neat sketch of this truss the simplest truss at the same time you have very interesting results for these members, you can draw a line sketch I have the luxury of showing that as members. And you should identify the support conditions on one hand it is supported by a pin joint or a hinged joint and you have on the other side to allow for temperature variations supported on rollers.



And you have these members are at equi distance; a is the span length for each of the segments and this is also symmetrically loaded. I have 2 kN acting at the joint, I have another 2 kilo Newton acting at this joint and you would try to find out the forces in all these members by the method of joints.

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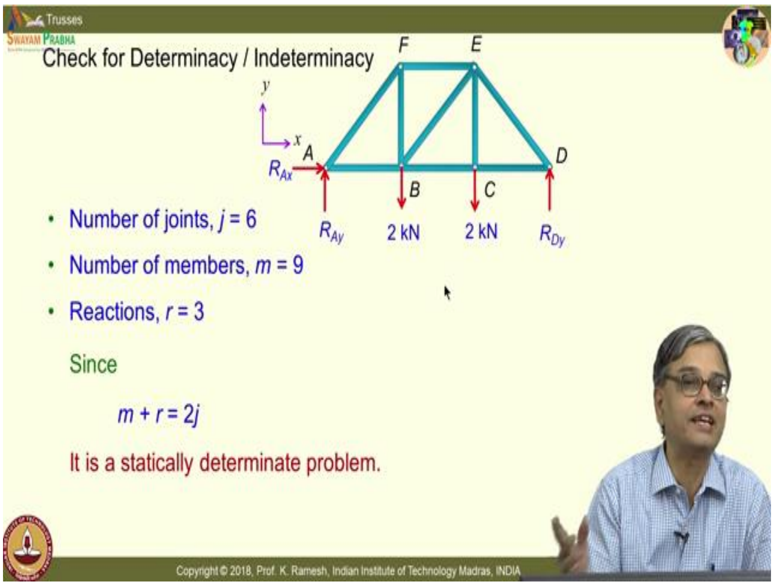
And first step is to draw the free body diagram show the unknown forces and known forces, I can put the known forces first 2 kN and 2 kN; I should replace the support at A and D as appropriate force interaction. And A is a pinned joint. So, I have two components because I do not know what is the direction of the force.

So, I have unknown as R_{Ax} and R_{Ay} at D it is a roller support I know the direction of interaction and this is R_{Dy}. And see that I have the reference coordinate system for the problem though I have luxury to draw like this; you can simply draw as a line diagram putting these known by definitely put these circles indicating that no member is continuous through the joints, you know you have to keep in mind.



Truss is constructed in such a manner no member is continuous through the joints, if a member is continuous through the joint the member will behave like a beam and it will bend, we do not want that to happen. So, you have to recognize that this is collection of two force members bound together that is a very nice definition.

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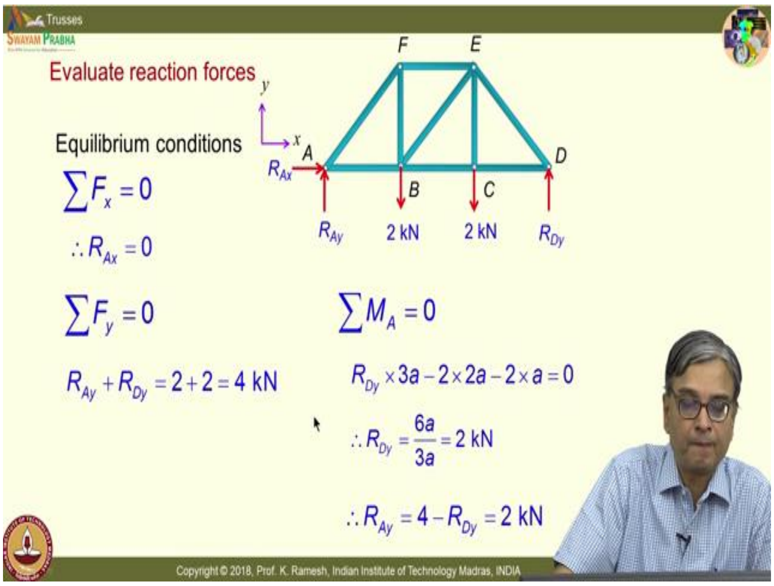
And the next step is to find out the unknown forces, but before that we will also check whether the problem is solvable by using the equations of statics. So, you check for determinacy or indeterminacy you will say how many joints are there? I have 1 2 3 4 5 6 joints and members are 1 2 3 4 5 6 7 8 9 members. I have unknown reaction is three. And does it satisfy m + r = 2 j?

It does satisfy m + r = 2 j. Since m + r = 2 j, I can say that this is a statically determinate problem. It’s more of a training for you to get the jargon in the subject you have to identify whether the problem is statically determinate or indeterminate. See usually a truss has only triangle members; and one support is hinged, other support is on roller invariably it will be a statically determinate problem.



If the supports are both the supports are hinged, then you have a difficulty in it because I have 4 unknowns and if I have cross brace members then the number of members and number of equations will not be sufficient for you to solve number of unknowns will be more then it becomes statically indeterminate.

