Computational Fluid Dynamics Using Finite Volume Method
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Lecture – 40
Finite Volume Method for Fluid Flow Calculations: SIMPLE algorithm for Colocated Mesh

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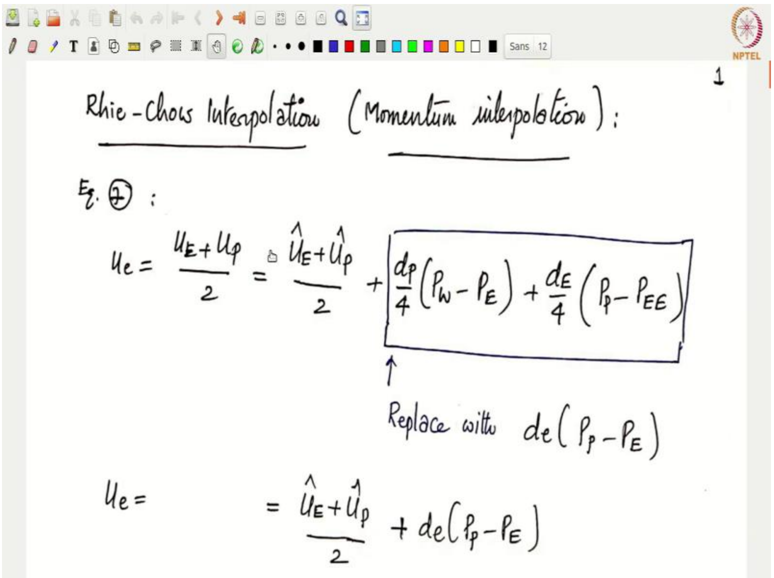

Hello everyone, let us gets started. So, welcome to another lecture as part of our ME6151 Computational Heat and Fluid Flow course. So, in the last lecture, we looked at a couple of versions of simple algorithm that is simple revise, and simple corrected right.

And there after we kind of saw how to extend the staggered grid to basically to curvilinear meshes or to unstructured meshes, and we realize that it was not very easy; although there were several fixes that are available in the literature, but it was not very straightforward to do, an extension of a staggered grid approach for unstructured meshes or for that matter even convenient meshes ok.

So, as a result, we decided that the best solution to move forward was basically to get back or revert back to the colocated approach where essentially you store the velocities as well as the pressures at the cell centroids ok. Now, however, we know that if you switch back to a colocated approach, then this pressure velocity checker boarding would come into play, and that is where we saw how the equation support checker boarding in the context of colocated storage right, so that is what we saw in the last class.

So, one of the fixes for this to avoid pressure velocity checker boarding is basically to do something known as momentum interpolation or Rhie-Chow interpolation. This was suggested by Rhie and Chow in about the early 1980s, so that is what we are going to see today ok.

So, basically the idea is if you have let us say if you have a momentum equation for u_E and momentum equation for u_P, then the face value assuming you have a two-dimensional uniform mesh, then the face value u east can be written as a linear average or arithmetic average of u_E and u_P upon 2.

So, this can also be written as in terms of hat velocities as $\hat{u}_E$ plus $\hat{u}_P$ by 2. Remember $\hat{u}_E$ basically has $\sum \mathrm{a_{nb}} \mathrm{u}_{nb}$plus b upon the coefficient that is coming from u_E or u_P right essentially for a east will be there for $\hat{u}_E$, and in the denominator a_P will be there in the definition of u_P hat ok.

So, we already saw this expression in the last class essentially the face velocity using a arithmetic average is either u_E plus u_P by 2, or in terms of the right hand side values. It will be $\hat{u}_E$ plus $\hat{u}_P$ by 2 plus you have this extra pressure terms that is d_P by 4 times $P_W - P_E$ plus d_E by 4 times $P_P - P_{EE}$ right. Essentially each of these terms come from each of the pressure gradients coming in the equation for u_P and in the equation for u_E right. I think we realize that.

Now, the idea is essentially what Rhie-Chow interpolation or momentum interpolation suggest is that because this particular pressure term is only dependent on the non-contiguous pressures. That means, it is dependent on P_W and P_E right, which are not contiguous because there is P_P in between which is not taken into account.

And similarly the other pressure difference P_P and P_{EE} is also non-contiguous or it is kind of alternate cell values, not the adjacent cell values, because in between you have P_E which is not taken into consideration ok.

As a result what momentum interpolation suggest is you replace this pressure terms with something that is contiguous ok, that means, you replace these two terms with d little e times $P_P - P_E$ that means your final equation in terms of hat velocities would be $\hat{u}_E$ plus $\hat{u}_E$ hat by 2 plus this entire thing is replaced with d_e times $P_P - P_E$.

