

Engineering Mechanics
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Module – 01
Statics
Lecture - 09
Trusses - III

Let us, continue our discussion on Trusses.

Determination of member forces

Consider equilibrium of joint A

$$\sum F_y = 0$$

$$2 + F_{AF} \sin 45^\circ = 0 \quad \therefore F_{AF} = 2.83 \text{ kN}$$

$$\sum F_x = 0$$

$$F_{AB} + F_{AF} \cos 45^\circ = 0 \quad \therefore F_{AB} = 2 \text{ kN}$$

F_{AF}	2.83 kN	C
F_{AB}	2 kN	T

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And in the last class, we had discussed method of joints and I have said that, I would prefer cutting and isolating a joint like this; this becomes easier when, you want to associate the final answer of these forces as tension or compression

in the members. It becomes very clear. When, I look the member like this, if I have a force acting away from the joint, it indicates tension and the mathematics tells me, if I have assumed it in a particular way, the algebra tells me whether, I should retain that or change its orientation. If I have a force towards the joint, it is also very clear the member is under compression.

Or I said, many books simply isolate a joint as a circle and then, indicate the forces. Algebraically, you will get the same numbers, you will not make any mistakes there, but when you evaluate the forces and try to associate whether, it is a tension or compression, you have to follow a convention. Because, when, I take a joint like this; suppose, you start thinking, how the pin joint behaves then, you would put tension like this, but if I have to associate the direction, I will have to put these forces at the joint only in the direction of the members, all that restriction is not there when, I simply isolate a joint and put the forces acting away from the member. It is very clear that, this is tension even when you perceive it.

So, if I start my forces like this, my mathematics when, I get the answer as negative, you can also associate that as compression. So, it has a slight advantage from physical visualization and secondly you do not worry about, how the joint has been made. So, the

Method of Sections

Determine the internal force in member FE using just one equation by the method of sections.

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choice is yours, which is the way that you would like to solve the problem in trusses.

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Next, we analyze the same truss by another method called method of sections, the focus is to learn the methodology. So, I have

taken the same truss and the question is you have to find out the internal force only in one member FE. I do not have to find out forces in all members and the member is specified and the question also says use just one equation by the method of sections. We have already looked at this truss, we have done the free body diagram, we have also

checked for determinacy and in determinacy all that I am not going to repeat.

Selection of the section

Section x-x can be of any Convenient shape.

It should completely separate the truss.

Can't cut just a member or two!

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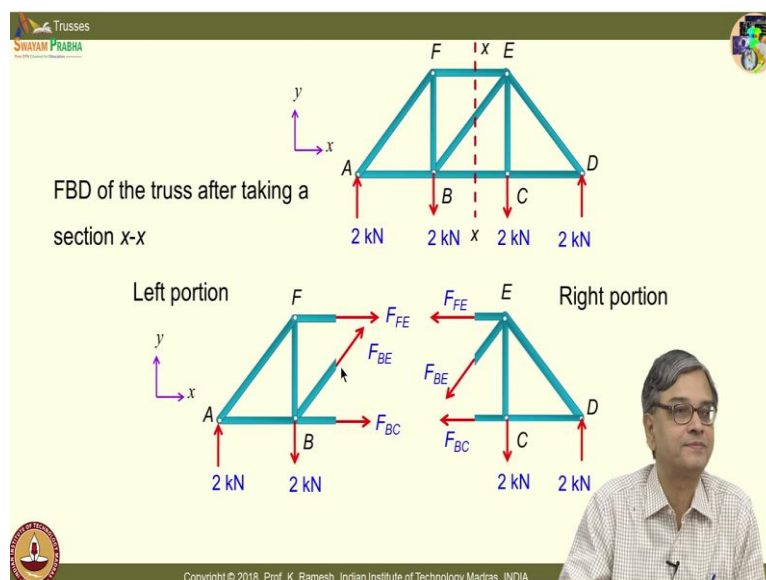
One of the key aspects here is, I have to pass an imaginary section and cut the truss into two. How do I take the section? When, I take a section it should completely cut the truss. I

cannot cut one member or I trim the member when, you say method of section simply go on trim one member, that is not acceptable. If I put a cut, that cut should pass only once in a member. And usually books will give a straight line cut like this and when I isolate

this, I should put the force interaction of these members indicated as forces acting away from the member, which would also say that, I am assuming it as tension, if my algebra gives this as negative, I would understand that, this is giving a compressive force of the member.

Now the question is you have to come out of the mental block, how do I make a cut. It need not be a vertical line; it can be an inclined line. Here again, I indicate the force as F_{FE} , F_{BE} , F_{BC} and so on. Wherever, I cut the member, the internal force remains same because, I have considered this as a 2-force member. No matter, where I cut the internal force remains same and I can also take a generic cut because, soon we are going to see one ore problem where, the challenge is how to take an appropriate section. In fact, that would require a curved line. So, you have to visualize when, I take a section the only restriction is, I should completely cut that truss and isolate, I cannot go and cut the member FE and B E alone and say that, I have done a method of section no, you have to separate it into two.

