Computational Fluid Dynamics Using Finite Volume Method

Prof. Kameswararao Anupindi
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture – 18

Finite Volume Method for Diffusion Equation: Steady diffusion in polar and axisymmetric coordinates

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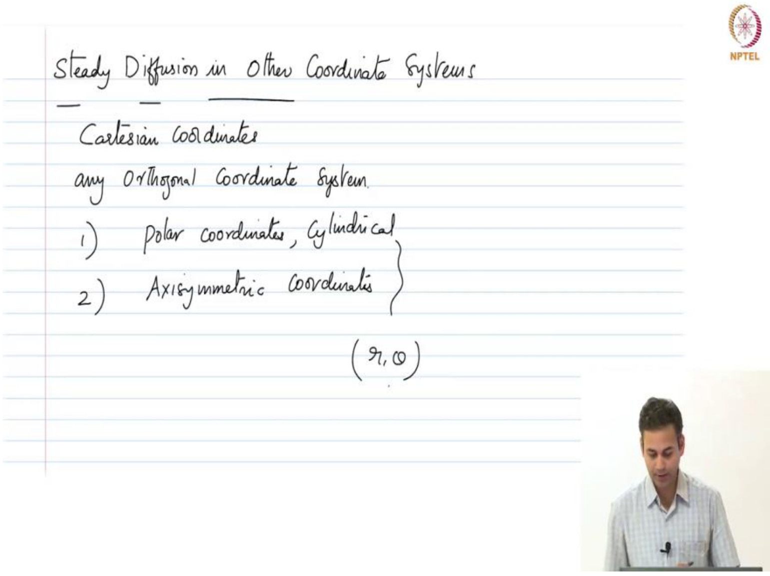

Good morning. Let us get started. So, in today's lecture we are going to look at Steady diffusion in other coordinate systems ok. So, this is a steady diffusion in other coordinate systems. So, till now we have developed steady diffusion only in Cartesian coordinates, right. So, in the Cartesian coordinates; however, the method that we have developed is applicable in any orthogonal coordinate system ok.

So, couple of examples of this orthogonal coordinate systems, we are going to look at are the polar coordinates or the cylindrical coordinates right. And the other example, we are going to look at is axisymmetric coordinates fine. So, by we are going to kind of go into each of these examples and then see, how we have to kind of modify the method that we have developed.

So, that we can adapt to these two coordinate systems that are orthogonal that have an orthogonal coordinate system orthogonal axis. Now, let me first take up the polar coordinates or the r theta coordinates ok this is also the r theta coordinates and develop this method.

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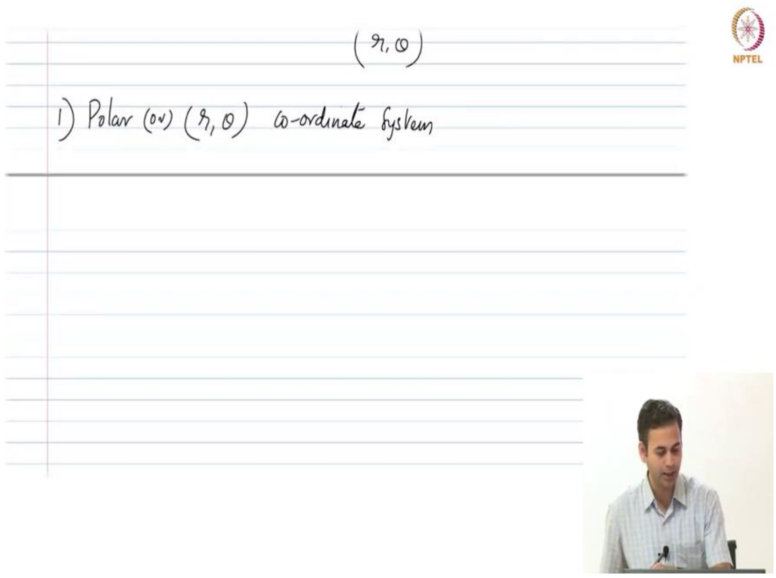

So, we been looking at the polar or the r theta coordinate system. So, first thing is how does the domain look like in r and theta coordinates? Ok. So, we have to kind of draw the domain.

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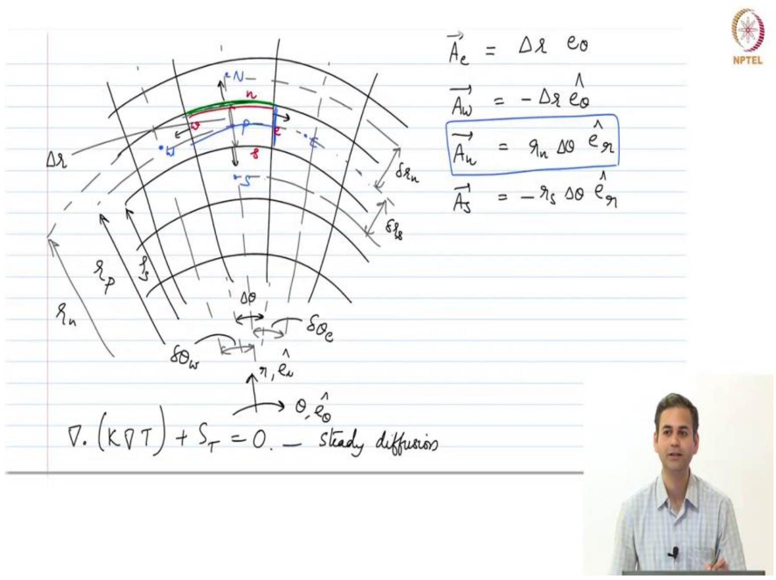

Basically we have; so, I have kind of drawn some cells here this is let us say the r direction is in this direction and theta is in this direction ok. And the unit vectors that we have are e r hat, in the direction of the r and e theta hat in the direction of the theta ok.

So, let us say we want to solve for diffusion in a circular geometry or if you have an angular region, we want to solve diffusion in there, then how do we go about this? But we know that the system itself is an orthonormal system ok. So, let us introduce the cell centroids which I would like to call it as let us call this as P; this is P cell, this is east cell, this is west, this is north and this is south ok.