Now, of course, you see that this is basically a difference in pressure for adjacent cell values unlike this difference in pressure which are for non-adjacent cell values right for non-contiguous cell values, whereas, this one is now for the adjacent cell values.

Of course, one consequence we see right away is that, because of this fix we what we say is basically we do not have the we do not have this pressure checker boarding anymore, because now instead of P_W and P_E you have a dependency on P_P and P_E, as a result the pressure checker boarding would be gone ok.

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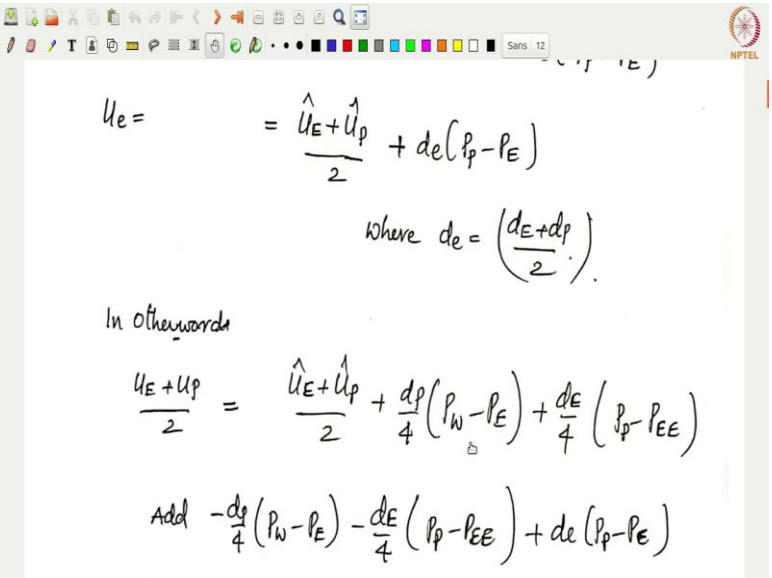

So, let us see in terms of in terms of the u_E and u_P instead of this $\hat{u}_E$ and $\hat{u}_P$ that means what we have is basically if you look at this equation, so we have just replaced something with something else. That means, if you if I consider this equation again, so what I have is I have u_E plus u_P by 2 that is on the left hand side. And on the right hand side, what I have is $\hat{u}_E$ plus $\hat{u}_P$ by 2 plus d_P by 4, $P_W - P_E$ plus d east by 4, $P_P - P_{EE}$ right this is what we have.

Now, of course, that means, the momentum interpretation says that you replace this guy with something else, so that means, what we are doing is basically we are subtracting this alternate pressure terms, and we are adding in addition pressure term ok.

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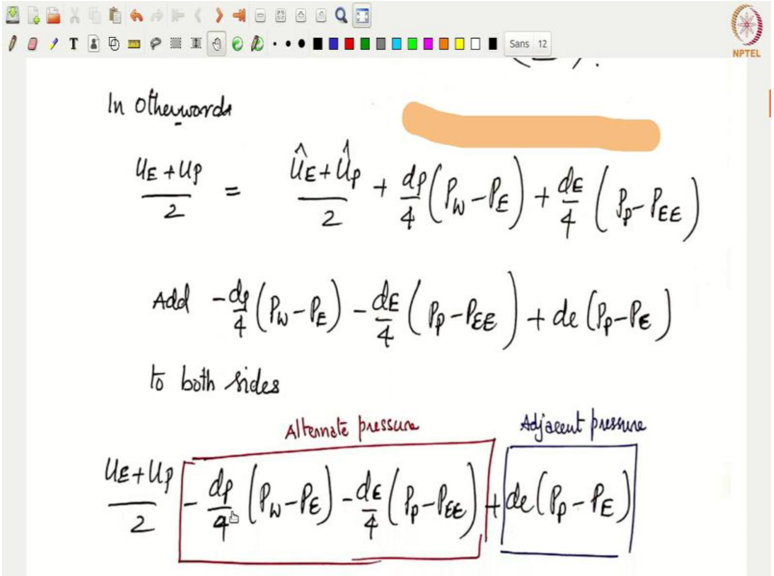

That means, on the both sides if I subtract these two terms that is basically d_P by 4, $P_W - P_P$ east and d_E by 4, $P_P - P_{EE}$ subtract it on both sides then this guy this guy goes to 0 right this guy going to be there right essentially. Then essentially this entire term would go away, and then you end up with a minus of these two on the left hand side, and you add plus d_e, $P_P - P_E$ on both sides ok, so essentially you subtract or you add this minus quantity and you add this plus quantity on both sides ok.

