Basics of Materials Engineering
Prof. Ratna Kumar Annabattula
Department of Mechanical Engineering
Indian Institute of Technology, Madras

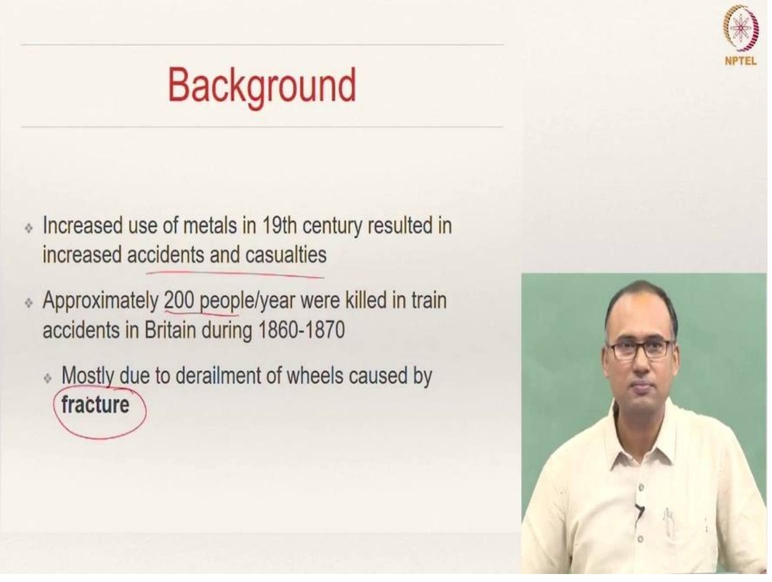
Lecture - 38
Introduction to Fracture Mechanics

The next module that we are going to look at is a very basic introduction to fracture mechanics. We are in no way trying to do a detailed study of fracture mechanics because this subject itself is a separate field.

We are only going to have a gentle introduction as this introduction is required for us to do the fatigue failure theories, because the concept of fatigue failure theories assumes the presence of a crack and then, we will be talking about crack growth and so on and hence, we need to understand what happens when there is a crack in a material.

Until now, when we were talking about static failure theories, we have assumed that there is no crack in the material. When we say crack, we are not actually talking about cracks at the atomistic scale, rather we are talking about a macroscopic crack which can be visualized. These are the kind of cracks that actually can alter the behavior of the structure when there are no cracks present. The science of such bodies which have cracks in them and their mechanical behavior is called fracture mechanics.

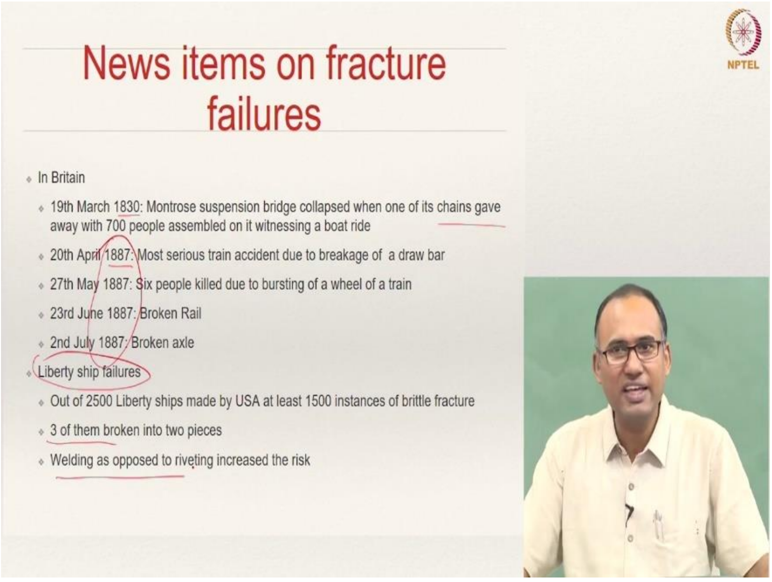
(Refer Slide Time: 01:43)



The advent of fracture mechanics or the need for fracture mechanics primarily arised after industrial revolution. When people started using the metals in large quantities and you see machine components or structures made of these materials almost all walks of life and particularly with the increased use of these metals around 19th century, people have observed increased number of accidents and causalities.

In UK alone, approximately 200 people per year were killed on train accidents. The early railway was there in UK and also in mainland Europe. So, in UK alone, they had reported 200 accidents per year particularly around 1880's and so on. Most of these failures are due to some fracture in the wheels or axles and so on. So, the main reason for them was the fracture.

(Refer Slide Time: 03:05)



There were certain news items around that time in Britain. In 1830, a suspension bridge in Montrose collapsed when one of its chains gave away, while there were about 700 people standing on it and watching a boat ride. The year 1887 seems to be a dangerous year as far as train travel was concerned.

On 20th April, they had most serious train accident due to breakage of a draw bar in the train and then, one month later, one of the wheels got burst off; that means, there was a crack and then the wheel gave in. Another month later, there was a rail that was broken. A month later, there was an axle that was broken.

These accidents were happening very frequently, every month one accident you can imagine, right? Today it is obviously not possible to ask people to travel by train if you have something like that, right? And there were not only train accidents, there were also other kind of accidents. During World War, the liberty ship failure is one of the very saddening stories, but it is also one of the most critical problems that actually led to the study of fracture mechanics or fractured bodies in detail.

