Basics of Materials Engineering
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Lecture - 25
Mechanical Properties of Materials
(Concept of Stress Tensor)

What have we done so far? We started with looking at the structure of crystalline materials and then we have discussed different types of crystal structures and then during the time, we said there are different ways of representing crystallographic points, directions and planes; both in FCC -- cubic and HCP structures.

We have discussed how one should calculate planar density or linear density. We have studied how one should identify close packed planes and close packed directions. We have also studied the X ray diffraction and how one would find out the crystal structure by doing the X ray diffraction.

These are the concepts that we have done in *Structure*s and then we have started *Imperfections*. In the *Structures* module, we tried to be idealistic saying that every lattice point will have an atom. We relaxed this constraint when we went ahead with the *Imperfections*, and we said that there are going to be imperfections in materials and we have discussed about different kinds of imperfections; point defects, line defects, surface defects and volume defects and how these defects lead to plastic deformation in materials, and how the close packed directions and close packed planes are responsible for easy motion of dislocations in them.

That is how we have shown the crystalline materials prefer to have dislocations movement, in those close packed planes and close packed directions. We have also calculated something called resolved shear stress and we have written a condition for plastic deformation when the resolved shear stress becomes equal to critical resolved shear stress.

We have discussed that depending on the change in direction, the Schmid factor which is the parameter that decides whether or not there is going to be slip on a particular slip system. All that you need to do is you need to maximize the Schmid factor on a particular slip system so as to initiate slip in the material.

We have discussed about other kinds of defects and then strengthening mechanisms. In the next module, what we are going to study is how one would actually go about measuring the mechanical properties. Let us say these mechanisms are at play in a material, when you are deforming. Then, experimentally how one would actually find out yield strength of the material?

How one would find out the elastic stiffness of the material, or Young's modulus of the material. When the material is made smaller and smaller, it becomes harder. What do you mean by hard? Is hardness same as strong? Can you define strength to be same as hardness of the material? Are they different? What are the different properties, particularly the mechanical properties, that we should be aware of in order to understand the mechanical behavior of the materials?

This is something that we are going to do in this module. Primarily what will we do is, how do we actually go about measuring them. We will spend a lot of time understanding how to measure elastic properties and the yield strength of the material using uniaxial tensile test. We will also talk about how do you measure hardness of a material. These are the two things that we will do in this module.

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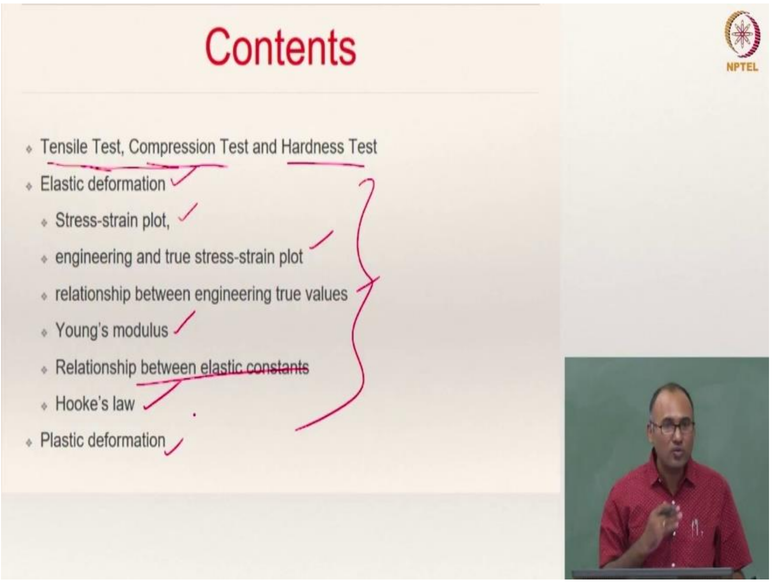

These are the contents. We will talk about tensile test, compression test and hardness test. When we are talking about tensile test and compression test, we will talk about elastic deformation. In that, we will look into the stress-strain plot and then we define something called engineering stress, engineering strain, true stress and true strain and how to plot them.

