Basics of Materials Engineering
Prof. Ratna Kumar Annabattula
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture – 37
Static Failure Theories (Notches and Stress Concentration)

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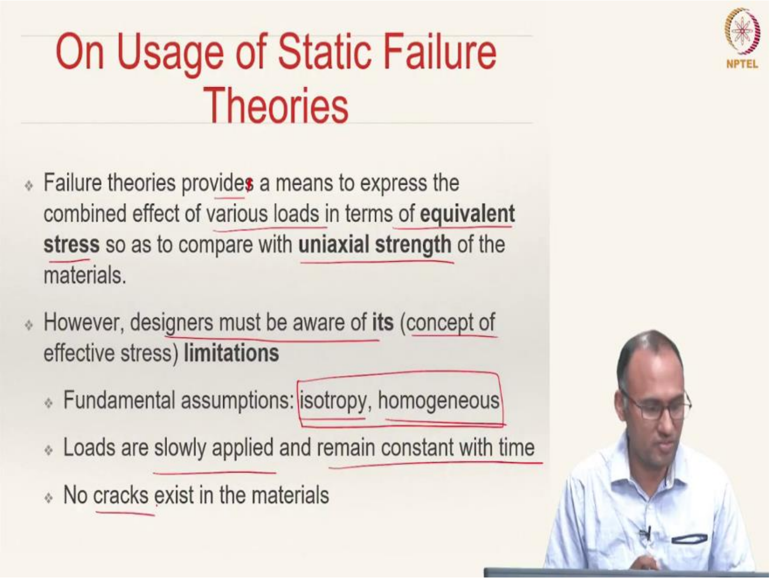

Let us now comment on the usage of static failure theories. We have previously discussed the failure theories for ductile materials and brittle materials. When we are using these theories, we should be careful and we should always keep in mind the limitations of these failure theories.

As we have discussed, the failure theories provide a means to express the combined effect of various loads in terms of an equivalent stress so as to compare with uniaxial strength of the materials. That is what we have done, right? In the case of von Mises stress, we have calculated an equivalent one-dimensional stress from the complex 3D state of stress.

And then, we compared that with the uniaxial tensile strength to say whether it is failing or not. However, designers must be aware of its limitations. There are certain fundamental assumptions, when we are deriving these failure theories. What are those fundamental assumptions? We assumed the material to be isotropic and homogeneous. However, the real materials are not perfectly isotropic and homogeneous.

But the underlying failure theory that we have defined has these assumptions. Hence, you would expect some variations from your estimations with the experimental data. The second important thing is that we have assumed the loads are applied quasi-statically and they remain constant with time; they do not change with time.

Only in those situations, the static failure theories are applicable, otherwise they cannot be applied. Another important assumption when we have derived these static failure theories is that there are no cracks present in the material.

If there are cracks, you will not be able to discard the presence of the cracks and go about prescribing the same design based on the failure theories that we have discussed. You need to account for the existence of the cracks in the materials; that is one of the most important aspects that one needs to keep in mind, when employing the static failure theories that we have learned so far in the real design process.

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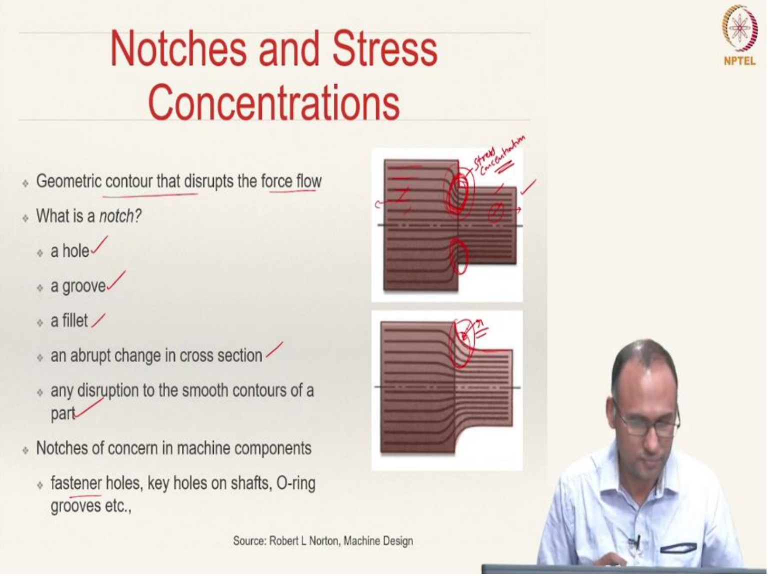

