Basics of Materials Engineering
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Lecture – 44
Fatigue Failure of Materials
(Rotating Beam Bending Test, Estimated S–N diagram)

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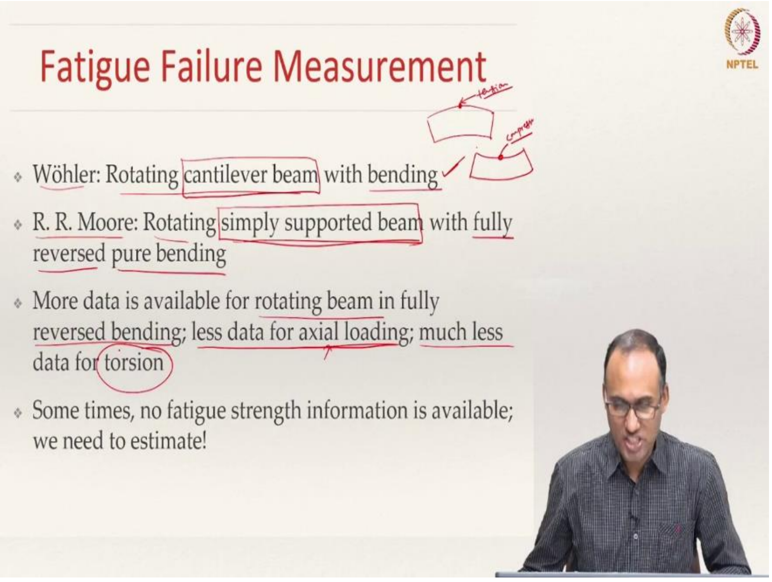

So, how did people make the measurements of failure and stress amplitude and the number of cycles to failure? What are the ways that we can actually do these measurements experimentally?

So, as we have said August Wohler did the experiments on a rotating cantilever beam with bending. However, most of the data that we are having today is not a fully reversed loading, please keep that in mind because it may not actually give you pure bending.

The rotating cantilever beam with fully reversed loading, although he says that it is bending it is not necessarily pure bending. So, it is not really a one-dimensional state of stress. However, most of the data that we have in the literature is generated due to an improvised experiment from Wohler experiment called R. R. Moore's rotating beam bending test or rotating simply supported beam.

Instead of a cantilever beam that Wohler used, Moore proposed rotating simply supported beam with fully reversed pure bending. He managed to get a scenario where you can subject the material to pure bending; that means, it is only normal state of stress, there are no other stresses in the material. As a result, you actually have a one-dimensional state of stress; that is very important.

Because you remember, when we have studied static failure theories, we have calculated the yield strength of a material using uniaxial tensile test. And, that is the material property that we have used even when you are dealing with multiaxial loading.

Similarly, in the case of fatigue failure R. R. Moore's rotating beam bending test ensures that there is a pure bending -- fully reversed pure bending applied. And, in literature we have a lot of data available for rotating beam in fully reversed bending. The fully reversed bending test is one of the popular tests and reasonably easier tests compared to other tests.

Compared to rotating beam bending data, we have less data for axial loading; that means, when we are saying rotating beam bending, you take a beam and you apply a pure moment and both positive negative. What does it mean? You are actually bending this way and unbending that way.

You imagine when you are bending this way. So, you have a component bent like that and in the next cycle this is bent this way. This material point now is experiencing tension and the same material point is now experiencing compression.

This is what we mean by time varying loading, right? The material is changing nature of stress at the same point from tension to compression; that is what we have plotted in the time varying stress state in the previous slides. So, you can create pure bending cyclic loading.

Similarly, you can also do a uniaxial push-pull test which is also repeating. You can have a fully reversed cyclic loading alternating between tension and compression.

That kind of an experiment is not so easy and there is less data available for such an experiment and there is also not so much of data available for torsion. You can also apply torsional load in a cyclic fashion, but the amount of data available for axial loading and torsional loading is much less compared to pure bending load.

Sometimes, for certain scenarios, the fatigue strength information is simply not available; for certain materials it is not available or for certain loading scenarios it is not available. Then you need to estimate this, when the fatigue strength information itself is not available to us. We will discuss how do we go about doing this estimation.