The relation between engineering and true components, Young's modulus of elasticity and the relation between the elastic constants. Then, we define something called Hooke's law. These are all in the elastic range and then we will get into plastic deformation. We know why plastic deformation happens in the material already; all of us know why plastic deformation happens in a material. But when you are doing an experiment, what will you see as an output of your experiment? How do you understand this? That is what we are trying to do.

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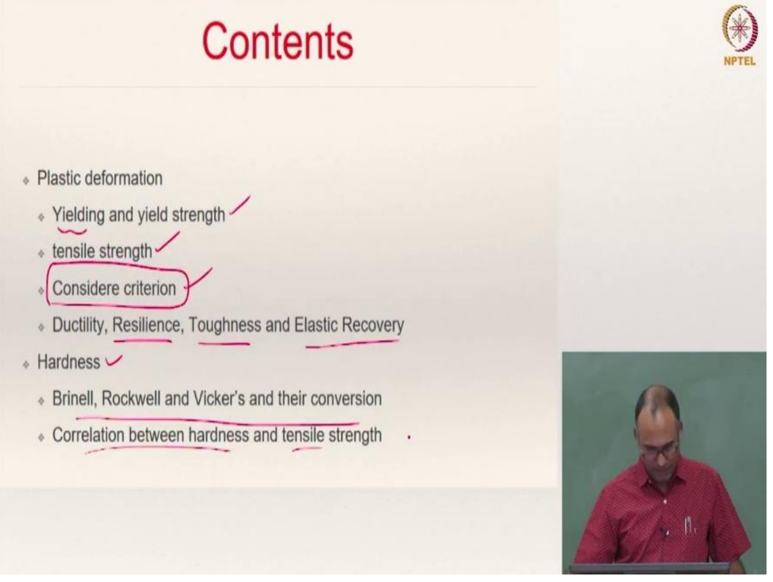

In the plastic deformation, we will talk about the concept of yielding and the yield strength. Then we will talk about tensile strength and during this discussion, we will dwell upon something called Considere’s criterion; we will talk about it when we encounter that and then we define other properties. I have been using this word called ductility quite often, but how do you actually go about defining the property called ductility?

What do you mean by ductility of the material? We also have to define something called resilience, toughness and elastic recovery; these are the properties of the materials and how do you go about defining them.

We define another property called hardness and how one actually measures this hardness. There are different methods to measure the hardness and we also have some correlation between hardness and tensile strength. So, if we measure hardness, we can estimate the tensile strength, for certain materials.

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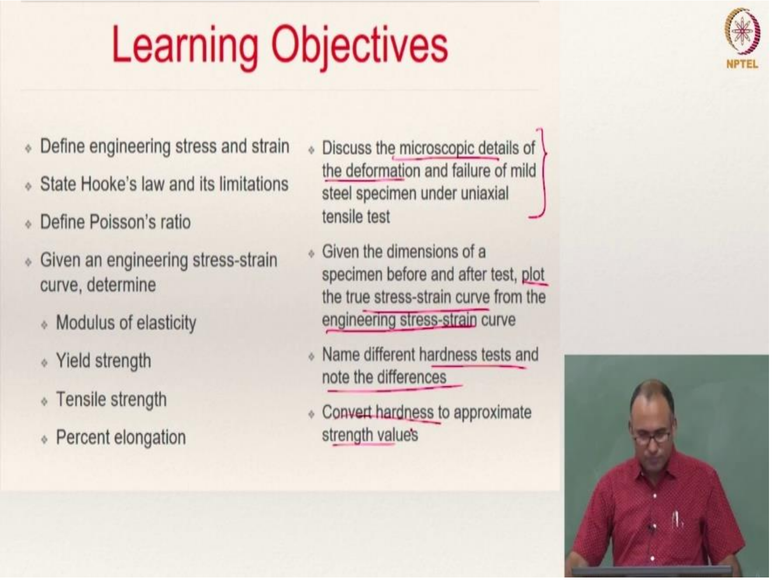

These are your learning objectives; so, please go through them. Most important thing is you should be able to connect the microscopic details of deformation that we have discussed in the last module to the stress-strain curve that you are seeing in the uniaxial tension test.

