Computational Fluid Dynamics Using Finite Volume Method
Prof. Kameswararao Anupindi
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture - 35
Finite Volume Method for Fluid Flow
Calculations: The staggered grid approach

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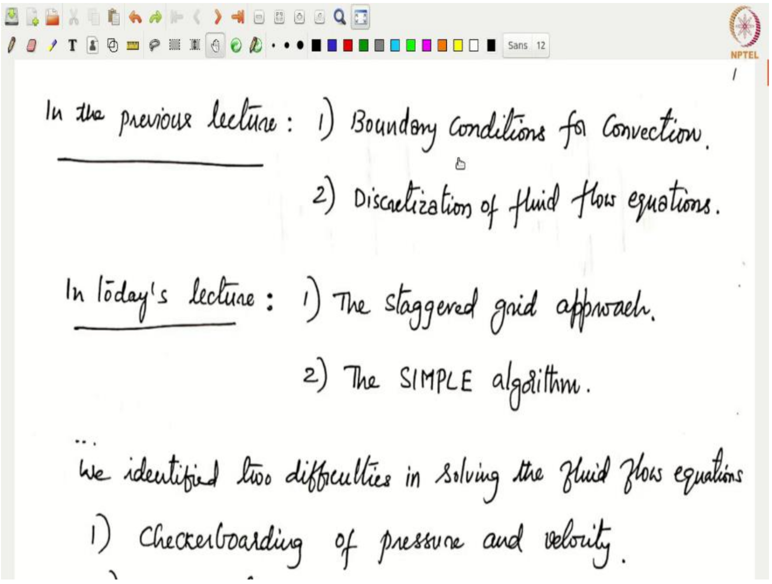

Hello, everyone. Let us get started. So, welcome to another lecture as part of our ME6151 Computational heat and Fluid Flow course. So, in the last lecture we looked at boundary conditions for convection that is essentially we looked at distinguish the in-flow, out flow boundaries and geometric boundary such as walls.

And, we also started looking at the final chapter that is basically the discretization or solution of a fluid flow equations, right and we looked at couple of difficulties that we kind of encounter in the solution of fluid flow equations which are basically there is no equation for pressure right in the solution of the fluid flow equations.

Because we have one equation for u, one equation for v whereas when we turn down to the other equation that is continuity equation then we realize that the continuity equation is also an equation for velocities, but not for pressure ok. So, that is one difficulty. The other difficulty is that we get the checker boarded pattern that can be supported by the velocity and pressure fields because in the equation for cell P we never got a term that kind of corresponds to the cell P, right.

For example, there was no u_P or v_P in the equations for momentum in the equations for continuity equation and similarly there is no piece of p in the equation for the pressure gradient in the momentum equations right. So, these are the couple of difficulties that we face and that is where we kind of stopped our previous lecture.

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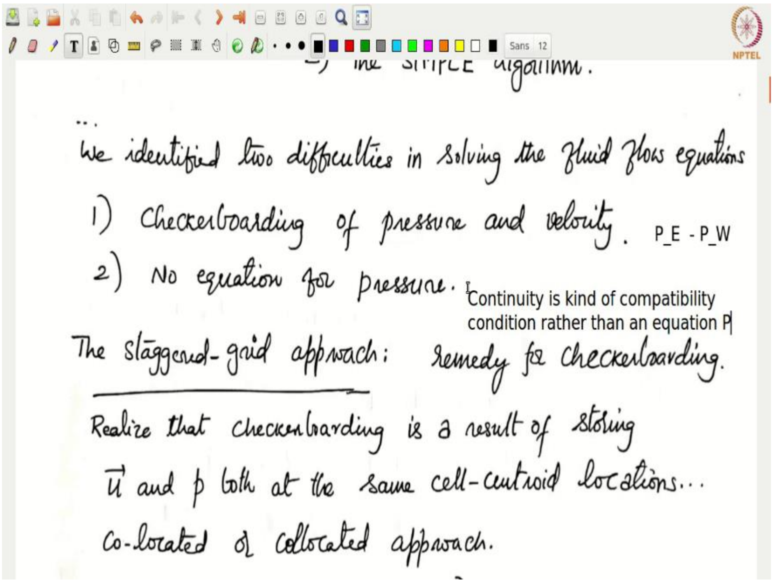

So, in today's lecture we are going to look at a remedy for the pressure checker boarding or velocity checker boarding, essentially how do we decouple this kind of checker boarding such that they do not together contribute to divergence and sustenance of the oscillations in the solution.

So, we are going to look at a famous remedy that is known as the staggered grid approach and after that we are going to look at the simple algorithm, which is again a very famous algorithm for the solution of in compressible fluid flow equations, alright.

So, essentially in the last lecture we identified two difficulties in solving or in the solution of fluid flow equations those are namely the checker boarding of pressure and velocity because both of them are $2\Delta x$ apart right, we got terms like P_E and P_W, right. We never had a P_P term in there; similarly, the velocities also always had v_e and u_e and u_w right and things like that ok.

So, checker boarding of pressure is one issue and velocity is one issue and then the other thing was there is no equation for pressure right because the continuity equation is kind of a compatibility condition, right rather than an equation right rather than equation for pressure right.

It is not the equation for pressure rather it is kind of a more of a compatibility condition because you have this thing like where $\nabla \cdot \rho\vec{u} = 0$which is more of a consistency check that the velocity field is divergence free or not alright. So, these are the two difficulties. So, one of the difficulties is basically the checker boarding of pressure and velocity this can be remedied by switching to something known as a staggered grid approach ok.

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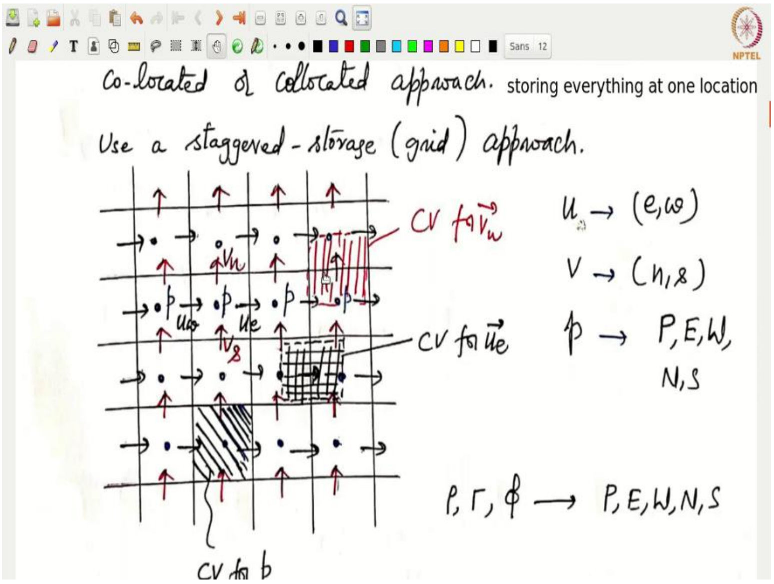

So, this is a very good remedy for avoiding checker boarding of pressure and velocity fields in the solution. So, we realized that we were actually getting checker boarding in the first place because of the way we have stored the velocities and pressures, right.

We have stored all the both the components of velocity that is u and v as well as the pressure. We have stored all of these at the cell centroids only, right. So, the cell centroid that is p capital E capital W and so on. We stored all the physical quantities that we wanted to solve.

So, u, v, p and any other scalar ϕ we have stored all of these at the cell centroid P. In fact that is a reason because when we wanted to have the face values of these quantities we never have a face value stored. So, for the face values we used linear arithmetic average and then that is where this checker boarding was coming into play right because it was supported by things that are parameters that are $2\Delta x$, apart right. They are away by between east to west right ok.

So, this approach in the literature is known as co-located or collocated approach which is basically storing everything or all the solution variables at one location right that is one location P or something like that ok, but rather we would not use this approach as of now. We will switch to a staggered grid approach because this will avoid the pressure checker boarding and velocity checker boarding ok.

So, here is a schematic of how should we store the different solution variables that is u, v, P ok. So, here we show a particular grid. Now, this may look very combustion, but there is pattern in this thing. So, essentially you have the these are the main cells that are shown with squares here. Now, the circles here the dots here indicate the cell centroids of these primary cells.

So, let us say we have one cell at the cell centroid that is denoted with this filled circle that is where we store the pressure ok. So, pressure or any other scalar such as density or any other scalar ϕ and the deficient coefficient gamma all these things are all stored at P, capital E, capital W, capital N and South S at the cell centroid. This is the primary cell, ok. This is the same as what we had or why have been working with till now.

Now, what we do is instead of storing the velocities also at the cell centroid both u and v, we kind of shift them by half cell. So, we go we store the horizontal arrows here denote the u_e and the vertical arrows here denote the v_n or v_s ok. So, essentially the horizontal vectors here denote the u component of velocity and the vertical ones denote the v component or y component of velocity that is v ok. So, now, these are shifted.

So, u v shifted from this cell centroid by half Δx and v is shifted by from the cell centroid by half Δy ok. So, that is where we store we want to store the velocities which are basically staggered from the cell centroids ok. So, now, this is where all the u_e will be stored or u_w and the vertical arrows is where all the v_n and v_s will be stored.