So far, we have looked at the static failure theories wherein, the material is assumed to be isotropic, homogeneous and there are no cracks. We also did not consider the geometric discontinuities or geometric variations within the body on which the load is applied. One needs to also consider the fact that the real machine components will have notches and grooves and so on and how do you go about taking those things into account, when you are doing the design. So, usually the geometric contours that disrupt the force flow are called notches or stress risers.

What is a notch? How do we go about defining a notch? A notch is anything that is either a hole, a groove, a fillet, an abrupt change in cross section or any disruption to the smooth contours of a part. All these things can be classified as notches. For instance, let us take an example here.

Imagine a step shaft with a sudden change in the cross section and imagine these lines represent the streamlines that you have studied in fluid mechanics. The flow is going to be constricted in this region to adjust to the new change in cross section; that is what happens when you are studying fluid flow.

You will have local turbulence in some sense. Even if the far-field flow is laminar, because of the change in cross section, you will have local turbulence. Similarly, when you are applying a load on the material, there will be stress lines. These can be seen as stress lines and the same load is applied, but here you have a larger area of cross section to resist that load.

Here, you have a smaller area of cross section and hence, the stress in this region is going to be a little higher than in this region. When there is a geometric discontinuity, the stress lines also have to conform to the new geometry and as a result, similar to the case of streamlines that you have studied in fluid flow, the stress lines also start coming together here, in order to confirm to the new geometry in the next section.

These stress contours meeting close to each other is what we call stress concentration. That means, there is a sudden rise of stress here due to geometric disruption

What are the different kinds of notches that we would see in machine components? Most of the machine components will have fasteners, the holes drilled to keep the fasteners, key holes on shafts, O-ring grooves etc, there are so many. I think you cannot find any machine component without a notch.

Notches are a part and parcel of the real life of machine components. Now, if you see this geometry, here the stress concentration is going to be really high. Primarily because you have a sudden jump in the cross section; but what if you actually reduce the severity of this jump; rather than having a sudden jump, you actually provide a groove or fillet, wherein the stress concentration is eased out by gradual change of the cross section.

In this region, comparatively you will have less stress concentration because you have a gradual change. So, the stress concentration really depends on the radius of this fillet in some sense.

When you have a machine component with such geometric disruptions, you need to account for increase in the stress in that region compared to the far-field applied stress.

Whenever you are designing a material, you are only looking at the maximum stress in the material or the component, and that maximum stress should not go beyond your ultimate strength or yield strength of the material depending upon whether it is a ductile material or brittle material.

When you have these geometric disruptions, typically you are going to have a stress rise in this region and that is what is called stress concentration. There is going to be increased stress in that region. So, if you do not take that into account, you would be underestimating the stresses in the material which is going to be detrimental for your design.