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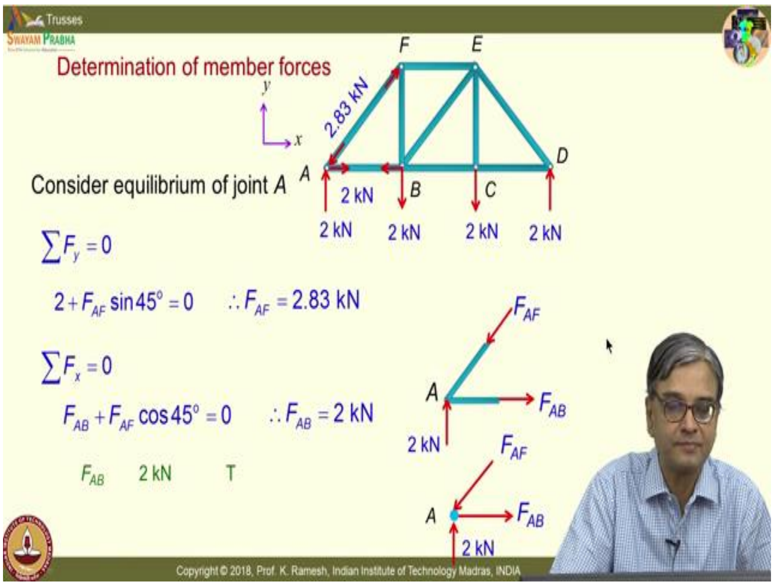
So, it's better for every truss when you solve them initially check for determinacy and indeterminacy. And I can easily find out these forces you know if you are clever by inspection you can write the results you have to graduate to that level because I have symmetry, I have given you the clue I have loads only acting on the vertical direction.

So, even without getting into solving this $F_x = 0$ or $F_y = 0$, you can clearly say what should be the force interaction at A and what should be the force interaction at B. Let us take the advantages of mathematics. So, when as $F_x = 0$ I obviously, get the x component of force R_{Ax} is 0. And when I say $F_y = 0$. I will have $R_{Ay} + R_{Dy} = 4$ kN. And you can also find out the moment about any point here we are finding out the moment about A. So, when I find out the moment about A, I get this as

$R_{Dy} \times 3a - 2 \times 2a - 2 \times a = 0$ that gives me $\therefore R_{Dy} = \frac{6a}{3a} = 2 \text{ kN}$

And I told you earlier the problem is symmetric very simple, even by inspection after you solve several problems you can afford to write the reactions mentally doing all these calculations.



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So, I would get R_{Ay} is also 2 kN. So, I have the complete free body diagram now, the unknowns are determined and I have these as the force interaction at A and D. Now we will have to find out the member forces by the method of joints, I can start from either joint A or D both are equally applicable. Let me start from joint A because I know for sure that I have 2 members, I have 2 unknowns. So, I isolate the joint by an imaginary cut and I have already told you the scientific community took quite a bit of time to understand this imaginary cut it is was not obvious and simple.

Now, I have the luxury of all these animations; I can show this member cut and put it like this and what I would do is, I would assume the forces on these members in a particular manner. I have this as acting in this direction F_{AB} away from the joint and I have also assumed one member AF force acting away from the joint. And I have told you do not have to worry about this if you do the mathematics properly, the mathematics will guide you whether your assumed direction is correct or not. So, I get this as

$$\sum F_y = 0$$

$$2 + F_{AF}\sin 45^\circ = 0$$

from the geometry of truss, you can find out this angle is 45 and I have

$$\sum F_x = 0$$
$$F_{AB} + F_{AF}\cos 45^\circ = 0$$

So, when I solve this, I get this $F_{AF} = -2.83\ \text{kN}$

So, that indicates my assumed direction that I started is not correct the answer is negative. So, I have to reverse the direction of force F_{AF} and F_{AB} if I look at, that turns out to be 2kN. So, whatever the assumption in F_{AB} is correct. So, this is about how do you handle the free body of an isolated joint; ultimately what we want is we want to find out whether the member is in tension or compression.

So, we have to learn how to interpret these results as tension or compression on the members that require some training and imagination fine. And let me first put the correct direction of force F_{AF}. See if I see this as a member that is cut then it is easy to comprehend. Then when I have force acting away from the member this could create a tension and when I have the force interaction like this could create a compression its very clear and simple fine. But many books what they do is they do not isolate the joint like this they isolate the pin. The moment you take it as a pin the results will be still similar, but you have to be careful and follow a discipline so that you interpret what is tension and compression.

Because ultimate interest is to find out whether the truss member is in tension or compression that is also required for you to visualize what will be the deformed shape. So, they just put this as a pin; pin is put as a circle and I have two kilo Newton's like this and I have the final force system like this. This would require further discussion we will do that discussion on the other hand if I isolated like assigning tension or compression is fairly simple. But you know many of these engineering practices, as come from the field there are ways in which technicians who work in the field, they have not gone through engineering mechanics course, but they will understand certain things and some notations have been used.