Now, of course, because we are using finite volume approach we need to have a volume associated with every variable. We do not have any issue with the storage of P because the

P is at the primary cell centroid as a result this is this serves as a control volume for the integration or for applying the Gauss-divergence theorem for as far as pressure or any other scalar ϕ is concerned ok, whereas we need to now identify a control volume for u_e.

So, u_e is basically staggered so such that the control volume which has to be centered around this u_e. So, u_e has to be at the cell centroid of the particular cell. So, the control volume is now shown here with by hatching here. So, here we see that cv is basically for u_e is shown here and similarly the control volume for v_n is also shown here with red color and strips ok.

So, we have these checks is for u, u e bar inclined lines is for pressure and the vertical lines in red color is for the control volume for the v_n ok. So, that means, u is stored on the faces east west and so on; v, the y component of the velocity is stored on the north and south faces and so on and pressure and any other scalar that you want to solve ϕ or gamma or the density all these things are all as usual stored at the original centroids that is P, E, capital E, capital W, N and S ok.

So, that is the staggered grid storage or staggered grid approach. Now, this is not unique, but we are going to use this particular way of storing as far as the course is concerned. But, if you look in the literature there are many other ways of staggering.

For example, instead of storing u on the east face people might store u parallel to the x-axis; that means, they may show u here and v here that is also possible that is also a particular way of staggering and there are also few other half staggering approaches where u and v are stored at one location and pressure is another location ok.

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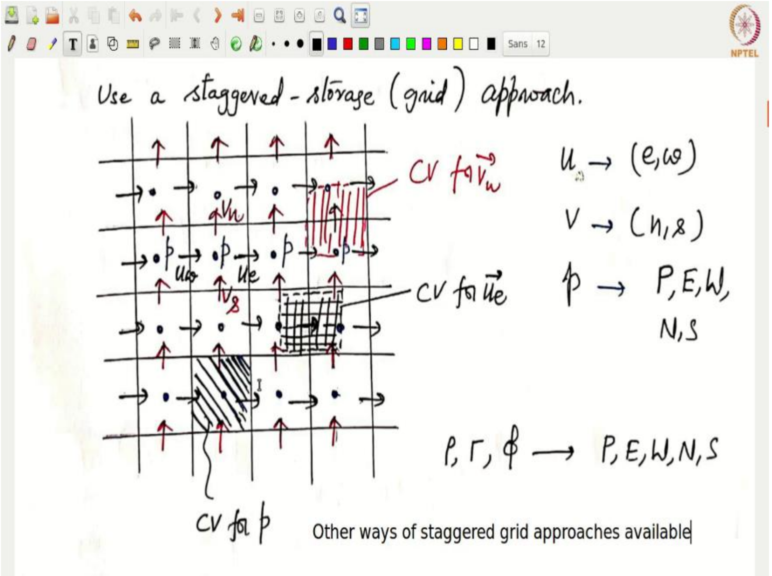

So, there are few other ways of other ways of staggered grid approaches available but, we will not be looking at all of them available in the literature. But, we will not look at all of them we will only concentrate on this particular way of staggering ok, alright.

Now, again one more question that pops up in mind is that so, shall I create three different meshes for solving one problem? No, you will not create three different meshes, you will create only one mesh, that is basically your primary mesh for the primary cells. And, everything else will be kind of only while you write the algorithm or while you write the code you will know that it is basically stored at half way through these cell centroids ok. So, that is what you would use.

Only in the process of coding you will have this, but not physically you will not create three different grids ok, alright. But, of course, when you store it you would have to count how many east and west faces are there, accordingly you have to allocate the storage for u, v and the P ok. So, that needs to be done, alright. So, let us move on. So, we have now introduced staggered grid approach let us look at a particular cell p and see how does this look, ok.

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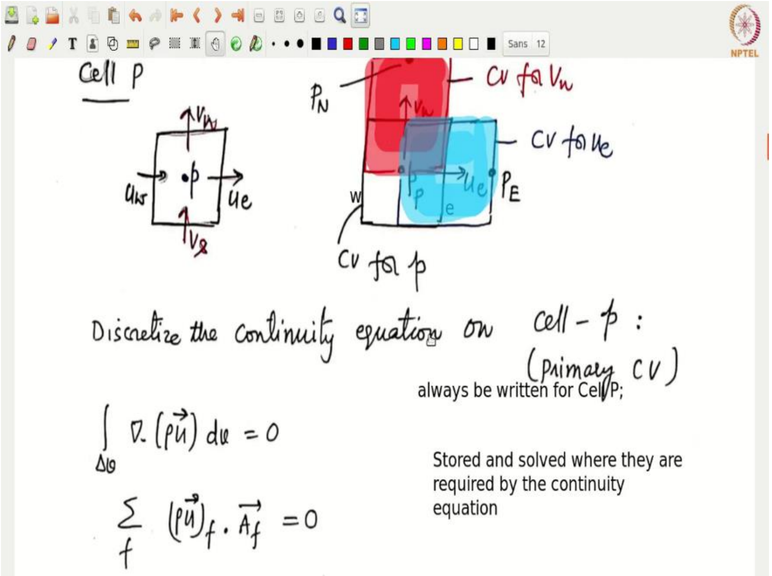

So, essentially the p if we if I plot only the cell p ok, this is what we have. Essentially we have P the pressure stored in the cell centroid and the horizontal x component of velocity that is u_e and u_w on the faces of the cell centroid, and on the north and south faces we have v_n and v_s stored ok, alright.

Of course, if I again draw all the three control volumes together this is what we get essentially we have the p cell that is shown here in black then we have u_e stored on the face which has this control volume for u_e that is kind of shown here and v_n is on the north face of the p cell and this is where you have the control volume for the north face, ok.

Now, of course, we know that the if you show me uniform mesh, then essentially your P or even if it is not uniform mesh, but as long as it is Cartesian essentially what you have is your P_E will coincide with the with your east face of the u_e cell, right because this is where you will have P_E, similarly P_N would be coinciding with the north face of the v_n cell, ok. I hope you see that.

Essentially, what I mean is that when you write any equation for $\overrightarrow{u_e}$, the east and west faces would be capital E and capital P. They will be aligned with that. Similarly, the north and south faces for $\overrightarrow{v_n}$ bar equation would be capital north and capital south whereas the east and west faces for the p cell are the same as before.

These are little e and little w right, these are essentially your little e and this is your little w whereas, when we write it for the other quantities that is for u_e and v_n, the faces are actually now capital E and capital P, capital north and capital P ok.

Now, of course, we are not talking about west and then south because essentially once you write it here you can of course, write the same thing for this guy and write the same thing for here and so on. So, we do not have to write equations for u_w and v_s ok.

They will come out to be the same as u_e and v_n; essentially, they are shifted by one cell in x and y directions. Or you can think of it like essentially for every p cell you will have one u_e and one v_n cell as well ok, that way everything kind of balances out, alright.

Then so, the idea is now we have to again discretize the continuity equation and again of course, discretize the momentum equations on this staggered grid kind of approach and see if the velocity and pressure in this particular pattern would they support the pressure and velocity checker boarding or not, ok? So, that is the question.

So, if we look at the continuity equation for cell p ok, now remember that continuity will always be written for the primary cell for the cell P only it will always written no matter whether it is a staggered grid or a regular grid ok. The continuity will be written only for the cell P ok, the primary cell, alright. Now, if we write the continuity equation then we have $\nabla \cdot \rho\vec{u} = 0$.

So, that means we are going to if you apply Gauss divergence theorem then you will get $\rho\vec{u} \cdot \overrightarrow{A_f} = 0$, right? Now, what about the faces? The faces for this cell are again same as before this is east, west and north and south. So, we have $(\rho u)_e \Delta y$ minus $(\rho u)_w \Delta y$ plus $(\rho v)_n \Delta x$ minus $(\rho v)_s \Delta x$.

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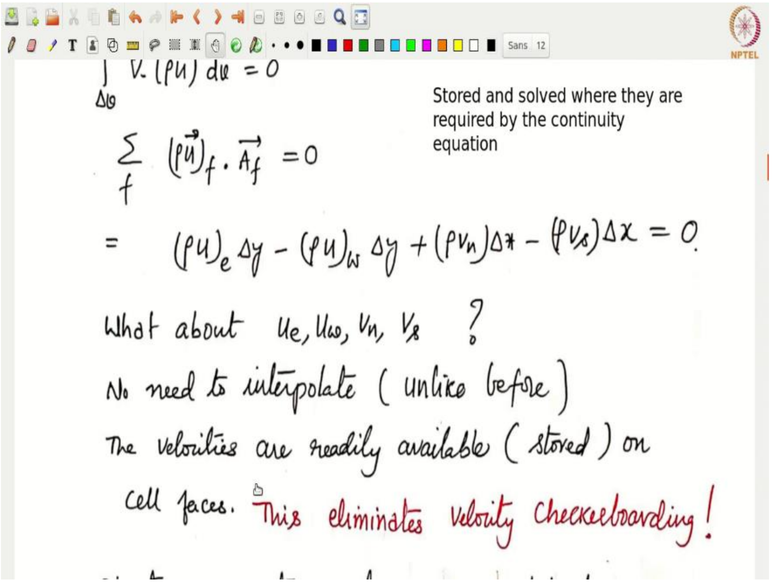

But, now what about u_e? So, u_e do we have a value stored on the east face or do we need to interpolate? Because we have already staggered it, we do not have to interpolate any more right, because u_e is already stored and solved at the east face and the west face. Similarly, v_n and v_s are essentially they are stored and they solved where they are required by the continuity equation, right.