So, when I take the section like this, I would have force interaction represented like this and I could also, have another line like this, this is also going to give me a similar free body diagram. So, I could use any one of these free body diagrams for me to apply the



method of sections. They are one and the same. The focus of this is to indicate, I could have even a curve like this to cut a truss. So, I am prompting you, when you have a problem later, if you can have a vary cut, if it going to simplify the problem; go ahead and choose it. Method of section does not say you

have to put a straight line. Now, let us solve the problem, for solving I will simply take a vertical cut and then separate this into two.

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So, I could write the free body of the left portion or I could also write the free body of the right portion, whichever is convenient to you, you can do that, there is no hard and fast rule, which one you should use in a given problem, which would simplify your mathematical calculation, that you choose it. And we have also seen in the question, I should find out the force F_{FE} by using just one equation. What is the difference between method of joints and method of sections? When I isolate a joint, I get a concurrent force system, I can write only two equations of equilibrium because, the moment equation is automatically satisfied. On the other hand, when I use method of sections the force system is not concurrent.

So, I can use

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

I can use all the three equations and do not jump to use all the three equations, without

The moment centre should be selected judiciously such that the maximum unknown member forces are nullified.

Taking moment about B

$$\sum M_B = 0$$

$$R_{Ay} \times a + F_{FE} \times a = 0$$

$$F_{FE} = 2 \text{ kN}$$

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pondering about, how to handle the problem because, in the examinations you know I would give a problem and we also indicate, what is the maximum mark you can get for that problem, the mark is indicative of the time that you should try to spend on a problem.

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If you think that, you are spending more time on it, then you have missed the crux of the problem in solving. Here the problem is very straightforward and I am sure everyone would be able to see, what is the equation I should use? Simply take the moment about,

the point B, the force as F_{BE} and F_{BC} do not contribute anything. I can directly write one equation and get the value of force F_{FE} and this gives me as -2 kN.

And when you say minus, we have already taken a clever way of representing the forces, even without visualization you could conclude, that the force is compressive on the member. But I would urge you to visualize because, the whole course is you have to be

Trusses
SWATAM PRABHA

How to model weight of members

- Usually self weight of the members are small compared to the external load.
- Self weight can introduce a small bending of the members.
- Successive rivets can also support a small moment at the joints.
- Such effects are generally neglected in the analysis.
- In certain instances such as snow fall on roof tops, one may need to account for the weight of snow.

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prompt at to visualize what happens in actual situation, when the loads are applied. Take mathematics as a support as a verification tool, which, should go hand in hand with your visualization. So, we have -2 kN and the direction is also changed and I finally put the value as 2 kN

compressive.

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Then we move on to another important aspect, which we have ignored, how to model weight of members? Just modelling the weight of member may not be a real issue. See, if we go to western countries where, you have very cold temperatures and you have usually snow falls on all the roofs, snow could be very heavy and depending on the intensity, you may have to design the roof truss. So, that you take into account effect of snow in some way. And as I said the self weight of the members are small compared to the external load and why we have carefree avoided self-weight? Actually, self-weight can introduce a small bending of the members which, we have never considered it for simplicity in the analysis.

And if you look at, how the joints are made, if I put rivets one after the other, they can also suppose support a moment and such effects are generally neglected in the analysis. You get a result which, is reasonably acceptable, that is why that approximation is valid, if it is not acceptable, you cannot make an approximation, the approximation should be

acceptable for a given application. As I said when, there is a snowfall on rooftops one may need to account for the weight of snow. And here again, we are going to make an approximation.

Trusses
Sriyam Prashna

- How to account for the weight/snow deposit on the members?
- Lumping of the weight of the members does improve the estimation of average axial forces (tension/compression) acting in the members.

It is only an approximation.

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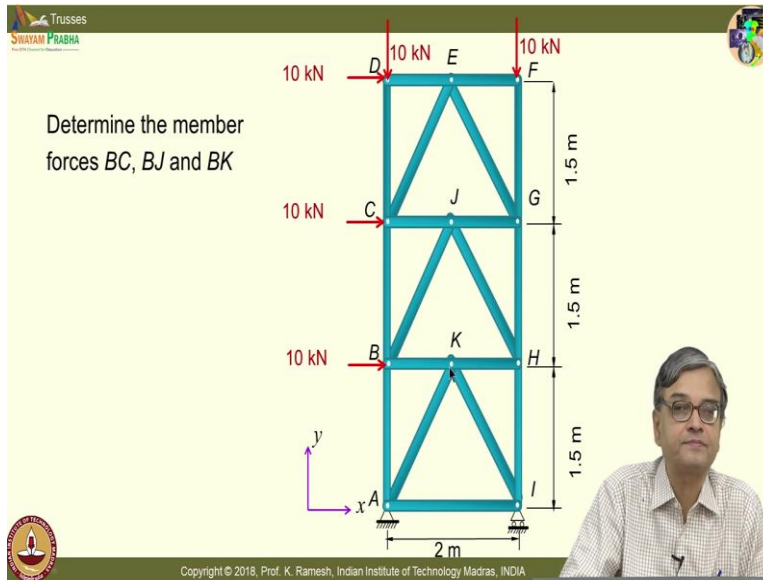
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Suppose, I look at that, this is the weight, you could visualize the body force acting on each volume element of the body or you could also consider a distributed force due to snow, either of the two you can handle it in a similar fashion. And what would;

we normally do; we replace the weight through the centroid and indicate by the weight; this weight could accommodate only the member weight or it could be member weight plus snow or it could be just snow. How do we take it in our analysis? We have already seen, I can analyze a truss, only if the loads are applied at the joints; no member is continuous through the joint.