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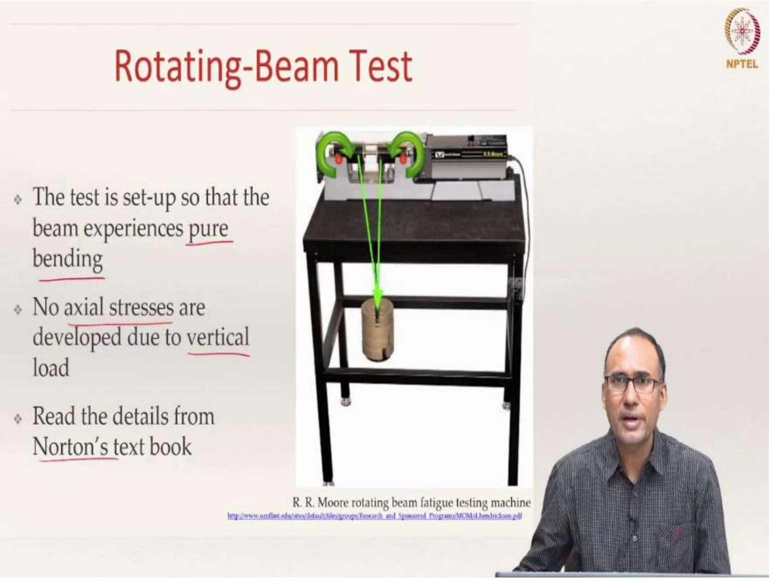

So, this is a schematic that shows how a rotating beam bending test set-up look like and the set-up is prepared in such a way that the beam experiences pure bending i.e., only bending stresses. The normal stresses prevail in the material, there are no other stresses so as to give a one-dimensional loading scenario.

No axial stresses develop due to the vertical load; that is how the test set-up is prepared. Please read Norton's textbook on understanding the rotating beam bending test in more detail.

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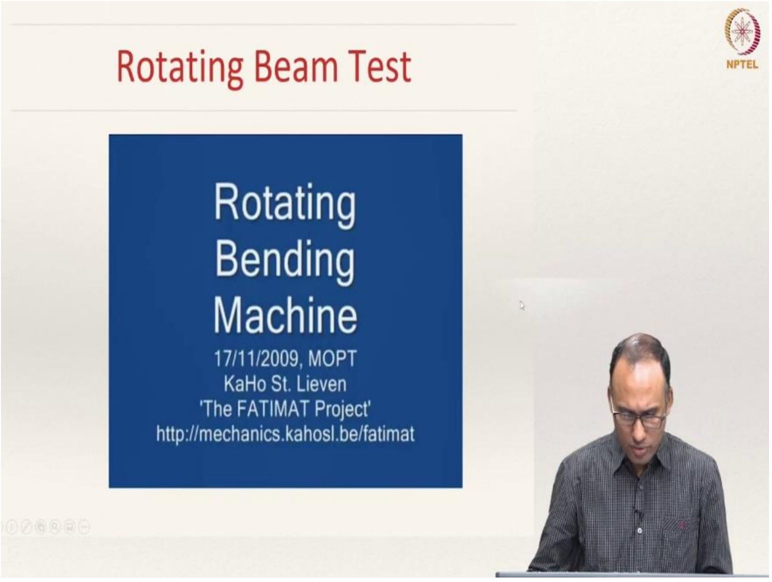

Here is a video in which you will see how this test is done. So, I have taken from this particular website in Belgium.

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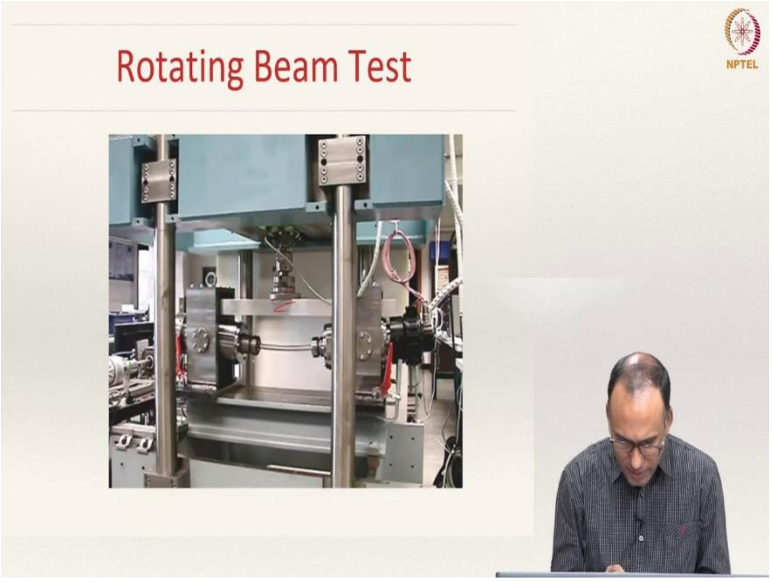

Here, you see that as the component at the top is pushed, the beam bends while rotating.