That is what we have and the radii for example for example, let us say we want to calculate what is this radii. So, this is some r P ok, that is the radius for the pth cell right, which is also of course, same as the r for capital E and r for capital W and so on ok.

We also have these names for the faces right. So, we said this will be our east face similar to the previous developments, this is west and this is north and this is south faces ok. And then we also need to know what would be the radial extent of the cell right, that would be we would call it as delta r ok. So, this is our delta r and we also need to know what will be our azimuthal extent right. So, this is we call it as delta theta ok.

So, this is my delta theta which is the azimuthal extent for the cell P and delta r is the radial extent for this l P ok. Similarly, we need to identify what will be the distance between the east cell centroid and the P cell centroid like in the Cartesian coordinates, we had delta x e delta x w delta y north and delta by south. Similarly, we have here what will be the angle between the P cell and the east cell centroid? So, that is this angle ok.

So, this angle would be will call it as delta theta east ok that is connecting the cell centroid e with the cell centroid P fine. Similarly, we will call this angle which is between the P and the w as delta theta w ok. So, those are the angles. Then we also need to know what will be a radial distance between the cell centroid of north cell and the P cell.

That means, we want to know what will be this distance right, we want to know what is this distance and of course, we also want to know what will be this distance right. These are similar to the notations before we will call it as del r north and del r south ok. So, we have now identified all of them is that clear. Of course, what will be the radius for the north face; what will be the radius for the north face? That will be r sub n ok.

So, if you want to calculate what is the radius for this guy, this would be r sub n; this should be r sub n that is for the nth phase and similarly r sub little s r sub s for the south face ok; so, that we can comfortably have those values as well. So, this is r s that is connecting the south face and the north face with the centroid fine. So, let us move on our governing equation is still the same that is del dot K grad phi plus ah del dot K grad T plus some S T equals 0.

So, we will write it down once you, they make a note of all these things ok. So, what is your governing equation? Governing equation is I would use the T ok, because we already have theta here I would avoid the phi for a for this particular problems.

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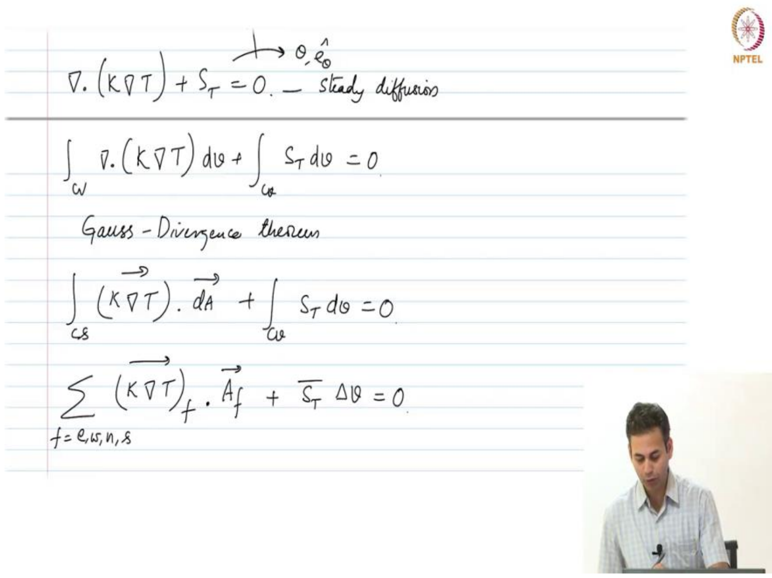

So, we will say $\nabla.(K\nabla T) + S_T = 0$ where K has replace gamma and T has replaced phi and S T has replaced S phi ok. So, that is what we have this is our steady diffusion equation that we are working with all right. So, what is the first step? First step is to integrate this on the control volume that is for the P cell. So, that will be $\int_{cv} \nabla.(K\nabla T)dv + \int_{cv} S_T dv = 0$ ok, that is what we have.

Then next step is to invoke gauss divergence theorem and convert the volume integral into a surface integral right. So, that would be $\int_{cs} \overrightarrow{(K\nabla T)}.\overrightarrow{dA} + \int_{cv} S_T dv = 0$ Now, we are going to introduce a couple of assumptions first one is the source value, we can calculate at the cell centroid.

And then say that that reminds the same for the entire cell, that is one approximation the other one is that the K grad T value on the faces, can be assume to be the face centroid value that prevails over the entire face right. We are making these two assumptions and I converting this line integral into a discrete summation, that is K grad T on the particular phase f dot A f.

And this summation here is f over the faces east, west, north and south that is for all the phases of the control volume plus, we have also introduce an average value, that is S T bar times delta v equals 0, alright. Now, we have introduced these metrics the delta volume for the cell and the A f bar right, these are the vectors the phase area vectors that are pointing outside the control volume. So, we need to go back and see what will be our east west north south, for these values on the east west north south faces for the area vector.

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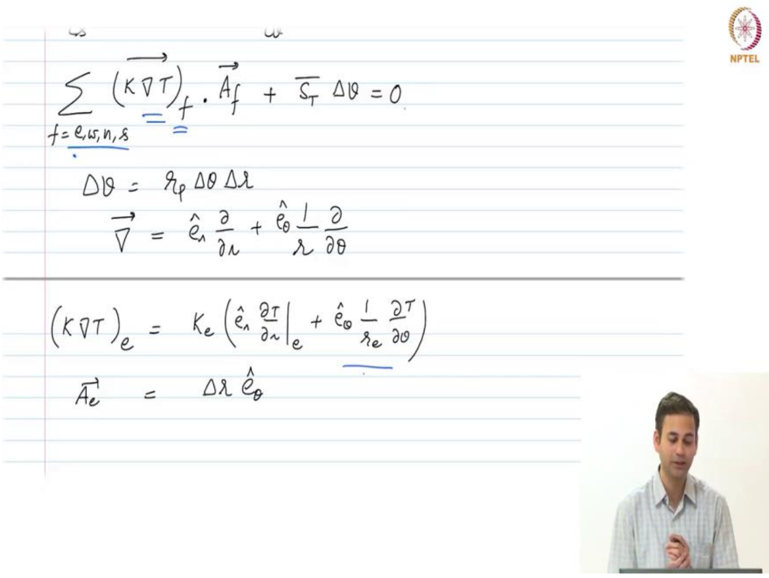

So, what is delta v here, how much would be delta v, for this for the P cell? $\Delta v = r_P \Delta\theta \Delta r$

Student: r P is the radius.

r P times.

