Basics of Materials Engineering

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Lecture – 40
Fracture Mechanics (Crack Resistance, Stress Intensity Factor, Fracture Toughness)

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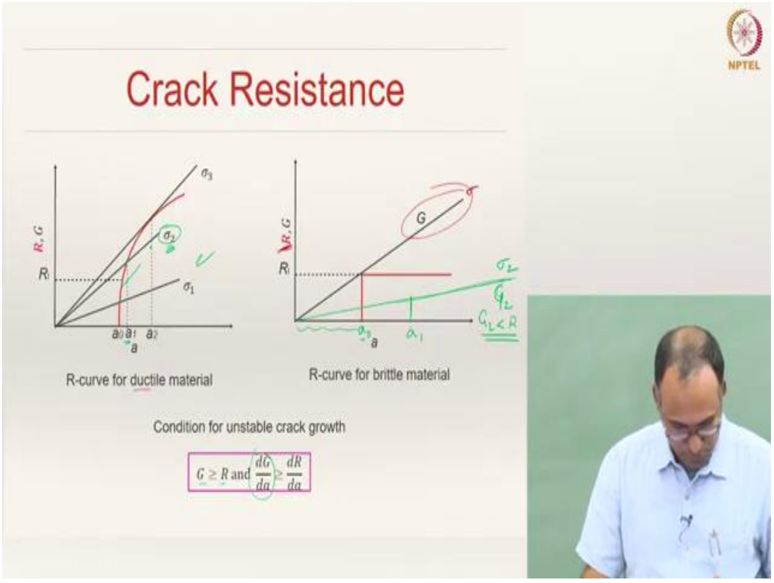

Let us now talk about the concept of crack resistance. Here you have two curves; the left-hand side is the crack resistance curve also called as R-curve, for a ductile material and the right-hand side you have an R-curve for a brittle material. The R-curve is a material property, it is like the critical energy release rate. It represents the resistance offered by the material for the crack growth. For a brittle material, the critical energy release rate does not change as crack grows, it is constant.

Let us say this is a_0. If the crack length is below a_0, that crack does not propagate. Let us say this is a loading curve which gives you energy release rate G corresponding to an applied load σ.

Let us say you have another loading curve, corresponding to an applied load σ_2 that corresponds to the evolution of G. We have calculated G, you remember? The energy release rate G can be calculated and let us assume that G varies like that, for a given σ_2.

For instance, you take a crack here. For this particular crack length, you have to see whether crack is going to catastrophically propagate or not; this is your crack resistance curve. The crack propagates only when $G > R$, where R is the resistance which is a material property.

Here it is less than that and hence it will not propagate, R curve is shown in red. I should not have chosen red colour; let me see if I can change my colour. So, let us say this is a_0 and let me draw this curve; the first black curve is corresponding to one particular load giving rise to that G evolution.

Let us say, this is load σ_2, that gives me another G evolution as a function of the crack length. For any material and for any value of G, if the crack length is less than a_0, there is no resistance; you do not see the red curve. So, if the crack length is less than a_0, the crack will never propagate i.e., it is not a critical crack anymore.

If crack length is greater than a_0, then will the crack always propagate? Not necessary. If the applied load is so low that the G value, say G_2 is always less than R, the crack will not propagate. So, even here you may have an increased length of the crack say a_1, but the crack will not catastrophically propagate.

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Yes. Is that clear? But here the point is the resistance is not changing, the R-curve is a constant curve. It is typical in ductile materials due to the plasticity that is prevalent ahead of the crack tip, that the resistance changes as a function of crack length.

As the crack grows, the crack tip becomes blunter. If the crack tip becomes blunt, then you have more resistance to crack growth. And that is why, here the red curve in this figure shows an increased R-curve. The crack resistance curve is increasing, that means, the resistance is increasing.

Now, you see, if I am applying a load σ_1, up to a_0 of course, there is no crack growth at all. But this is when the crack actually starts growing. As soon as you cross this, immediately the resistance is higher and hence, it will be there at a_1 and then it will not grow.

Now, when this is σ_2, you see that the crack starts growing up to a_1 and then it stops, right? The initial crack length is a_0. When you are applying load σ_2 and when this intersects the G curve, that is when $G = R$, and hence crack has to propagate.

So, the crack suddenly propagates from a_0 and when the crack reaches the length a_1, under the load σ_2, R increases and hence it cannot catastrophically propagate. It starts from a_0 and grows up to a_1 and stops there because beyond a_1, $R > G_2$.