The original design of liberty ships was in UK, but then USA had to make these ships in large quantities within a very short period. The original design had riveted joints. But if you have to do the riveted joints of the hull structure, it takes longer time. So, the designers or engineers in the United States took a call to opt for a welding joint rather than a riveted joint.

About 2500 ships were made of which just by the small design change, actually a small manufacturing process change from riveting to welding. In operation, there were about 1500 instances of brittle fracture and most severe of them is 3 of them actually broke into 2 pieces, and these are all warships.

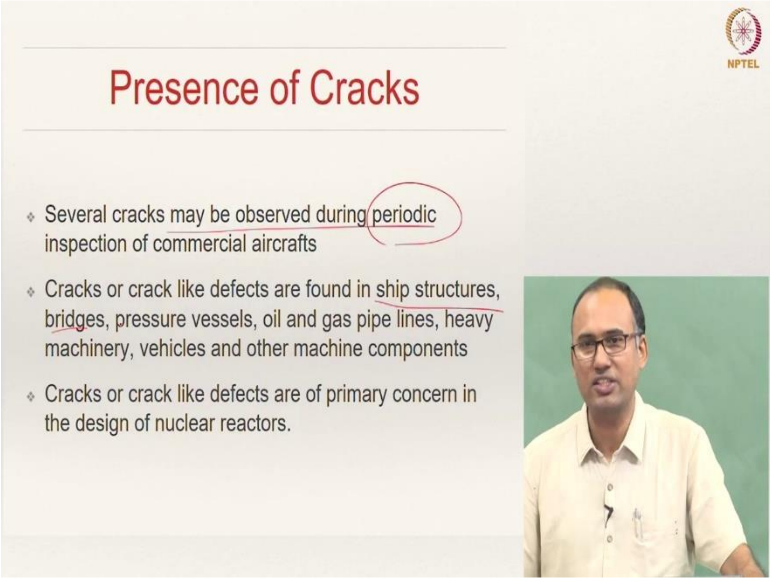
So, three of them actually broke up into two pieces and later people found out the reason was the welding process that was adopted as opposed to riveting process. And it is not really completely welding process, because when you have welding, you are prone to have these micro-cracks and they may grow in size and then, you may have pores. So, there are possibilities for you to have initial imperfections like air holes and so on. They might actually grow in size to a large crack and then, eventually, they may break, right? But there were also other reasons why it broke.

It is not only that, people really did not think about something called ductile-brittle transition. A material which is ductile at room temperature becomes brittle at very low temperature. That is something that people did not clearly pay attention to because during the World War these ships were sailing to Europe.

In Atlantic, the temperatures during winter can drop down significantly, right? They did not really think about these ductile-brittle transitions; that could also be another reason why the cracks became critical. So, we will see what do we mean by crack becoming critical.

All these issues forced the scientific community to pay more attention to the mechanical behavior of the structures which have cracks in them. If at all there is a crack, now you are required to find out is this crack ok to live with or not. That is how the birth of fracture mechanics took place.

(Refer Slide Time: 08:24)



You have presence of cracks in almost every body. Usually, the commercial aircrafts are inspected periodically for these cracks and several times, you may observe several cracks during periodic inspection and then, you will decide whether you need to fix the crack or not.

If I tell you that there is a crack on the aircraft that you are about to board, but it is ok you can board, what will you do? You better cancel your trip, right? But the beauty of this field of fracture mechanics is that it can actually predict. I have a crack and I know what is the load this structure is going to be subjected to, then one can actually predict whether the crack that is there is going to be a dangerous crack or not, or that is going to be active or is it going to grow or not.

If you are able to predict that, then you can say that ok I can have a little bit more life to this component before I can actually fix it. So, these periodic inspections are extremely critical for components of aircraft structures. They observe using some sort of non-destructive testing such as ultrasonic waves, guided waves which people send through the structure and then, see if there is a crack. Then the ultrasonic guided wave reflects since you know the speed of sound in the material, you can see the size and position of the crack.

And then, you can sort of figure out whether that is a critical crack or not. There are several startups at IIT Madras which use this technology to study for instance pipelines; very important problem. You probably heard of DeTect Technologies and so on right; all these are startups from your seniors.

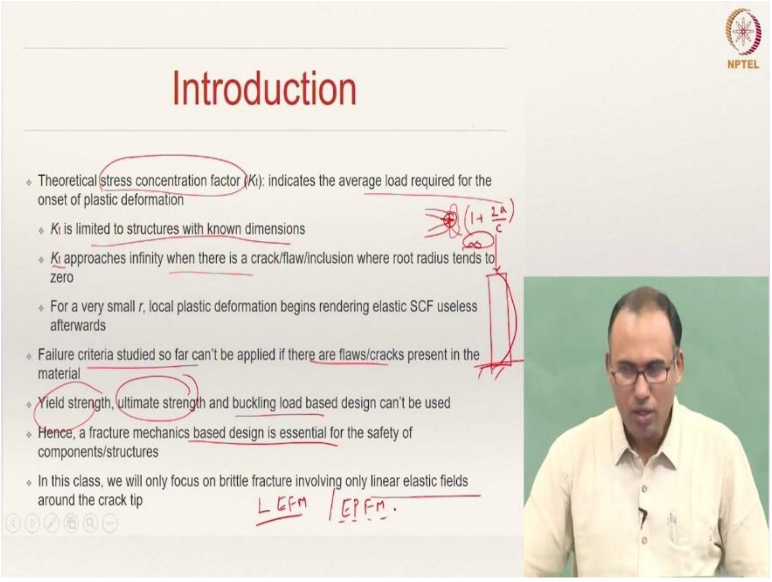
These are the kind of technologies that one can use. These are called non- destructive testing methods because you do not have to spoil your specimen which you are testing. You also have these cracks or crack like defects. When we say crack, not all of them are cracks, some of them are crack like defects like notches or these things may eventually become cracks ok or holes and so on.