Student: Delta theta.

Delta theta times.

Student: Delta r.

would be the volume of this particular cell that is for the P cell. So, that would be I would write it as r P times delta theta is delta theta times delta r right. So, this is the volume for the P cell right everybody agrees ok. Now, if I go back what will be the area vectors? Can we take a look at the area vectors? So, A east will be in this direction right.

Similarly, A west will be in this direction A north would be in this and A south would be in this, I am not exactly drawing at the face centroid that is just to you know avoid overlaying on the previous letters here ok. So, what is what would be A east, A east bar would be how much? A east bar is basically.

Student: (Refer Time: 10:38).

What will be the A east bar magnitude?

Student: Delta r right.

Delta r right similar to delta y we have now here delta r. Now, what is the direction which it is pointing?

Student: (Refer Time: 10:49).

It is pointing in the e hat T theta right e hat theta ok that is where the direction in which it is pointing. What will be A west bar?

Student: (Refer Time: 10:58).

That will be minus delta r times e theta hat ok, that is pointing in the negative e theta direction ok. What about A north? A north is basically.

Student: (Refer Time: 11:12).

This phase magnitude right and it is pointing in the positive r direction. So, what will be A north bar?

Student: r (Refer Time: 11:22).

r P or ok.

Student: r n r P.

r P or r n.

Student: r n.

It should be rn right because; we are looking at a phase that is at a location radius on the north face. So, this is r sub n times delta theta, that will give you the length of that surface of the north face and it is pointing in which direction?

Student: e r hat.

e r hat its pointing in e r hat and what about a south bar? That will be minus.

Student: r s.

r s its not r n right because, we are looking at a smaller surface. Now, this is r s times delta theta times e r hat right. Is that correct?

Student: Yes.

You are looking at the south face right, it is in the downward direction right north is upward ok.

Student: (Refer Time: 12:10).

Sorry e r r e n oh e r what it is e r I think not written clearly here, you want the subscript right. This is r; r fine, this is e r hat oh we do not have e n anywhere right, the only things we have are e r hat and e theta hat fine ok. Of course, now we have not looked at what is the definition for the del operator right, that is what is remaining there because, we need the del operator in evaluating the gradients here right. So, we need the del operator. So, what would be del bar in cylindrical coordinates or polar coordinates? It should be e r hat partial partial r plus e theta hat one upon r partial partial theta ok.

Of course if you have a three dimensional volume right, if you have a cylinder then you have to add e z hat partial partial z right or if you want to call it as x it will be i hat partial partial x right that extra derivative. You have to add otherwise this is the definition for nabla in cylindrical coordinates ok. So, far fine questions till now on the area vectors no easy right everyone understands very good. Let us move on then and let us move on with the calculating these values.

So, what will be K grad T east would look like? $(K\nabla T)_e = K_e\left(e_r^{\wedge}\ \frac{\partial T}{\partial r}\Big|_e + e_\theta{}^{\wedge}\ \frac{1}{r_e}\frac{\partial T}{\partial \theta}\right.$K; partial partial theta is what it would look like. Is that correct? K grad T on the face e right, we have to evaluate f on these faces on the east, west, north, south.

So, I first calculated for east ok. So, that would be K grad e would be K e that is gamma e similar to gamma e diffusion evaluated on the face times. We have a grad T that is e r hat partial T partial r on the east face plus e theta hat 1 by r, this r would be should be evaluated where? On the face that is r little e times, I think I have missed nicely the temperature.

So, partial d partial theta ok. So, that is what we have evaluated on the east face fine ok. Now, what would be A e bar we have just written it down. A e bar was how much? Delta r e theta hat ok. Now, if you take a dot product or inner product of these two quantities what term survives out of the two? Only the.

Student: Second one.

Second one because you have e theta dot e theta dot right, e theta hat dot e theta hat that is the only one that survives, the first term does not come into play. Why is this happening? This is happening, because we have chosen a.

Student: Orthogonal.

Orthogonal coordinate system ok, that has to be kept in mind if it is not orthogonal, then we will have terms coming from both of them right. So, that is the philosophy here right you have a orthogonal coordinate system you only get gradients survive in the direction in which you are taking the you know the dot products ok.

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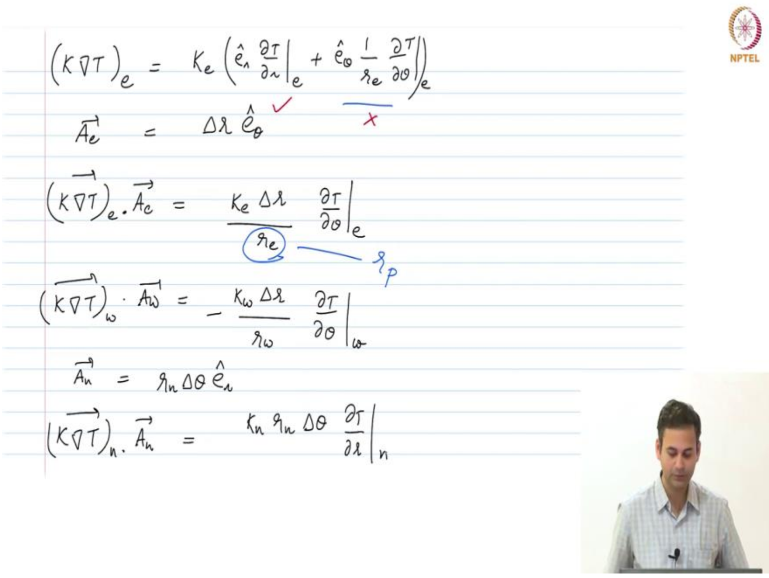

So that means, I can rewrite this as so, what will be now$\overrightarrow{(K\nabla T)}_e.\overrightarrow{Ae} = \frac{K_e \Delta r}{r_e}\frac{\partial T}{\partial \theta}|_e$ How much is this? This will be K e right only the second term survives K e.