So, its better as engineers you also learn those notations and symbolism. So, we have already seen that F_{AB} is in tension its value 2 kN and it is a convention in truss analysis to put these arrows on the members. Let me put the members forces I have first put the force in F_{AF} member AF, I have put the force in member AB. I have just translated the forces away from the joint, I have put a similar one at B away from the joint. And we have looked at from the logic of analysing the free body that the remember AB was in tension, but if I look at the arrows like this what is impression you get? It appears as if it is in compression this you have to live with it; you have to understand. Finally, when they represent the results of truss in a diagram, civil engineers do it like this. And if I look at here the member is in compression I could see very clearly in this sketch and I have just translated identifying that this force is towards the join. The same thing is put here and we have also seen when I have to analyse it by method of joints, I should go from one joint to the other in a consecutive fashion. Once you have determine the force I have already said when the interconnected rigid bodies, once you have determine the force in one body you have no choice in the other body, you have to put it from Newton's third law equal and opposite you cannot assume for both separately, if you have assumed for one, the other one is automatically fixed so when I have to solve this, I have to go from one joint to another joint systematically and the reference point is I look at the joint whether the force is towards the joint or away from the joint. That is a way they will look at it and you can easily see away from the joint it introduces a tension, towards the joint it introduces a compression. Please clarify your understanding on this you can go wrong in your sitting in a squeeze and then, trying to answer then will start thinking when you put the arrows on the diagram it looks compression, but sir said it is tension. You should not start thinking there you should right now digest the concept.

Now, what we will do is we will look at what happens in a pin we have already seen no truss has a pin jointed construction, except the Chennai central station has a truss I think it should be there for the last 100 years. And you have that completely on pin joint, all other trusses are its only idealization. But let us go back and see what was the interaction between a pin and the bar in which you have a hole where you insert the pin ok, let us go back and see.

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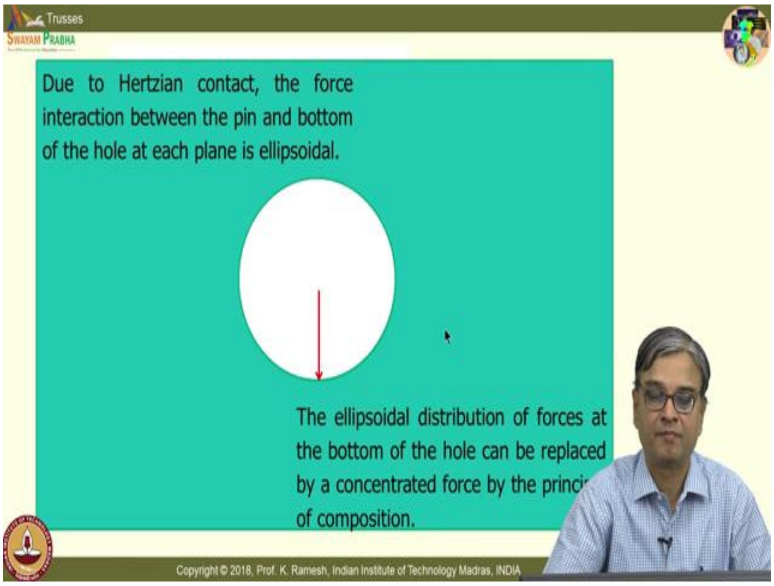
We have seen this earlier we have put an imaginary section to cut this and when you see the cross section at that stage you will have a clearance between the pin and hole. This is exaggerated, but you will always have a clearance however small it is.

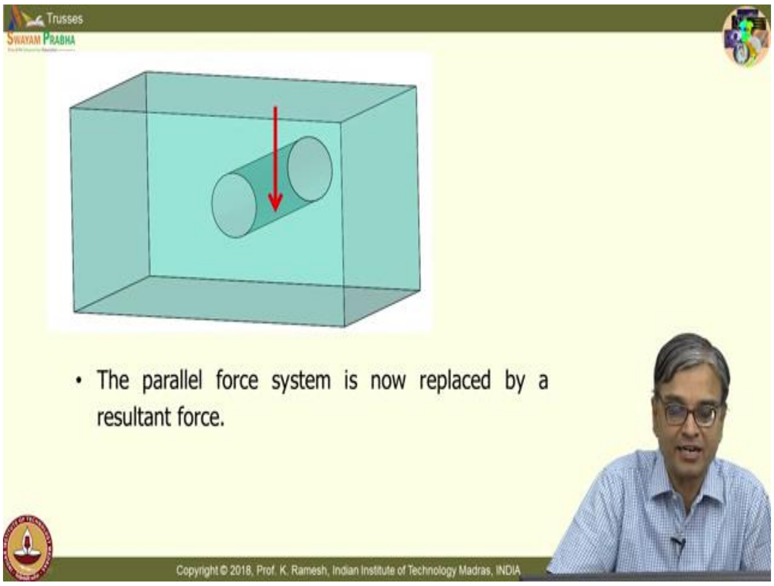


And in this case, we have seen from the previous experience there was weight which was pulling this pin down. So, you have force interaction taking place on the bottom surface, suppose the force is acting opposite then the pin will move up and touch this place and you will have clearance at the bottom. Just because you say body is rigid, body does not behave in a rigid manner all bodies are deformable which is the concept that I wanted to drive in your head ok, because it's very important. Practical structures have only distributed forces.