Suppose, I have this load to be considered in totality, you will have a force like this and this would introduce a bending. Instead, what is accepted in engineering analysis is, you lump half of this weight on this joint and lump half of this weight on this joint; this kind of an approach is acceptable in engineering analysis because, it simplifies your mathematical calculations; yet it gives an improved estimate of member forces. So, I caution that, this is only an approximation. Suppose, I want to accommodate the weight of this member, I could put forces like this; suppose, I want to accommodate the weight of this member, I should put the forces like this; suppose, I want to accommodate the weight of this member, I have to put the forces like this. On the other hand, if I have the snow, you have to be careful, which of the members are loaded, and this you can carry on and complete for all other joints. So, this is called lumping of the weight of the members and it does improve the estimation of average axial forces.

So, it is acceptable. You still maintain your simplicity of your analysis and slightly improve your estimates.



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And now, we will move on to solving another problem; you have already looked at the method of joints as well as method of sections; you have a truss like this, make a neat sketch of it; you have this supported on a hinges or pin joint at one end and a

roller support at the other end.

And the question is, I have to find out the member forces of only 3 members. I have the member BC; I have the member BJ and I have the member BK. The choice is yours, whether, you want to use method of joints or method of sections; you should use them judiciously either one of it or combination of this and ultimately get this member forces as efficiently as possible. Even before, we solve the problem, let us set a target that we would use for each unknown one equation to solve. Can we set that as a target for us to aid our thinking?

So, I am going to greatly reduce my mathematical calculation effort and I will look for, which way, I should have the strategy to solve the problem. So, my recommendation is, when you look at a problem in your exams ponder about, how to go about using your mathematics. Do not jump on immediately, you start from one corner and then take consume half an hour of the examination completely on your solving only one problem, do not do that mistake. You have to decide on a strategy. So, now I have set the goal, I have 3 member forces to be determined, we will try to find out whether, we could solve them, by just three equations.

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And I would like you, to get the habit of looking at, whether the problem is solvable

Trusses
Srinivas Prabha

Check for Determinacy / Indeterminacy

- Number of joints, $j = 11$
- Number of members, $m = 19$
- Reactions, $r = 3$

Since

$$m + r = 2j$$

It is a statically determinate problem.

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from equations of statics or not. So, we will check for determinacy or indeterminacy.

So, you have to get the number of joints, you could count them as 11 and I have groups of 6. So, 3 into 6, 18 plus 1 19 members I have. And since, I have a pinned joint

at one end and roller support at one end; you have just three reactions and if I check, $m + r = 2j$. It is very obvious in this case, see, if I have a truss, which has essentially triangular shape in every section of it, no cross braced members and the two supports; one of them is a pinned joint and other one is a roller support, invariably it will be a

Trusses
Srinivas Prabha

Solution

Forces in specified members of truss has to be determined.
Method of sections would be an appropriate choice.

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statically determinate problem, but it is better, when you are learning the subject check this for each of the problems you solve, you will get the training to do that.

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Now, the next job is, should we find out the reactions or should we directly go and find out the member forces. I said that, I have set the goal as, I need to have only three equations to solve for three unknowns. So, that indirectly implies, can we avoid finding out the reactions. See, when I discuss the first problem, I said it is desirable to find out the reactions, it is a context based; if I have to

find out all the member forces find out the reactions and go from one joint to another joint and then, estimate all the forces. In this case, it is desirable that I look at the method of sections and the first aspect that you have to pay attention is, what is the kind of section that is appropriate for this problem?

Make an attempt on it; make an attempt on it, the whole problem depends on a nice section that you will have to select. See, normally, in method of sections I have said that, I can have three equations, when I separate it. So, cut a section, that will have only three unknowns; I can give a hint, I relax that condition. You can have more number of unknowns also. So, that is a clue for you to arrive at a section and I have already trained you, that a section can be a curved line. So, the second clue is it is not a straight line. It is a curved line. You have to choose the type of section appropriately. The moment you take the right section, the problem is solved. The crux in this problem is, how to identify a suitable section, which would help me to determine the unknowns. So, you have to bring in one side cut the section; how do I take the moments; can I find out some nice locations, which would minimize the calculation because, when I say a body is in equilibrium, I can take moment about any point, I can take the moment on any of the points on the body or even outside the body and then satisfy. So, you would use all these principles in trying to do that.

Let me ask a question, we have to find out the forces on member BC, BJ and BK. Can I take a section and cut these three numbers? Do not do that, because by cutting only these 3 members, I am not going to separate the truss into 2, whatever section I take, I should be able to analyse, that as one free body, I should be able to analyse the other cut as another free body, which should be completely separated. Has anyone of you figured out, what is the section that you can think of? I said that, you do not have to have a straight line, it has to be a wavy and I have also relaxed another condition, it can also cut more members more than 3, to be specific, you can cut 4 members. Let me, give the other clue also.