Student: Delta r.

Delta r upon.

Student: r east.

.

Student: (Refer Time: 15:58).

Now, similarly can you tell me what would be$\overrightarrow{(K\nabla T)}_w.\overrightarrow{Aw} = -\frac{K_w \Delta r}{r_w}\frac{\partial T}{\partial \theta}|_w$.

Student: Delta r.

Delta r.

Student: (Refer Time: 16:16).

Upon.

Student: r w.

would, there be a plus or minus or something.

Student: Minus.

There will be a minus right there will be a minus fine. Now, let me look at what is a north bar. What was A north bar?$\overrightarrow{A_n} = r_n \Delta\theta \hat{e_r}$ If you go backSo, let me put that back in here. So, this is which direction? This is r n delta theta e r hat right that is what we have. Now, what would be then $\overrightarrow{(K\nabla T)}_n . \overrightarrow{A_n} = K_n r_n \Delta\theta \frac{\partial T}{\partial r}|_n$? Oh how is re different from r p? Ok. So, it is not different.

So, the question is how is this r sub little e different from r capital P? Right. It is not different it is the same thing I just wrote it e because, we are evaluated on the face essentially you are correct. So, all the radii for P and E and W are all the same. Only its different for r north and r south only to make the distinction, I have written r sub e and r sub w, but it is same as r P for the P cell right. Other questions? Ok. We understand all this fine, then what about K grad T n dot a n bar? How much is this? So, which term now survives out of the 2?

Student: (Refer Time: 17:47).

This one survives right when you are doing in the north and this would not survive. So, what will be the value?

Student: K north.

K north.

Student: r n.

R n.

Student: Delta theta.

Delta theta ok.

Student: Partial T.

Partial T.

Student: Partial r

Partial r evaluated on.

Student: North face.

North face right that is K n r n delta theta times partial T partial r on the north face correct fine ok.

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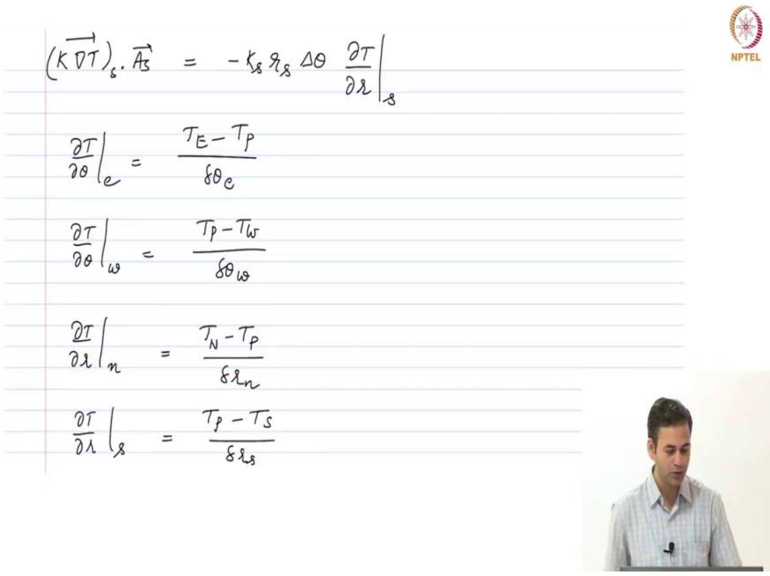

Then what would be $\overrightarrow{(K\nabla T)_s}.\overrightarrow{A_s} = -K_s r_s \Delta\theta \frac{\partial T}{\partial r}|_s$ dot A south bar?

Student: Minus.

Minus ok.

Student: K s.

K s.

Student: r s.

r s.

Student: Delta theta.

Student: Partial T.

Student: South.

On the south face ok. So, we have now got all the expressions evaluated right, but of course, now we have these partial derivatives for which what we will do we will make a linear profile assumption. Assume that the temperature varies linearly between the cell centroids and all the directions then of course, I can write partial T partial theta in the east direction as what? So, we are evaluating at partial T partial.

Student: Theta.

Theta in the east direction. So, we are evaluating this on this particular face right; on this face right. How do I calculate that using linear profile assumption it will be temperature at the east cell?

Student: Minus.

Minus temperature at the P cell divided by what will be the distance, we are travelling?

Student: (Refer Time: 19:29).

Delta theta.

Student: (Refer Time: 19:32).

East right.

Student: (Refer Time: 19:35).

Not r because we are we are not doing 1 by r, we are doing only partial d partial theta right. I am not doing 1 by r partial T partial theta in which case I would have had an r, there right. I just have partial d partial theta. So, what will this be?

Student: T east. $\frac{\partial T}{\partial \theta}\Big|_e = \frac{T_E - T_P}{\delta \theta_e}$

T east minus T P upon.

Student: Delta theta.

Student: East.

that is all right its only the traveling theta direction right because, the r factor has already been taken into account into the grad del term right in the del term right in the nabla fine ok. What will be $\frac{\partial T}{\partial \theta}|_w = \frac{T_P - T_W}{\delta \theta_w}$ partial T partial theta on the west face?

Student: T P minus.

Student: T west.

Student: (Refer Time: 20:19).

Student: w.

that is basically the angular distance delta theta w, between the P cell and the west cell fine ok. Then what about we have two more terms, that is partial T partial r on the north face that is the gradient in the radial direction right; radial temperature gradient on the north face. So, we are talking about calculating the gradient on this particular face right, what will that be?

Student: T n.$\frac{\partial T}{\partial \theta}|_n = \frac{T_N - T_P}{\delta r_n}$

Student: T P

Student: Delta r n.

that is basically the distance in the r direction. So, this would be come out to be T P minus T capital D T N minus T P right ok this will be T N minus T P upon delta r n right fine. Then what will be partial T partial r in the south direction? $\frac{\partial T}{\partial r}|_s = \frac{T_P - T_S}{\delta r_s}$

Student: T P minus.