They are usually found in structures like ship structures, bridges, pressure vessels, oil and gas pipelines, heavy machinery, vehicles, automobiles, ground vehicles and then several other machine components. And no matter what, you will always have these defects. As we have been discussing, defects of all kinds at all scales are basic part of every structure. You cannot say that I will not use a structure which has a small crack. It is probably ok to use it, provided you know that the crack is not going to grow.

One of the primary concerns in the design of nuclear reactors is that these cracks or crack like defects can actually be very catastrophic. If there is a crack in a nuclear reactor, it is not only the fracture of the reactor or breakage of the reactor, but what happens afterwards; the radiation effects and so on. So, the fracture mechanics principles are extremely important for designing these kinds of critical structures; whether it is an aircraft or a nuclear reactor or an automobile.

In some places, it is extremely important that you should design the structures for fracture. Before the evolution of fracture mechanics or before the introduction of fracture mechanics, people were only designing structures based on strength and deformation like what we have done until last class. Only after the evolution of fracture mechanics, it was possible for us to design structures for fracture.

(Refer Slide Time: 12:58)



In the last class, we have talked about something called stress concentration factor, right? When you have a crack like defect, a hole -- we have seen the example of an elliptic hole, we have also seen the example of a notched bar. There we have defined something called stress concentration factor.

This basically indicates the average load required for the onset of plastic deformation, right? Because we have used it for ductile failure so, when we are comparing in the design process, we are comparing our stress induced is equal to stress concentration factor times the nominal stress that is the far field stress that you have applied.

And hence, K_t is limited to structures with known dimensions because you should know the dimension. You probably remember our charts for K_t wherein depending upon the values of the ratios $\frac{D}{d}$ and $\frac{r}{d}$, you will have different K_t values.

In the case of an elliptic crack, we have seen the stress concentration factor

$$K_t = 1 + 2\frac{a}{c}$$

But you can also write that it in terms of the curvature, right? When you do that, there the radius of curvature goes into denominator.

When the radius of curvature becomes 0, then you have a sharp crack and the stress at the tip of the notch will be infinite.

As opposed to when we have seen an elliptic crack, the stress concentration factor $K_t = 1 + 2\frac{a}{c}$, right? While if this happens to be a sharp crack, then this becomes infinity. Because you can actually write this in terms of a$\sqrt{r}$ and r is the radius of the crack.

The stress concentration factor K_t approaches infinity when there is a crack/flaw/ inclusion where$\sqrt{r} \longrightarrow 0$. The stress concentration factor is predicting infinite stress, but real materials will not have infinite stress because there are some limitations, we will see what are they.

The reason why real materials will not have infinite stress is that because you cannot have exactly infinitely sharp crack. Firstly, if you have a really sharp crack, even with the slightest of externally applied load, you will impart plastic deformation here.

The moment you have plastic deformation, the stress-state around the crack tip will make the crack blunt. The moment you have a small force, it will already get into plastic deformation and as a result, your crack becomes blunt and as a result, you cannot have infinite stress; that is one thing.

Secondly, you cannot have zero radius because at the atomic scale, the radius cannot be smaller than the atom radius. No matter what, you do not have atoms of 0 size. So, no matter what you, cannot have an atomically sharp crack. You will always have some radius that you need to work with. Mathematically it is possible to have zero radius, but reality is far from what we describe mathematically.

The failure criteria studied so far cannot be applied if there are cracks or flaws. As we have already discussed, one of the important assumptions when we are writing static failure theories was that there is no crack present in the material. Hence, you cannot base your design on yield strength or ultimate strength. There is also another failure mode in materials called buckling failure which we have not discussed. In your strength of materials class, you probably have done column buckling, right? Have you done? Some of you have done, some of you have not?

