Basics of Materials Engineering
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Lecture - 39
Fracture Mechanics (Energy Release Rate)

In the last class, we looked at the Griffith's analysis. If you apply a load on a member, while holding it with the grips and then insert a slit, as you continue to increase the length of the slit, at a particular length, the slit zips through and the crack propagates suddenly. Then, we have looked at the reason for creation of the surface: the strain energy that is present in the material is released and that is what is responsible for the creation of the two new surfaces.

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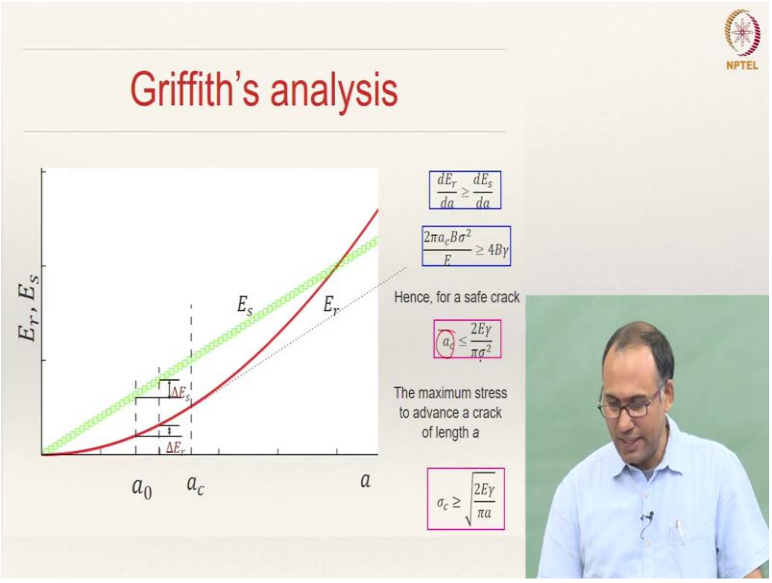

Then, we looked at the condition for the safe crack given by,

$$a_c \leq \frac{2E\gamma}{\pi\sigma^2}$$

If the slit length that you have introduced is less than this value, then the slit does not suddenly propagate or expand. But the moment this condition is not met, i.e., as soon as the length of the crack is larger than this value, then the crack suddenly zips through.

Because, then you meet the condition $\frac{dE_r}{da} > \frac{dE_s}{da}$.

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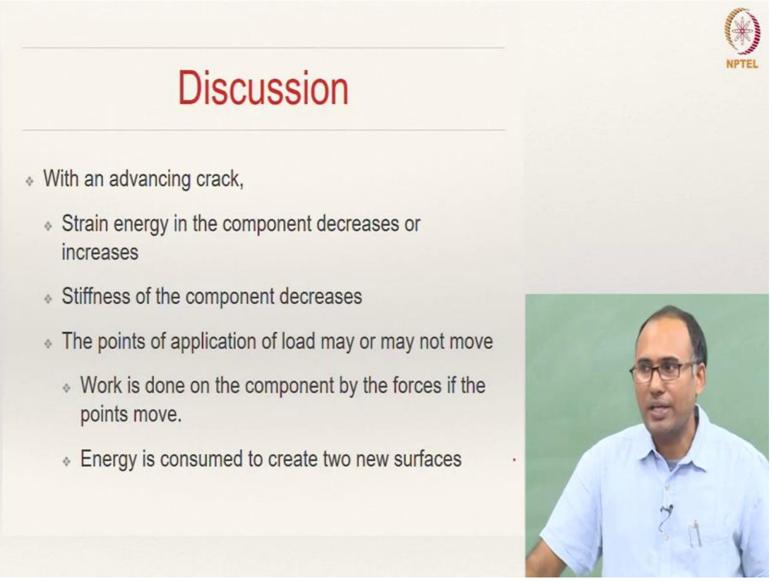

When we are having a body with a crack present, the strain energy in the component; it can either increase or decrease depending upon the kind of boundary conditions that we have, when the crack is advancing in the body.