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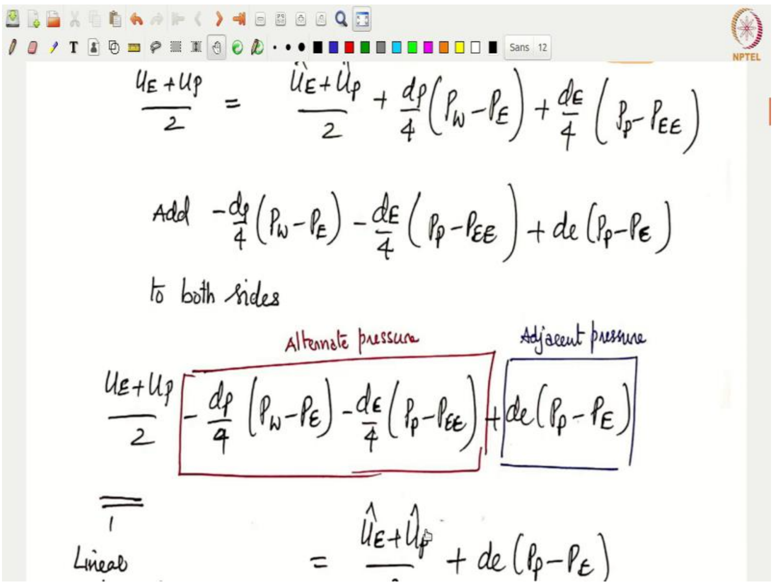

Then what we end up is basically on the left hand side you will get u_E plus u_P by 2 minus d_P by 4 right $P_W - P_E$ minus d_E by 4 $P_P - P_{EE}$ plus you get d_e times $P_P - P_E$. And on the right hand side, you would basically get u east hat plus $\hat{u}_P$ by 2 plus this guy gets cancelled with this minus quantity. Then what you end up with plus d_e, $P_P - P_E$ ok.

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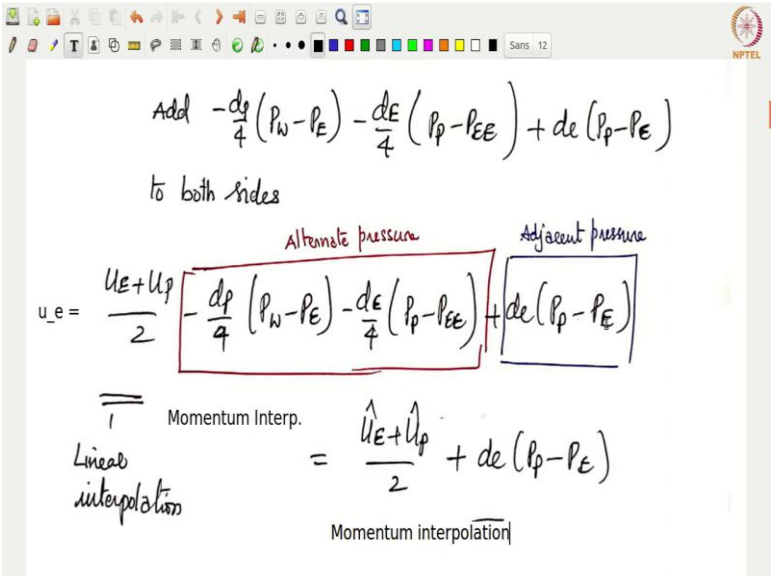

Now, essentially these are both terms are again now equal to what we have as the modified velocity for u_e ok. So, now, what we see is that if it were just u_E plus u_P by 2, then this is called as a linear interpolation right, then this is a linear interpolation. But what we have done is well if it is just u_E plus u_E by 2 minus d P of this guy minus d east of this guy, then this entire thing would be your u little east right.

Essentially, if we go back here, so essentially u_E plus u_P by 2 would be linear interpolation ok. Now, instead of linear interpolation, we have essentially two more terms that are coming into play that is minus of this guy minus of this d_E by 4, $P_P - P_{EE}$, and then we have plus d_e times $P_P - P_E$ ok. So, this is basically your adjacent pressure term. Now, these two terms together make this convert this linear interpolation into something known as momentum interpolation ok.