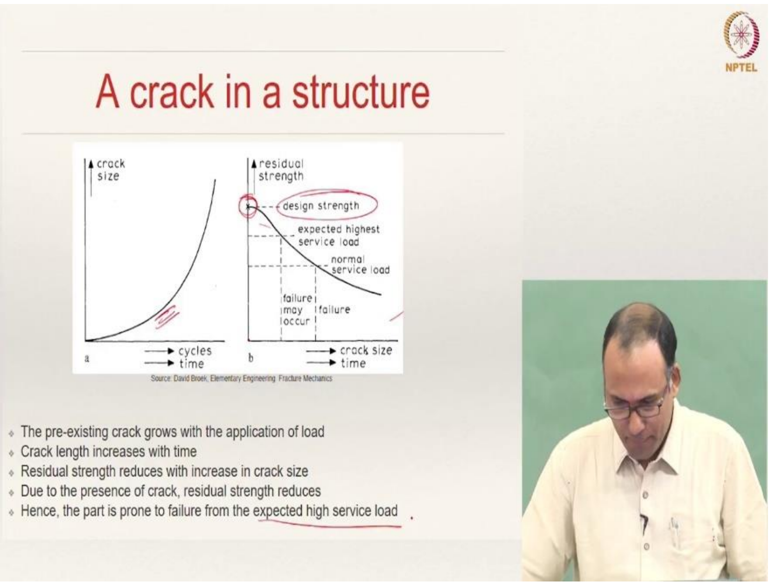
So, if you have a column and if you apply an axial load, then at some load, that column can buckle like that. You are applying an axial load and column buckles out-of-plane; that is displacement is coming this way, does that make sense? It makes sense? Isn't it violating Newton's law? If you applying a load in one direction, how can you have displacement in the other direction? Is it violating Newton's law or not?

Think about it ok. So, you can have failures like that; that is also not acceptable, because you have designed this column to be straight and if it bends like this, it is not going to serve the function for which it is designed. When you have compressive loads in the members, you also need to worry about buckling along with other failure modes.

The moment there are cracks, you cannot base your design just on these things, you should be worried about failure. So, hence, a fracture mechanics-based design is essential for the safety of any components or structures that we are defining. In this class, we assume that the stress-state around the crack tip is only linear elastic. We are not going to consider any plasticity whatsoever because that involves other complications.

We are only focusing on stress fields having linear elastic fields around the crack tip. Such an area of fracture mechanics is called Linear Elastic Fracture Mechanics (LEFM). If we have plasticity around the crack tip, then it is called Elasto-Plastic Fracture Mechanics (EPFM).

(Refer Slide Time: 19:44)



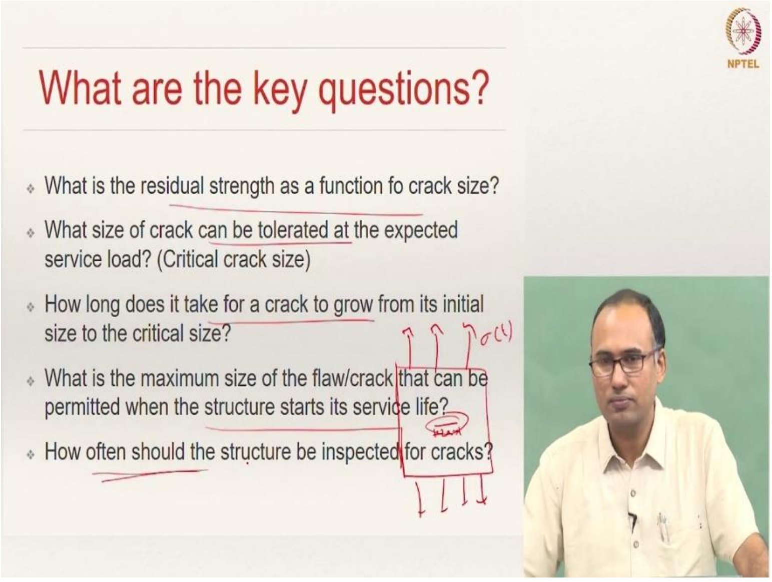
Let us say there is a pre-existing crack in a material. In the figure on the left-hand side, on the y-axis, you have crack size, on the x-axis, you have time or number of cycles of load. You are applying a cyclic load which is a time-varying load. Then, the pre-existing crack grows with the application of load. As you increase the load, the pre-existing crack can grow or with the time it can grow. So, as a result, the crack length increases with time.

Initially, you may have a small crack, but because of the application of the load, the crack length increases as the function of time. The moment you have a structure with some pre-crack, it will have some strength; that means, the load carrying capacity. The moment the length of the crack increases, what happens to the strength of the structure? It reduces; the residual strength comes down as your crack length increases.

The figure on the right-hand side is showing that when there is no crack, this is the design strength, right? So, at time equal to zero, there is no crack or you may have one crack and then the material has some strength like that. As the time is increasing, the crack length is increasing here. As the crack length is increasing, the residual strength is decreasing; because the length of the crack is increasing the amount of available area to resist the load is decreasing.

So, the part is prone to failure. This is the design strength of the part. But because of the fact that you are applying the load, the length of the crack is increasing. As a result, the residual strength is decreasing and hence, the part might actually fail before it reaches the design load. There may be a pre-mature failure of the component than the expected high service load.

(Refer Slide Time: 22:01)



What are the key questions to ask having understood how a crack behaves? We said as you increase the length of the crack, the residual strength of the component reduces. What is the residual strength as a function of crack size? Can you actually relate these two things? The second question to ask is what size of a crack can be tolerated at the expected service load.

If I have a certain crack size and if the applied load is below a certain threshold load, then it may be ok to have the crack there, because the crack is not going to grow and the material is still able to withstand the load that you are applying. So, the question to ask is, what is the critical size of the crack that can be tolerated at the expected service load. And then, as we have discussed, as a function of time, your crack grows. So, the question to ask is how long does it take for a crack to grow from its initial size to the critical size.