Essentially then we do not have to interpolate anymore right unlike before because earlier when we got u_e we had to interpolate as their arithmetic average as u_E and u_P by 2 and that landed us into trouble right because then we ended up with essentially velocity field supporting our checker boarded pattern right.

So, therefore, a by staggering the velocities what we see is the velocities are readily available and stored and solved where they are required on the cell faces for the primary cell and this kind of now staggering eliminates velocity checker boarding right because you do not have to interpolate anymore. And, we are essentially talking about east minus west and north minus south, basically these are know the primary these are now adjacent values right.

So, if u_e is here u_w will be here. So, essentially the neighbor is basically u_e and u_w as a result you cannot have checker boarding, right. Earlier u_e and u_w are $2\Delta x$ apart now they are only Δx apart right ok, alright.

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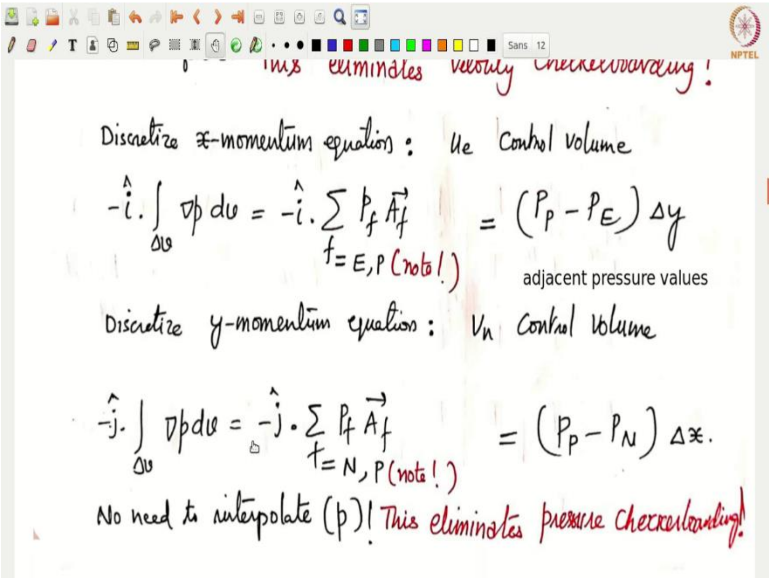

Then, let us look at what happens to the momentum equation. So, let us look at the discrete x momentum equation. Now, where do we will solve the momentum equation we are writing an equation for x component of velocity that is u_e. So, we need to solve momentum equation on the u_e control volume right; that means, we solve momentum equation x momentum equation on this control volume and y momentum equation on this control volume ok.

That means, x momentum equation will be solved on essentially on this control volume and y momentum will be solved on this control volume ok. So, essentially it is slightly bigger. So, this is where we will solve the y momentum equation and this is where we will solve the x momentum equation ok, right and on the primary cell we have solved for the continuity equation, right, ok.

Then let us go and see right the x momentum equation. Now, again we are not writing the entire equation which we will write little later. As of now we will look at only the pressure gradient term because we want to see if this staggering supports the pressure checker boarding or not, ok.

So, if you look at the equation essentially what we have is the pressure gradient is $-\frac{\partial P}{\partial x}$ in the x momentum equation and we can write this as $(-\hat{\mathrm{i}} \cdot \nabla P)\, dV$, right. Again, if you apply gradient theorem then we can convert this as $-\hat{\imath} \cdot \sum \mathrm{P_f}\overrightarrow{\mathrm{A_f}}$.

Now, what about the faces here? The phases are not little e little w little north little south; rather the faces are now actually capital E and capital P. Do you see that because we are now writing an equation for the blue cell for the u_e cell and what will what are the faces for the cell?

If this face is capital E, this face is capital P; this face of course, is still the little north and little south, but because we do not we have our A_f because we are taking a dot product with $-\hat{\imath}$ the only term that survive are the $\overrightarrow{A_f}$ bar that are aligned in the x direction. As a result only E and P will survive. So, we do not have to worry about the little north and the little south for this particular case.

So, and this would basically be your evaluate to because A capital E would become $\Delta y\hat{\imath}$ right because there is a minus it gets pushed to the negative value and A capital P would evaluate to $-\Delta y\hat{\imath}$and that would make it plus. So, as a result what we get? Out of this summation is basically $(P_P - P_E)$ times Δy ok.

So, now, you already see that the pressures are now the pressure gradient uses adjacent pressure values, right. We do not have $(P_W - P_P)$. $(P_w - P_e)$ right which eventually lead to $(P_W - P_E)$ like what we had before in the collocated approach. So, right now here we have $(P_P - P_E)$.

So, as a result if you have checker boarding, then it will automatically be seen by the momentum equation as a nonzero pressure gradient and it will not it will be felt right unlike before where such a pressure checker boarding was not felt with the momentum equations as a pressure gradient right it was felt by the momentum equation as 0 pressure gradient ok, alright.

Now, if you look at the y-momentum equation, this has now of course, all other components are there, but then what we have is $-\hat{\jmath} \cdot \nabla P\, dV$. So, this basically gives you again applying gradient theorem we get $P_f\overrightarrow{A_f}$ dotted with $-\hat{\jmath}$ now what are the faces for the y-momentum equation? Essentially, we are talking about V n control volume ok.

So, v_n control volume what are the faces? So, this is the control volume, what is this face? This face is basically capital N, this face is capital P; of course, these faces are again little e and little w, but they will not survive in the equation because we are taking a dot product with $-\hat{\jmath}$, ok. So, capital N and capital P are the faces for the v_n controlled volume. So, that means, what we have is basically this term $-\hat{j} \cdot \sum P_f \overrightarrow{A_f}$ will evaluate to $(P_P - P_N)$ times Δx ok, alright.

Now, again we see that we already have P_P and P_N available and there also adjacent values, as a result we do not have to interpolate p and this also eliminates pressure checker boarding ok.

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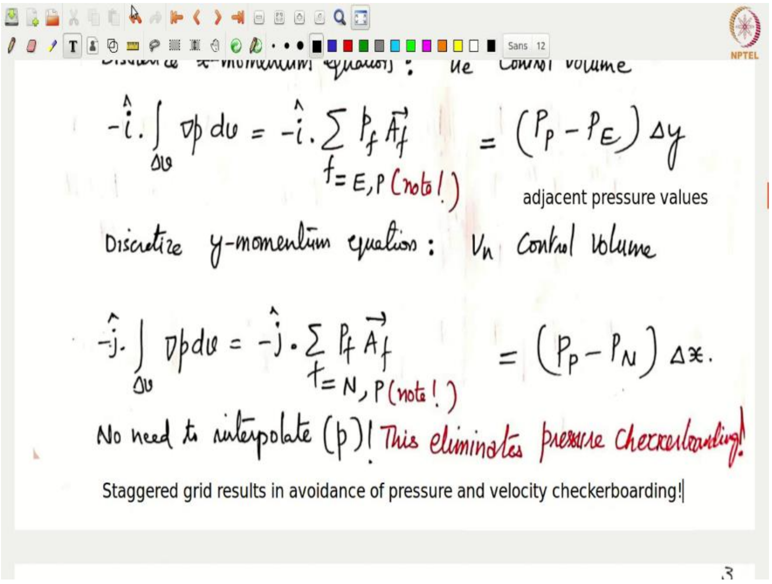

As a result what we can say is the use of staggered mesh or grid results in avoidance of pressure and velocity checker boarding ok, that is what we see from the equations, alright. So, this is only one fix for our problem, we have another issue also to look at that is basically the actions of an equation for pressure, ok. Before we do that let us look at the full equation essentially the complete set of x and y momentum equations discretized on these staggered cells ok.

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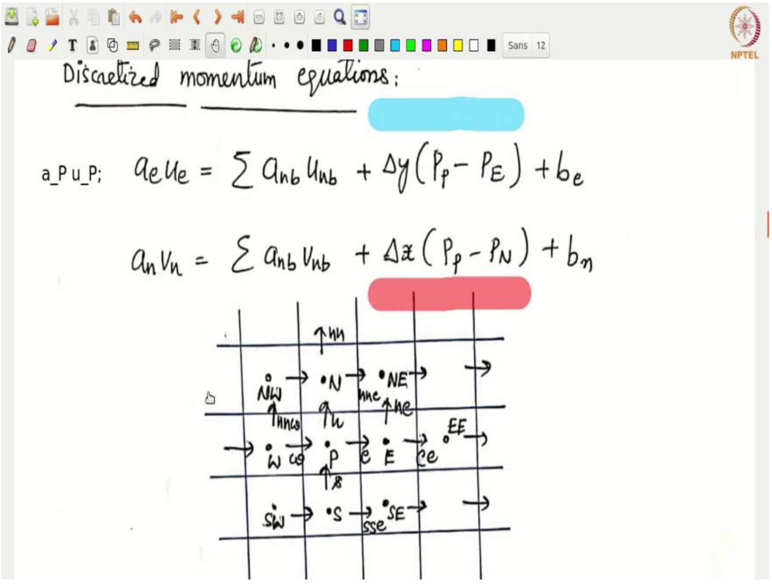

That means, discrete momentum equations are we can of course, write the full discrete equation, but one thing one difference is that up till now we were writing $a_P u_P$ equals $\sum \mathrm{a_{nb}} \mathrm{u}_{nb}$ plus b, but now because we are writing for u_e we will like to call our p cell as now our east face. So, the coefficient also will changed to $a_e u_e$ is our essentially similar to our a_P similar to $a_P u_P$ ok, but it is not the same because now, because u_e is not stored at the cell centroid for primary cell ok.