So, now, these three terms together is basically momentum interpolation ok. Now, that is if you talk in terms of u east and up now if you talk in terms of $\hat{u}_E$ and $\hat{u}_P$ terms, then your u_e little east is basically $\hat{u}_E$ plus $\hat{u}_P$ by 2 plus d_e times $P_P - P_E$ ok. So, this is the expression for momentum interpolation. And I we have two expressions here. One is this guy, the other one is this one.

The first one here is in terms of east and west east and P. And the second one here is in terms of hat velocities for east and P ok. So, both are one and the same, but we will be most of the times using this hat velocity based definition.

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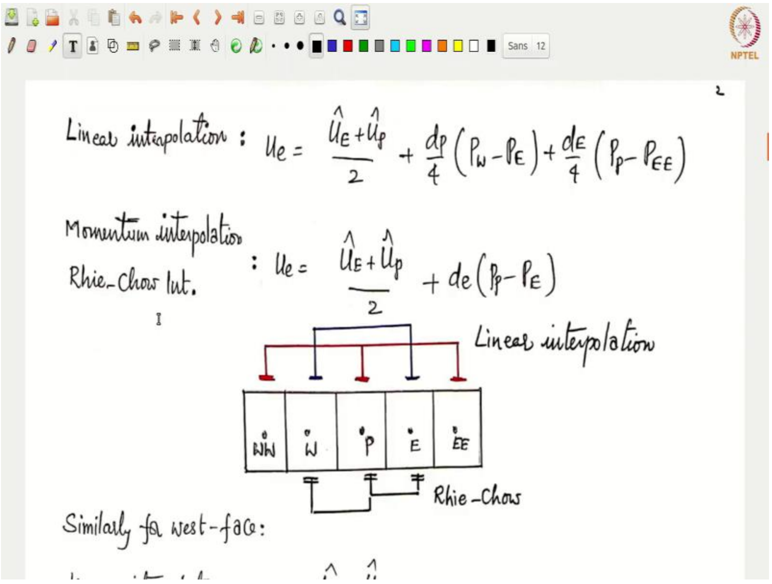

So, this is momentum interpolation in terms of hat velocities and this entire thing that is u_E plus u_P by 2 minus d_P by 4 times $P_W - P_E$ minus d_E by 4 $P_P - P_{EE}$ plus d_e times $P_P - P_E$ is basically your momentum interpolated face velocity in terms of east and P values fine. I hope this is this part is clear alright. Then now we know what is basically in linear interpolation, what is momentum interpolation.

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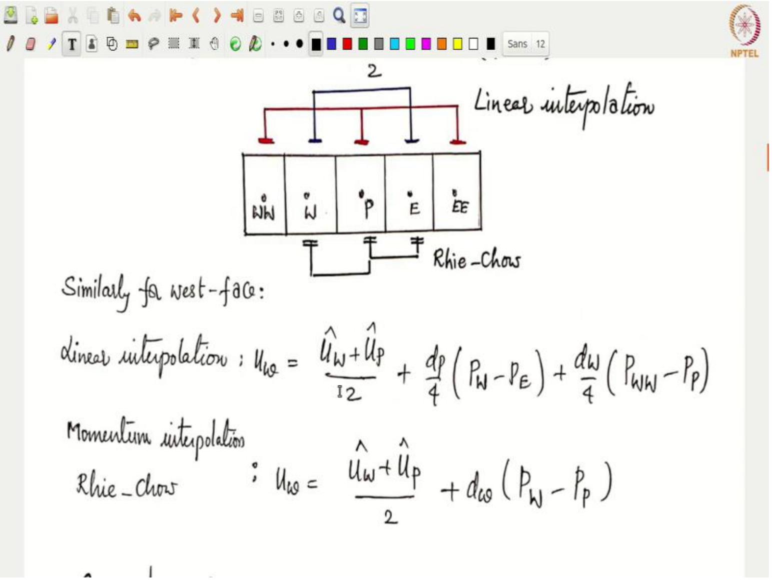

That means, if it for linear interpolation, then your u_e equals would be is this value right $\hat{u}_E$ plus $\hat{u}_P$ by 2. This is basically what we had from here right. This is basically this value. That is nothing but $\hat{u}_E$ plus $\hat{u}_P$ by 2 plus d_P by 4, $P_W - P_E$ plus d_E by 4, $P_P - P_{EE}$ ok.

Now, if it is momentum interpolation, then we know that u_e is basically u capped, $\hat{u}_E$ plus $\hat{u}_P$ by 2 plus d_e times $P_P - P_E$ ok. Now, similarly for west values, we can write this as similarly for west values we can write the linear interpolation as u_w equals $\hat{u}_W$ plus $\hat{u}_P$ by 2 plus d_P by 4 times $P_W - P_E$ plus d_W by 4 times $P_W - P_P$ ok.