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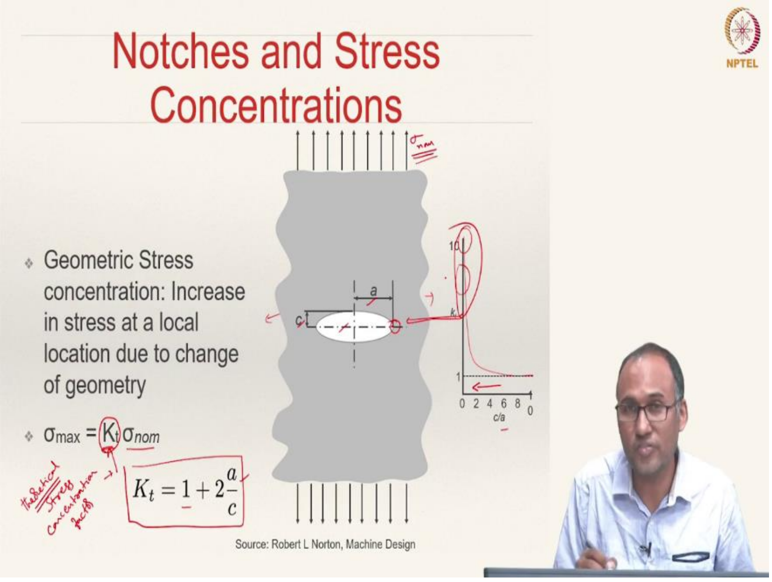

Let us take one example. If you have an infinite body -- so, here this shape actually means that the body is pretty long and you have applied a far-field stress, far away from an elliptic hole. This elliptic hole is described by semi-major axis a and semi-minor axis c. Then, the maximum stress near the notch i.e., the elliptic hole boundary is given as,

$$\sigma_{\max} = K_t \sigma_{\text{nom}}$$

σ_{nom} represents the far-field applied stress. If you do not have the elliptic hole, in this point you would expect the stress to be same as σ_{nom} because whatever is applied will be there at each and every cross section.

But the presence of the hole or the geometry discontinuity is going to increase the stress at this region and the factor by which it is increased is called, stress concentration factor, given by K_t. In some terminologies, it is also called theoretical stress concentration factor.

For an elliptic hole,

$$K_t = 1 + 2\frac{a}{c}$$

If it is a circular hole, $a = c$, and $K_t = 3$. The stress concentration factor for a circular hole is 3; that means, if you are applying a far-field stress of 100 MPa and if you have a circular hole, near the hole boundary, you would experience 300 MPa of stress. If your yield strength is say, 250 MPa and you are applying a far-field stress 100 MPa, if you do not take into account of the stress concentration, then you would think that you are actually safe because your factor of safety is 2.5

But if you have a hole, the maximum stress in the material is 300 MPa which is much larger than your yield strength, implying that the part is not safe anymore. Hence, taking into account of the presence of notches and the stress concentration due to their geometry is one of the most important aspects in the design

Here, you can see as you make a larger and c smaller, the elliptic crack eventually becomes a sharp crack. As the value $\frac{c}{a}$ decreases, i.e., as the crack gets sharper, the stress concentration factor increases.

In the limit a tends to very large number and c tends to a very small number, then you may have extremely high stress which is going to be dangerous for the real design. So, you cannot really choose a to be pretty large compared to c, otherwise you will have very high stress concentration factor. This is one of the design considerations one needs to take while designing components.

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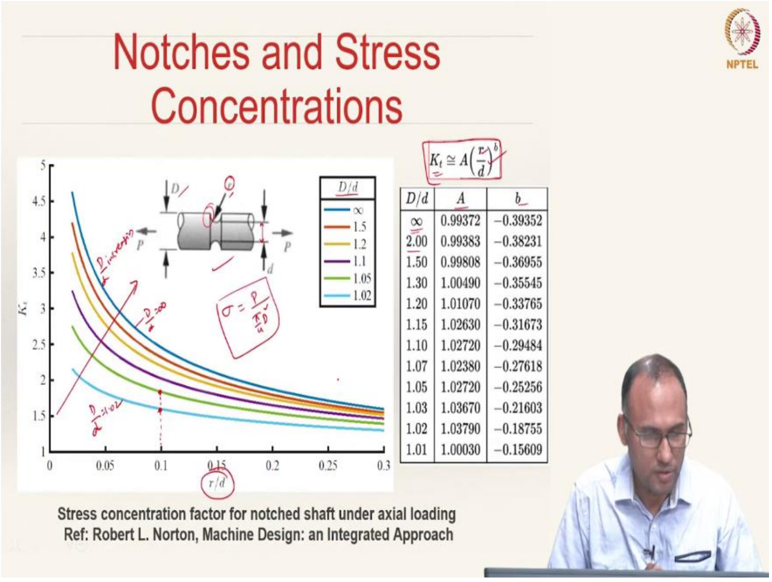