When the G curve becomes tangent to the R curve, that is when you will have critical crack growth. Because the rate of change of G with respect to a becomes larger than rate of change of R with respect to a beyond that point. And that is when the crack suddenly propagates; that means, that is when you will have brittle fracture and the ductile material breaks like a brittle material beyond this point.

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So, the conditions for unstable crack growth are

$$G > R$$

$$\frac{dG}{da} > \frac{dR}{da}$$

That is typically the case in ductile materials. For brittle materials, the R-curve does not change and the resistance will not increase, as the crack length increases.

That is precisely the reason why whenever you throw a stone on a glass, the glass breaks all of a sudden, right? If you have a small crack, it zips through. That is because you have no increased R value. But, if you have a reinforcement to the glass, for instance, typically the car windshields have some reinforcement within.

They do not break like the glass that you would break on your window. The crack propagates up to certain distance and then it stops. Although, from the design perspective, you have to replace the windshield, but it is not going to break like the window glass breaks; that is because, there is some reinforcement which is causes the R-curve of that material is to increase as the crack is growing.

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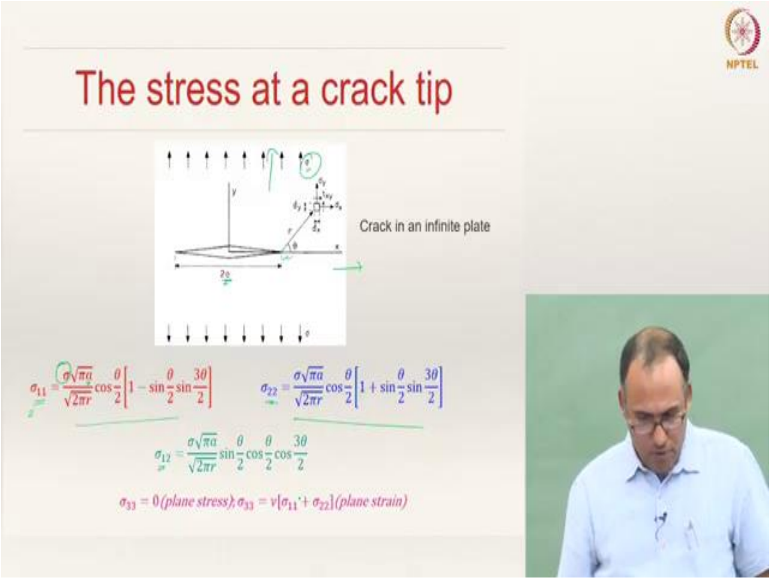

How does the stress-state ahead of a crack tip look like? You can do an elasticity course and derive this formula, but in this class; we are not doing that, I am only giving you the formula. Let us say this is the crack in an infinite body subjected to far-field stress σ and the crack length is $2a$.

If you define a polar coordinate system at the crack tip, at any position r, θ, the stresses σ_{11}, σ_{22} and σ_{12}, given by these equations clearly depend on the size of the crack a, far-field applied stress σ and the state of stress changes from crack tip to the faraway. What happens near the crack tip? What is the value of r near the crack tip?

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The value of r near the crack tip is 0 and as a result $\sigma_{11} \rightarrow \infty$. So, you would predict infinite stresses ahead of the crack tip. If you are predicting infinite stresses ahead of the crack tip, as we have discussed in the previous class, if you take a material, if you already have a crack you really do not need to apply a far-field stress. The material would break apart just by mere blowing.

But that is not what is going to happen in the real material because real materials are not going to have sharp cracks. You are going to have some crack blunting. But this solution is for perfectly sharp crack. The moment you have plasticity, you will have a local plastic deformation, and then there will be local crack blunting; so, you need to take that into account when we are doing that. However, this is a solution assuming only linear elasticity.