Student: South.

South fine, alright. So, then what do we do? Of course, we need to introduce a model for our source term right.

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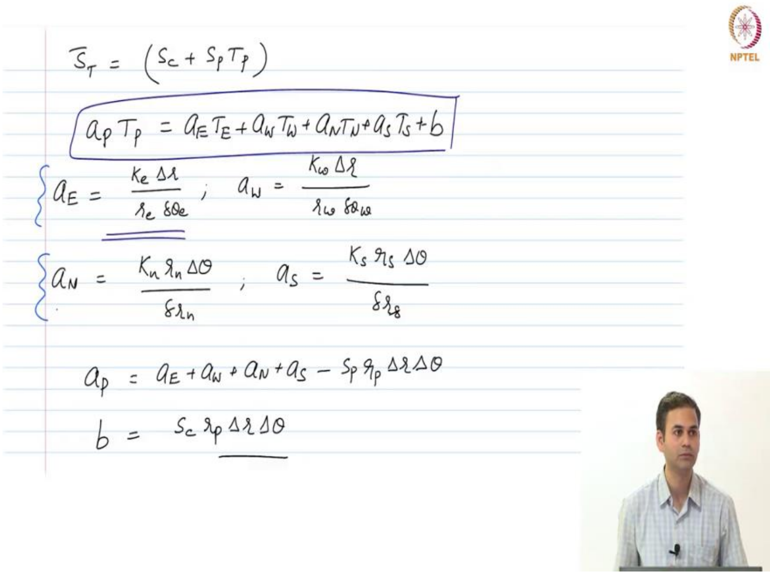

We have this $\vec{S}_T = (S_C + S_P T_P)$ which we will call it as right, that is how I will write it as and delta v is r P delta r delta theta right that we already have fine ok. Now, can we plug in all these things back into the original equation, which is basically this equation right and see and rearrange them send all the coefficients of T P or the T P terms to the right hand side leave everything else.

The east west north south on the left hand side and can we write it in a form that is acceptable to us in terms of finite volume method. So, I would rearrange that and if you do the algebra you will get $a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b$. Then can you tell me what will be a east?

Student: That should be.

A east what will what terms will go into A east terms coming from here right. $a_E = \frac{K_e \Delta r}{r_e \delta \theta_e}$ The K grad T dot A e these terms will go. So, what will A east will contain?

Student: K e.

Student: r.

Student: Delta theta So, essentially that delta theta e will also go into the coefficient right because, we are talking about coefficient of temperature at the east cell centroid T e right. So, then can you tell me what will be that? a e should be K e.

Student: Delta r.

K e delta r.

Student: Upon r e.

Upon r e.

Student: Delta theta e.

Delta theta e right fine. So, this will go into coefficient for T E and also it will go into coefficient for T P on the other right hand side fine. So, similarly what will be a west? $a_w = \frac{K_w \Delta r}{r_w \delta \theta_w}$

Student: K w.

. What will be a north? $a_N = \frac{K_n r_n \Delta\theta}{\delta r_n}$

Student: K n r n; K n r

Student: Delta theta.

Studen

Upon delta r n, this factor in the denominator is coming because of the temperature gradient right delta r n is coming, because of that. And then what will be a south? $a_S = \frac{K_s r_s \Delta\theta}{\delta r_s}$

Student: (Refer Time: 24:21).

Minus.

Student: No. So, we can calculate all of these ok, then what will be our a P term? $a_P = a_E + a_w + a_N + a_S - S_P r_p \Delta r \Delta\theta$

Student: (Refer Time: 24:41).

Student: Minus.

Student: (Refer Time: 24:50).

Student: Delta r.. What will be the term b?

Student: (Refer Time: 25:02) delta (Refer Time: 25:02).

What will b? $b = S_C r_P \Delta r \Delta\theta$

Student: S c.

Student: r P.

Student: Delta.

So, does not matter they are looking at a cylindrical or a Cartesian you get the same form right, for the working equation and of course, the coefficients would change accordingly fine. So, we have only looked at a steady diffusion in polar coordinates. Questions till now?

Student: (Refer Time: 25:32) orthogonal coordinate system.

Yes so, the question is we are getting the same form because, we are considering the orthogonal coordinate system that is true right. Because, we have orthogonal coordinate system we get well what I mean by same form is basically, we are looking at the this equation ok. We could arrange all of them and of course, the coefficients also are you do not have anything else right, you do not have the non orthogonality components come into play at the moment ok. We will look at them how does the non orthogonality come into play little later.

Student: (Refer Time: 26:07).

Yes.

Student: Can you scroll up?

Can you scroll up? Ok. You want to note it down.

Student: (Refer Time: 26:16).

Little more up.

Student: Delta delta e should be delta r e (Refer Time: 26:24).

No we are not that is what. So, the question is why do not I have an r here r delta e theta e? Right. So, what is the temperature? Temperature this is partial T partial theta right, the gradient is with respect to theta not with respect to, if this were 1 by r partial T partial theta, then I would have written as r e delta theta e right. Do you see that we are calculating the temperature gradient with respect to azimuthal angle. We are looking at partial T partial theta not partial T upon r partial theta right.

If that is the case then the distance will come into play right. Now, the distance is basically in terms of the angular positions right. So, that is why we do not have the r in here fine other questions.

Student: (Refer Time: 27:15).

Yes.

Student: (Refer Time: 27:20) here (Refer Time: 27:21) should be.

a E and what else?

Student: a.

a east and dimensionally very good. So, a east and a north are not matching dimensionally right, that is because we are working with r theta coordinates right. Are they not matching?

Student: They are matching.

They are matching right.

Student: Yes.

Because, delta theta is anyway in radians right. So, no issues other questions.

Student: Theta linear and (Refer Time: 28:00).

Right so ok. So, that is all the question or.

Student: Then linear kind of (Refer Time: 28:10).