The stress concentration factor depends on the geometry and the loading. Consider a shaft subjected to an axial load with a diameter D with a groove radius r. If the applied load is P, then the nominal stress,

$$\sigma = \frac{P}{\frac{\pi D^2}{4}}$$

Here, you will have higher stress. The value of the stress concentration factor depends on r. For different $\frac{D}{d}$ values, you can show the stress concentration factor K_t by this function for various $\frac{r}{d}$ values. For different $\frac{D}{d}$ ratios, A and b will change.

This is $\frac{D}{d} = \infty$ and this is $\frac{D}{d} = 1.02$. So, $\frac{D}{d}$ is increasing in this direction. For each $\frac{D}{d}$ value you know the stress concentration factor for a given $\frac{r}{d}$ value.

If $\frac{D}{d}$ increases, for the same $\frac{r}{d}$ value, the stress concentration factor increases. Usually, all these charts are available in design data handbooks and designers usually refer to these charts while choosing what is the stress concentration factor. You do not remember these values; you actually refer to the data that is available in the design data handbooks.

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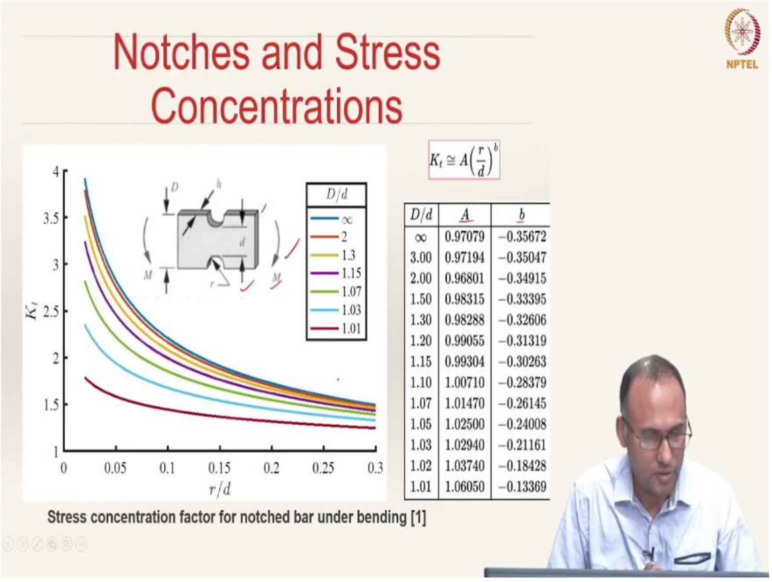

Similarly, if you are applying a bending moment, not on a circular cross section, but a rectangular cross section with these geometry dimensions, again you need to come up with new values of A and b and then, depending upon the load you have these concentration factors.

These concentration factors ranges are different compared to here, keep that in mind. So, like that, for various kinds of loading scenarios and geometry scenarios, people have come up with these stress concentration factors and they have been tabulated or graphed for our convenience.

All that we need to do is you need to pull out the appropriate table or chart which needs to be applied for a given purpose or a given design scenario. So, I have only shown 2, but there are several other tables and charts that are available in the design data handbooks and you should refer to them while solving the problems.