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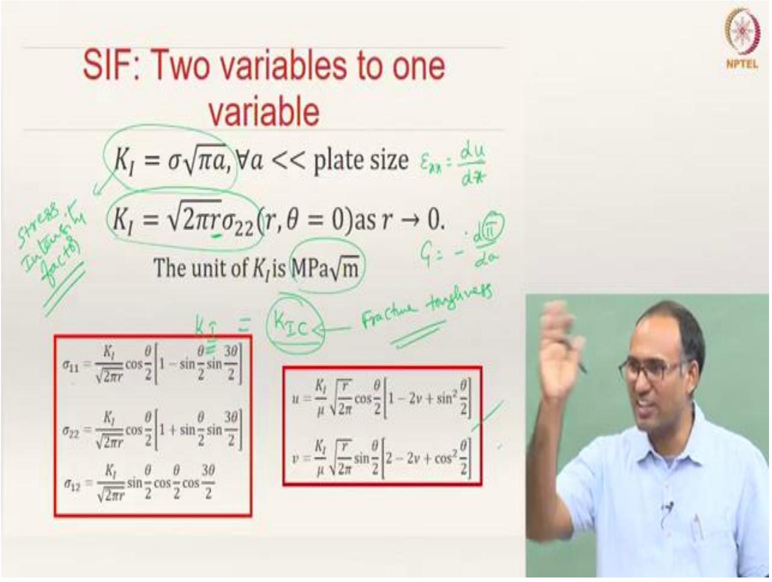

The stresses depend on the applied stress σ and the crack size a. You can put these two variables together and define a new variable K_1, called as the stress intensity factor, given by,

$$K_1 = \sigma\sqrt{\pi a}$$

The SI units of this parameter are MPa$\sqrt{\text{m}}$. K_1 sort of represents the vulnerability of a crack, i.e., how vulnerable a crack is for propagation.

When the stress intensity factor at the crack tip reaches some critical value, that is when crack propagates. The condition is given by.

$$K_1 = K_{1c}$$

The subscript 1 indicates mode 1. Similarly, you will have mode 2 stress intensity factor, mode 3 stress intensity factor; and they will not be same as mode 1 stress intensity factor.

The crack propagates when the above condition is met, until then crack does not propagate. Similar to G_{1c}, which is the critical energy release rate, K_{1c}, called as the critical stress intensity factor or also the fracture toughness, is a material property.

Now, the stresses σ_{11}, σ_{22} and σ_{12} can be written in terms of K_1. The far-field stress is applied perpendicular to the crack's face and hence this is mode 1 loading. The displacements ahead of the crack tip can also be defined in this way. How does one write the stress-strain relations?

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu\left(\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}\right)$$

You can calculate strains by knowing stresses. What will you get if you integrate strains?

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What is strain? How do you define ϵ_{xx}?

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For small strains, it is one-dimensional, $\epsilon_{xx} = \frac{du}{dx}$, where u is the displacement in x direction. Isn't it the definition of strain? This is the proper way to define strain. How will you get displacement? You integrate strains to get the displacements and apply boundary conditions, right?

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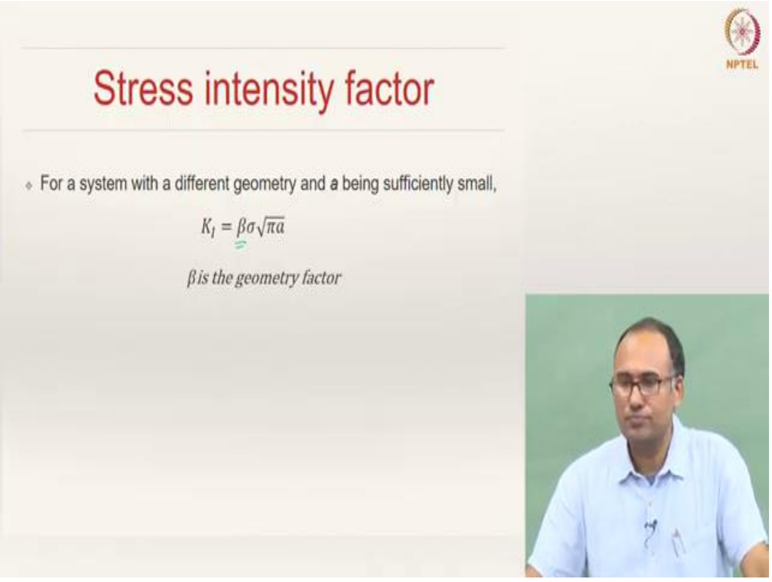

However, the stress intensity factor that we have defined strongly depends on the geometry. K_1 that we have defined is true only for infinite plate geometry; that means, the size of the crack $2a$ is much larger than the size of the plate. But it is not necessary to have that sort of a condition all the time.