Because in a problem like this; we can have multiple ways of solving it. There is no one standard way to solve a problem. You have to apply your mind and then find out, what it is and I say that, I can cut 4 members; that means, 4 unknowns; how do I solve 4 unknowns with only three possible equations? It is still possible, if you take the point about, which you take the moment cleverly. So, the focus in taking this problem is to pay

attention on strategies to solve a problem. You know, how to in general apply a method of joints or method of section, you also know the equilibrium equations. So, you have to

Trusses Solution
Judiciously selecting the upper section such that it is in equilibrium

The diagram shows a truss with nodes A through I. A section cut is taken through members CJ, GJ, and GH. The upper part of the truss is shown as a free body with forces F_{CB} , F_{CJ} , F_{GJ} , and F_{GH} at the cut. External forces of 10 kN are applied at nodes D, E, F, and C. Dimensions are 2m for the top chord and 1.5m for the vertical height of the section.

weigh between what would you select and which equation you would ultimately use for solving. We have spent sufficient time on this.

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Let me show the section. I will take a section like this and what I am going to do

is, I will separate this. So, when I look at this free body first thing is, I do not have to find out the reactions to solve this free body. I could solve the problem by solving the upper part or the lower part. I cleverly take the upper part because, I do not want to determine the reactions, because if the question is only to find out forces in 3 members, but here, I have the force F_{CB} , F_{CJ} , F_{GJ} , F_{GH} ; they are 4 in number. From the principle of statics, I can write only three equations, but if I take a point about which, I take a moment, I can straight away knock off 3 forces; can you see that?

If moment equilibrium is considered about point G (point on the section), the contribution of forces F_{JG} , F_{CJ} and F_{GH} zero since their line of action pass through G.

$\sum M_G = 0$

$$F_{BC} \times 2 + 10 \times 2 - 10 \times 1.5 = 0$$

$$F_{BC} = 2.5 \text{ kN}$$

So, the focus is, you think and solve the problem. Do not simply take a section and do it in a hurry. So, in my questions the mark is an indicator, how much time, I expect you to spend; it is an indirect guidance for you to solve a problem. If you are taking more time, then you have

missed the cuts of the problem. So, revisit it and then, see how you can do this. So, I will

complete the free body here; I have also put the known forces and it is very clear, if I take moment about G; I have my problem is set and I can directly find out what is the force in member BC. Is the idea clear?

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It is a very nice problem, that is what I said, to learn a subject, select few key problems, understand the methodology thoroughly, learn the concepts, you can solve any problem in that chapter. So, if I do this, I get

$$F_{BC} \times 2 + 10 \times 2 - 10 \times 1.5 = 0$$

You have to do this carefully; you have to do this carefully, when I take moment about the point G; these three and this one, they do not contribute to the bending moment. So, I have these two forces, only contribute to this and this unknown force and this force also does not contribute to bending moment.

So, if I take this, you should be very careful in applying this condition, looking at all the known and unknown forces. Do not miss out any one of them. So, we have been able to find out the force F_{DC} and I get this as, -2.5 kN. So, I indicate it on the diagram and we will also reverse it and then, put the sign and this indicates that the member BC is under compression. Then, we have to find out the member forces BJ and BK. How do we go about? I have taken one section, see, these sections are imaginary. So, they do not have a mental block, I have taken the problem, I put one section, solved it, I should not use method of sections again, there is nothing like that. I can take another imaginary cut and determine the member force

And here, I say, you can also cut 4 members and the clue is, it can be a straight cut. Can you guess, which cut will you select? Because in method of sections you have to identify, what is the section I should take, because you also want to solve a problem in a reasonable length of time.

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I would simply put a horizontal cut, here again, I have 1 2 3 4; even though I cut 4

If moment about H (point off the section) is considered, the only unknown to be evaluated is F_{BJ}

$\sum M_H = 0$

$F_{BJ} \times \cos 56.3^\circ \times 1.5 + F_{BJ} \sin 56.3^\circ \times 1 - 2.5 \times 2 = 0$

$\therefore F_{BJ} = 18.02 \text{ kN}$

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members, I have determined the member force here. Tell me about which point should I take the moment? Let me, complete the free body diagram, I should show the known and unknown forces clearly. And please note, that I have indicated the force correctly on the member BC, because, I

have already determined its magnitude as well as direction and that is, indicated correctly.

I can easily find out for the member force F_{BJ} and you have to tell me, which should be the point of taking the moment; because, it is a nice training, it is nothing but, equilibrium rigid bodies and you will learn many of important concepts, even while solving a problem on truss. I can take moment about any point, in the previous case, we took point lying on the truss; the clue now, is it is lying outside the section at an appropriate place. If you look at the truss, it will still lie on the truss, but from my section is concerned, it is still outside the section; have you figured out what is that we will take.

Student: (Refer Time: 29:44).