So, at the joints you will have an elliptical distribution, which I would replace it by composition as a concentrated force for convenience in analysis. In reality the boy body is deformable you idealize it as a rigid member for you to calculate the forces, it helps you are not making a serious error in the process.

And the next level course you consider the body as deformable and find out its resistance and decide what is the material I should use? What should be the cross-section minimum I should have? So, that it instant with the force and so on and so forth. So, I actually have an elliptical distribution and this is replaced by composition as a concentrated force. And I have already alerted you this interaction is dictated by how the pin is moved in the hole ok? So, let me take up two members like this and then I have inserted the pin fine suppose I hold it like this and I actually push the bottom member left then the pin will touch the bottom.

Suppose I put the bottom member at the right side, the pin will touch the right side of the hole you should understand this. So, when the member is in tension or compression the contact what happens at the pin and the hole is different it is not one and the same.



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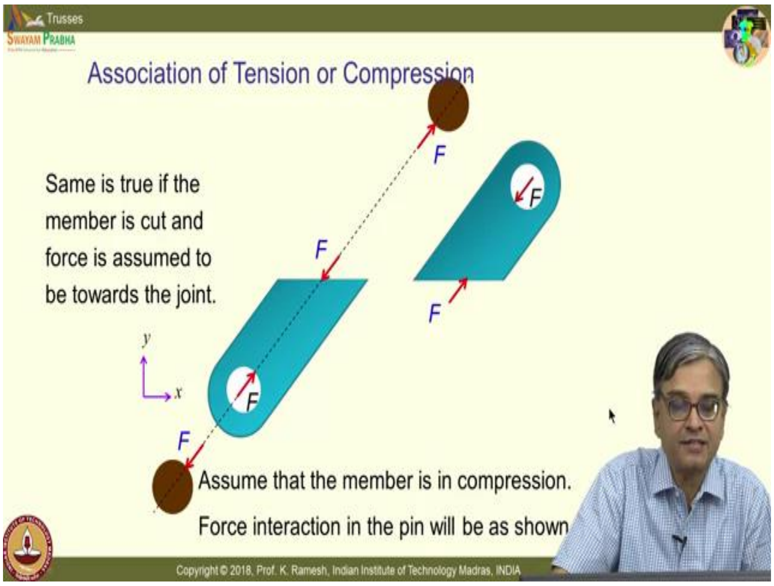
So, this is what we had seen that I had a distributed force on the length of the pin which is replaced by a concentrated force.



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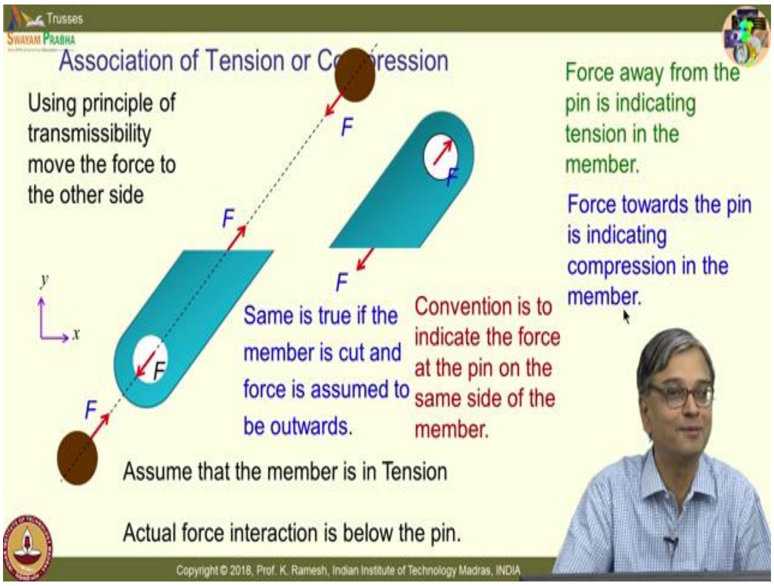
Now we would look at just take one member and I have two pins and let me take out the pins and let me first take the member is in compression fine. So, I could have a force interaction acting at this stage at this place and by Newton's third law what would be the force interaction on the pin? It would be equal and opposite. I will have the force like this and if I look at what happens in this pin, I would have equal and opposite there. And you could see very clearly if I have forces like this; the member is under compression and the interaction between the pin and the hole takes place at this end of the hole.

Suppose I cut this member what happens? I cut this member I separate them and I put the interaction then again, I see that the member is in compression. In fact, I had isolated the joint by cutting the member and when I cut the member, I have a place where I will put the force interaction and what we find is, the direction of the force is towards the joint. So, when I use the pin for my reference in this case there is no confusion, you see where the interaction takes place you have the force when it is towards the joint it ultimately produces compression in the member, same is the case when I cut the member.



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Let us look at the other case suppose the member is under tension because the other members have to pull this member. So, when it pulls where would be the force interaction, the force interaction has to happen here at the bottom of the hole when I say the member is under tension, so I would have the force interaction. And ideally where would be the force interaction be on the pin, it has to happen only in on this surface because that is what is in actual contact with this. So, that is where you will have. No, we have a nice advantage I have principle of transmissibility, I can always move the force conveniently to the other side and I can say that this is what are the interactions takes place.