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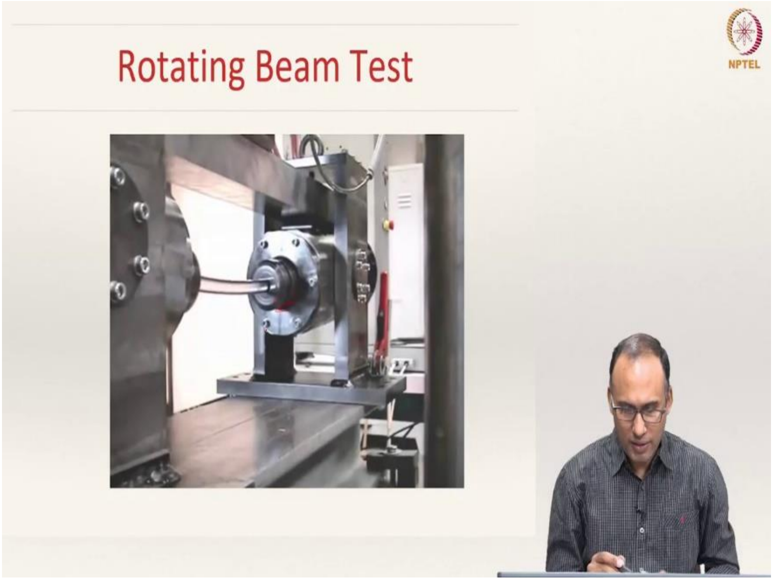

While it is rotating you are applying this load and the material is experiencing repeated loading at a given material point. That is how typically a rotating beam bending test is done.

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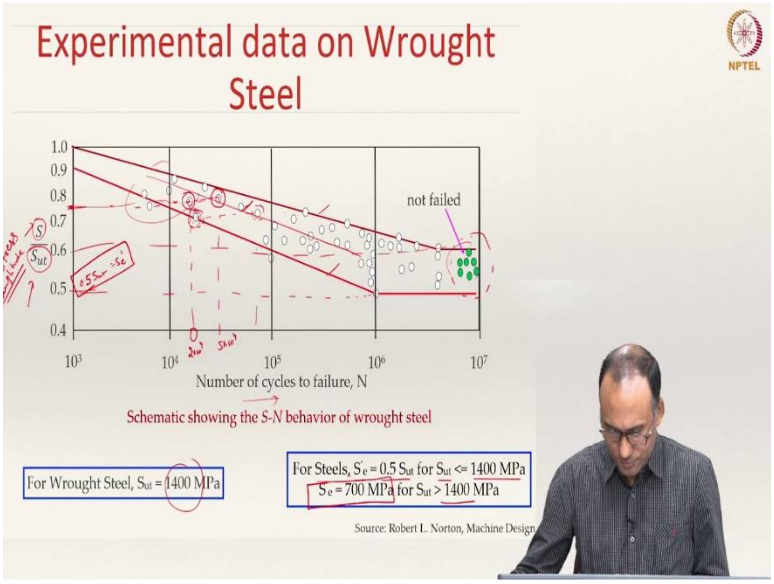

Here I am showing experimental data on wrought steel which shows an endurance limit. On the y -axis, I have plotted $\frac{S}{S_{ut}}$. I have normalized the stress amplitude S with the ultimate strength.

If you see the S-N diagram, here it should start asS_{ut}. That is where it starts and let us not focus on these lines now, just focus on the data points shown in the white circles. On the x-axis you have number of cycles and on y-axis, stress amplitude; both are in log-log scale.

It is only a schematic. I have prepared the schematic from the experimental data. So, the data points are more or less taken from the real experiments from the literature.

A very interesting scenario can be observed; for instance, these are all the specimens of wrought steel whose ultimate strength is about 1400 MPa. They are all specimens from the same batch; each specimen is different, but they are from the same batch.

And, you would normally expect a small variation in the properties when you do a tensile strength. But the variation is not expected to be very high if all the specimens are cut from the same batch. Here all the specimens are cut from the same batch.

However, if you see a specimen, for instance you take this sample and this sample. Both of them are subjected to the same normalised stress amplitude say 0.75. But this specimen gave a life of say 2×10^4 cycles whereas, this specimen has given say 5×10^4 cycles.

So, the number of cycles to failure or the life that it is giving are different from one specimen to another, although both of them are subjected to same stress amplitude. And, you see that here it is subjected to a higher stress amplitude.

Now, you look at this point; it is subjected to a lower stress amplitude and hence, you would expect a higher life, right? We do not expect this data point to be here, you would have expected this to be somewhere here, because the stress amplitude is less. But here it is showing premature failure, while, at the same stress amplitude the other data point is showing a higher life.

If you would look at these data points, it is very difficult to conclusively say anything about the behaviour. However, these data points fall under these two bands of the brown line and the red line.

So, there is lot of noise in the data; that means, there is lot of standard deviation for the data and this is typical of any fatigue experiment. Why is that? As we have discussed, the fatigue failure has three different stages - crack initiation, propagation and final failure.

The crack initiation stage depends on the internal microstructure of the component. Although the microstructure is the same, the micro-crack structure that is prevailing in specimen 1 and specimen 2 is not necessarily be the same. So, your micro-crack distribution could be very different from one specimen to another specimen.