So, as a result our discrete equation will read $a_e u_e = \sum \mathrm{a_{nb}} \mathrm{u}_{nb} + \mathrm{b_e}$ plus $\Delta y(P_P - P_E)$. This is what we just saw before, right. This term is what we just saw before right, because of the integration, ok. Similarly, the other term is basically the y-momentum equation which will read as $a_P v_P$, but instead we are storing the velocities on the north face.

So, this will read as $a_n v_n = \sum \mathrm{a_{nb}} \mathrm{v}_{nb} + \mathrm{b_n}$ and we have our $\Delta x(P_P - P_N)$ ok. So, this is the pressure gradient term that evaluates alright. So, far so good, now we have the discrete momentum equations and also the discrete continuity equation, but of course, we still do not have any equation for pressure ok. Now, what about the neighboring values here $\mathrm{a_{nb}} \mathrm{u}_{nb}$ here see in the context of collocated approach these u_{nb} where east, west, north, south and so on.

So, we have done that all the way through, but now because we have shifted these things the u_{nb} are also now different they are not E capital E capital W capital N and south right. For example, if you look at $a_e u_e$ essentially we are talking about this guy, so, what will be the neighbors for this guy? This guy will be neighbors will be if you were to call the face here as between capital E and capital east east as little e e, then the neighbors are little e e little w little north north east and little south south east.

So, these are the u and v's that we have to use while writing this equation ok. Similarly the $a_{nb}v_{nb}$ we have to use while writing the equation for north that is v_n is basically is north north and south and north east and north north west, ok. So, essentially these four guys is what we have to use, fine. So, that is what we have.

So, essentially the neighbors a and b for v equation are this guy. This guy, this guy and this guy and for u equation the neighbors are this is the east east, west north north east and south south east ok. So, those are the neighbors. Of course, on the p cell again the neighbors are east west north south that is same as before there is no change in that, alright. So, that is where, but what about now a nb? These a nb values are they same between this and this?

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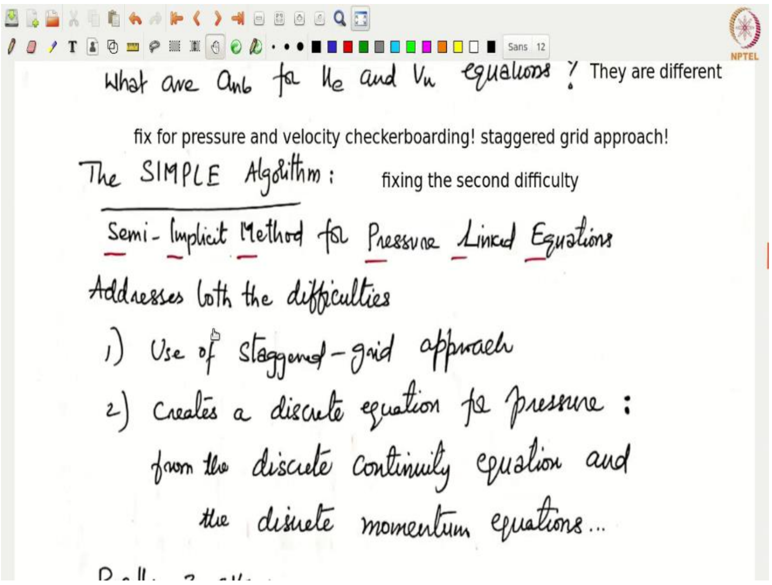

They are not the same because now because of the staggering these values a_{nb}'s are not the same right, essentially they are different ok. They are not the same; they are different ok. So, essentially between the two equations u_e and v_n equations in the a_{nb}'s are different because the way the discretization are there written for different cells ok, alright.

So, that kind of makes us with a fix for the pressure and velocity checker boarding ok this is basically use staggered grid approach that is basically helps us fix the first difficulty that we have in solving the incompressible fluid flow equations, alright.

Now, in order to look at the other difficulty, we look at this famous algorithm which is known as the simple algorithm which of course, does not indicate the level of difficulty ok; the simple that does not mean the level of difficulty simple basically stands for semi implicit method for pressure linked equations, ok. We will see what is semi implicit in this method little later.

So, essentially this is a very famous algorithm that is used in order to solve for incompressible fluid flow equations, ok. So, this will also help us with fixing basically the second difficulty that we have that is basically the absence of any equation for pressure ok. So, what simple method does is, it addresses the both the difficulties.

So, it uses a staggered grid approach, thereby it allows us to work with pressure and velocity fields without them becoming checker boarded and it also creates a discrete equation for pressure from the discrete continuity equation and the discrete momentum equations that we have ok. So, essentially it creates an equations of pressure and the way it basically does is what we are going to see next, alright.

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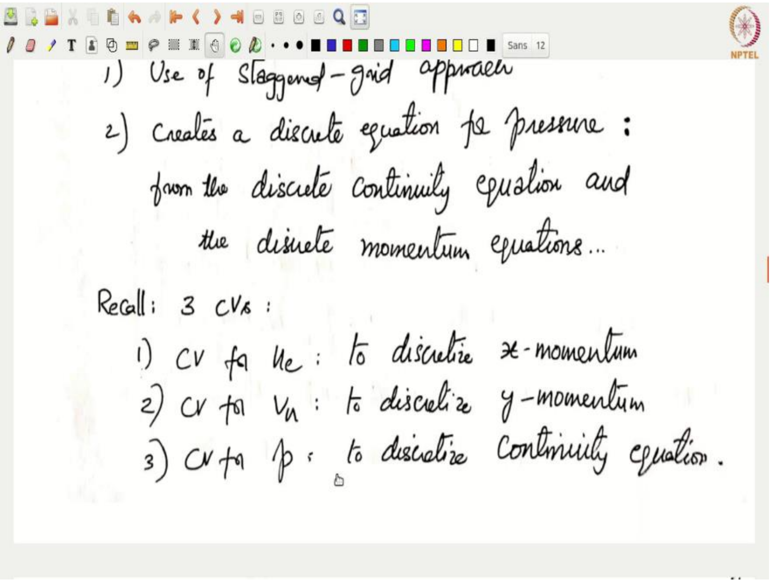

So, it is a good idea to recall that we have three control volumes – the primary control volume is basically control volume for pressure, storing the pressure and the cell centroids of the main control volumes and this is where we discretize our continuity equation always. Now, we have a staggered control volume that is staggered by half Δx that is basically for storing the x-component of velocity that is u_e and this is used to discretize the x-momentum equation.

Similarly, the v_n the y-component of velocity is also staggered from p by half Δy and this will be used to right the discrete y-momentum equation ok. So, that is basically just to recall the existence of the three different control volumes, alright.

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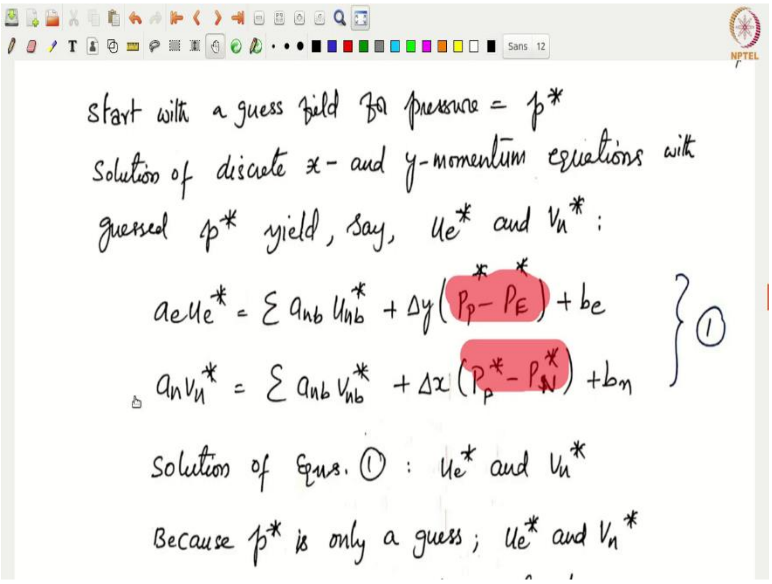

Now, let us get started with the simple algorithm. So, basically what we have is we start with a guess field for pressure ok. So, essentially you guess what is the pressure field if I had to denote the guess of the pressure as P^* and then I would we would like to solve discrete x and y-momentum equations on the staggered grid using this guessed pressure ok.

That means, we will write $a_e u_e = \sum \mathrm{a_{nb}} \mathrm{u}_{nb} + \Delta y(P_P^* - P_E^*) + \mathrm{b_e}$ ok. Similarly, $a_n v_n = \sum \mathrm{a_{nb}} \mathrm{v}_{nb} + \Delta x(P_P^* - P_N^*) + \mathrm{b_n}$ ok. So, what we have done is we have just made a guess for our pressure values and using these pressure values, now we are solving for this x and y momentum equations, and the solution thus obtain by solving this system are now denoted with star values here ok.