Why I do this? I have to do this if I do it for a pin because if I put the force on the other side, the force is towards the joint my interpretation will collapse. Only if I say it is away from the joint it is tension and the convention is indicating the force at the pin on the same side of the member that is what you have to notice. All that confusion does not come effect at the member, but somehow many books simply put the joint as a circle and put the force as without providing a satisfactory explanation.

You need to have a satisfactory explanation for you to comprehend what I am doing? An engineering is one profession where you have several conventions, we follow conventions if you take out conventions from engineering, engineering will collapse; because my interpretational will be faulty. If I put the force all though force interaction takes place only here physically, I do this because we want to make our life simple in drawing the sketch simply want to put a circle.

On the other hand, I isolate a joint I have no difficulty in interpreting the force direction, in my teaching I would isolate the joint. As I told you earlier when you learn the subject you should not be afraid of solving problem from any book. If some books follow this you should also know what they do it how to do it yourself. Let me continue this and then look at what happens in this pin? And I can argue very easily that the member is under tension you have force interaction here.

Now, I cut the member, separate it, when I put the force interaction that will also indicate that the member is in the tension. So, cutting the member is very easy to associate the direction of the force to tension or compression; do not confuse it while drawing the free body diagram. Free body diagram if you get positive negative that does not indicate tension or compression that only says the force direction you have taken originally is correct if it is positive; originally what you have taken is wrong if it is negative.

Those positive and negative are not indicators of tension or compression in the member you have to associate, the force is tension when it is away from the pin a force is compression when it is towards the pin. This you have to be very clear because, when you solve a problem in truss you are expected to finally, summarize your results indicating which are the members under tension? Which are the members under compression? And which are 0 force members?

Do not neglect 0, when you want to get a job you want to get many 0s after 1 or even 5; because the values are salaries are sky rocketing in these days 0 has no value independently, but in its association, it has lot of value. Similarly, you cannot remove a 0-force member from a truss the truss would collapse, 0 is very important you do not like your 0 from your salary to be removed.

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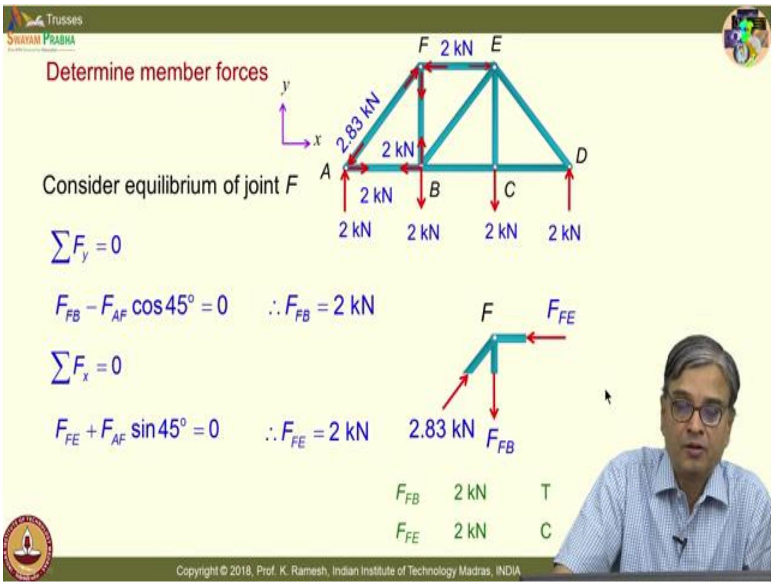
So, now I have indicated these forces and I have also brought your attention, when I show the force away from the joint and towards the joint, I interpret this as tension I interpret this as compression. And even before you do that you must also physically see when I take the member and push, I mean pull it down I would; obviously, expect a member like AB to have tension. I would like you to associate whatever the numbers you get to the physical reality. Do not say I have done the mathematics has given this it is tension compression do not learn the subject like that. You also feel like the truss get yourself stressed ok.



Associate tension compression for this member is difficult, but of course, creating tension for this member is rather easy to do. Now which member which joint can I go and do next if I go to B, I have 1 2 and 3 unknowns, if I go to f I have only 2 unknowns. So, I would go to joint F, but when I go to joint F mind you that I have already indicated a force in member AF following our convention this is under compression towards the joint is compression. You have no choice of changing this you have to start with this fact and solve the other member forces do not bring in your arbitrary assumptions here.

So, I take an isolation of joint *F* which I can do it nicely here and indicate the forces, when I put the forces F_{AF}, I would put it like this; mind you I am not putting any sign here. I have already indicated the sign in my direction I just put the value, this is how you have to handle the free body diagram of the joint. I do not know the forces on members F_{FB} and member F_{FE}. I put the force away from the joint usually you take the tension and if the result is negative you also can say that it is compression, but that is only a chance fine.