Because of the fact that the micro-crack distribution is very different, the position where this micro-crack is there, may be subjected to a higher stress in specimen 1 and in the other specimen, it may be subjected to a lower stress. As a result, both of them are giving you completely different lives.

However, if you see the big picture, you would see that general trend is this way and that is what people have observed. Although there is a lot of scatter, you are able to put them under a band and then you could have given a mathematical expression to represent this trend.

The green circles represent the wrought steel samples that have never failed; beyond 10^6 cycles, they never failed and that is precisely what we said endurance limit, right? The material will not fail below a certain stress amplitude for instance the specimen corresponding to this data point; below certain stress amplitude, they never fail and that is what we call endurance limit of the material.

The endurance limit S_e' is typically taken as $0.5S_{ut}$. This is how typical experimental data for a specimen subjected to fully reversed bending load looks like.

So, for steels $S_e' = 0.5S_{ut}$, if $S_{ut} \leq 1400$ MPa. If the ultimate tensile strength happens to be a greater than 1400 MPa, then $S_e' = 700$ MPa; that is the general guideline.

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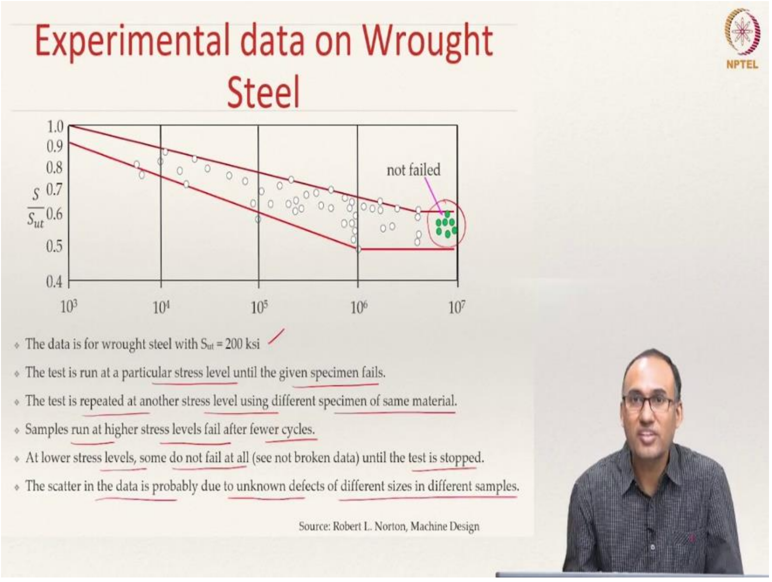

As we have discussed, this is the data given for the wrought steel with ultimate strength 200 ksi or 1400 MPa and the test is run at a particular stress level until the given specimen fails. The test is repeated at another stress level using different specimen of the same material.

Samples run at higher stress levels fail after fewer cycles. At lower stress levels, some do not fail at all until the test is stopped. The test here is stopped at 10^7 cycles. The scatter in the data is probably due to unknown defects of different sizes in different samples as I have already discussed.

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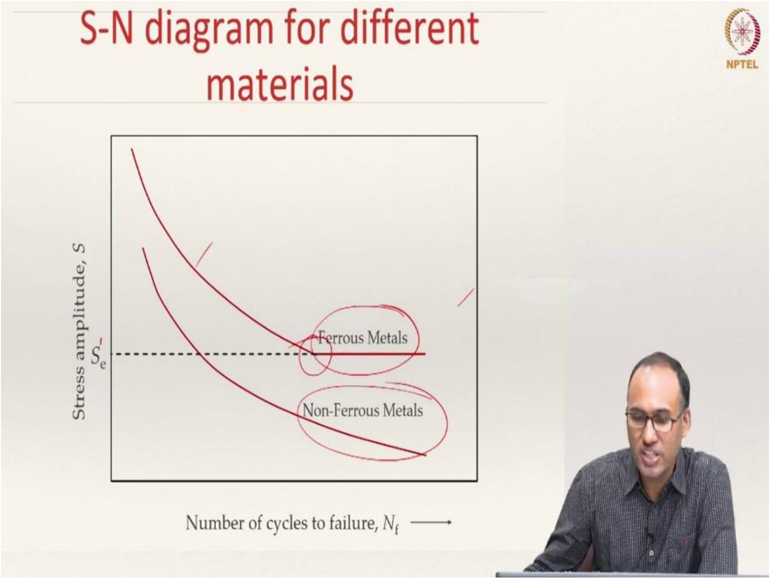

How does the S-N diagram look like for different materials? The S-N diagram on a linear scale looks something like this. This is for ferrous materials which show endurance limit; this is for non-ferrous materials which do not show endurance limit. It is a power law kind of a relation. Only when you plot that in a log-log scale, then you would see it is a straight line.