So, u_e^*, v_n^* are basically the velocities they are not simply the guessed velocities they are the velocities that are obtained by solving this system of equations using guessed pressure P^*, ok. How do you solve for these two equations? How do you solve for the first equation at least? Do you need to you make a guess for u^*, right if you use Gauss – Seidel you have to iterate basically is not it? Because this is kind of a implicit equation, right.

You have a system of linear equations where b is known and P^* is guess is known, but although you made a guess for u^* you need to converge this equations right you need to convert using Gauss-Seidel and you need to converge the v_n also using Gauss-Seidel, ok.

Again, if you kind of read through whatever we have discussed what does a_{nb} contain? a_{nb} contains u guess values of u s as well, is not it? Because this contains D and F the diffusion and the convection and the convection uses the existing values of u, alright and v.

So, essentially these equations are coupled you can see this a v contain u and v and this a v contains u and v right coming from the D and F that we have discussed before in the context of convection diffusion equations, ok. So, that means, these two equations are actually coupled and they are also now linearized, right. So, we have made them linearized from the non-linear v that we have ok, ok. So, you may have two kind of counter up on this two equations and see how what will be the values of a_{nb} here, alright.

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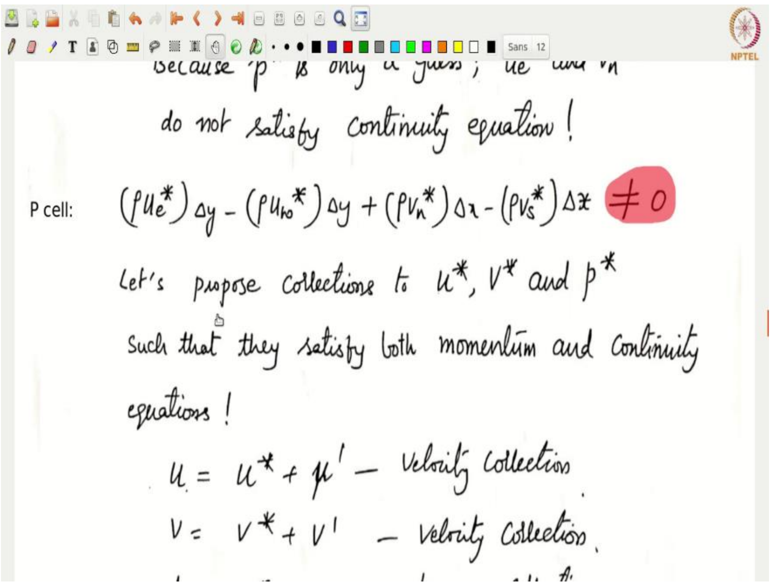

Now, so, we need to solve for these two equations using Gauss-Seidel or line by line TDMA and eventually we get the converged solution of the equations one ok. So, the converged solution is basically u_e^*, v_n^* ok. This is obtained using P^* values using the guessed pressure, alright.

Now, because P^* is only a guess value this is only a guess value u_e^* and v_n^* will of course, satisfy the momentum linearized momentum equations 1, because they are solved for convergence. However, they will not satisfy continuity equation unless we are extremely lucky right that we guessed the correct pressure in the first place itself which we are not.

So, the u_e^*, v_n^* obtained using the guessed pressure will not satisfy continuity equations ok; that means, if we were to go back and discretize the continuity equation on the primary cell right on the p cell, then the continuity equation will read as $(\rho u_e^*)\Delta y - (\rho u_w^*)\Delta y + (\rho v_n^*)\Delta x - (\rho v_s^*)\Delta x = 0$, right.

This is not equal to 0 because the u_e^* and v_n^* only satisfy the momentum equation for a guessed pressure linearized momentum equations for a guessed pressure, but they will not satisfy the continuity equations. So, as a result there will be some mass imbalance. So, mass conservation will not be satisfied by the stared values that come out of the solutions of the 1 equations at 1 ok, is that clear, alright.

So, now, what we look for is basically we know that these velocities the u_e^* and v_n^* and the P^*, they are not correct, right and they will not satisfy momentum continuity equation. As a result we propose corrections for these guessed values, ok; that means we want to correct the u_e^* v_n^* and P^* such that they satisfy both the momentum and the continuity equations.

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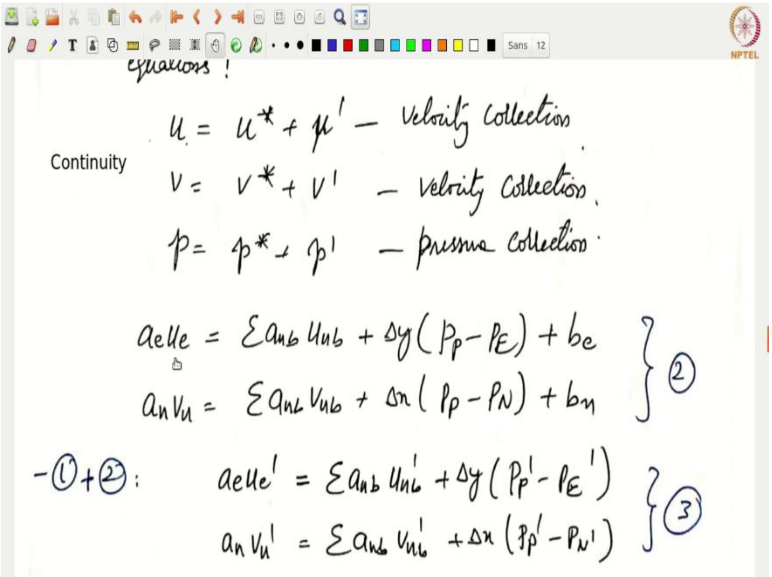

That means, we want to introduce some correction for u^*. So, this is P^*; P^* is the guessed value. We want to correct it using P' which we do not know what is this value or how to obtain. We only know we guessed something, but now we hope to correct this with some pressure P' such that we get a pressure p. This hopefully will be the correct pressure.

And, similarly we want to correct the u_e^* and v_n^* these are obtained from the solution of momentum equations with a guessed P^*, ok. These also we want to correct them such that these corrected velocities here u and v satisfy continuity equation, ok. So, we want to add these corrections u' v' such that the $u_e = u_e^* + u_e'$ and $v_n = v_n^* + v_n'$ satisfy the conx`tinuity equation, ok.

Because the star value do not themselves satisfy right, but we hope u and v if you plug it in here will satisfy the continuity equation and we want to find such corrections here ok, alright. Now, of course, we also hope that u and v for the correct pressure will also satisfy the momentum equation.

So, I can write the original momentum equations that is basically the discrete momentum equation for x and y components of velocity as $a_e u_e = \sum \mathrm{a_{nb}} \mathrm{u}_{nb} + \Delta y (P_P - P_E) + \mathrm{b_e}$ and then we have $a_n v_n = \sum \mathrm{a_{nb}} \mathrm{v}_{nb} + \Delta x (P_P - P_N) + \mathrm{b_n}$ where this pressure is the corrected pressure and the velocities are the corrected velocities. And, if you solve for this u will kind of converge to some value, ok.

Now, this is basically the same equation set we had before only thing is that we removed the stars everywhere, ok. Now, if I were to subtract the star equations from the non starred equations; that means, if you want to do 2 minus 1 for each of these x and y momentum equations separately, then what we get is the equation set 3 which is basically your, $a_e u_e$ minus $a_e u_e^*$.

If you take a_e common then you get $a_e u_e'$ equals similarly on the right hand side what you get is $\sum \mathrm{a_{nb}} \mathrm{u}_{nb}' + \Delta y(P_P' - P_E') + \mathrm{b_e}$ the body forced term gets cancelled, ok. So, as a result now we have an equation set 3 which is $a_n v_n' = \sum \mathrm{a_{nb}} \mathrm{v}_{nb}' + \Delta x(P_P' - P_N') + \mathrm{b_n}$ ok.

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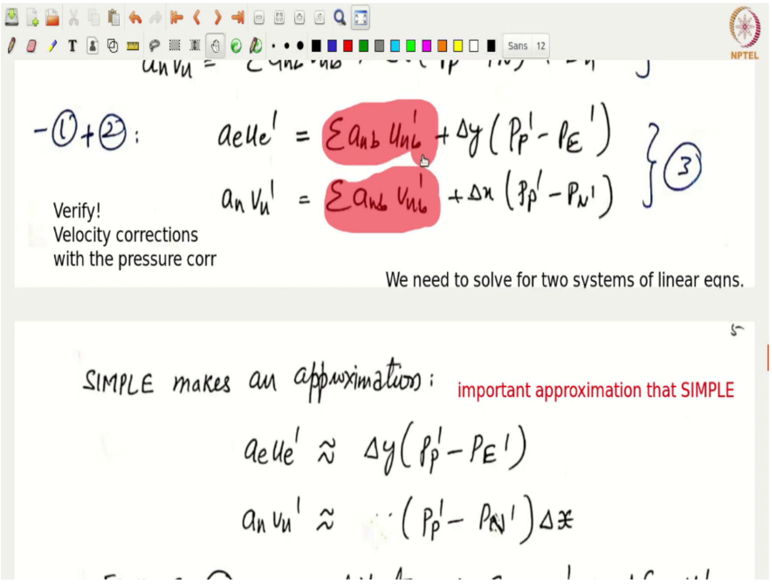

So, you need to verify how we got these equations. Basically subtract the equation set 1 from equation set 2, ok. Now, we got a very nice equation very important equation this is basically an equation that is relating the velocity corrections that we proposed with the pressure corrections, is not it? With the pressure corrections, with the pressure corrections ok. So, that is a very important equation.