Your tension and compression have to be associated with reference to the joint how the forces you can also make the algebra equalant to that, but if you miss that algebra finally, look at the direction of the forces. So, I get

$$\sum F_y = 0$$

$$F_{FB} - F_{AF}\cos 45^\circ = 0$$

$$F_{FB} = 2\text{ kN}$$

So, this direction is correct and you can say that the member F_{FB} is in tension which also coincides with when I pull the truss down the member is pulled it is under tension. It goes with your physical appreciation of the problem and I have

$$\sum F_x = 0$$

$$F_{FE} + F_{AF}\sin 45^\circ = 0$$

$$F_{FE} = -2\text{ kN}$$

I get this as negative. So, my assumption is wrong so I have to reverse this direction and this is how the joint is in equilibrium. And now we can summarise the forces like this F_{FB} is 2 kN, T denotes tension and F_{FE} 2 kN indicates compression. From this how do we proceed next I can go to joint B because I know what is the force here I have only two unknowns. I can also start from joint D. I would start from joint D before that you know we have also indicated what is the force in the member FE like this, I put these arrows with reference to the joint.

I have 2 kN acting on this and I have 2 kN acting on this member this member is in tension, but if you look at the arrows. It gives me an impression that this is another compression do not get confused have a clarity on interpreting the symbolism you are practicing symbolism here.

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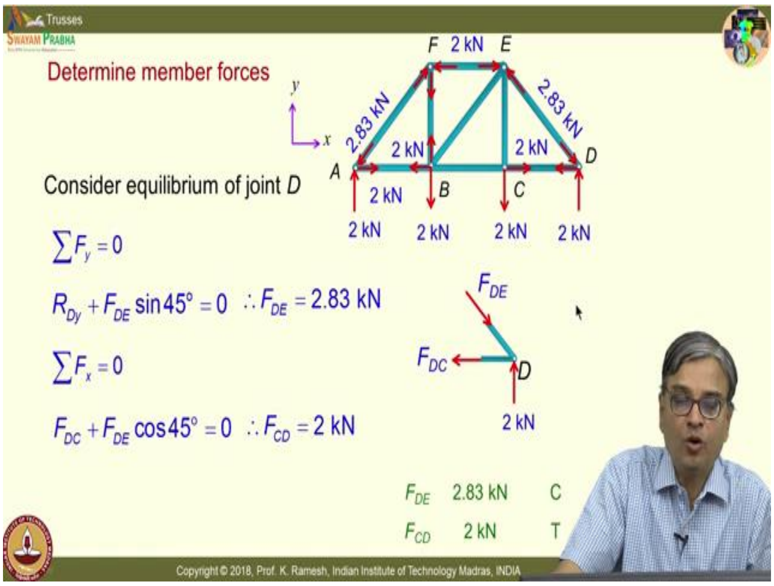
I will go and consider the equilibrium of joint D again I isolate this, see in fact, whatever



you study in trusses it is nothing, but equilibrium or rigid bodies much simpler in complexity. And this chapter is one everybody can score full marks very simple and that should be your learning goals, we should not have any problem solving a problem on truss I isolate this joint. So, when I do not know the force, I put it away from the joint.

So, if I use this kind of symbolism in drawing the free body, I can also associate my positive and negative value of the answer to tension and compression choice is yours whichever is convenient to you practice that. But final and full proof interpretation is you should refer with respect to the joints, rather is away from the joint is tension towards the joint it is compression. So, I write the equations

$$\sum F_y = 0$$

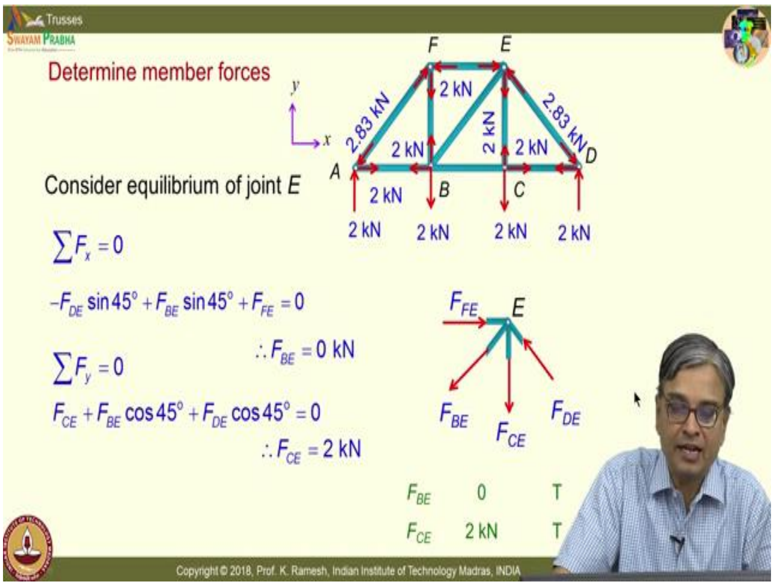
$$R_{Dy} + F_{DE}\sin 45^\circ = 0 \quad F_{DE} = -2.83 \text{ kN}$$

$$\sum F_x = 0$$

$$F_{DC} + F_{DE}\cos 45^\circ = 0 \quad \therefore F_{CD} = 2 \text{ kN}$$

So, this gives me F_{DE} is -2.83 so; that means, the member is under compression and F_{CD} is under tension so which I would indicate on the truss also.