Nevertheless, whenever you are having a crack in the body, the stiffness of the body reduces. As you increase the length of the crack, the stiffness of the body reduces. That means, the amount of force that is required to create unit displacement is going to go down, right? That is what we mean by stiffness. And sometimes, wherever you are applying the load; that means the point of application of the load can actually move or stay remain. If the point of application of the load moves, then you have work done on the system.

If the point where you have applied load does not move, then there is no work done, right? When we are generating two new surfaces, that is the creation of the crack, then you are consuming some energy. That energy must be either coming from the strain energy or the external work done or both.

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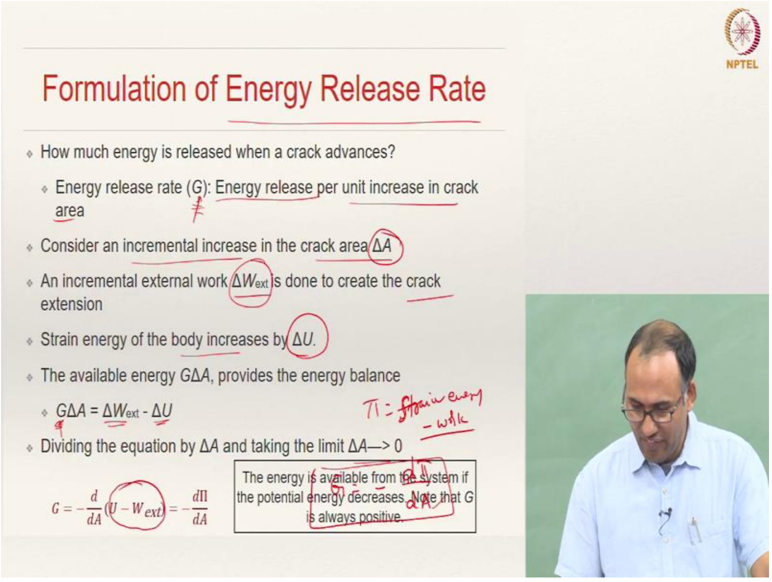

Now we look at what do we mean by this Energy Release Rate. A very important component that you need to be aware of whenever we are talking about crack growth in a material. So, the energy release rate is usually represented by word G in honour of Griffin.

Whenever you are in a fracture mechanics class, G is the energy release rate. What is the definition of energy release rate? Energy release per unit increase in crack area.

Let us now consider an incremental increase in the crack area to be ΔA. It is the increase in the crack area and an incremental external work will be ΔW_{ext}. W_{ext} is the external work. The subscript *ext* means external work that is done to create the crack extension.

When you are doing some work on the body, then you are obviously imparting some strain energy to the body. So, the strain energy of the body during this process increases by an amount ΔU. Then you can write the balance.

$$G\Delta A = \Delta W_{\text{ext}} - \Delta U$$

A part of the external work that you are putting in, is going into increasing the strain energy of the body and a part of it is going to crack propagation and that is what is responsible for this energy release rate.

And now if you divide the both sides with ΔA and take the limit $\Delta A \longrightarrow 0$, then you get,

$$G = -\frac{d}{dA}(U - W_{\text{ext}}) = -\frac{d\Pi}{dA}$$

The difference between the strain energy and external work is the potential energy Π of the body.

That is the relation between the energy release rate and potential energy of the system and A is the crack area. Please note that the energy release rate G is always positive. Hence,

$$\frac{d\Pi}{dA} < 0$$

The energy is available from the system if the potential energy decreases. Only when the potential energy decreases, that is when the energy is available for crack growth, because G is always positive. That is negative means potential energy is decreasing. If it is increasing, then it will be positive isn't it? So, that is the condition.