And the corresponding momentum interpolated Rhie-Chow interpolation would be basically $\hat{u}_W$ plus $\hat{u}_P$ by 2 plus this non-contiguous pressure terms will be dropped and a contiguous pressure term that is d little west times $P_W - P_E$ would be P_P would be added right. So, this is your momentum interpolated value for the west velocity ok.

That means, if you look at these pressures that we have here, that means, the if we are using linear interpolation, then the terms we have are P_W and P_E, that means these two cells connected and shown using here using blue lines, these are non-contiguous. Similarly P P_E, that is P_E and there is a P_{WW} that is coming here, so P_P P_{EE} P_{WW}, so all these three are again non-contiguous.

But if they are of the same value as that of these two could be of a different value, then essentially this supports a pressure kind of checker boarding, which we do not want to have that is why we are not introducing the linear interpolation that we thought we would use before rather we would work with something that is known as momentum interpolation which has basically P_P and P_E that is this term, and this this value in the u_e.

And similarly P_W and P_P that is this cell this cell value and this cell value that is basically the adjacent pressure values that is what will be used in the calculation of face velocities in Rhie-Chow interpolation ok. So, that is the difference.

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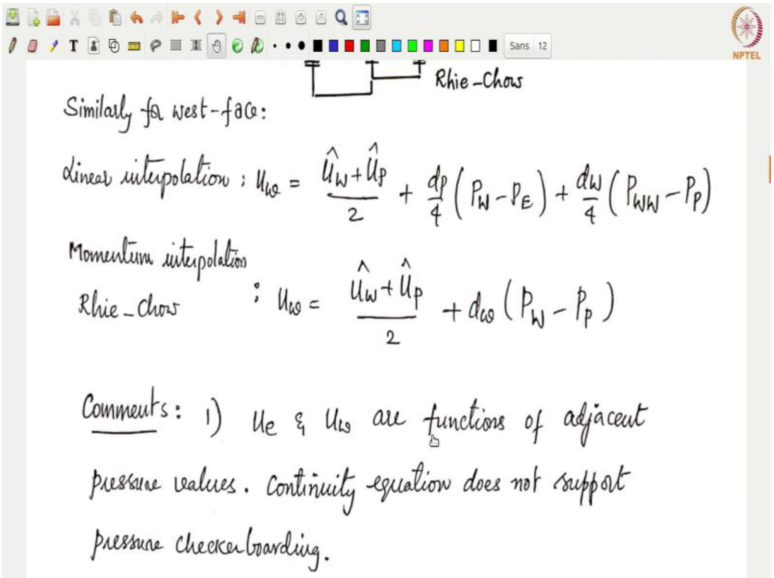

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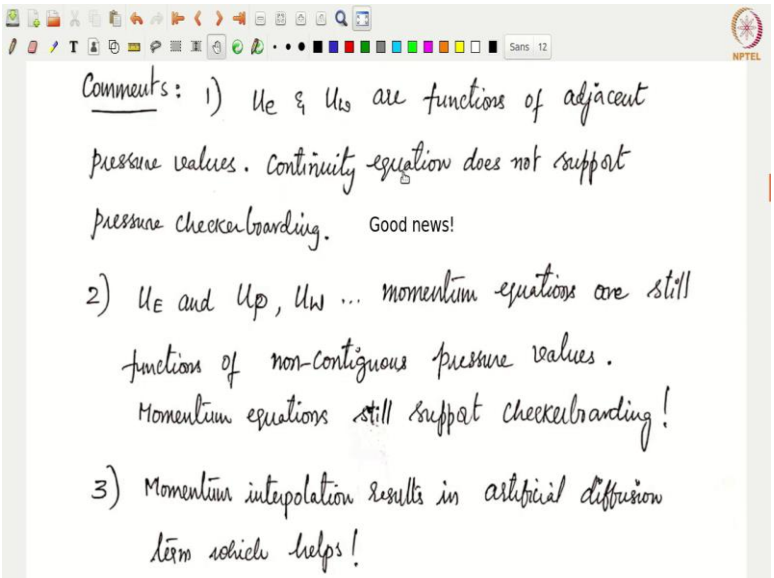

Now, this may kind of look rather arbitrary like why did we basically just replace something some pressure gradient with something else. But this is actually not arbitrary this is basically similar to what a staggered grid approach does ok. But only thing is that here we do not have a grid that we create physically like we do in these staggered grid approach, rather using the interpolation we have now created something which will depend on addition pressures ok.