Yeah, very good. So, I will take H because, I have these 2 unknown forces do not contribute moment about this point and from geometry you have to find out this angle; see you are given this height as 1.5 meters and this is symmetric. So, it is 1 meter. So, I can find out what is this angle. Some computation you may have to do and I would urge that, some next class onwards please bring your calculators and you could verify, whether my arithmetic is alright and if I put $\sum M_H = 0$ I get the equation like this; I get this as

$$F_{BJ} \times \cos 56.3^\circ \times 1.5 + F_{BJ} \sin 56.3^\circ \times 1 - 2.5 \times 2$$

$$10 \times 1.5 - 10 \times 3 + 10 \times 2 = 0$$

So, while writing this equation, accommodate both unknown and known forces carefully. It is like solving any other isolated section; here the section is, cut on the member and you are accommodating for all the forces and this has to remain in equilibrium. So, with this, I am in a position to evaluate F_{BJ} by another equation and that turns out to be 18.02

kN. Please verify that, this arithmetic is alright.

Now, what is left, I have to find out force in member BK; is in a restriction that, I start with method of sections, I should not touch method of joints; there is nothing like that; you utilize both these methods judiciously to solve a given problem; your goal is you

are learnt methodologies, apply the methods to solve a given problem. So, the next clue is, I would isolate a joint and find out the force in member BK and in this case, it also becomes very obvious, which joint should I isolate.

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I put the free body diagram of this; see the methodology, I have adopted in isolating a joint is so convenient physically, you can appreciate what it is and there is no great difference between, what I do for method of section and for what I do for method of joints.

So, it is much more convenient to handle that way. And this is fairly straightforward and would be in a position to find out, what is the value of the force F_{BK} and you get this, as -25 kN and you put this, towards the joint and we have determined the forces on three

members, using just three equations and judiciously, using a combination of; method of sections and method of joints. I would urge you, to take this kind of an approach in

Trusses
SWAYAM PRABHA
Method of Sections

- A section of the truss is considered as a subsystem for the analysis.
- A suitable imaginary section (which need not be a straight line and can be an arbitrarily drawn curve) is passed through the truss to separate it into two sub-systems.
- The force system acting on each of these sub-systems is non-concurrent and so **three independent equations** can be written.
- In this regard, while choosing a section of the truss not more than three members whose forces are unknown are cut.

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minimizing your mathematics. I can also solve the problem, I method of joints, going from one joint to another joint, take half an hour to 45 minutes, still I will get the same answer. And this just summarises what you have done in method of sections.

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So, we take a suitable imaginary section, which need not be a straight line and force system acting on this is non-concurrent. So, I can use three independent equations. So, generally you choose a section, where three members whose forces are unknown are cut,

Trusses
SWAYAM PRABHA

- In case of more than three unknowns in the section, additional sections or joints may be necessary.
- The moment equilibrium condition is used to a great advantage in the method of sections and through proper choice of moment centers, the unknowns can be determined conveniently.
- It is convenient to arbitrarily assign the direction as positive for an unknown force by showing arrows pointing away from the section.
- This convention indicates that at the end of the calculation if the algebraic sign of the force is positive, the member is subjected to tension, else compression.

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but we have seen a special case, where we had more unknowns, but we utilize the equilibrium equations appropriately; particularly the moment equilibrium.

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So, we could still get the answer, even with 4 members that means, one of the members will be able to calculate, still you have the difficulty; if I have to find out for all the unknowns, then it may not be sufficient. So, you should look at in that context and we have also discussed on what should be the sign convention I should use.

So that, I am able to associate the member force as tension or compression on the actual truss member.

Trusses
SWAYAM PRAKASH

Identification of Zero force members

By using

$$\sum F_y = 0$$

it can be found that F_{AD} is zero.

F_{AD} as well as F_{AB} are zero!

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Then we move on to another important aspect, how do I identify zero force members? I have a situation like this, at joint A, I do not have a force acting at joint A; external force, I have only members connected here. Can you

tell me, which is the zero-force member? It is obvious; I have no vertical component of force, acting on the joint externally, so; obviously, if I put $\sum F_y = 0$ I get F_{AD} should go to 0. So, even by inspection you can identify, what is the zero-force member. And people ask me, what do I do with this, zero force members? I said, from the stability point of view, you cannot remove the zero force member and you should also recognize for the given loading; this member is not carrying any load, but you know very well, I always have these structures available in the open because you see trusses are used for stadium and bridges and so on.

In many of these application, you also have forces due to wind, that keep changing the direction; for the given loading this member transmits zero force, for another loading this member may carry a way force, from that point of view also, you should not remove zero force members, but particularly from stability point of view, you cannot remove even for a given loading, when the member is transmitting zero force, they should be retained.

Suppose, I have a joint like this, I have F_{AB} and F_{AD} forces acting like this and there is no external force acting at joint A, which are zero force members; both of them are, this is to remove your mental block; we took one example of a truss in that, so happened one member was zero force; in a truss, you can have several members carrying, no load ok;

several members may be zero force members. Then, we move on to another aspect how do we handle statically indeterminate trusses? see the idea is.

Trusses
SWAYAM PRABHA
Analysis of Simple Statically Indeterminate Trusses

- Types
 - External Redundancy: More external supports than are necessary to ensure a stable equilibrium configuration.
 - If both supports are pin jointed then the truss has external redundancy.