So, you have a component and let me say I have a crack here, I am applying a far-field load. This crack length is let us say $2a$. Now, the question is when applying far-field load, how long does it take because this is a time varying load σ as a function of t. So, how much time do I need to wait so that, the length of the crack increases to a critical size? What do we mean by critical size?

Once you reach the critical size, the crack suddenly zips through the entire structure; that means, you do not have any time. The moment it reaches critical size, it will break in a brittle manner. Until then, you do not have a problem.

How long does it take for the crack to grow from its initial size to the critical size? Given a structure and given a load, what is the maximum size of the flaw or a crack that can be permitted when the structure starts its service life? You have designed the structure, but you found a crack there and what is the maximum size of the crack that you can permit when it is starting its service load?

That means, you know what is the maximum load that this structure is going to be subjected to. If you know the load that this structure is going to be subjected to, then you can say that ok if the size of the crack is 1 mm, I am fine, if it is 1.5 mm also fine, if it is 2 mm, you have to stop, right?

For instance, take the example of an aircraft. You have cracks to begin with and you said that ok for that service load, the crack is fine; this is not a dangerous crack. But then, we know that as time progresses, as aircraft make several trips, your crack is going to grow in size. So, then, you need to verify whether the aircraft has cracks which are critical. But then, will you do this check with each and every flight?

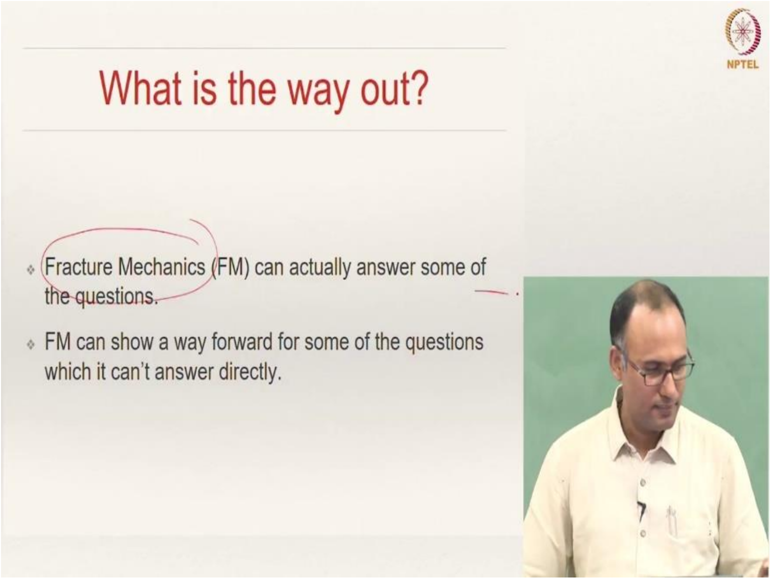
Then, airline companies will be making lot of negative profit, right? So, you should be able to prescribe how long should I wait from one check to another check. What is the time period from one check to another check or how often should I inspect the structure?

When the aircraft is flying, you know the load history. It is not going to be constant load isn't it? If it is a smooth flight, no turbulence whatsoever, then we know what is the load history from the time of take-off to landing.

But let us say, the aircraft encountered some severe turbulence. This is not the service load, and you have some load which is beyond what you expect. You need to take that into account if it is serious turbulence or ok with acceptable turbulence. Then based on that, you prescribe how many times in a month or 2 months, you need to send the aircraft for evaluation; whether the crack sizes are well within the limit or not.

So. the time should also be prescribed. These are the questions that one need to answer when we are designing something for failure or fracture.

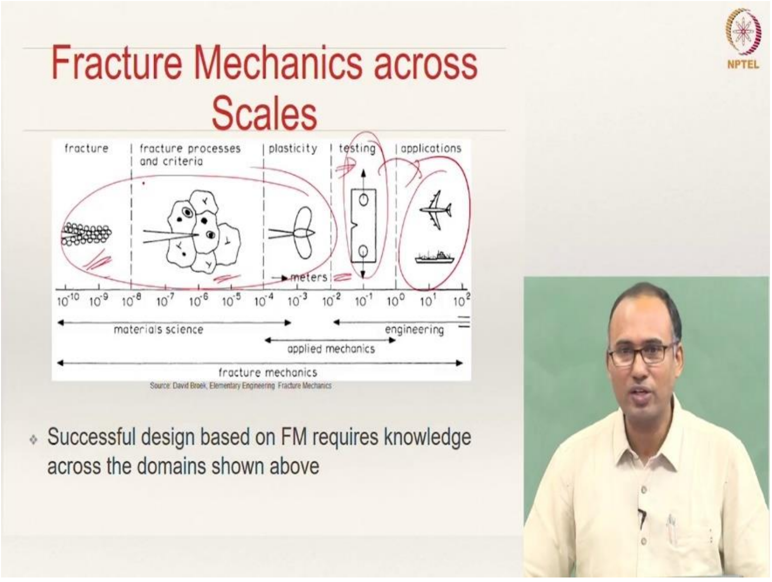
(Refer Slide Time: 27:31)



The questions that we have asked so far did not even arise when we were doing the design based on static failure theories, isn't it? Because there was no crack to begin with. But then, in order to answer the questions that we have just now posed, fracture mechanics will help us.