So, I have reversed the direction of force I have to reverse the direction of force I have reverse the direction of force and I will change this as F_{DE} = 2.83 noting well it is towards the joint. I will also indicate that truss these forces I have this as 2 kN away from the joint this is towards the joint.



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Now what I will do is I can move to several other joints let me go to joint E, isolate the joint and put the member forces here I already know two forces. So, I have no choice other than putting the directions correctly, so these two directions are known I am putting these two directions. For the unknown directions I put the values away from the joint for convenience.

So, when I write this equilibrium equation

$$\sum F_x = 0$$

$$-F_{DE}\sin 45^\circ + F_{BE}\sin 45^\circ + F_{FE} = 0$$

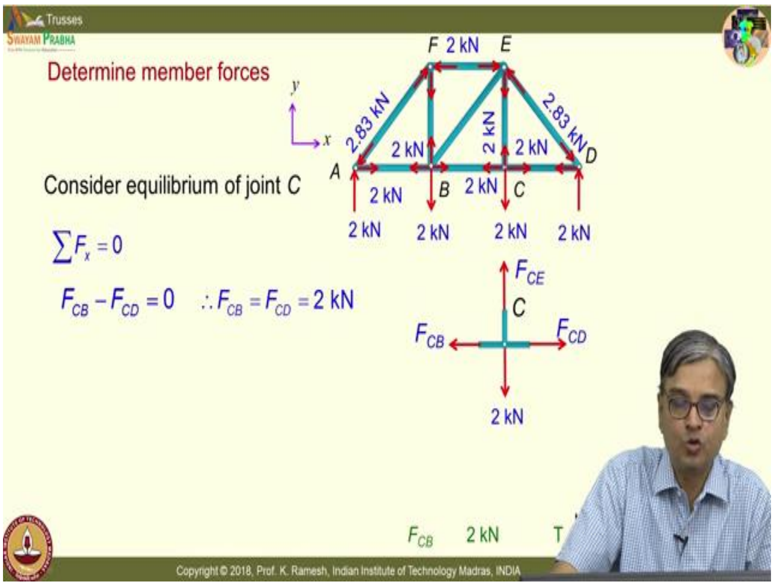
$$F_{BE} = 0 \text{ kN}$$

see this is slightly incline imagine that this is along the length of the member ok. Sketch is not absolutely clear on this and I get a surprising answer, I get the force in member BE as 0. This is the reason why I chose this truss I have a small truss which can be analysed comfortably, it has members which are under tension, which are under compression. And there are also members which carry no force they are part and parcel of the truss they cannot be removed.

In fact, you will also see towards the end of the lecture on trusses how to identify 0-force members, that takes your life sample in calculation you cannot remove the 0-force member for stability of the truss 0 force member should exist. Just because it does not transfer any force, I cannot remove the member that is what I meant when I say 0 force members, you may say I have designed the truss to support loads this is not carrying any load it is waste I will optimize.

You should not do extreme optimisation see god is very claver god has given you two kidneys, he has not optimized that you have only one kidney. And we see very often people have kidney problem and they donate one kidney and then they get that kidney from others and they survive, but he has given you only one heart he has done optimization judiciously.

So, do not get into the bandwagon that I want to optimise everything and removes zero force members foolishly don't do that. So, I have this force F_{CE} as 2 kN that is under tension and I have also marked as force and I have force on member BE as 0.



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Now I have to go to joint B and solve the member force BC I have taken the joint C both are equally possible for my slide I have taken joint C indicate the forces that you do not know in an arbitrary direction we take it away from the joint. I know the force F_{CE} I have already determined what is F_{CD} I do not know what force F_{CD} is it is very easy to calculate F_{CD} minus F_{CB} equal to 0. So, I get this as 2 kN and you know very well that is away from the joint so this is under tension. And I have also indicated it on the truss and F_{CD} is 2 kN tension.