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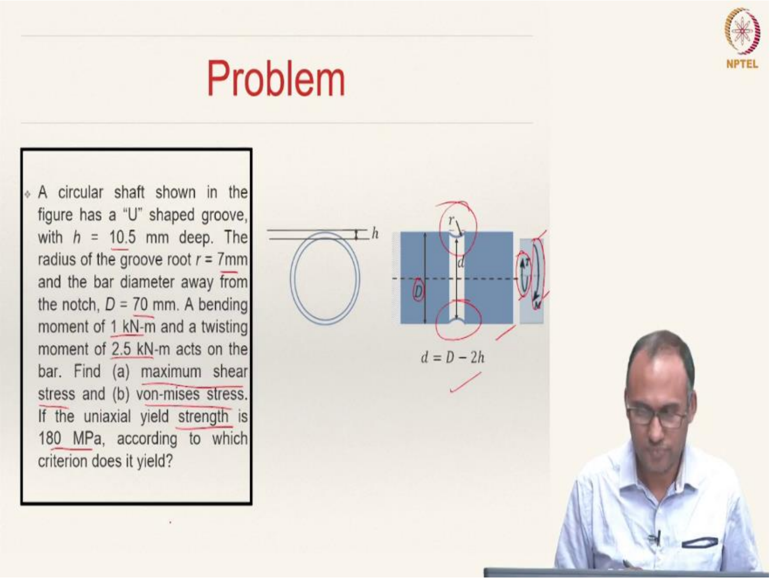

Let us solve one problem. Here you have a circular shaft of diameter d and then, the "U" shaped groove with $h = 10.5$ mm; that means, the groove diameter is 10.5 mm deep. The radius of the groove is 7 mm and the actual diameter is $D = 70$ mm. On this shaft, there is a bending moment as well as twisting moment; there are two loads that are there.

So, a bending moment of 1 kN-m and a twisting moment of 2.5 kN-m act on the bar. Now, find the maximum shear stress and von Mises stress for this problem; it is a ductile material. And if the uniaxial yield strength is 180 MPa, according to which criteria does it fail? That means, whether maximum shear stress theory or von Mises theory/distortion energy theory; according to which of these criteria the material will fail, under these loading scenarios.

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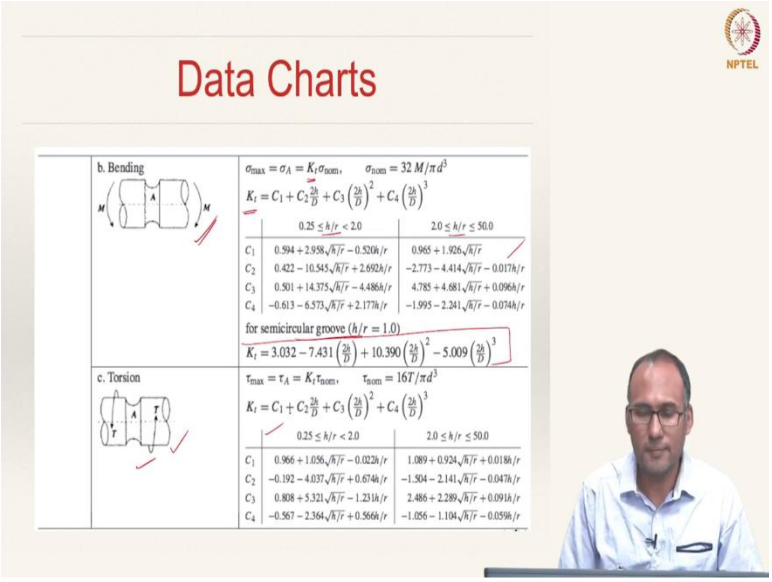

The material is subjected to both bending and twisting, but we know that the stress concentration factor changes depending on the type of load that is being applied and hence, you need to look for stress concentration factor due to bending separately, due to twisting separately.

The stress concentration factor due to bending is given by this formula for different values of $\frac{h}{r}$ and we know D. So, you can actually calculate from this formula for semi-circular groove $\frac{h}{r} = 1$, otherwise you will use this formula, ok? And this is K_t. Similarly, for torsion, you will do the same thing.