You can answer some of the questions, but not all of them. For some other questions, it cannot give you direct answers, but it can show you a way to find the answers.

(Refer Slide Time: 28:17)

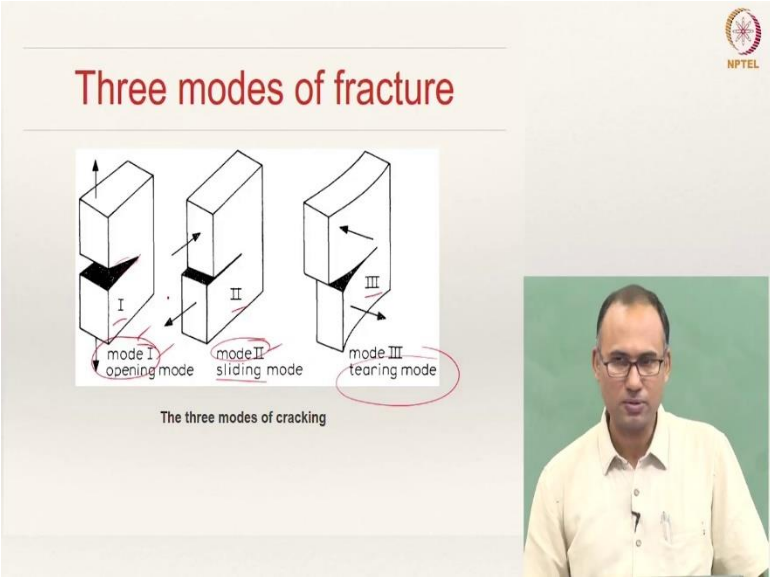


You can study fracture behavior at different scales and many times as engineers, we are interested in designing structures at the macroscopic scale for safety against fracture failure. However, the microscopic mechanism for fracture starts at this scale, but you do not necessarily study how a crack is propagating by looking at an FCC crystal structure and each and every atom and how the crack is propagating; that is the job of somebody else. If you are designing structures, components at that small length scales, then you need to worry about those mechanisms.

Many times, we develop theories which would homogenize the information that is observed or the mechanisms that are observed at these scales to a reasonable length scale -- at the laboratory testing scale and then, based on this information, these structures will be designed. But the information that we are prescribing here actually come from the microscopic mechanisms happening at these length scales. But we will not use those phenomena directly, but indirectly through some parameters.

However, you need to know what is happening at the microscopic scale. What does fracture mean? Breakage of bonds between atoms, right? So, you are creating two new surfaces. Until then, there was no new surface and then, the moment a crack propagates, a new surface forms; that means, you have broken the bonds. That is the fundamental mechanism. But what is responsible for that is something that will become clear depending upon the kind of the material that you are using, depending on the kind of the microstructure that you have for that particular material and so on.

(Refer Slide Time: 30:24)



When we are studying fracture, you have three modes of fracture: mode I, mode II, mode III. Mode I is also called opening mode. Let us say this is the crack face and the load is applied perpendicular to the crack face. Because the load tends to open the crack surfaces, it is called opening mode. Mode II is called shearing mode or sliding mode. The load is parallel to the surface you can see. Mode III is the tearing mode.

In this class, we will mainly focus on mode I, but in general, you can have all three modes present. If a crack is expanding or propagating, it is possible that all the three modes of mechanisms are present and such a fracture is called mixed-mode fracture. But here, we are only going to focus on mode I fracture. If you take a complete fracture mechanics course, several of them are available on this campus, then you will study what is the effect of having mode II and mode III also alongside mode I fracture.

(Refer Slide Time: 32:08)

This is an interesting experiment called as the Griffith's experiment. He is the father of fracture mechanics for us. So, he did this very interesting experiment. You can also do this experiment yourselves. You have a sheet of paper which is unstretched and then, you stretch it by applying certain force and then, fix it; that means, hold the grips there. Can you do this third experiment?

You can do this experiment using paper or using a polymer sheet; that is much easier to observe. So, you have stretched it and then are holding the grips; that means it is a displacement-controlled experiment where you are not allowing any further displacement. Is that clear?

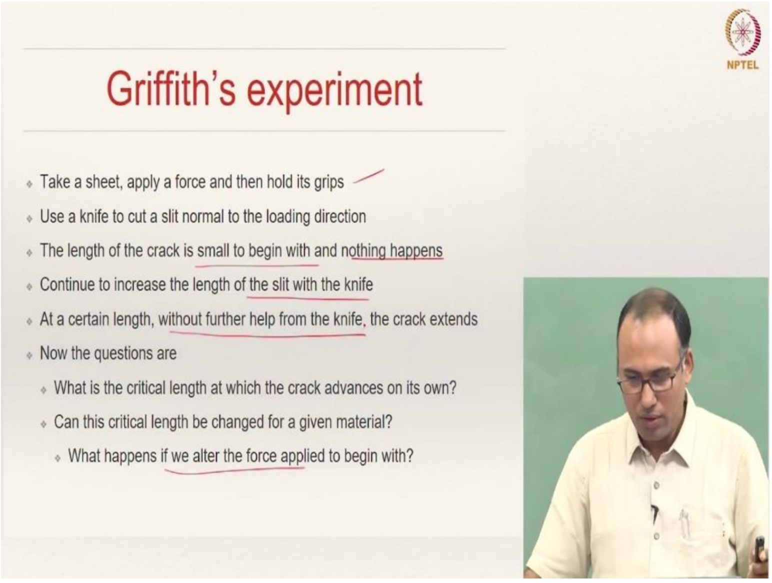
Now, use a scissors or a knife and make a small slit in the center somewhere here. You can do that probably with a paper also. Apply load, make sure that it is not breaking and hold it, ask your friend to hold it. It is very difficult to control the displacement, but let say that your friend is having a feedback loop allowing him or her to actually control that. And then make a small slit may be using a knife. Then, what happens is the system will not break. You probably have done this with polythene sheets when you were a kid. If you have not done that, do it today.