Then, you need to account for the violation of the constraints that the size of the crack being much smaller than the plate size. If the size of the crack is comparable to the plate size, then you have to add a geometric factor to the definition of K_1 and that geometry factor is given by β, i.e.,

$$K_1 = \beta\sigma\sqrt{\pi a}$$

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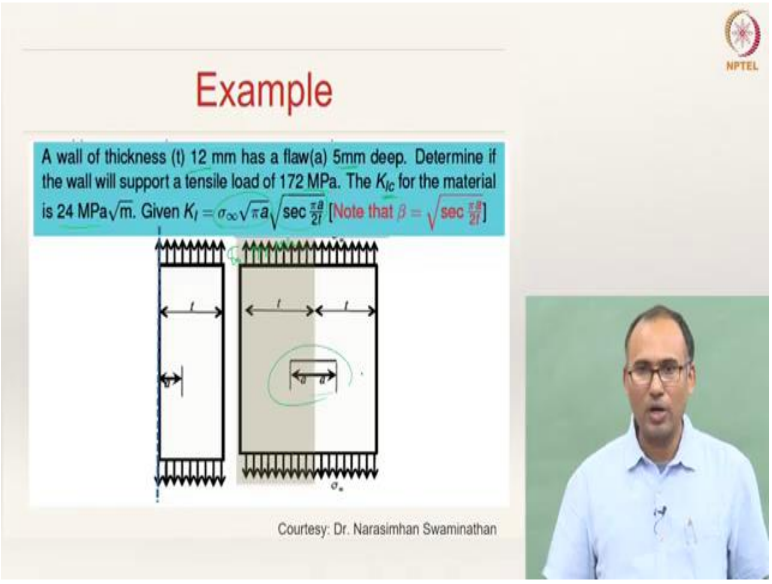

Here is an example problem. So, you have a wall of thickness 12 mm which has flaw size that is 5 mm deep. So, you have a crack 5 mm deep; you need to determine if the wall can support a tensile load $\sigma_\infty = 172$ MPa; that means whether the crack will propagate or not, given $K_{1c} = 24$ MPa$\sqrt{m}$. Normally, $K_1 = \sigma\sqrt{\pi a}$, but here the flaw is 5 mm and thickness is 12 mm; so, they are comparable.

So, you cannot neglect the geometry effects. The geometry effect is given by,

$$\beta = \sqrt{\sec\frac{\pi a}{2t}}$$

The geometric factor needs to be included while calculating K_1. So, the formula that we have is for crack length of $2a$, right? So, you consider the symmetry and then you will have to add that one. So, this is exactly similar to what the problem that we have looked at, when we have defined K_1.

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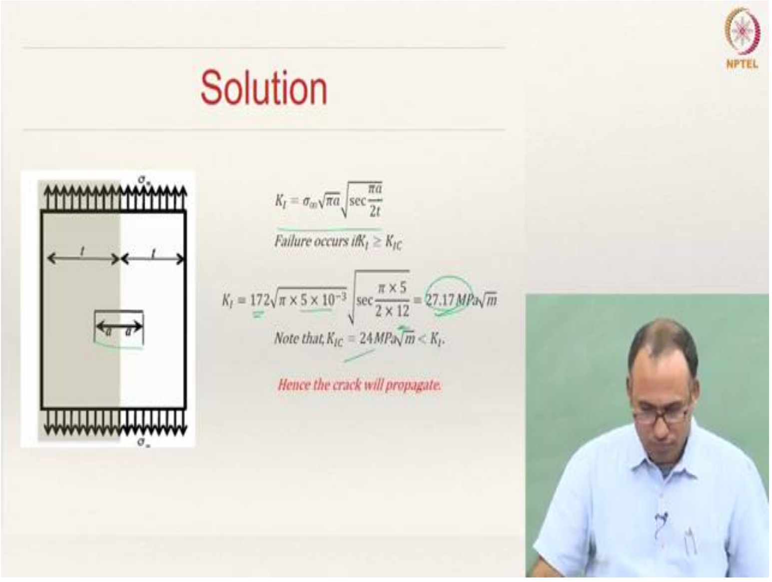

This is the expression for K_1. Failure occurs if K_1 is greater than K_{1c}. So, now we need to calculate K_1. 172 MPa is the far-field stress, $2a$ is total distance, but a is the flaw size. We see that $K_1 > K_{1c}$ and hence the crack propagates. So, the wall cannot support that load.