Solvable if external loading is simple.

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If I have to analyse, beyond the purview of statics, I have to bring in deformation in my analysis and we said that we are going to analyse bodies as rigid; nevertheless, certain class of problems, you can still handle. One class is; if I have external

redundancy, if I have more external supports, than are necessary to ensure a stable equilibrium configuration; I cannot solve using the principle of statics. Suppose the loading is very simple, I have taken a same truss, which we have analysed it earlier. We had pin joint here, originally, we had a roller support; now, I have replaced by a pin joint, but I still retain the same vertical force; I can still solve this problem, even though it has external redundancy.

If the loading is simple, if there is no horizontal component, I can make a reasonable

Trusses
SWAYAM PRABHA
Analysis of Simple Statically Indeterminate Trusses

- Types
 - Internal Redundancy: more members than are necessary to prevent collapse.
 - Cross braced structures are of this type.

Solvable if diagonal members are flexible cables.

Used in structures that have to bear wind loads that changes its direction.

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assumption that, there are no horizontal interaction of forces at these two supports; I can still solve the problem. So, this is one class of problems you can handle. Other important class of problem is when I have internal redundancy.

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I have more members than, what is actually necessary. You know we have seen the failure of Tacoma Narrows bridge and I said, one of the problems that precipitated was forces due to wind. Then people learnt that, they have to do something for the wind forces and we saw cross bracing in many of the practical bridges; you had seen in the first class, when we discussed, how trusses are constructed.

So, I have a cross braced structures and what I have here is, you should see clearly that, there is no circle put here. So, the member is continuous; they are not joined here and what I bring in is, the wind changes direction. So, depending on the wind force, it will deflect in a particular manner and we have already seen that, these cross braced members are much lighter than the actual numbers of the truss and I said, we have put a string in one of the classes and shown that, it simply buckles; if it is subjected to compression. So, it can take only tensile forces. So, I can use this, if I consider that the diagonal members of flexible cables; if they could be flexible or behave in a sense flexible from the overall truss point of view, I have thin members doing this.

So, even though, I have this member, I would ignore the presence of this member and determine the force as if this member is not present, but you have to verify later by taking that this member is present, the other member is absent and find out whether, the force is indeed tension or compression. If it is compression, your original assumption is correct. We have also taken up a problem and then, understand this and I have already

The cross braced members can support only tension and incapable of supporting compression.

Determine forces in members CE and DF.

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said that, these are necessary to bare wind loads because, they keep changing direction, you do not know, which way the wind will blow.

So, I should have cross braced members. So, with when, the wind force direction changes, one other member will come

into play, structure will be safe, the bridge will not collapse two days back, there was a

storm and one other bridge connecting France and Italy has collapsed. So, do not think that, failures happen in the past, they can happen even now.

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So, you have to design it properly. So, I have another truss, which has cross brace members; in this illustration I have shown it thick, but we have seen in all the earlier actual structure of a truss, there were shown deliberately as thin. So, imagine that, these

are flexible, but I am making an assumption that they are flexible in doing it both are equally good. And I also have modelling of the wind load for this application; on that day wind was blowing from right to left.

Solution

FBD of the truss is shown as below.

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So, what I can do is, I will have the free body diagram, where I will have the support forces; now, I have four unknowns at the supports; I have the wind forces like this and I

Check for Determinacy / Indeterminacy

- Number of joints, $j = 8$
- Number of members, $m = 16$
- Reactions, $r = 4$

Since $m + r > 2j$

It is a statically indeterminate problem.

The degree of redundancy is 4.

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will have to do whether, the problem is statically determinate or indeterminate; I should find out the number of joints and number of members and number of unknowns.

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So, when I do that, I will find that, this is a statically indeterminate problem and I will have, you can do that,

calculation; you I will have the number of unknowns are 4, which I will not be in a position to find out, but I can still solve for this problem, by making an assumption under the given action of these wind forces that, deflection could be like this.

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Cross braced members cannot bear compressive load.
The presence of CE member is neglected
In view of the loading conditions let the deflection shape be assumed as

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So, what I would do is, I would make this member absent in my free body and estimate the member forces; what I assume is, only this member is present and it is taking tension and the deformation of the truss element is like this, that

comes from physics of the problems. So, I bring in additional information from the deformed picture; you have to visualise, what way the deformation could be. I assume it like this and this is reasonable because of the wind loads are very clearly specified, if they are not specified, then you may have to verify by the other way of deformation and

find out, which is under tension and which is under compression.

Consider equilibrium of joint E

$$\sum F_x = 0$$

$$-F_{DE} - 10 = 0$$

$$\therefore F_{DE} = -10 \text{ kN}$$

$$\sum F_y = 0 \quad F_{EF} + 30 = 0$$

$$\therefore F_{EF} = -30 \text{ kN}$$

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So, I do this equilibrium of this joint. So, that, I do not have this member present in my analysis. So, I can directly find out, what is the force in the member F_{DE} and you could get this,

these are the compression.