You have seen your uniaxial tension test in your applied mechanics lab and you have seen the yield point. And then, upper yield point, lower yield point; I do not know if you have observed, but how these individual points can be interpreted with respect to the microscopic mechanisms at play at the atom scale that we have discussed until now? We should be able to connect these two things.

If you are given the dimensions of a specimen before and after the test, you should be able to plot the true stress-strain curve from the engineering stress-strain curve. You are given an engineering stress-strain curve and also the initial and final dimensions of the part. Then, you should be able to generate the true stress-strain curve; that you should have done already in the lab.

And name different hardness tests and note the differences; what are the different kinds of hardness test and the differences. And how one can convert the hardness value to approximate strength.

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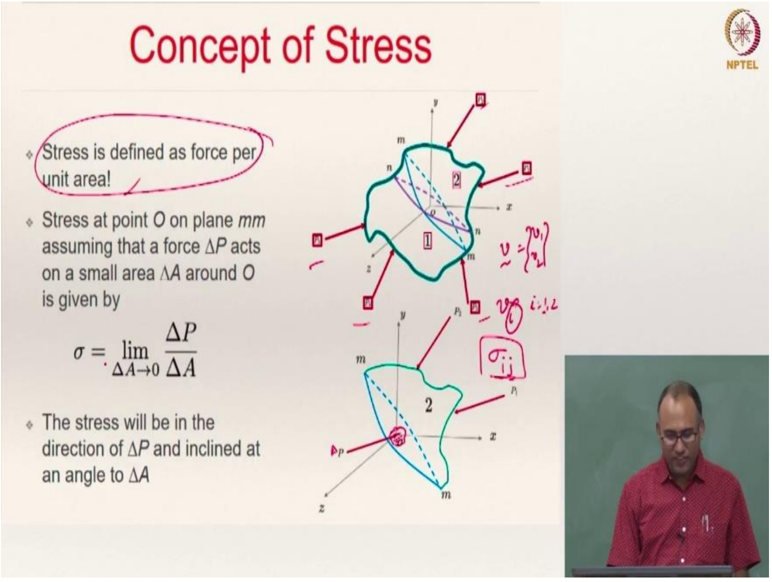

What do you mean by stress? The way that I have defined here is a naive definition of stress; defining stress as force per unit area. It is ok, but it is not really the right way to define stress.

When you are applying load on a material, then the material offers some resistance. The resistance offered by the material per unit area is what you call stress. But then this is a little tricky. Does it give you an impression that it is a scalar quantity or a vector quantity? Is stress, scalar or vector?

It is a tensor, right? All of you are familiar with the concept of tensor, right? Stress is a second order tensor; a vector is a first order tensor; scalar is a zeroth order tensor. What do you mean by this order?

The number of indices that are required to describe the components of this quantity, represents the order of your tensor. For instance, a scalar is only a magnitude and does not require anything else; so, there is no need for an index.

That means, it’s a zeroth order tensor. Let us say you have velocity in 2D for instance. How many components will it have? If you have a frame of reference with x axis and y

axis, you have a component of this vector along *x* and *y* axes. You need two components to describe this vector; the projection of this vector on to *x* axis, the projection of this vector on to *y* axis.

If this is my vector, this is v_1, v_2. I can describe this as v_i, where $i = \{1, 2\}$. I just need one index to represent the components of my vector. What is the second order tensor? For instance, let us talk about stress; it requires two indices; let us say σ is your stress, σ_{ij} is the right way to represent the stress tensor.