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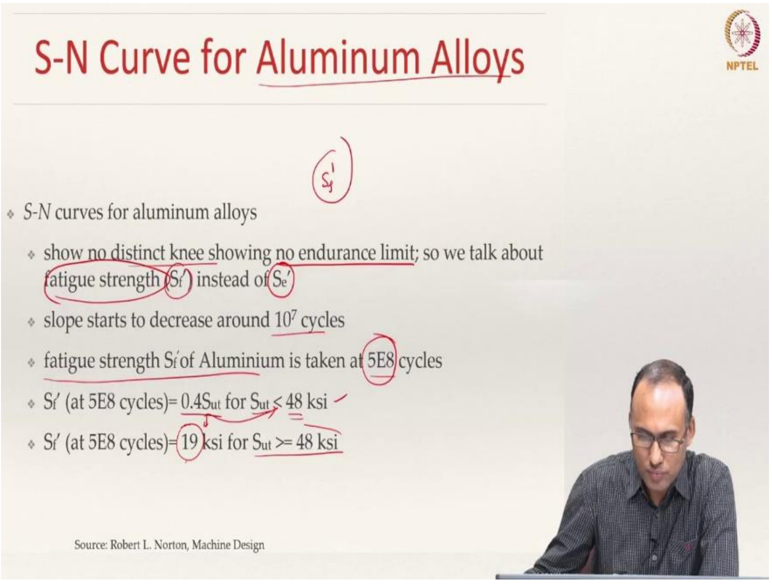

The S-N curve for aluminium alloys which are non-ferrous, do not show a knee. What do we mean by knee? Something like this; you do not see a knee, meaning there is no endurance limit for these materials.

For aluminium alloys, we should use fatigue strength rather than the endurance limit. The fatigue strength is denoted by $S_f{}'$ and endurance limit is denoted by $S_e{}'$.

You can see that the slope starts to decrease beyond 10^7 cycles. Since aluminium alloys do not have endurance limit, you define something called fatigue strength. How do we define the fatigue strength of the aluminium alloys? The fatigue strength of aluminium alloys is defined at 5×10^8 cycles.

The endurance limit of steel is defined at 1 million cycles. At 1 million cycles, what is the stress that it gives, i.e.,$S_e' = 0.5S_{ut}$ is the endurance limit. For aluminium alloys, we look at 5×10^8 cycles, and $S_f' = 0.4S_{ut}$ if $S_{ut} < 48$ ksi. If $S_{ut} > 48$ ksi, then $S_f' = 19$ ksi.

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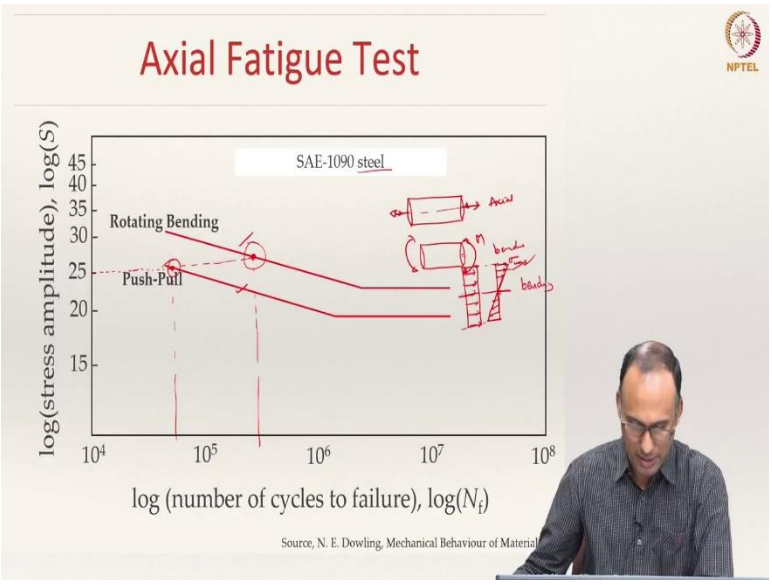

So, that is about the rotating beam bending. What happens in axial fatigue test? Let us compare the fatigue failure lines on a log-log scale for a rotating bending test and a push-pull test i.e., an axial test done on a SAE 1090 steel.

This is the curve obtained from the rotating beam bending and this, from the push-pull test. Suppose you are subjecting this specimen to a particular stress amplitude, a push-pull test gives me this much life whereas the fully reversed bending scenario gives me longer life compared to push-pull test.

Why does that happen? If you look at the stress state through the thickness, let us assume that this is a solid shaft, the stress state at the neutral axis is 0 and the outer layer is maximum; that is how the stress state changes and this is the $\sigma_{\max}$ in bending.

Because the amplitude is same, the magnitude will be the same and this is the neutral axis and the stress state is going to be -- because this part is under tension and that part is compression whereas here, the stress state is going to be same everywhere as the entire cross-section is subjected to axial load.