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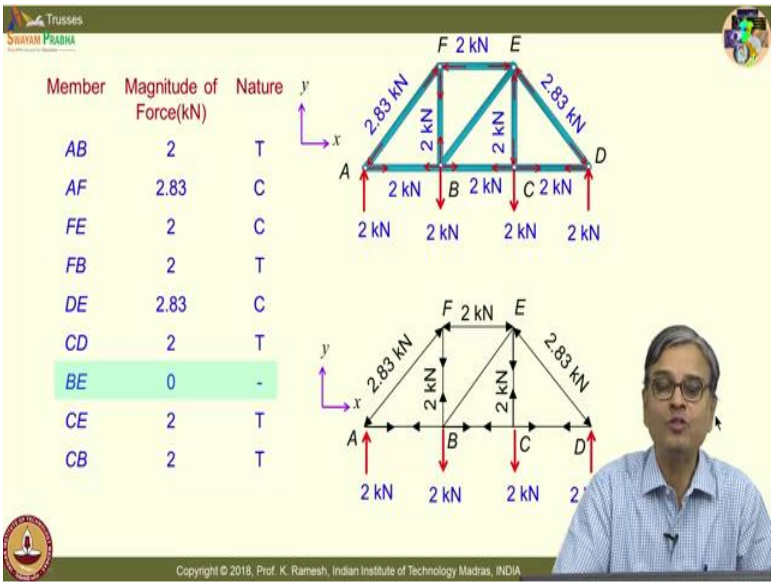
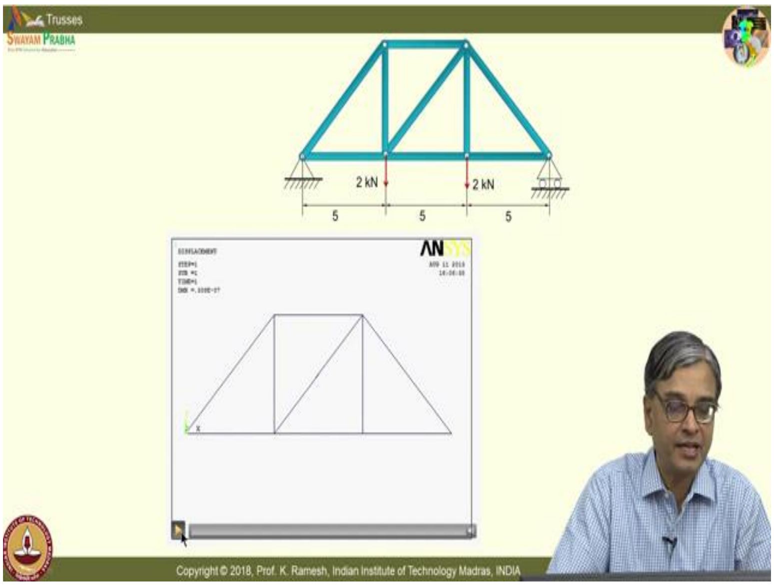
And this is how the final result is normally summarised in a truss analysis. So, I have members AB, AF and so on and so forth and you have the values. And you also have the nature of the forces whether it is tension or compression and you also have which member is under 0 force is mark separately? I have drawn this sketch like this I expect you to draw the sketch like this because it is easier for you to draw a line sketch rather than putting a bar in place of a line and keep it in your mind because I put all the forces with respect to the joint if I see this as physically compression on the member it is actually tension.



If I see what is physically tension on the member is actually compression this is a symbolism. And you know this truss my student has solved it by a final element analysis.



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Because the idea is, I would like you to see what would be the deformed picture how the truss has deformed under the action of these forces.

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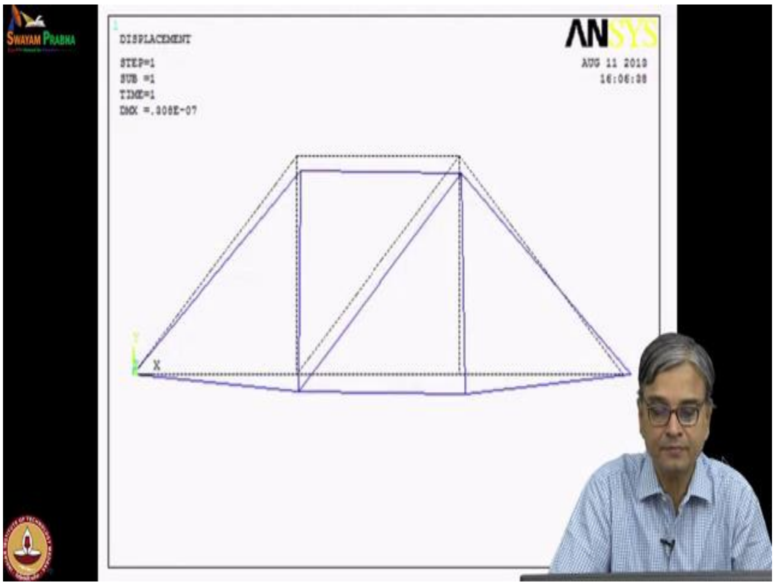
And you watch the animation and the load is increased and you would observe very interesting things on the animation. First observation is you have provision of roller is



essential because this edge because of the members being elongated or contracted, it is re located at distance away.

So, your roller support should allow it to move it that way and if you have a clear visualisation you look at this member which is a 0-force member; it remains in length same it has not all till. And you would see this member has elongated and this member as contracted and you find the need for one support to be hinged other support to be roller is very clearly brought out in this animation. And this is a real-life animation, no approximations are involved other than what we have done as a pin joint. So, it understands the members as deformable which you normally do in a next level course, it is better to know how the deformed picture of the truss can be.

I think in this class we have seen systematically what is a method of joints, we have determined the member forces. I personally feel you isolate a joint by an imaginary cut you do not have to worry about how the joint has been made it can be a riveted joint, it can be a bolted joint, it can even be welded. And you will naturally put the force interaction on the member only on the member there is no need that you have to put the forces on the pin when I isolate separately in the direction, in the on the side of the member all that confusion will not come, when you have a force away from the joint it is tension, when you have a force towards the joint it is compression. Interpretation also becomes simple. And you isolate a joint where you have two unknowns, you also have truss in special cases if I have more number of unknowns a limited number can be handled not all generic situations. And you have to go consecutively from one joint to another, once you are determining the forces of a number forming the other joint you have to put the corrected force and then carry on with it. Thank you.