So, why are we making a linear assumption a linear profile assumption along the theta direction also? Can there be any other assumptions; can there be any other assumptions?

Student: Yes.

Yes we can go for a second-order quadratic expression and things like that which we are not considering at the moment right. In fact, its usually not considered by most of the solvers as such, but it can be made as well ok. If you want to increase the order of accuracy, then you have to go for a quadratic fit here right.

Now, the idea is essentially you take this cells small enough that linear assumption would be sufficient as such right. So, in the context of higher order schemes which we will get to probably, once we get to convection we will look at higher order schemes in that context. We will see how to have other than linear fits for the dependent variables ok.

Other questions. No clear, fine alright. So, then very good. So, one question I have is what will happen at the r equal to 0? Will you still get a cell that looks like this; will you still get a geometry that looks like this r equal to 0 what will be the cells? Then just adjacent to r equal to 0 how do, they look like? They look like more like sectors right.

And how do you modify this entire thing? Because, whatever I have discussed is now like an orthogonal coordinate system, right. Now, do you need can you handle that? If you have r equal to 0 the cells that contain r equal to 0, right.

Student: Obviously, will (Refer Time: 29:56).

So, what will happen r S will be?

Student: 0.

0, in which case what will happen to the cell? Student: (Refer Time: 30:04).

Cells will become more like sectors. So, will the still the same formulation work or will not work? Maybe that is something for you to think ok.

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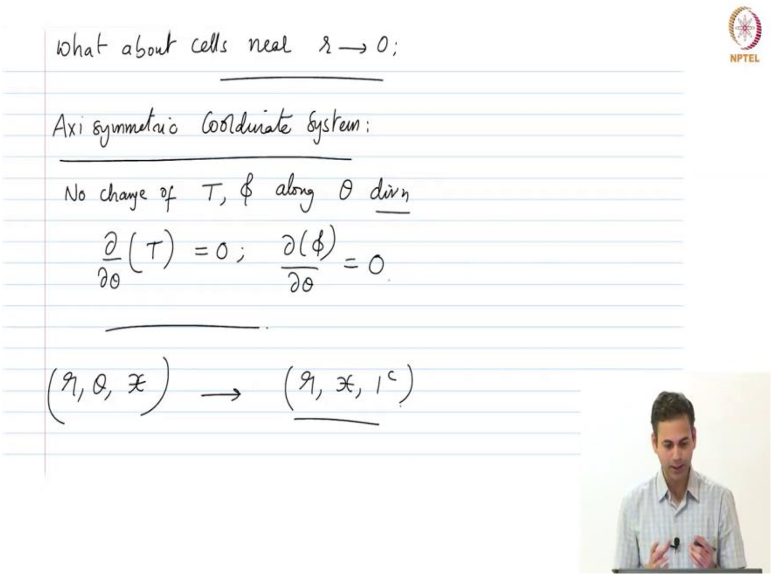

So, what about cells near r tends to 0 right. How do, they look like what will happen to the equations all these things is for you to think and see, if we can still work with this or not, alright. Let us move on to the other example of an orthogonal coordinate system, which is an axi symmetric coordinate system ok. So, we are looking at axisymmetric coordinate system fine.

So, axisymmetric coordinate system is something specific to engineering as such is not it because, its not in the mathematics as you learn it right. Because you only learn

Cartesian cylindrical and spherical coordinate, but when you come to engineering you have this thing called axisymmetric right. What is axisymmetry mean?

Student: (Refer Time: 31:26).

Independent of theta right. So, something so what is independent of theta, what quantity?

Student: (Refer Time: 31:29).

The dependent variable right. The dependent variable if it is independent of theta; that means there is no variation of the dependent variable in the theta direction right. Then ok, then why do we care? Something is not varying in theta direction. So, what does the simplicity that theta offers?

Student: The derivatives along the (Refer Time: 31:45).

The derivatives along the direction are 0 ok. So, essentially what we mean is that no change of the dependent variable that is either T or phi along theta direction ok. Here we are saying that theta is our azimuthal direction. Then the derivatives of the theta of the phi are equal to 0 or if our dependent variable is phi, then partial phi partial theta equal to 0 ok. Then, what does the simplicity or advantage that it offers, why should we have a special case for axisymmetry?

Student: 3 dimensional problem.

So, essentially the 3 dimensional problem now reduces to a?

Student: 2 dimensional problem.

2 dimensional problem right. Essentially earlier you had r theta x, if you call it as the original problem. Then you can only get away by solving what coordinates r and?

Student: x.

x and what about theta? You would solve it by like one radian ok. So, essentially you would consider one radian extent in the theta direction and then you its only solve for r and x, that is good. That means, we have reduced the complexity by one dimension, which is a great thing. What will be these problems usually that you encounter in reality in applications? Any applications where do you see this thing, what kind of geometries you will have?

Student: (Refer Time: 33:06).

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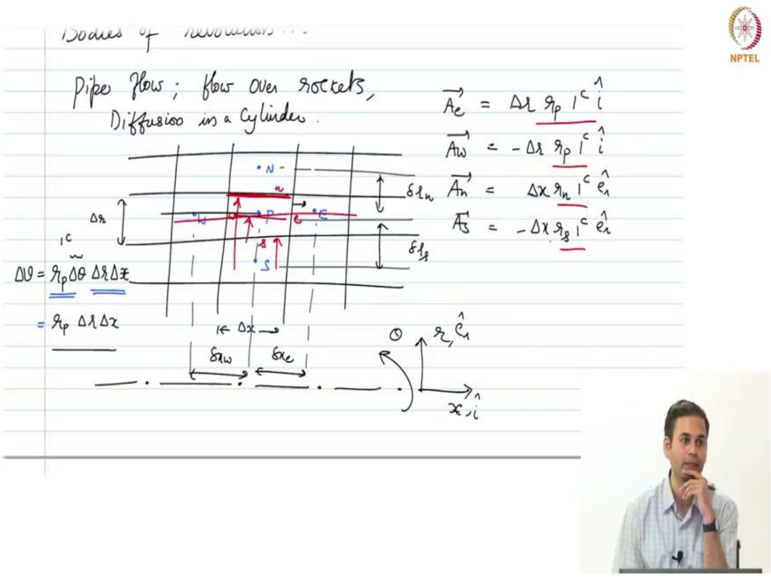

Anything that is bodies of revolution right. You have diffusion conduction flow all these things in bodies of revolution right. Then you can hopefully work with them in axisymmetric coordinates because, it offers reduction in dimension. So, what are these bodies of revolution examples?