Now, how do we solve for this equation? Let us say if P' is guessed, how do we solve for this equation? Do we need to solve for a system or can we directly get the answer? We need to solve for a system right even for this so, we need to solve for again 2 systems right of a linear equations that is a lot of work and also as a result we do not want to solve for this.

So, what the simple method proposes is, this is the biggest or the most important approximation that simple proposes ok. So, what it says is neglect the contribution of the neglect the contribution of the $\mathrm{a_{nb}u'_{nb}}$, make this 0 and $\mathrm{a_{nb}v'_{nb}}$ in these equations, ok. That means, we are writing approximating $a_e u'_e$, the velocity correction purely in terms of the pressure correction ok.

Similarly, $a_n v'_n$ equals approximated to $\Delta x(P'_P - P'_N)$; that means, essentially if we want to write these two equations by making these two to go 0 as $a_e u'_e$ prime approximates $\Delta y(P'_P - P'_E)$ and $a_n v'_n$ approximates $\Delta x(P'_P - P'_N)$, ok. This is an approximation we will see the consequence of this approximation little later ok.

That means, what does this step has implied? This basically has what it has implied is that the burden of correcting the velocities is completely placed on pressures right because now the velocity corrections are gone. So, the neighboring velocities do not correct the velocities for the primary cell, rather the velocity correction is completely has to be corrected using the pressure corrections, right.

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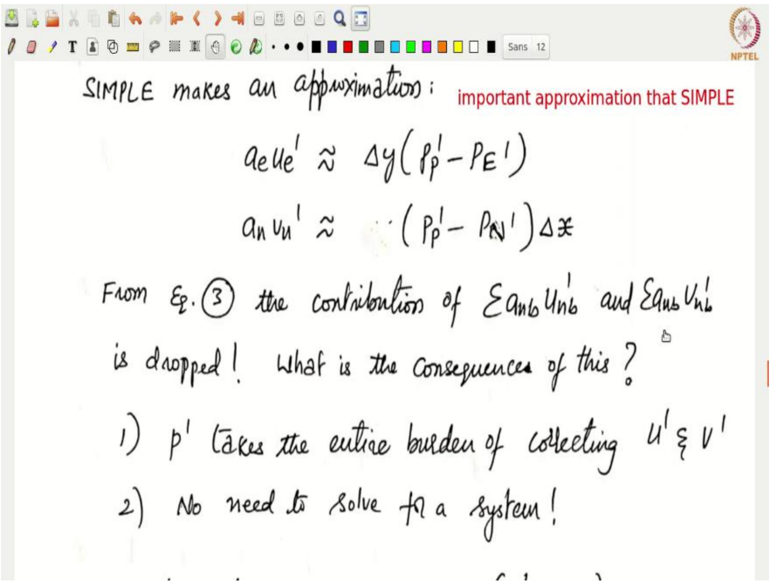

Because their contribution is now nullified ok; that means, from equation 3 essentially the contribution of $\sum \mathrm{a_{nb}u'_{nb}}$ and $\sum \mathrm{a_{nb}v'_{nb}}$ is dropped.

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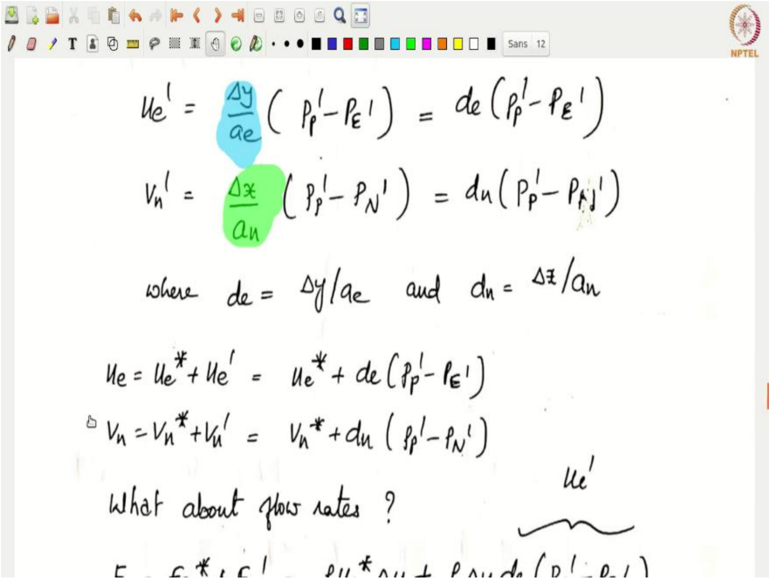

That means, the consequence of this is the P' takes the entire burden of correcting the face velocity corrections that is u_e' and v_n'. Of course, we also have a good news basically, because we have removed these terms we do not have to solve for a system because if we know the pressure corrections you can directly plug it in because a_e is known we can directly calculate what is u_e' and v_n' ok. So, that is a good news. We do not have to solve for a system anymore.

Then of course, I can write now what is u e prime. u e prime would be $\Delta y/a_e$ times $(P_P' - P_E')$ ok. So, that is $\Delta y/a_e(P_P' - P_E')$. So, if you want to denote $\Delta y/a_e$ with another constant which we will like to call it as d_e we will call it as d east. So, $d_e(P_P' - P_E')$.

Similarly, v_n' equals what we have here is that v_n' equals $\Delta x/a_n(P_P' - P_N')$ that is what we have $v_n' = \Delta x/a_n(P_P' - P_N')$ which you would like to call it as a d_n, ok. So, then we have 2 constants d_e and d_n and we have essentially got an equation for velocity corrections on the faces in terms of adjacent pressure corrections on the cell centroids ok. So, this these are our what equations? These are velocity correction pressure correction relations, ok.

Now, of course, we know that our total velocity the corrected velocity we denote it with using $u_e = u_e^* + u_e'$ right because this correction is what we want to add to the u_e^* values eventually, such that this u_e satisfies continuity. Similarly, v_n^* we want to add v_n' such that it satisfies continuity we get the corrected velocity v_n ok; that means, u_e^* is coming out of the equation set 1 after the solution of equation set 1.

And, then if we add u_e' to this u_e^* plus u_e' is basically $d_e(P_P' - P_E')$; similarly, v_n equals v_n^* plus $d_n(P_P' - P_N')$, ok. These are the corrected velocities.

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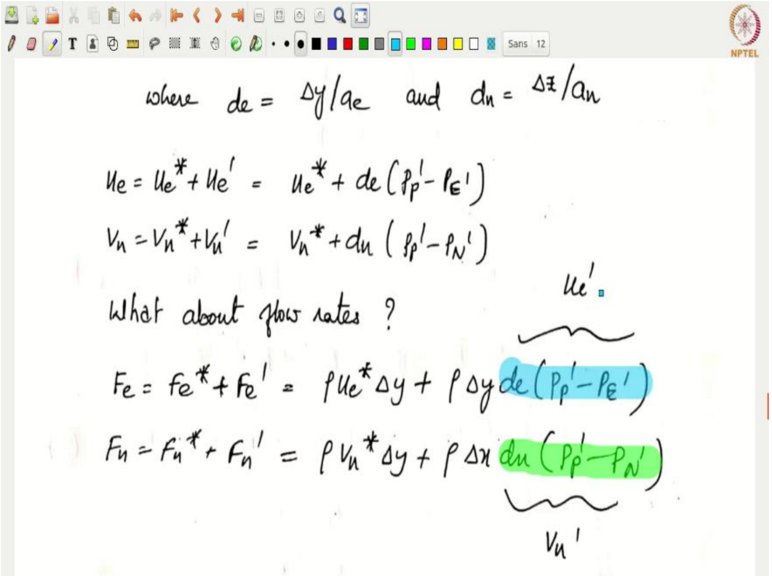

Now, what about the flow rates the flow rates are basically F_e is basically the flow rate through the east face using the corrected velocity. This can be written as also $F_e = F_e^* + F_e'$ is $(\rho u_e^*)\Delta y$ plus F e prime would be rho u e prime delta y. So, I have written instead of u e prime we have written rho delta y u e prime, but u e prime we can substitute as d e times P P prime minus P east prime, right from here.

Similarly, F_n would be the flow rate on the north face this can be written as the star value plus the correction where the star value of the flow rate is rho v n star delta y plus the correction is rho delta x times. v n prime, where for v n prime we can substitute d n times P P prime minus P north prime just like we have substituted for d east times P P prime minus P north prime for u e prime ok, alright.