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And I find out the equilibrium of joint D. The focus here is, I would essentially get the

Trusses
SHAYAN PABHA

30 kN

30 kN

30 kN

10 kN

10 kN

10 kN

2 m

2 m

2 m

2 m

2 m

Consider equilibrium of joint D

$$\sum F_x = 0$$

$$-F_{DF} \cos 45^\circ - 10 = 0$$

$$\therefore F_{DF} = 10\sqrt{2} \text{ kN}$$

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value of the force acting on the member DF as positive, if my result is positive, what I have started is ok, but still it is not completely verified; for the given loading it is so simple, I am able to visualise the deformation. Suppose, I also have some loads acting from right to left, but I have major load

acting from this direction.

You may not be able to visualize clearly. So, you may have to verify both the cases and find out, which is the actual deformation for the given loading; that, I will leave it as an exercise to you.

Trusses
SHAYAN PABHA

Role of Computers

- When number of members increase by hundreds solving them using the method of joints with hand calculation will take a lot of time.
- With the advent of digital computers, the procedure for evaluation of not only member forces but also the stresses and deflections can be computerised.
- The force displacement relations for each member of the truss is expressed in a matrix form.
- Subsequently, individual matrices of members are assembled appropriately to obtain a global matrix equation.

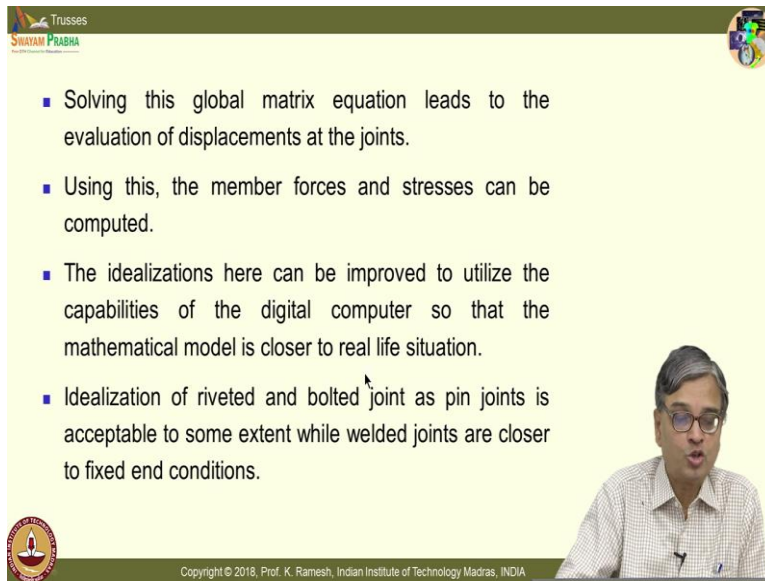
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You may wonder with the advent of computers; how do we go on do the analysis? We have seen bridges having hundreds of members; you can't solve them manually, it will take a lot of time and we have also made consciously several simplified

assumptions, to solve a problem and what is done in computer based analysis is the force displacement relations for each member is expressed in a matrix form.

Then you form a global matrix, there is a methodology called finite element methods and



Trusses

- Solving this global matrix equation leads to the evaluation of displacements at the joints.
- Using this, the member forces and stresses can be computed.
- The idealizations here can be improved to utilize the capabilities of the digital computer so that the mathematical model is closer to real life situation.
- Idealization of riveted and bolted joint as pin joints is acceptable to some extent while welded joints are closer to fixed end conditions.

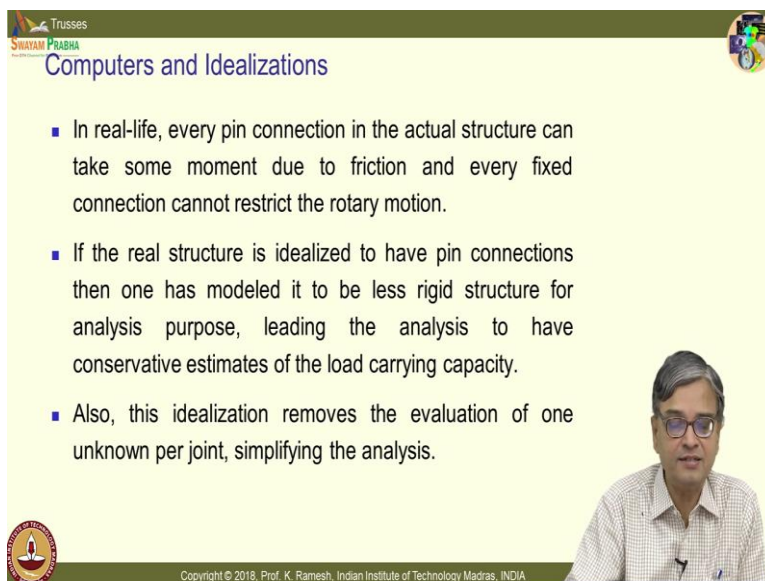
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you have matrix theory of structural analysis; it was started base civil engineers, then now, it is popular as a finite element method.

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And you can solve the global matrix and get the evaluation of displacement

at the joints, from the displacements find out the member forces and also find out the stresses and you can also improve the idealizations because, we have said that this is a pin joint and I have always been saying in reality the joints are very complex.