So, let us make some comments and see how thus, how does this momentum interpolated face velocities enable us to have a non-checker boarded solution ok. That means, the u little e and u w are functions of adjacent pressure values, yes that we know, basically it only depends on $P_W - P_P$ or $P_E - P_P$ that is good.

Then the are functions of adjacent pressure values. So, that means, if we use these face velocities that is u_e and u_w in the continuity equation, then the continuity equation does not support pressure checker boarding which is a good news right, because we do not want to have pressure checker boarding.

However, if you look at the momentum equations that those are basically again for u_E, u_P, u_W and so on, these equations still have pressure values that are similar to this right. You will have $P_W - P_E$ into d_P into Δy by 2 right, you will have a term like that which will still contain non-contiguous pressure values.

So, the momentum equations will still support checker boarding, but that is not a problem because if the momentum equation support checker boarding, and the continuity equation do not support checker boarding, then we will not have this pressure velocity checker boarding sustained towards the end of the solution right. This will be eliminated somewhere because both the continuity and the momentum equations have to support checker boarding such that it will sustain it will continue to be there ok.

So, as a result, in the colocated approach the continuity equation is now modified such that the pressure checker boarding is avoided. Whereas the momentum equations may still support checker boarding, but we do not care about it, because the in the final solution we would not have checker boarding because both the equations do not support it ok. One of the equations does not support it; that means, it will not be there ok.

Essentially, we can show that this kind of a momentum interpolation that is basically replacing the non-contiguous pressure with some contiguous pressure difference values results in something like an artificial diffusion which we saw before.

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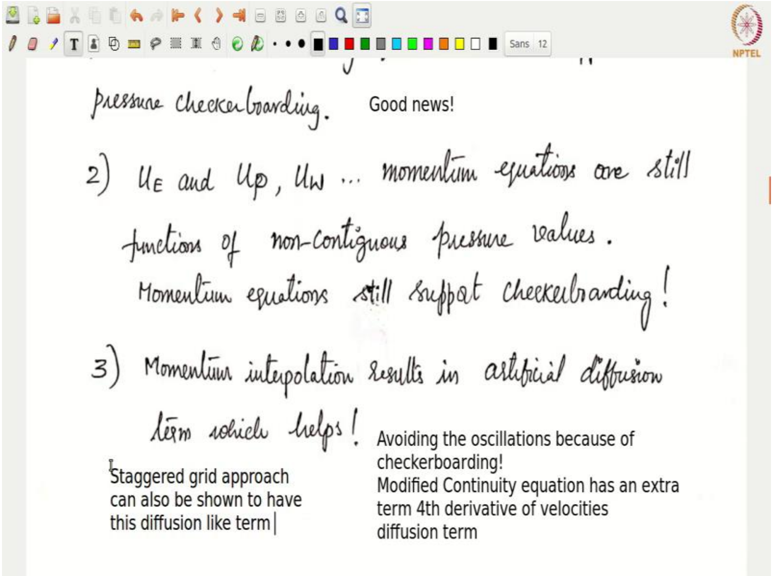

And that is what that helps in essentially avoiding the oscillations that show because of checker boarded ok, because of checker boarding ok. That means, essentially you can show that the modified continuity equation is basically has an extra term that goes as fourth derivative of velocities ok. So, that means, you would have fourth partial derivative of velocity if you will have something like partial to the power 4 u by partial u to the power 4 which is more like a artificial diffusion term.

And of course, even the staggered grid approach can staggered grid approach can also be shown to have a shown to have this diffusion like term ok. Essentially, we are essentially achieving a staggered grid approach, but without really creating a grid and the corresponding physical values with it rather we are now doing everything through some kind of an interpolation that is known as in the iteration as in the momentum interpolation or Rhie Chow interpolation alright that is good.

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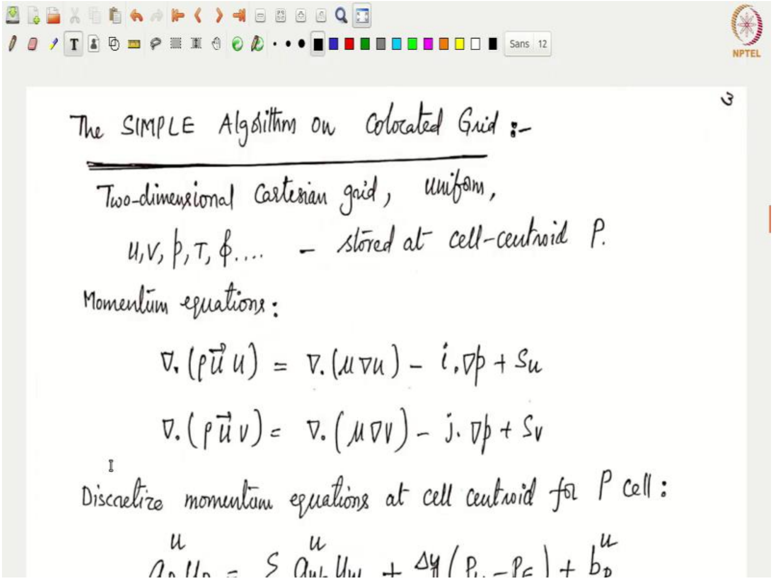