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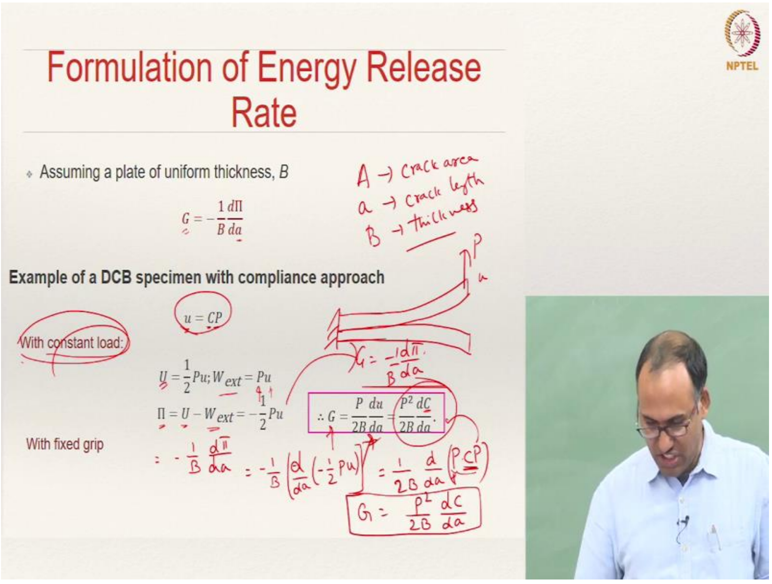

Let us now assume a plate of uniform thickness B. Then

$$G = -\frac{1}{B}\frac{d\Pi}{da}$$

Note that A is the crack area and a is the crack length. B is the thickness of the body. Let us consider the example of a DCB i.e., double cantilever beam specimen, which looks something like this.

We are taking the example of a DCB specimen with compliance approach. We will look at what the compliance approach means. If you know the compliance of the material to be C, and if the applied load is P, the displacement u corresponding to that, is given by,

$$u = CP$$

Compliance is the inverse of stiffness. Is that clear? Let us say you have applied constant load. The load is fixed. Then your strain energy can be written as

$$U = \frac{1}{2}Pu$$

The external work is

$$W_{\text{ext}} = Pu$$

Now I can write,

$$\Pi = U - W_{\text{ext}} = -\frac{1}{2}Pu$$

$$G = -\frac{1}{B}\frac{d\Pi}{da} = -\frac{1}{B}\frac{d}{da}\left(-\frac{1}{2}Pu\right) = \frac{1}{2B}\frac{d}{da}(P \times CP) = \frac{P^2}{2B}\frac{dC}{da}$$

Since the applied load P is constant, it can be brought out of the differentiation as shown above. If P is not constant, I could not bring it out right because the condition is with the constant load is what we are trying to do. So, the energy release rate when I have a constant load is given as,

$$G = \frac{P^2}{2B}\frac{dC}{da}$$

Now let us fix the displacement instead of fixing the load. Fixed grip means the displacement is fixed. That means, u is constant and as the crack advances, P changes because the reaction force is changing. So, can you find the expression for G when you have fixed grip?

So, when we have fixed grip, what happens to U and W_{ext}? What is W_{ext} when you have fixed grip? What do you mean by fixed grip? The point is not moving, right? You are holding the displacement. That means, wherever you are applying the load the position at which you are applying the load that is fixed in space. That means, there is no displacement associated with that force.