Student: (Refer Time: 33:29).

Pipe ok, conduction ok, this is basically let us say pipe flow right. We want to only look at pi flow with r and x coordinates right ok. What other things?

Student: Heat.

Heat exchanges essentially these are all pipe flows ok, what about flow on a let us say on a missile or something right, anything that is of body of axisymmetric thing right its body of revolution right. So, essentially flow over kind of missiles or anything like that and the conduction in again a cylinder right.

If you have diffusion in cylinder all these things will come into play. So, these are basically flow over, let us say rockets or something and also diffusion in a cylinder; in a cylinder fine. So, these are all examples fine. Then let me draw a domain for this and also discretize it. So, we are talking about domain. This is my domain and of course, this is different from; this is different from a 2 dimensional domain because, now we have what?

Student: (Refer Time: 34:56).

We have an axis right; basically this is revolved around the axis such that you get that full 3D domain right, if you take a cylinder or something. So; that means, my coordinates are what this is my r sorry, this is my x and this is my this is the r coordinate and what about theta? Theta is in this direction right, this is theta right that is fine.

So, then let me also name this l. So, this is P cell this is east, this is west, this is north and this is south again we have x and r direction. So, I would call this distance of the P P cell as delta x and the y extent of the P cell as delta r ok. And similar to before we have east face, west face, north face and the south faces ok. We have all these things then what about the distance between the P cell and the east cell? What do we call it as delta x e and this would be delta x w ok. And what about the north cell and the P cell? This would be how much?

Student: Delta r.

Delta r.

Student: North.

North and this would be.

Student: Delta r south.

Delta r south fine ok. Now, I have one basic question we have just named r as y right, how is it different from a 2 dimensional situation? I just drew a rectangular cells right a Cartesian cell and told you that ok, this is not a Cartesian coordinate system instead of y I just replaced with r. So, what is the difference between axisymmetric domain and a 2 dimensional domain, how is this different?

Student: (Refer Time: 36:59).

r.

Student: Cannot be negative.

r cannot be negative ok. So, r is not negative whereas, y can be negative fine, let us not worry about a let us say we are talking about a 2 dimensional system in the positive y coordinate. Is that the same as axisymmetric? Somebody else had an answer here.

Student: Area is (Refer Time: 37:20) area if.

Area is different that is right. So, why area is different?

Student: (Refer Time: 37:24).

So, we are talking about one radian right, we are talking about a 3 D thing. So; that means, we are talking about all this entire mesh, I have drawn is actually sweeped in the theta direction by one radian right.

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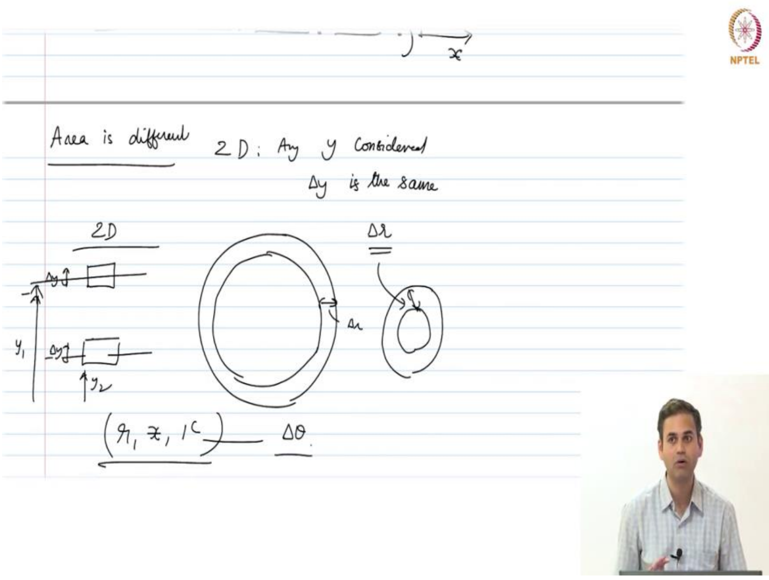

So, we have this cylindrical cells; that means, the area is different; area is different. That means, if it were a 2 dimensional domain, then does not matter any y I consider right, any y considered will have the same area as long as delta y is the same right.

For example: I consider a 2 D domain; if I consider a cell by this delta y or this delta y the same, but at any different location. So, let us say this is at y 1 this is at y 2, right. So, for these two the area is always delta y times minus i hat right. Whereas, if I have a r theta coordinates what will happen to the area? So; that means, you are looking at a let us say a pipe flow right.

Now, if I look at the same delta r delta r is the same, but the area included would be larger if you are at a higher radius right. That means, I consider this is my same delta r right, but I look at another case where my mean radius is smaller, but still this is also the same delta r, but what will be the area included by these two?

It is different because you have r times delta theta right r times theta so; that means, if you have a uniform flow through a pipe and; that means, the if you kind of make the pipe into two sections from 0 to capital R, that you make into two sections 0 to r by 2 and r by 2 to capital R. Then, if we have the same velocity let us say mean velocity then, which of these will have a higher flow rate?

Student: Outer.

The outer section will have higher flow rate although, you have divided into two halves right in terms of gradient; that means, the area is coming into play right that is what is the main difference between the axisymmetry and the 2 dimensional things ok. So, that is the difference and that is what we have to consider while generating the area vectors and the volume for these cells ok. So that means, we are talking about axisymmetry, but we are actually talking about a 3 dimensional system with r x and theta that is one radian ok, we are actually developing this fine.

So, one radian is equal to delta theta, fine. Questions till now? That is the main difference clear or right. So, basically if I were to draw it again if I take a 2 D cell right.