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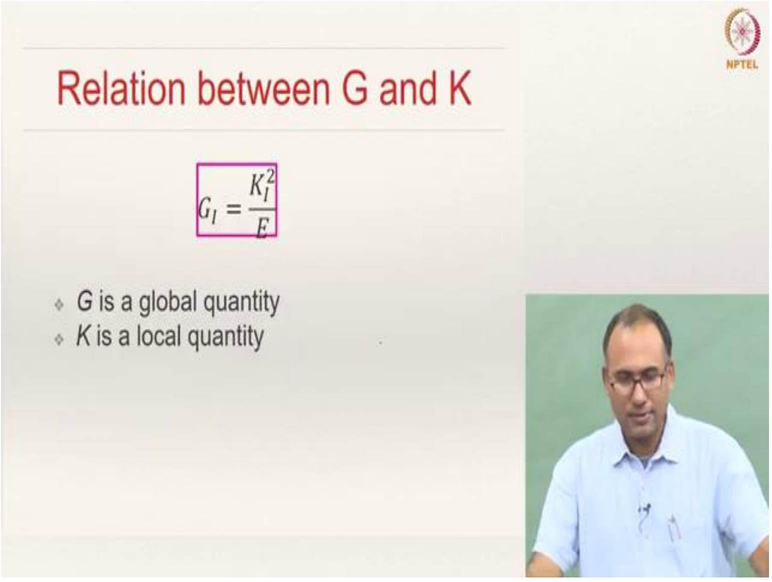

We have talked about two quantities; energy release rate and stress intensity factor. So, there is a relation between energy release rate and stress intensity factor given by,

$$G_1 = \frac{K_1^2}{E}$$

G_1 is the energy release rate, K_1 is the stress intensity factor, but where did we define the stress intensity factor? For a given crack length; so that means, it is a local quantity.

When we are talking about energy release rate, we are talking about the total energy of the entire component and hence it is a global quantity. How K that we have defined at the crack is evolving, that changes the way that G evolves. So, K and G are directly correlated. Any questions?

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When we are talking about K_1, you are talking about the crack tip. The alternative definition of K_1 is given by,

$$K_1 = \sqrt{2\pi r}\sigma_{22}\ (r, \theta = 0) \text{ as r} \longrightarrow 0$$

That means, we are actually looking close to the crack tip. So, we are actually evaluating the behaviour of the crack locally. Whereas, G is telling you the energy; energy is the total quantity of the body that we are talking about. So, what is the total energy release rate?

The energy release rate is calculated from total strain energy and the work potential; that is how we have defined. So, we have written $G = -\frac{d\pi}{da}$, where π is the total energy of the component; that is why the G is called a global quantity, K is called a local quantity.

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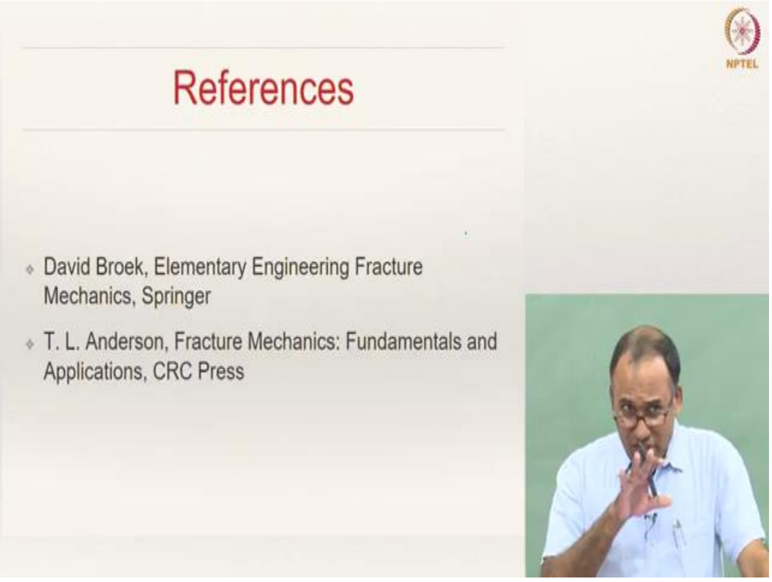

The purpose of this module and fracture mechanics is in no way to give you complete understanding of fracture mechanics, but to educate you on the two concepts: the strain energy release rate and stress intensity factor, and to make sure that you understand that the crack propagates when the stress intensity factor exceeds the fracture toughness of the material or energy release rate exceeds the critical energy release rate; both are equivalent.

But if you want more knowledge about fracture mechanics, that can only be done by taking a serious course in that area, alright?