What is the work done? The external work done is 0 and hence,

$$\Pi = U - W_{\text{ext}} = \frac{1}{2}Pu - 0 = \frac{1}{2}Pu$$

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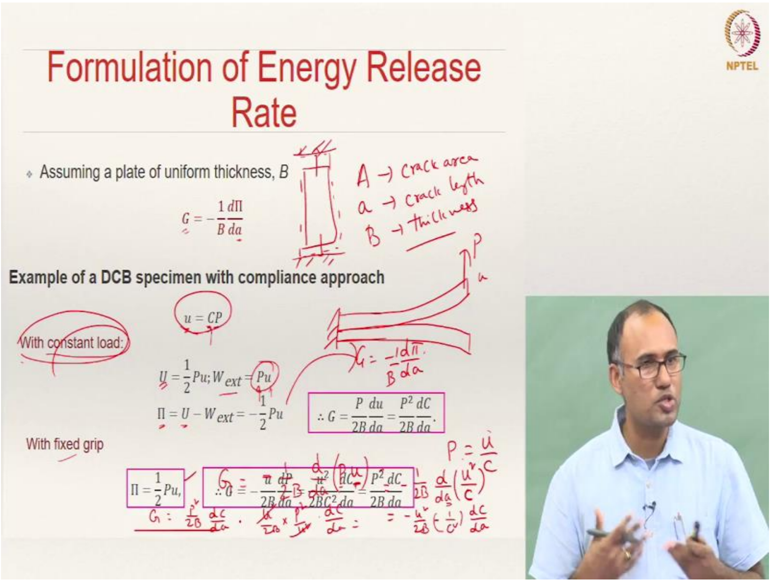

In this case, the displacement is constant and not the force. So, P should be written in terms of the displacement because you have to find the derivative with respect to P.

$$G = -\frac{1}{B}\frac{d\Pi}{da} = -\frac{1}{B}\frac{d}{da}\left(\frac{1}{2}Pu\right) = -\frac{1}{2B}\frac{d}{da}\left(\frac{u^2}{C}\right)$$

Student: Sir we do not have any displacement change?

Change in displacement is 0. You have applied some displacement and then you are holding there. So, the initial strain energy that you are putting in is $\frac{1}{2}Pu$ and u is not changing afterwards. So, you have a body and to this body, you have applied some displacement and there you are holding; that means at that displacement, you are fixing. That is what you mean by fixed grip experiment.

Imagine that now you have this body you are holding at the displacement and now you have a crack. If you are not changing the displacement, as the crack propagates, what happens to the force? So, when you are holding at this particular position, there will be some reaction force at this position, right?

Here you are prescribing displacement. So, at the ends you will have reaction force and as the crack is extending, what happens to that reaction force? It reduces because the stiffness is reducing. In the previous case, what happens? You have applied some force, you got some displacement and you are maintaining the force same. So, when you are maintaining the same force and the crack is propagating, what will happen to displacement?

It increases because, for the same force your stiffness of the component is decreasing as you are increasing the crack length.

Student: What is compliance?

Compliance is opposite of stiffness. Stiffness is force over displacement, right? Compliance is reciprocal of that. You can see here C is your compliance right, C is displacement by force. So, now what will be this one? So, as your crack length is changing here, will u change? It is a fixed grip case, so u is not changing. So, u can be brought out or not. Can I bring u out? We have,

$$G = -\frac{1}{2B}\frac{d}{da}\left(\frac{u^2}{C}\right) = -\frac{u^2}{2B}\frac{d}{da}\left(\frac{1}{C}\right) = -\frac{u^2}{2B} \times -\frac{1}{C^2} \times \frac{dC}{da} = \frac{1}{2B}\left(\frac{u^2}{C^2}\right)\frac{dC}{da}$$

Since $P = \frac{u}{C}$, the above equation can be simplified as,

$$G = \frac{P^2}{2B}\frac{dC}{da}$$

Whether I do constant grip or constant force, G is the same in both cases. Similarly, if you are given any other problem, you should be able to calculate what is the energy release rate. Is that clear?

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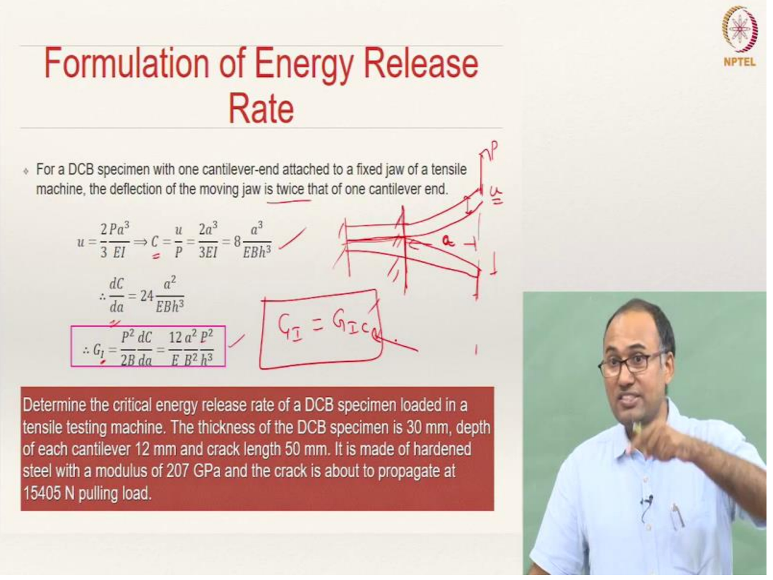