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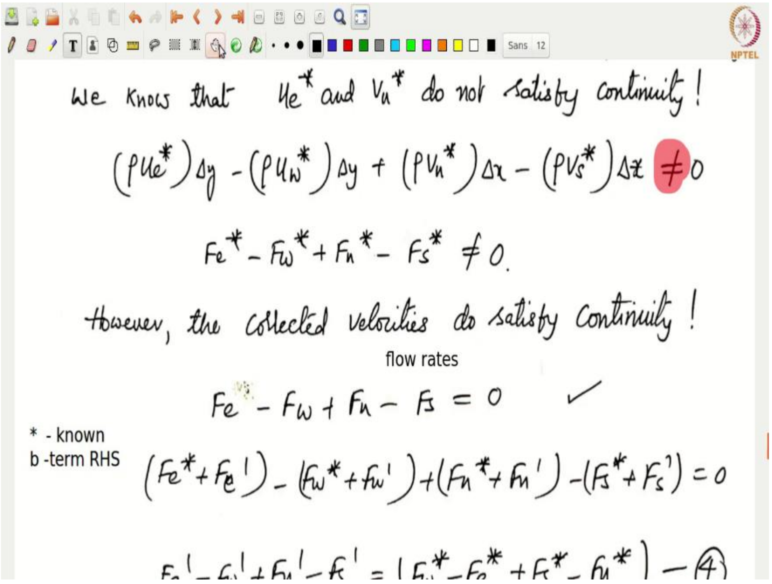

So, these are now known. We know that if we go back to the continuity equation the guessed values that came out of the guessed pressure values by solution of the momentum equations do not satisfy continuity right because we know that u_e^* and v_n^* do not satisfy continuity; that means, rho u e star delta y minus rho u w star delta y plus rho v n star delta x minus rho v s star delta x.

This is basically is not equal to 0, right. This is not equal to 0 because the u_e^* un star came out of momentum equations, but not out of continuity equation right out of a guessed pressure. If this pressure was correct then these would satisfy both momentum and continuity right ok; that means, if you were write in terms of flow rate these are F e star minus F w star plus F north star minus F south star is not equal to 0.

But, we miss to; we we wish to convert this not equal to 0 into an equal to 0 by adding the contribution of the corrected flow rates ok; that means, the corrected velocities or the corrected flow rate right the corrected flow rates would satisfy continuity equations. What are the corrected flow rates? Those are F e minus F w plus F north minus F south this will definitely satisfy continuity equation right.

In fact, we want to make the corrections for velocities and pressures such that the corrected velocity satisfy continuity equation ok. So, that is our objective. Essentially, find the corrections such that the corrected velocities satisfy continuity equations ok. Then we can split this F_e into F_e^* plus F_e' similarly F_w into F_w^* plus F_w', F_n into F_n^* plus F_n' and F_s into F_s^* plus F_s' ok.

Out of these, the star values are they known or unknown the star values are known or unknown? Are known, right; essentially we already solved u_e^* v_n^*. So, these are known these can go to which term this can go to the b term, right essentially the right hand side ok.

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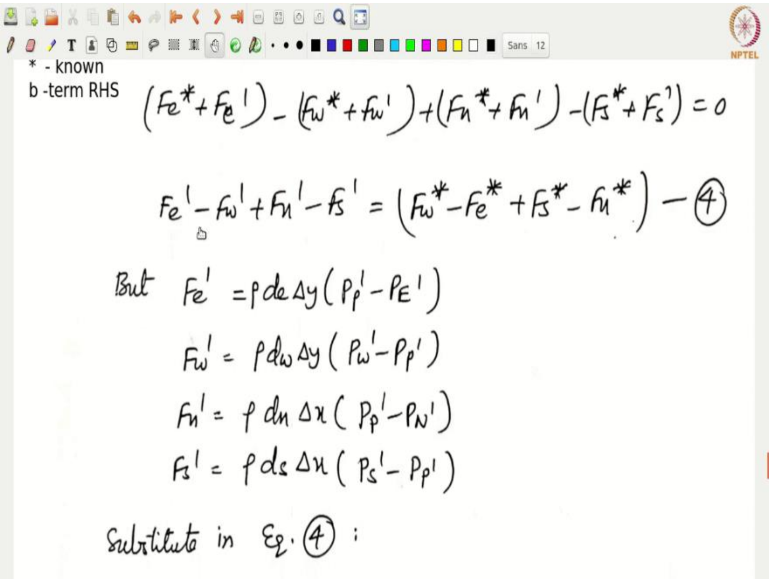

That means, we can send this star values to the right hand side. So, what we left on the left hand side is basically F e prime minus F w prime plus F north prime minus F south prime equals look at the change in sign here because now these are send to the right hand side. So, minus F w star becomes positive. So, what we get is $(F_w^* - F_e^* + F_s^* - F_n^*)$ ok. So, this is our now b e term which is known that is send to the right hand side.

But, what we are left with on the left hand side is basically the prime flow rates which of course, we can substitute in terms of our pressure corrections right because we can write F_e' as $\rho d_e u_e'$ right, but u_e' is $\Delta y(P_P' - P_E')$.

Similarly, we can write for F_w' as $\rho d_e u_w'$ which is nothing, but $\Delta y(P_W' - P_P')$; similarly for F_n' as $\rho d_n v_n'$ which we can write as $\rho d_n \Delta x(P_P' - P_N')$ and similarly, for F_s' as $\rho d_s \Delta x(P_S' - P_P')$ ok.

So, essentially we have used the velocity correction pressure correction equations right, which we have derived here right in substituting for u_e' in terms of pressure primes and v_n' in terms of the pressure primes and u_w and v_s ok.

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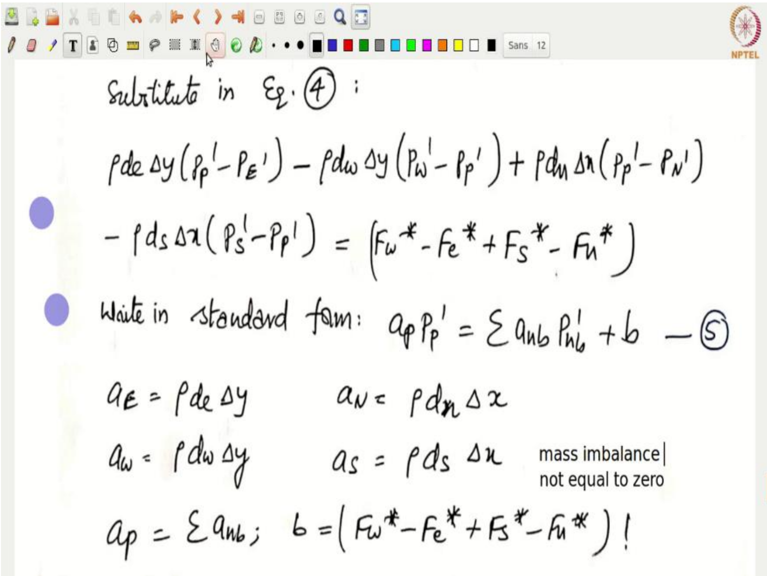

So, essentially we substitute for all these things then if you put these things back into equation 4 here, what you get is on the left hand side you get $\rho d_e \Delta y(P_P' - P_E')$ minus $\rho d_w \Delta y(P_W' - P_P')$ plus $\rho d_n \Delta x(P_P' - P_N')$ minus $\rho d_s \Delta x(P_S' - P_P')$ equals $(F_W^* - F_e^* + F_S^* - F_n^*)$ ok.

So far so good, now does it remind you of some equation that we have seen before? Yes, it does. It kind of looks like the diffusion equation we have looked before right. So, we can certainly write in the standard form where in we can write this as $a_P P_P'$ equals sigma a nb P nb prime plus b where a east. Now, only difference compared to the previous equations here is that in the previous equation we use to send the P_P' to the right hand side right all the a_P to the right hand side.

Here we will send all the neighbors to the right hand side because the b term is already here and P_P' will get positive coefficient. So, E prime, W prime, north prime and south prime would be sent to the right hand side such that all of them would become positive and the positive P_P' coefficients will retain or remain on the left hand side ok.

Then if we rearrange in our standard form that is $a_P P_P'$ equals $\sum a_{nb} P_{nb}'$ plus b our neighboring coefficients a_E would be would be what? Would be $\rho d_e \Delta y$. This minus is there, but it is gone to the right hand side. So, this will become $\rho d_e \Delta y$ similarly a_W would be $\rho d_w \Delta y$ and a_N would be $\rho d_n \Delta x$ and a_S would be $\rho d_s \Delta x$ ok. So, we got all these things.

What will be a_P? a_P would be again summation of a $\sum a_{nb}$. So, a_P equal to $\sum a_{nb}$ and what would be b? b is your $(F_w^* - F_e^* + F_s^* - F_n^*)$ star, ok. Now, essentially what is this quantity? This quantity is this equal to 0. This not equal to 0, right. If this were equal to 0, then our problem is solved our guess value satisfy continuity. They are out of momentum equation.

So, everything is solved, this is not equal to 0 and this also indicates the mass imbalance that is there in the problem right essentially in the domain this is the mass imbalance for the cell. So, for every cell this is the mass imbalance that we have; that means, at any moment if this b term goes to 0; that means, the mass imbalance for every cell is 0; that means, if we satisfy continuity equation by every cell.

Then this equation would become b equal to 0 for every cell right this would equation will transform into $a_P P_P' = \sum a_{nb} P_{nb}'$ with b equal to 0 everywhere, right.