Then now that we understand the momentum interpolation, we will now look at the how do we modify simple algorithm on colocated grids ok. So, if you remember we had two issues, one was the issue was the there was no equation for pressure, and then there was this problem with pressure velocity checker boarding. So, the pressure velocity checker boarding is now avoided using momentum interpolation, whereas the no equation for pressure still has to go through the simple algorithm ok.

So, now we modify our simple algorithm or rather we apply it for a colocated grid storage ok. So, let us consider a two-dimensional Cartesian grid. Again let us say that the mesh is uniform for simplicity, and we are using a colocated grid, that means, we are storing both the velocity components that is u, v as well as pressure as well as temperature and all the scale as everything is now stored at the cell centroid P ok.

So, everything is stored at the cell centroid P. And we are creating an additional velocities on the faces which we call u_e, u_w, but they are additional set of velocities ok, but everything is otherwise stored at the cell centroids.

Then let us look at the momentum equations. So, momentum equations are basically $\nabla \cdot (\rho \vec{u} u) = \nabla \cdot (\mu \nabla u) - \hat{\imath} \cdot \nabla P + S_u$. And the y component of momentum equation is $\nabla \cdot (\rho \vec{u} v) = \nabla \cdot (\mu \nabla v) - \hat{\jmath} \cdot \nabla P + S_v$ write we have a source term.

Now, let us discretize these momentum equations at the cell centroid for cell P ok, so that is going to give us basically $a_P^u u_P = \sum a_{nb}^u u_{nb} + \Delta y/2(P_W - P_E) + b_P^u$ ok. Now, this a_P^u would be different from the a_P that you would get for the y-momentum equation.

Because now we are discretizing both the cells at the same cell centroid let us call these a_P's with the superscript notation such that this will be different of course from the a_P coming in the y-momentum equation. Then let us call this all the coefficients in the x-momentum equation with a superscript u that is a_P^u, a_{nb}^u and b_P^u.

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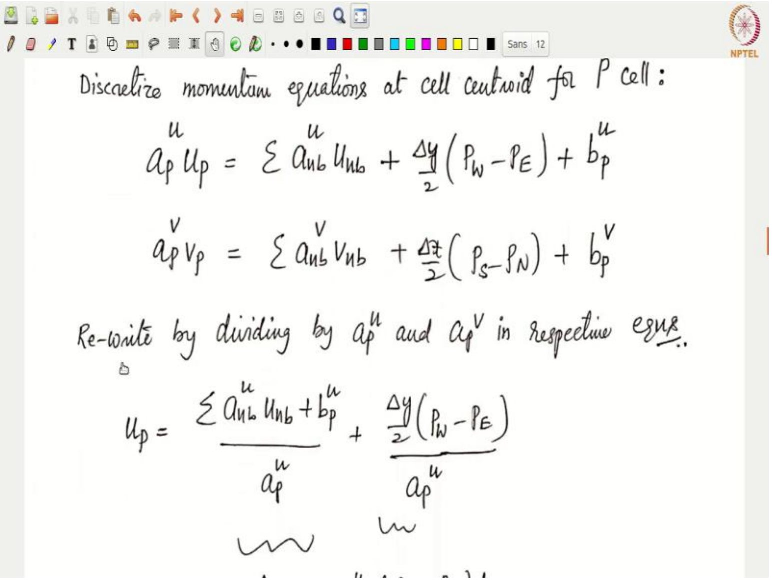

Similarly, all the coefficients that you get in the y-momentum equation with a superscript v ok, that is a_P^v, a_{nb}^v and b_P^v ok. So, that means, you have now the discrete momentum equations for cell centroid for both x and y-momentum equation.

Of course, what you realize is basically you see that you have pressure checker boarding that can be supported by the momentum equations ok. But we will use momentum interpolation when we discretize the continuity equation such that these pressure checker boarding would not be supported by the continuity equation ok.