So, for a DCB specimen with one cantilever end attached to a fixed jaw, that means the left-hand side is fixed. The deflection of the moving jaw is twice that of the cantilever end because you have double cantilever. If you have fixed this to one end, another end you are pulling.

So, the displacement that we are talking about is the twice the displacement because you have two cantilevers, this is one unit this is another unit. Both of them are moving symmetrically. The total displacement is twice that of the single cantilever displacement.

So, if you have a cantilever with an end load, what is the deflection of the cantilever? $\frac{Pl^3}{3EI}$, right? In this double cantilever, it is fixed here. There may be bond here, but you can see that this is like a fixed end, then this is a; this is the crack length.

Can you see that? If that is the crack length, then this displacement will be $\frac{Pa^3}{3EI}$, but you have two cantilevers the total displacement will be,

$$u = \frac{2}{3}\frac{Pa^3}{EI} \Rightarrow C = \frac{u}{P} = 8\frac{a^3}{EBh^3}$$

$$\therefore \frac{dC}{da} = 24\frac{a^2}{EBh^3}$$

$$\therefore G_I = \frac{P^2}{2B}\frac{dC}{da} = \frac{12}{E}\frac{a^2}{B^2}\frac{P^2}{h^3}$$

That is the energy release rate of a double cantilever with initial crack length a, applied external load P, elastic modulus E, out-of-plane thickness B. What is h? That is your h, right? Please note that I have put I here.

What is the reason for that one? That is what is called mode 1 energy release rate because it is an opening mode, right? We have seen three different kinds of modes. Mode 1 fracture, mode 2 facture and mode 3 fracture. Here it is clearly an opening mode and hence, it is written asG_I.

This is a problem wherein all the other dimensions are given and you are required to calculate the energy release rate. **Determine the critical energy release rate of a DCB specimen loaded in a tensile testing machine. The thickness of the specimen is 30 mm, depth of each cantilever is 20 mm, crack length is 50 mm. It is made of a hardened steel and the crack is about to propagate**. That means, this is the load at which the crack is ready to propagate.

The problem asks you to calculate critical energy release rate. Please pay attention here. G_I whatever you have calculated here is energy release rate and this is dependent on the load that you apply, geometry of the body and the crack length. For any given load, you have some energy release rate, but when does the crack propagate?

The crack starts propagating when the energy release rate in the material becomes equal to the critical energy release rate of the material. Critical energy release rate is a property of the material. Energy release rate is the response of the material to the applied load due to its geometry and applied load.

G_{IC} is called critical energy release rate. This is a material property. For each material, you will have a critical energy release rate. That means, mode 1 critical energy release rate. If the same material is loaded in mode 2, the critical energy release rate in mode 2 will not be same as the critical energy release rate in mode 1.

The critical energy release rate depends on the type of mode or mode of fracture; that is important to know. So, here he is saying that this is the force at which the material is about to give in; that means, that is the critical energy release rate; that is why the question asks you to calculate critical energy release rate because that is when the crack is ready to grow.

You can plug in these numbers and you will be able to figure out what this number is. What is the unit of energy release rate?

Student: (Refer Time: 25:16).

What is the definition of energy release rate?

Student: (Refer Time: 25:20).

Jm^{-2}, because energy released per unit area of the crack. So, it is not Watts. Energy released per unit time is Watt. It is not Watt; it is Jm^{-2}.