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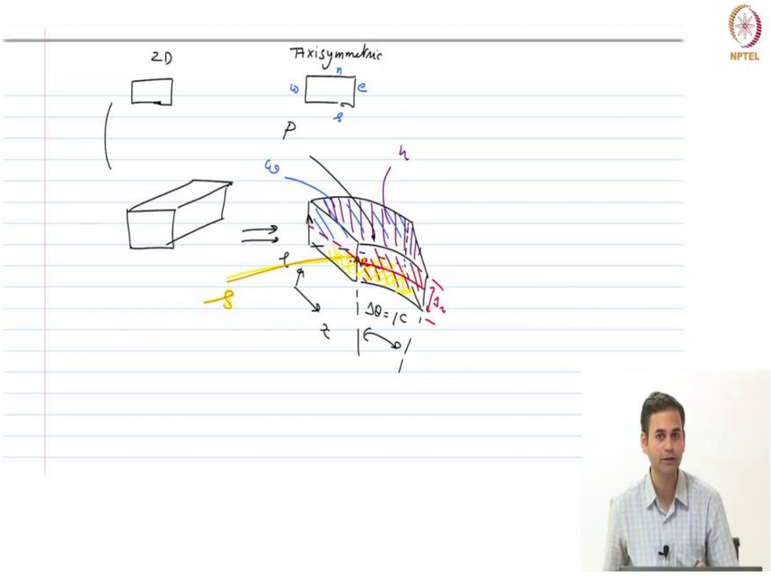

So, this is my 2 dimension situation, this is my axisymmetric situation, I have these two cells let us say both are of the same size ok, this is slightly bigger ok. So, in 2 D, when I sweep it to 3 dimensions I would get a geometry like this right. Whereas, here when I sweep it in theta direction what do I get?

Student: (Refer Time: 41:14).

I would get a something that goes like this right. Is not it? Because of the theta direction right I would maybe draw it little better ok. So, this is the fine. So, essentially the area differs as you are around fine ok. Now, this is what we have to use to develop all the area vectors, let us say this is my P cell, this is the P cell. And this is the delta theta which we would call it as 1 radian ok. And this is the x direction this is the r direction; r direction is along these lines fine, we are we are looking from this side right.

So, theta is out of plane ok. Then what is our east, west, north, south? This is our east right, that is what we said. When we say this is east face we are actually talking about this entire thing as the east face right. Similarly when we say west face we are talking about this entire thing as the west face right the back one, when we say north face we are talking about this entire thing as the north face right. And we say south face we are talking about the south face here right.

That is the south face right I think that we have see this is the south face right ok. So, when we said this is east, west, north, south all these faces are swept by one radian right, they are all swept one radian in the theta direction fine; which is not the case if you have 2 dimension situation right. Is that clear? You are able to see through then ok. Now, the tough part what will be the area vectors and the volumes for this particular case?

Student: (Refer Time: 43:34).

I will go back to the figure ok. Now, essentially you take this grad and then sweep it by theta in the other direction. So, then what will be; what will be the volume delta v for this for the P cell? We are actually talking about a nominally 2 dimensional domain, but it is actually cylindrical right with 1 radian in the delta theta direction. So, what will be the volume for the cell P cell?

Student: Delta v delta r.

What is the 2 dimensional volume 2 dimensional area?

Student: Delta r.

Delta r delta theta now how much that should be multiplied with, what will be the length in the theta direction?

Student: r P.

r P times delta theta, but delta theta itself is?

Student: 1 radian.

1 radian. So, this is basically r P times delta theta times delta r times delta x right, because delta r delta x is the 2 dimensional area times r P delta theta is the.

Student: (Refer Time: 44:35).

Length in the theta direction; so, this will come out to be how much because, we are considering delta theta equals 1 radian this will be r P delta r delta x fine. Is this clear to everybody, yeah this part? Ok. Then what about the area vectors now? What is the area A east? What will be A e? A east is in the direction is in this direction ok. Let us call the unit vectors as e r hat and i hat; i hat and e r hat. What will be the area vectors?

Student: (Refer Time: 45:22).

So, basically one way to look at is what will be the length of this line in the r direction?

Student: Delta r.

Delta r, now that has to be swept in the theta direction right. So, then that will be how much?

Student: Delta theta.

Delta r times.

Student: r P delta theta.

r P.

Student: Delta theta.

Delta theta is 1 radian right this is same as r little e fine. What would be A west? If I were to go back its basically here what will be this area? Right. We are talking about ah east face that is this the one in red right this will be delta r, that is delta r right times this r P that we have rotated by 1 radian right that is all the area ok. Now, what will be the west area? A w bar minus there should be a i hat here is not it i hat ok, that is A e bar what about A w bar?

Student: Minus delta (Refer Time: 46:28).

Minus delta r.

Student: r.

r p.

Student: 1 radian.

1 radian.

Student: i hat.

I hat fine ok. What about A north? A north we are actually looking at this guy right. What is this length?

Student: Delta x.

Delta x. Now how much do we have to sweep in the other direction?

Student: r n times.

r n times.

Student: 1 radian delta theta.

1 radian delta theta. So, this will be how much? This will be delta x times r little north 1 radian times that is all right.

Student: (Refer Time: 47:09).

e r hat ok. Then what about A south bar? No minus, minus delta x r south 1 radian e r hat fine.

Student: r north and r P delta face is different.

r P and r north are they different?

Student: Yes.

They are different right because, we are talking about r P would be this guy right is this one is r P, r north would be this guy and r south would be this much right. So, they are different right. Only r little e and r little w are equal to r capital P right, only anything on this particular radius would be the same right everybody else is different.

Now, what is the extra terms that we got here because of the axisymmetry? We got this r P and r n right, they were not there in the case of 2 D right in 2 D it would just read delta r delta x right that r factor r times delta theta is the one which is coming out extra, is not it? If you kind of remove these guys what will it be?

Student: 2 dimensional.

It will be 2 dimensional right that r times 1 radian is something that is extra in the axisymmetry, right. Of course, we have not defined the del operator till now, that has to be also defined fine.

Thank you.