You can clearly see that in the case of axial loading, more material is subjected to a larger amount of stress compared to pure bending scenario. Imagine you have a micro-crack network. There are more micro-cracks that are prone to experience higher state of stress compared to the beam bending test and that is the reason why this is much more severe compared to beam bending scenario.

That is why for the same stress amplitude, the axial loading will give a lower life compared to rotating beam bending test. That means, the axial fatigue is going to be more severe than pure bending fatigue.

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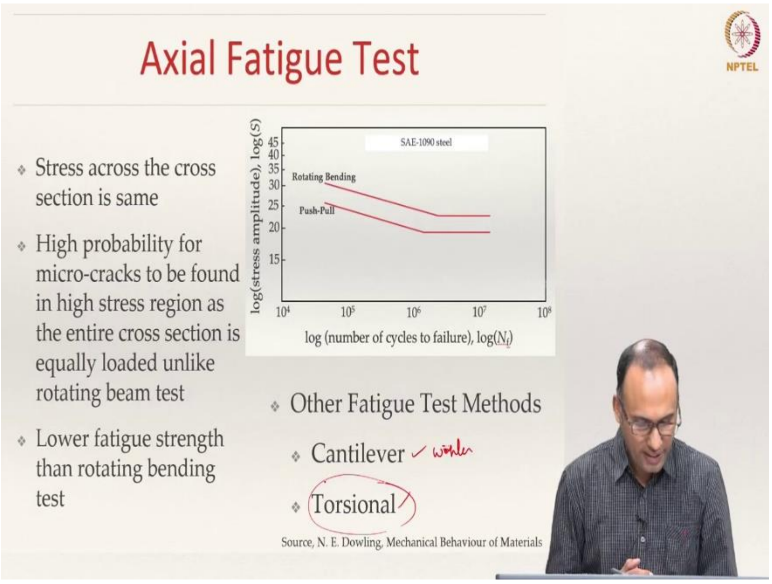

There are also other fatigue testing methods possible i.e., cantilever which was the first test that August Wohler did. You also have a torsional test, which is more complicated to do. However, you can find the data of these tests in the design data handbooks.

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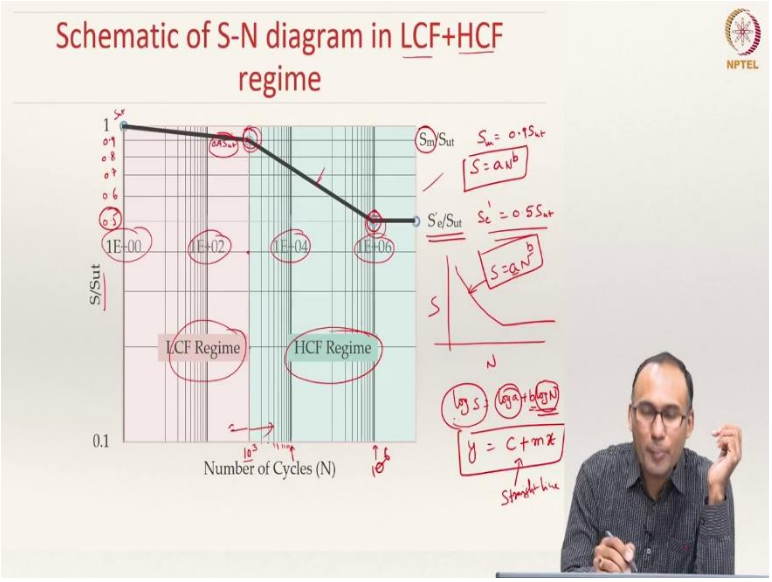

Let us look at the schematic of the S-N diagram in LCF as well as HCF regime. What do we mean by LCF regime? LCF represents cycles less than 10^3 and HCF is beyond 10^3 cycles.

So, here I am plotting 1E-0, 1E-2, 1E-4, 1E-6 right. So, here you have $\frac{S}{S_{ut}}$; this is one. So, this is actually S_{ut} and here it is 0.9, 0.8, 0.7, 0.6, 0.5 In the LCF regime, you have one line and it ends at $0.9S_{ut}$. And this is your 10^3 cycles.

So, this is 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000. This is your 10000 cycles. At 1000 cycles, you have $0.9S_{ut}$ and at 1 million cycles, you have $0.5S_{ut}$, that is $\frac{S_e'}{S_{ut}}$, and this one is what we callS_m. $S_m = 0.9\ S_{ut}$ and $S_e' = 0.5S_{ut}$, and that is the line that you will join.

On a normal plot it will be like this, right? This is our S-N. You can represent that as

$$S = aN^b$$

a and b are material constants. When you take log on both sides, you will see that it is a straight line, as it can be expressed as

$$\log_e S = b \log_e N + \log_e a$$

The above form is similar to the equation of a straight line,

$$y = mx + c$$

That is why it can be approximated as a straight line on a log-log scale. The region left to 10^3 cycles is called LCF regime, and the region right to 10^3 cycles is called HCF regime.