What does each of these indices represent? Here, this is very clear; this is the direction of the force the projection on to that plane, but what does this indicate? First of all, you need to have the direction; a scalar has only magnitude, vector has magnitude and direction, and second order tensor has magnitude, direction and what else?

Yeah, plane of action. The normal describes the plane; that is another direction. That is why you need two parameters; the direction of the quantity itself and the plane in which it is acting. If you take a stress tensor in 3D, total how many components will you have?

If you have a 3D stress state, how many components will be there? If you have a vector in 3 dimensions, how many components will be there? 3 components of the vector. If you have a second order tensor, how many components should it have? 3? Then it is vector.

9, right? Because I said σ_{ij}, $i = \{1,2\ ,3\}$. In a matrix form,

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Here we are given a naive definition because that serves the purpose, but you should keep in mind that your stress is actually a second order tensor which requires the direction and also the plane of action.

This is an arbitrary body which is subjected to the external loads P_1, P_2, P_3, P_4, P_5. If you cut open this body, you have the plane *mm*. Let us say this is the load ΔP, acting at point O, on an infinitesimal area ΔA. In the limit $\Delta A \rightarrow 0$, that is why it is called infinitesimal area, this stress is defined,

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A}$$

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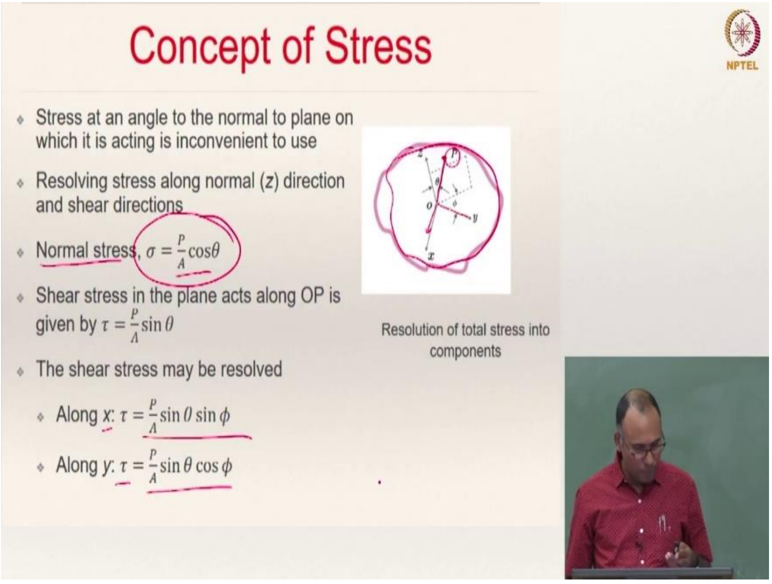

The stress direction need not be perpendicular to the plane or in the plane. In general, it can be at an angle to the normal to the plane. Let us say this is a plane, and let us say z is the normal to this plane.

The load P in general, be at an angle to the normal; sometimes it can be parallel to the normal or perpendicular to the normal; in the plane or out of the plane, but in general it is at an angle to the normal. Then, the normal stress which is the component along the normal direction can be written as,

$$\sigma = \frac{P}{A}\cos\theta \,,$$

because its direction is along the normal to the plane; that component of stress is called normal stress and that is where the name comes from. The normal is along the normal to the direction of the plane, on which it is acting.