Trusses

Computers and Idealizations

- In real-life, every pin connection in the actual structure can take some moment due to friction and every fixed connection cannot restrict the rotary motion.
- If the real structure is idealized to have pin connections then one has modeled it to be less rigid structure for analysis purpose, leading the analysis to have conservative estimates of the load carrying capacity.
- Also, this idealization removes the evaluation of one unknown per joint, simplifying the analysis.

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So, you can improve the idealization of these joints better in computer analysis and you should also keep in mind this, I have emphasized it several times, in real life every pin connection in the actual structure, can take some

moment due to friction. So, you can't idealize something as a true pin joint. If you look at fixed joint, are they really fixed? They can also allow some small rotary motion. So, it is always debatable; whether, a physical structure a joint or a support, does it actually behave like a pin joint or a fixed support. So, this is where I said, you have lower bound solution and upper bound solution. So, you have to do the analysis in either way and estimates the load carrying capacity and by making this as a pin joint we have removed one unknown; I have only two unknowns per joint, so that also simplifies the analysis.

Trusses
SWAYAM PRABHA
Computers and Idealizations

- Idealizing as a rigid connection leads to more complications in performing the analysis.
- Thus idealization has to be done with care with possible solutions from both analysis giving a boundary of the load carrying capacity.
- Computers allow advantage to be taken of the additional stiffness that comes from riveted, bolted or welded connections in the analysis.
- Analysis of pin-jointed trusses are now limited to engineering education.

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And you have to do the idealization with care and computers allow advantage to be taken of the additional stiffness, that comes from riveted bolted or welded connections and you should also keep in mind analysis of pin-jointed

trusses are now limited to engineering education. So, do not think that, actual trusses people ignore the stiffness introduced because of joints; these are efficiently modelled.

So, you do not have to worry about it.

Trusses
SWAYAM PRABHA

Max. Deflection = 0.0308 mm Max. Deflection = 0.0154 mm

- $E = 210 \text{ GPa}$
- $A = 10 \text{ mm}^2$

- $E = 210 \text{ GPa}$
- $A = 20 \text{ mm}^2$

Change of Cross Section

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And this, shows a nice animation of the trusses, because the whole idea is, I said, whatever you learn in engineering mechanics is a initial data; that you get for your next level course, the

next level course you have to find out, what is the torque section I should use? And here you have two trusses; one is having a cross section of 10 millimetre squared, another is double the cross section.

And you see the maximum deflection is 0.0308 mm. So, this is highly exaggerated. In another case, the deflection is only 0.0154 and if you look at this, Young's modulus, this is made of steel; it is 210 GPa and you could see very clearly, the deformation is more here and deformation is less here. Suppose, I change the material, I retain the cross section, but I change the material, what happens?

Trusses
SWAYAM PRABHA

Max. Deflection = 0.093 mm

- $E = 70 \text{ GPa}$
- $A = 10 \text{ mm}^2$

Change of Material

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I have now selected aluminium, which is 70 GPa and retain the 10 mm^2 as the cross-sectional area and you would see a large difference, which is seen in the deformed picture; I have a maximum deflection of 0.093. So, you get an idea, you

evaluate the member forces ultimately you have to select, what should be the cross section of the member and what should be the material, it should be made of fine; that is the ultimate purpose of it, the first step is to find out the member forces, that you learnt

Trusses
SWAYAM PRABHA

Why Trusses are efficient?

Truss member

Cross section is fully utilized in load transfer

Beam

Central core does not take any load

What is the typical variation of stress in these members to resist external transverse load?

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in this course and we have also made a statement the trusses are efficient. Don't you think that before we close our discussion on trusses, why they are efficient?

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See, truss is subjected to axial forces, it is subjected to either tension or compression. On the other hand, when the member behaves like a beam, it is bent; the way member resists the loads are different in tension as well as bending. And suppose, I take a generic section, imagine that, this is a long member, I have taken a section and if I look at, what is the resistance; it would appear like this for a truss member; from this you cannot conclude anything.

Suppose, we want to look at what happens if this behaved like a beam; the resistance at this cross section internal resistance, that is developed varies in a triangular fashion and

what happens in this, cross section is fully utilized in load transfer; the central core does not take any load, when the member is bent. So, that is the reason, why trusses are more efficient and you cannot conclude that I do not have load carrying in the beam; so, I should not have any beams.

It is also having a purpose. Even, if you look at God's creation, all your bones are hollow; when you walk up the stair and come down, you are tie bones are subjected to bending and torsion and you know very well, your haemoglobin is created only in bone marrow, bone marrow is very soft, bone is very hard. So, you should not have any load in the inner core that is why, god clearly cleverly made your body, where God has understood bending and torsion, much before as mechanics people we are trying to unravel.

So, in this chapter, we have looked at trusses, we have seen, how they are constructed; the key point is no member is continuous through a joint and loads are applied only at the joints. For analysis and to determine the forces we have learnt two methods; method of joints and method of sections. In a given problem, you could freely choose either one of the methods or a judicious combination to solve the problem as quickly as possible and we have also discussed at length, how to associate your member forces as tension or compression.

Thank you.