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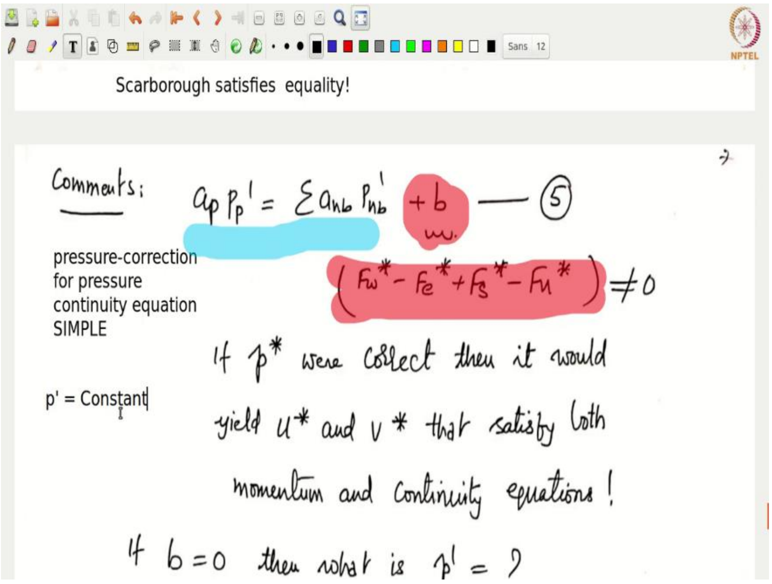

Now, what about Scarborough criteria for this? Scarborough criteria is it satisfied? Because we have to solve for a system here, is it not? This is an equation for a system of linear equation we have to solve for a pressure correction equation. So, we need to know whether Scarborough satisfied in equality or inequality? Equality right because equality condition right basically because a_P equal $\sum \mathrm{a_{nb}}$ and all the coefficients are positive. So, this will basically satisfies Scarborough inequality, but what about.

So, essentially let us make some comments in terms of what will happen or how do we solve this equation. Now, you see we have arrived at this equation, this is an equation for what? For pressure correction which is basically an equation for pressure itself, is not it? Instead of calling it a pressure correction is then equation for pressure, how did we get? This we got this from invoking continuity equation.

Now, see that is the idea of the simple method. You start off with continuity equation and by enforcing the guessed velocities to satisfy the continuity equation in terms of corrections we have arrived at a an equation for pressure ok. So, the continuity equation is transformed into an equation for pressure correction with the help of momentum equations ok. So, that is where the second difficulty that we have in the solution of incompressible flow problems is now addressed using this simple algorithm ok.

Of course, this is the right hand side is basically the mass denotes the mass imbalance, this term denotes the mass imbalance this is basically your b term. Now, the moment if your pressure guess were correct then it would yield u^* and v^* that satisfy both momentum and continuity equations, because if u^* and v^* satisfy if P^* was correct then u^* v^* would be also correct in the momentum equations and they will also give you a 0 here which would essentially give you a 0 here.

That means, that $a_P P_P'$ equal $\sum \mathrm{a_{nb}} \mathrm{P}_{nb}'$ prime, what is the solution for this equation? What would be the solution for $a_P P_P'$ prime $\sum \mathrm{a_{nb}} \mathrm{P}_{nb}'$ with b equal to 0? If b were 0, let us say you end up with a continuity satisfying flow field, what is the solution for $a_P P_P'$ prime equals $\sum \mathrm{a_{nb}} \mathrm{P}_{nb}'$?

Everywhere in the domain a solution would be basically of this equation if you solve using Gauss-Seidel you will get basically P' equals constant right that will be your solution. That satisfy this equation right because a_P equal sigma $\mathrm{a_{nb}}$ then P_P' everywhere will come out to be constant that is solution for this equation.

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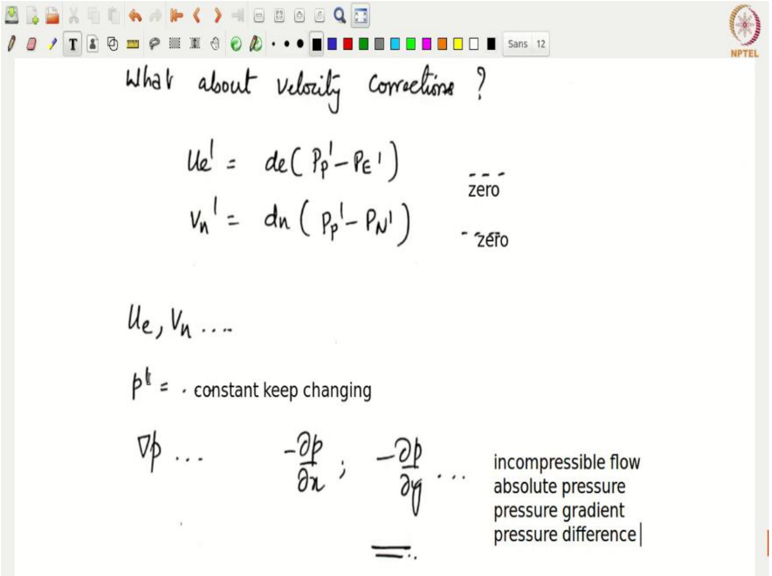

That means, if pressure correction is constant, then what will happen to velocity corrections? Velocity corrections would p prime is constant then velocity correction would go to this will go to 0; that means, we do not have to iterate any more right both the velocity corrections will go to 0 that means, we can say that if P' equal to constant we have arrived at a converged solution because velocity corrections also will go to 0.

Of course, this constant might keep changing because every times the prime iteration this might be changing, but P' equal to constant would tell you that for that particular iteration this is basically satisfies you have obtained a velocity field that satisfies continuity ok, but we can consider P' equal to constant everywhere as a converged pressure solution and that will give you also a converged velocity solution because u e prime and v n prime are also 0.

The velocity corrections are 0; that means, we can say that the we have obtained a converged u_e', v_n' and P' is basically becomes a constant. Now, we see that even if this constant value keeps changing that does not matter as long it is a constant because your ∇P is actually what matters in the incompressible flow equations right.

It is not the absolute pressure, it is not the absolute pressure that matters it is only the pressure gradient or the pressure difference that we have between two points that is what matters because in the equation you have $-\nabla P$ or $-\frac{\partial P}{\partial x}$ and $-\frac{\partial P}{\partial y}$, right and we do not have any equation like an ideal gas equation or an equation of state where the absolute value of pressure is related to the density ok.

So, as a result, the in the incompressible flow equations only the gradient of pressure that matters; so, as a result your p prime even if it keeps changing its as long as it is a constant that constant value might be anything, but your it will not affect your solution your essentially velocity corrections will go to 0 ok.

So, that is in a sense. But, now after these velocity corrections essentially your velocities the corrected velocities u and v would satisfy continuity equation, but they will not satisfy momentum equations right because now we have obtained a different pressure field and obtained a velocity field from these other equations.

So, essentially you have to go back and solve your momentum equations again with the updated pressure from there you have to obtain the velocity fields and keep doing that until you satisfy both momentum and continuity equations at the same time for the pressure and the velocities ok. So, that is the overall algorithm. We will look at the algorithm in a different lecture in the next lecture so, but this is the idea of the simple method ok.

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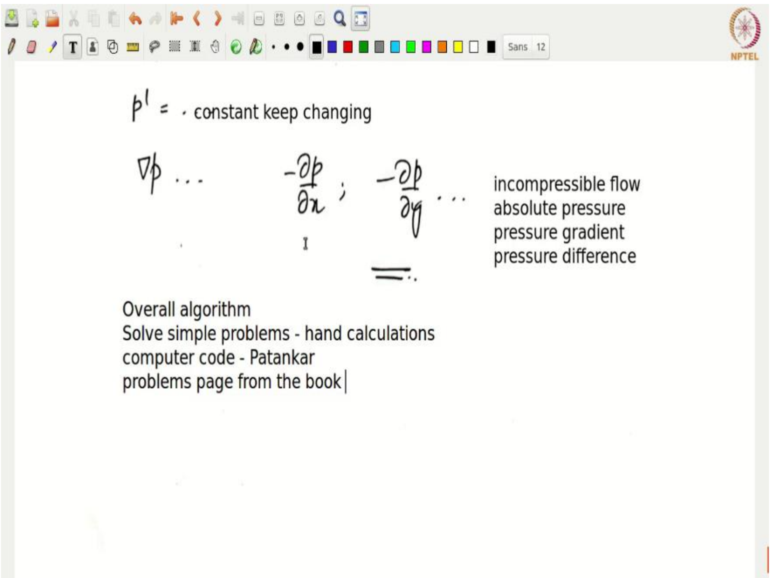

So, essentially we will look at the overall algorithm in the next class and also we will look at we will solve some simple problems both using hand calculations as well as using computer code. So, I am going to show you some small programs. These will be taken from the book by Patankar ok. So, I am going to hopefully upload the problems page from the book so that you can take a look and you can also go through this algorithm once.

So, that we can get started with the overall algorithm and the hand calculations for some simple problems in the next class ok. I will also see if I can post may be one assignment or something like that so that we can you know kind of get used to these, alright ok.

Thank you very much I am going to stop here. If you have any questions do let me know through e-mail and I will kind of try to get back to you as soon as possible ok.

Thank you.