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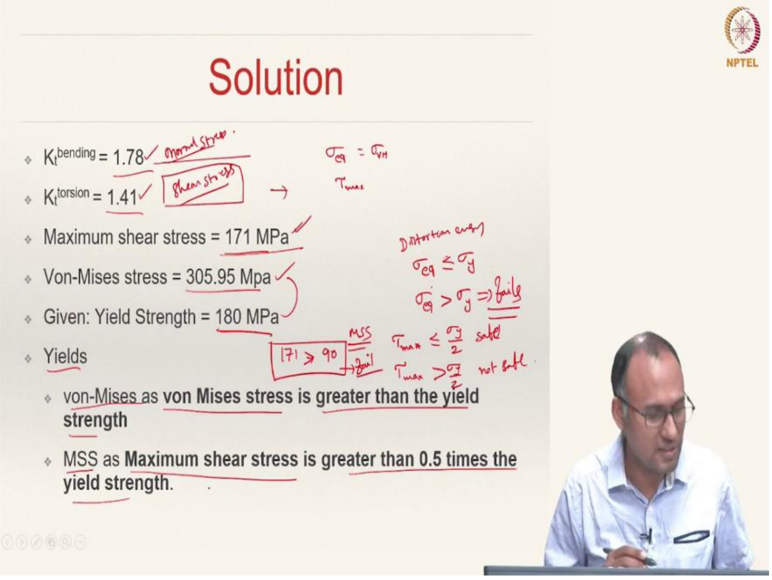

We see that,

$$K_t^{\text{bending}} = 1.78$$

$$K_t^{\text{torsion}} = 1.41$$

Then, you calculate the maximum shear stress and multiply that with K_t^{torsion}; that will be 171 MPa. And you calculate maximum bending stress from the bending formula and multiply that with 1.78; that will be 305.95 MPa.

So, this is normal stress, this is your shear stress, ok? So, this factor should be multiplied with the normal stress and this factor should be multiplied with the shear stress and then, calculate your equivalent stress.

According to von Mises theory, the material is safe, if the equivalent stress is less than or equal to yield strength of the material.

But here, you see that equivalent stress is greater than yield strength that implies the material fails; that is distortion energy theory. For maximum shear stress theory, what is the condition? $\tau_{\max}$ should be less than or equal to $\frac{\sigma_y}{2}$ for the material to be safe.

Here, we have

$$\tau_{\max} = 171 \text{ MPa} > \frac{\sigma_y}{2} = \frac{180}{2} = 90 \text{ MPa}$$

Hence, the material will yield based on both the failure theories.

So, you need to redesign the geometries as they are not good enough; that means, either you should reduce the load or you should redesign your system.

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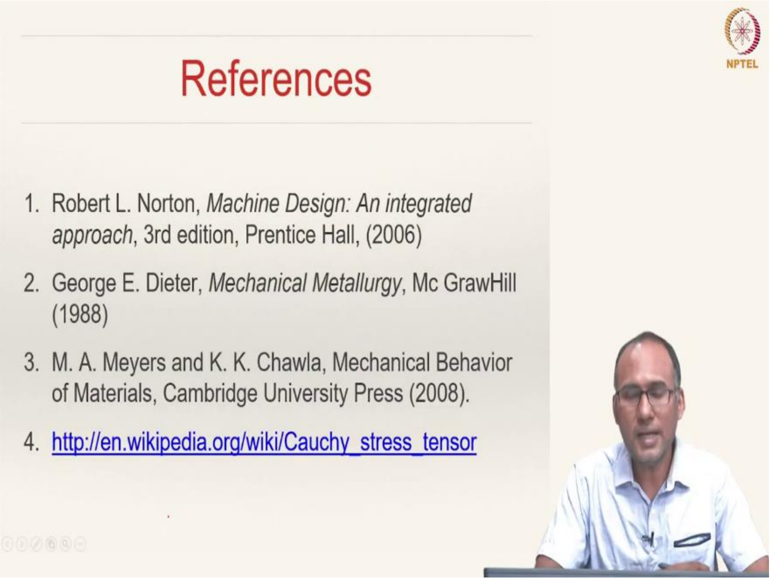

So, with that we complete the static failure theories and in the next class, we will look at fatigue failure theories, wherein the load applied on the material or a component is going to change as a function of time. Until now, one of the assumptions when we are dealing with static failure theories was the load does not change as a function of time. But what happens when the load changes as a function of time is something that we are going to look at in fatigue failure theories.

Thank you very much.