And then, it will have (i.e., the stress vector) two more components in the plane; they are called shear components. The component into the plane will be

Shear stress in the plane: $\tau = \frac{P}{A}\sin\theta$

Shear stress in the plane resolved into x and y components:

Along x: $\tau = \frac{P}{A} \sin\theta \; \sin\phi$

Along y: $\tau = \frac{P}{A} \sin\theta \; \cos\phi$

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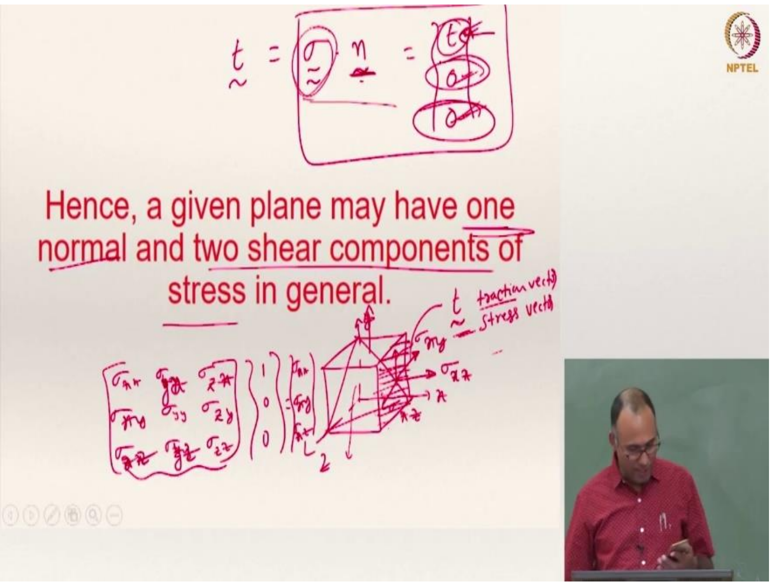

In general, given a plane, you may have one normal component and two shear components of stress. If you remember from the strength of material class, this infinitesimal element, and let us say, this is x axis and that is y axis and that is z axis. This is σ_{xx}, that is σ_{xy}, and this is σ_{xz}.

I am not drawing the components on the other planes; that you probably are familiar with. So, on a given plane, you have three components. In general, if you take these three components alone, they will have a vector and this vector is what you call stress vector or traction vector. Please do not get confused between stress tensor and stress vector; what is this stress vector?

On a plane you have three components of the stress acting, the resulting vector of these three components i.e., the normal and two shear components, will be at an angle to this plane and that vector is called traction vector or stress vector.

This is your stress tensor. If I want to find this traction vector, what is this plane? That is the traction vector on that plane, right? What is the normal to that plane? What is the normal vector? $\begin{Bmatrix}1\\0\\0\end{Bmatrix}$; so, if you operate this tensor on this vector, what will you get? You will get a vector right, what would that vector be? $\sigma_{xx}, \sigma_{xy}, \sigma_{xz}$; what did you get?

You got the traction vector on that plane, right? So, when you are operating a second order tensor, on a first order tensor, you will get a first order tensor.

$$\boldsymbol{t} = \boldsymbol{\sigma}.\boldsymbol{n}$$

Here, the normal vector to this plane is 1 0 0; so, when we operate this matrix on this normal, I should be getting traction vector acting on that plane and that is the traction vector that we got.

If you multiply the same matrix with this normal, you should get these components. Let us say this is my (111) plane, if I have to get the traction vector on this plane, all that I need to do is, multiply this stress tensor with $\begin{Bmatrix}1\\1\\1\end{Bmatrix}$, so that I get the traction acting on that particular plane. So, like that you can actually calculate traction acting on any plane.

Let us say you are multiplying this and you get a vector -- some value and 0 0; what is the meaning of that? Let us say you have found some normal for some plane in which if you are multiplying this, you are getting these values. What is the meaning of that? What are these two components? These two are shear stresses and this is normal stress.

On this plane, you do not have shear stresses. That plane is your principal plane because principal plane is the plane on which you only have normal stresses, shear stresses are zero. You can find out the principal stresses or principal planes using this relation.

You have a stress tensor; if you know the stress at any point, all that you need to do is you need to find a normal or a plane along which this becomes true i.e., you will only have normal component and the shear components will be zero.