Then, you ask your friend to continue to hold this and then increase the length of the slit. At some point, the length of this crack becomes sufficiently large, after which it suddenly breaks. You can feel that when you are doing this thought experiment.

Let us say the crack length is $2a$. What is happening now?

So, he is Griffith, he is the one responsible for us to understand design of materials based on fracture.

(Refer Slide Time: 35:03)

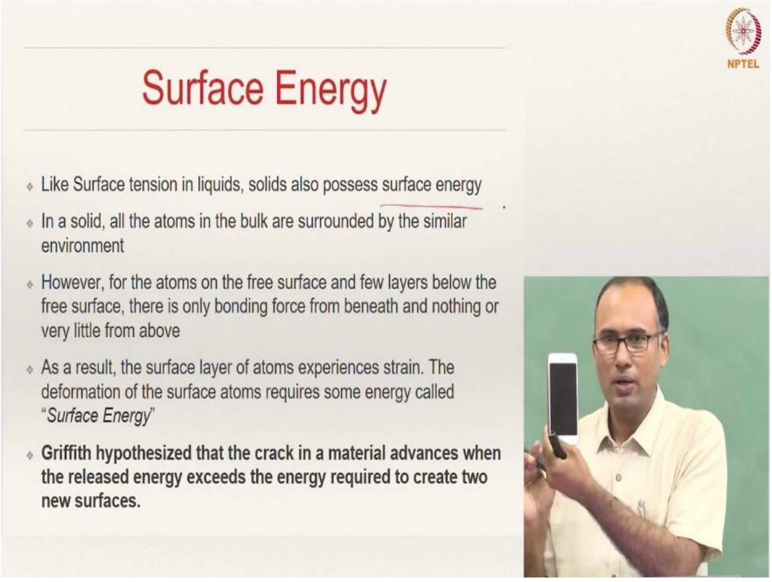


So, take a sheet, apply a force at the hold, then hold its grips and use a knife to cut a slit normal to the loading direction to ensure mode I, otherwise its little bit trickier, right? In the beginning, make sure that you do not cut a long slit, cut a small slit to begin with. So, the length of the crack is small to begin with and nothing happens. Continue to increase the length of the slit with the knife. At certain length, as we have discussed, without any further help from knife, the crack extends.

Now, the questions are: what is the critical length at which the crack advances on its own? Does this critical length change from material to material? Does it depend on the material? If you take a polymer sheet, a paper or a metal sheet, does this critical length change?

All of them are applied with the same force. If you take a material, does this critical length change as a function of the initial load that you have applied? What happens if we alter the force applied to begin with? These are the question that we need to answer.

(Refer Slide Time: 36:28)



What is happening when you are cutting a slit to the system? You probably heard of surface energy, right? Like surface tension for liquids, for solids we have something called surface energy. It is reasonably easy to understand.

Suppose, you take a material and look at the bulk; at the bulk take an atom. So, if I take this mobile phone and if I consider an atom at the center, not on the surface, but somewhere in the middle through the thickness, then that atom is surrounded by several atoms; that means it is the bond strength, the bond resistance offered by other atoms has some value.

But if you go from bulk to the surface, the surface atoms are primarily subjected to the bond forces atoms below them and above them there is nothing, right? As a result, there will be some strain, they can accommodate some strain because there is no bond above them. And the energy associated with that strain is what is called surface energy.

For the atoms on the free surface and few layers below the free surface, there is only bonding force from beneath and nothing or very little from above. If it is a surface layer, there is nothing above, and one layer below the surface there is little bit and so on. As a result, the surface layer of atoms experiences a strain and the deformation of the surface atoms requires some energy and that is called your surface energy.