We are going to focus our attention to HCF regime only. We are not going to do any analysis for low cycle fatigue regime. So, that is your LCF and that is your HCF regime and that is your endurance limit. So, this part is LCF and that part is your HCF.

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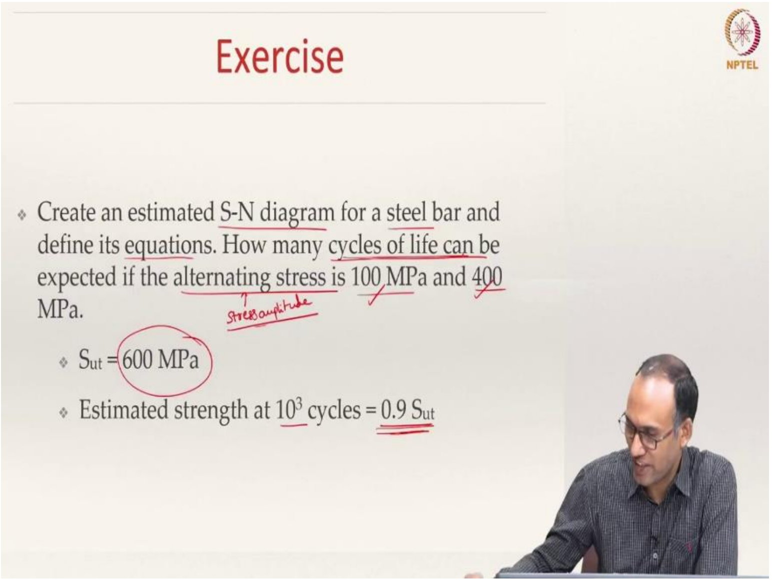

Let us now do this exercise and then be able to understand how to construct an estimated S-N diagram. Create an estimated S-N diagram for a steel bar and define its equations. How many cycles of life can be expected if the alternating stress -- so, alternating stress is another name for stress amplitude.

Alternating stress is 100 MPa and 400 MPa, what happens? How many cycles of life can be expected if the alternating stress is 100 MPa; how many cycles of life can be expected if alternating stress is 400 MPa?

The ultimate strength of the material is 600 MPa. The estimated strength at 10^3 cycles is $0.9S_{ut}$. Why is it required? If it is not given, then you assume it to be $0.9S_{ut}$.

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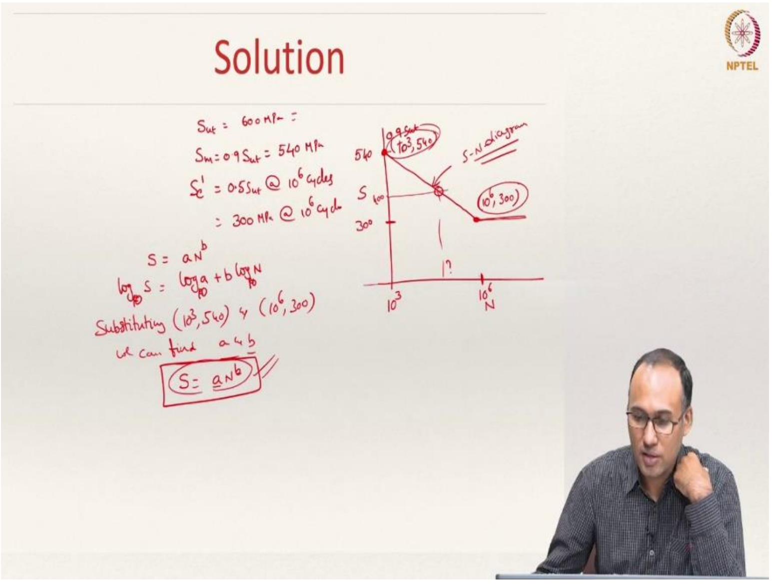

So, you have a system where $S_{ut} = 600$ MPa. When you are drawing the S-N diagram, at 1 cycle it is S_{ut}, but at 10^3 cycles it is 0.9 S_{ut}; that is $S_m = 0.9S_{ut} = 540$ MPa.

So, this corresponds to 10^3 cycles.$S_e' = 0.5S_{ut} = 300$ MPa, at 10^6 cycles; that is important; you need to know 2 points in order to draw a straight line.

Let us say this is my 10^6 cycles and this is 300 MPa. So, that is the second point, then you draw that line and that is the S-N diagram. And, after that 10^6 cycle it is supposed to be, because it is a steel, that is what it should be.