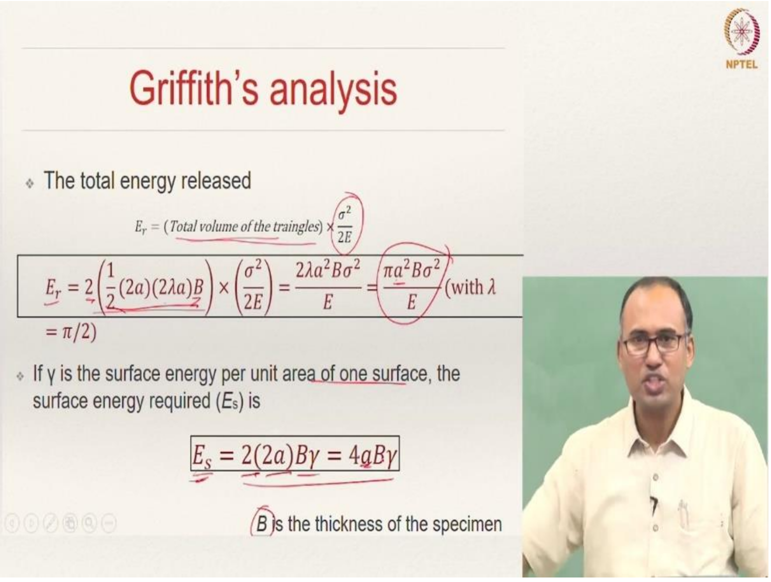
What Griffith hypothesized from the previous experiment is that the crack in a material advances when the released energy exceeds the energy required to create two new surfaces.

What do we mean by release energy? What Griffith hypothesized from this experiment is that when you are creating a slit, you are creating new surfaces, right? Whenever you are creating two new surfaces, you have to provide this additional energy.

How did you provide this additional energy? When you are applying the load, there is some strain energy stored in the material, because you have deformed the material.

When you are creating the slit, the material around the slit will release some strain energy and that strain energy will be used for creation of these two new surfaces. What is the extent of the material that I need to consider? Griffith hypothesized that take two triangles, one triangle above, one triangle below. and where this length is $2a$ and he says that this will be $\lambda \times 2a$, where λ is a constant. This is the volume of the material that will actually contribute to release of the strain energy. That energy will be utilized for the creation of the surfaces.

(Refer Slide Time: 39:35)



Since we are only considering linear elastic materials, the strain energy density i.e. the strain energy per unit volume of the material is $\frac{\sigma^2}{2E}$. We have done that in the previous modules. Let us assume that B is the out-of-plane thickness.

This is the volume of one triangle -- I should not be calling it triangle; the end view is triangle, but it is extruded in the direction having thickness B; it is like a prism.

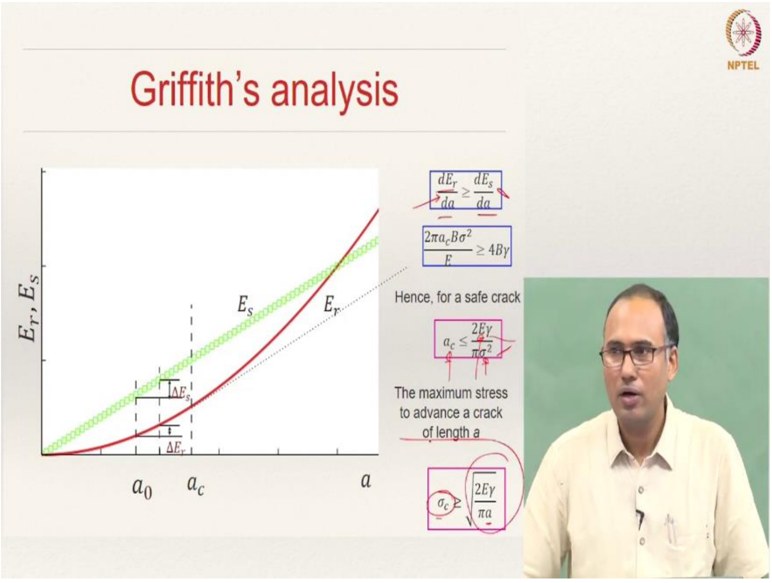
The energy released can be calculated as

$$E_r = 2\left(\frac{1}{2}(2a)(2\lambda a)B\right) \times \left(\frac{\sigma^2}{2E}\right) = \frac{\pi a^2 B\sigma^2}{E}\left(\text{with } \lambda = \frac{\pi}{2}\right)$$

If γ is the surface energy per unit area of one surface, how many surfaces we created? Two surfaces. So, the total surface energy is

$$E_s = 2(2a)B\gamma = 4aB\gamma$$

(Refer Slide Time: 41:15)



The released energy scales as a^2; that means, it is a parabola and surface energy is proportional to a; that means, it is a straight line. What Griffith said was, when the energy release rate becomes greater than the surface energy rate -- here when we say rate, it is not time rate, but it is energy released per unit crack length.

The crack propagates when

$$\frac{dE_r}{da} > \frac{dE_s}{da}$$

Firstly, E_r should be equal to E_s, and $\frac{dE_r}{da} > \frac{dE_s}{da}$.

You know the equations for E_r and E_s and you can actually show that for the crack to propagate,

$$\frac{2\pi a_c B\sigma^2}{E} \geq 4B\gamma$$

Hence, if you have to have a safe crack,

$$a_c \leq \frac{2E\gamma}{\pi\sigma^2}$$

Now, the questions that we have asked, does the critical length change from material to material? Yes, it does because it depends on the E and γ. Does it depend on the applied force? Yes, it does because the σ is the applied stress, right?

If you know the initial crack length, is it catastrophic? The maximum stress to advance a crack size is given by,

$$\sigma_c \geq \sqrt{\frac{2E\gamma}{\pi a}}$$

So, if I know my initial crack to be a, for that crack to be catastrophic, I should be having a stress greater than σ_c.