So, we know this curve is represented by $S = aN^b$. So, you can write,

$$\log_{10} S = b\log_{10} N + \log_{10} a$$

By substituting $(10^3, 540)$ and $(10^6, 300)$, we can find a and b. Then you can write $S = aN^b$ which is the failure diagram for this material. You would have found what is your stress life diagram. So, this is the equation that we need to figure out.

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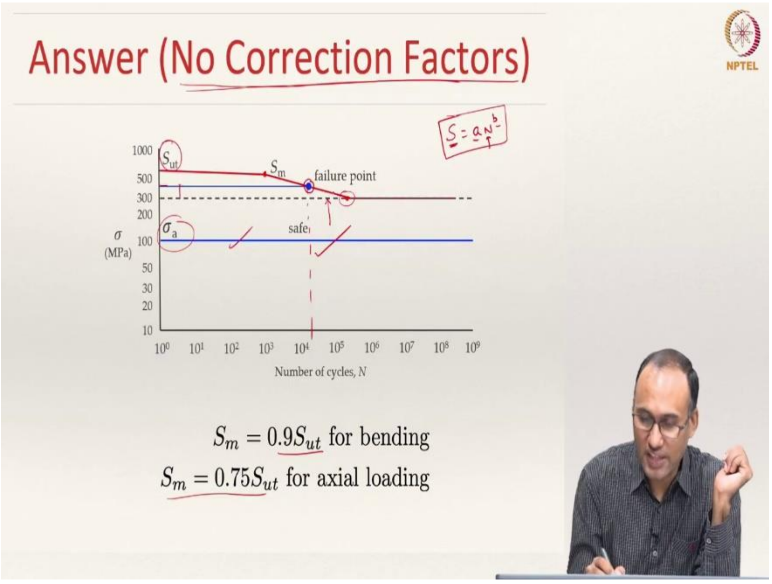

So, here is S_{ut} and this is $S_m = 540$ MPa and $S_e' = 300$ MPa.

$$S_m = 0.9\,S_{ut} \text{ for bending}$$

$$S_m = 0.75\,S_{ut} \text{ for axial loading}$$

We need to find the number of cycles of life that can be expected if the stress amplitude is 100 MPa. And you see that here, the stress amplitude is 100 MPa, which is below the endurance limit and hence this should be safe. But, if the stress amplitude is 400 MPa, it is above the endurance limit; this is 400 MPa; that means, this is the failure life and that should be our number of cycles.

You should be able to find out the number of cycles it will give. That can be found by plugging in $S = 400$ MPa in the equation $S = aN^b$ and thereby estimating N.

So, to do that, we have used these two boundary conditions to find the values of a and b because they are the material properties. By using this equation you will be able to figure out how many number of cycles that the component will last for $S = 400$ MPa.

So, you can actually find out the value. I would expect you to find out by looking at the figure graphically, if you have drawn this guy without actually solving an equation you will be able to predict this graphically, but by writing the equation you can also predict the value exactly.

Please note that, here I have used something called no correction factors; that means, I have not used any correction factors in estimating these values, right? So, where does this term correction factor come in? The reason why we need to be using correction factors is because the estimated S-N diagram -- this is what we call an estimated S-N diagram, where, if you know the ultimate strength of the material, if it is steel then if I know the ultimate strength of the material, then I can take this point as $0.9\,S_{ut}$ and this point as $0.5\,S_{ut}$ corresponding to 10^3 cycles and 10^6 cycles respectively. Then, I will be able to calculate a and b, in the equation $S = aN^b$.

However, $S_m = 0.9\,S_{ut}$, $S_e' = 0.5S_{ut}$ at 10^6 cycles are based on the experimental conditions proposed by R. R. Moore's rotating beam bending test. What are those conditions? For instance, R. R. Moore's rotating beam bending test is done for a specimen of a certain size; it is done on a circular cross-section of diameter say 8 mm.

If your machine component is not of the same size, then your defect concentration is going to be very different, right? Hence, you need to account for this change in size. As long as the size is less than or equal to 8 mm you do not have to worry about that size variation as these numbers have come in due to that size of the rotating beam bending test.

If you are dealing with a different size, you need to take into account of the variation in this size and again the experiment was done at room temperature. But what if the component in service is actually working at a different temperature. So, you need to account for this change in temperature as well.

Similarly, you know that you have a lot of scatter in the data when you have looked at the rotating beam bending test. The reliability of the data is taken to be 50 percent, but when you are doing the design calculations, if you want to have higher reliability you need to add some correction factor to that as well and so on.

So, if the component being designed is going to be different from the rotating beam bending test based on which we have constructed the S-N diagram, then you need to account for these changes through correction factors and that is something that we will look at in the next class that is about correction factors.

Here, we have not used any correction factors. But we will look at the same problem when you apply the correction factors and then we will see whether this 100 MPa will still be safe or not. When you are changing these correction factors, the endurance strength is going to change; that is what we are going to look at in the next class.

With that, I stop here today and thank you very much for your attention.