Then then what we do is let us divide with this coefficient for u_P that is let us divide with a_P throughout the equation, then we can rewrite by dividing a_P^u and a_P^v in the respective equations. I can write this as u_P equals $\sum a_{nb}^u u_{nb}$ plus this guy this is b_P^u upon a_P^u plus we have $\Delta y/2(P_W - P_E)$ upon a_P^u ok, so that is what we have.

Then we can call this entire quantity that is $\sum \mathrm{a}_{\mathrm{nb}}^{\mathrm{u}} \mathrm{u}_{\mathrm{nb}}$ plus b_P^u upon $\mathrm{a}_{\mathrm{P}}^{\mathrm{u}}$ all of them written for the u coefficients as some hat velocity right. So, this will be $\hat{u}_P$ plus again $\Delta y / \mathrm{a}_{\mathrm{P}}^{\mathrm{u}}$ let us call it as some $\mathrm{d}_{\mathrm{P}}^{\mathrm{u}}$ ok, so that means, $\Delta y / \mathrm{a}_{\mathrm{P}}^{\mathrm{u}}$ would be $\mathrm{d}_{\mathrm{P}}^{\mathrm{u}}$, then we have $P_W - P_E$ by 2 remaining as it is. So, this equation that is basically u_{P} equals $\hat{u}_P$ plus $\mathrm{d}_{\mathrm{P}}^{\mathrm{u}}$ times $P_W - P_E$ by 2.

This is nothing but you are basically you are discrete x-momentum equation right written in a different way right. Because essentially we just divided by a_P on the right hand side and we kind of plugged it together the terms for the source term and the neighboring coefficients ok. So, this is basically your momentum equation alright.

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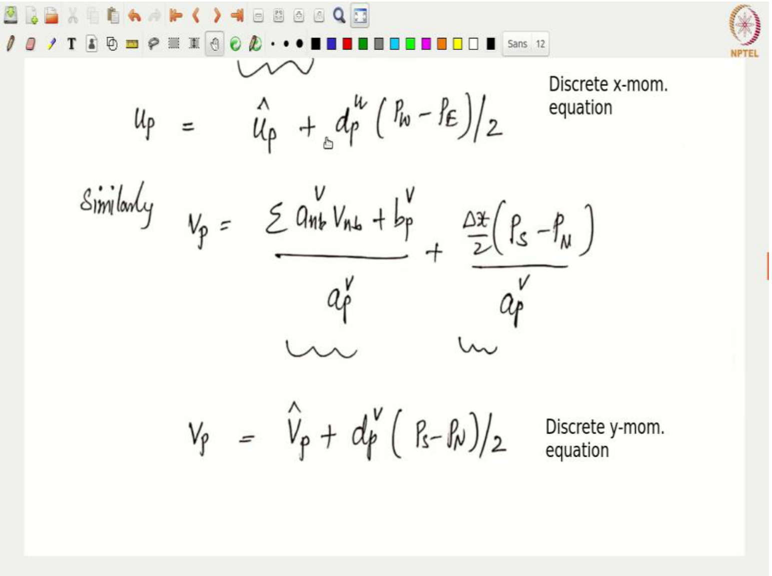

Then similarly let us write for the y-momentum equation that is $\mathrm{a}_{\mathrm{P}}^{\mathrm{v}} \mathrm{v}_{\mathrm{P}} = \sum \mathrm{a}_{\mathrm{nb}}^{\mathrm{v}} \mathrm{v}_{\mathrm{nb}} + \Delta x / 2 (P_S - P_N) + b_P^v$ by dividing with $\mathrm{a}_{\mathrm{P}}^{\mathrm{v}}$, then what we get is v_{P} equals $\sum \mathrm{a}_{\mathrm{nb}}^{\mathrm{v}} \mathrm{v}_{\mathrm{nb}}$ plus b_P^v upon $\mathrm{a}_{\mathrm{P}}^{\mathrm{v}}$ plus you have $\Delta x / 2 (P_S - P_N)$ by $\mathrm{a}_{\mathrm{P}}^{\mathrm{v}}$ ok.

And that means, v_{P} equals we can write again this entire quantity as $\hat{v}_P$ this is $\hat{v}_P$ and then $\Delta x / \mathrm{a}_{\mathrm{P}}^{\mathrm{v}}$ let us call this as some $\mathrm{d}_{\mathrm{P}}^{\mathrm{v}}$ ok. So, this is $\mathrm{d}_{\mathrm{P}}^{\mathrm{v}}$ times $(P_S - P_N)$ by 2 ok; that means, this is basically your discrete y-momentum equation ok.

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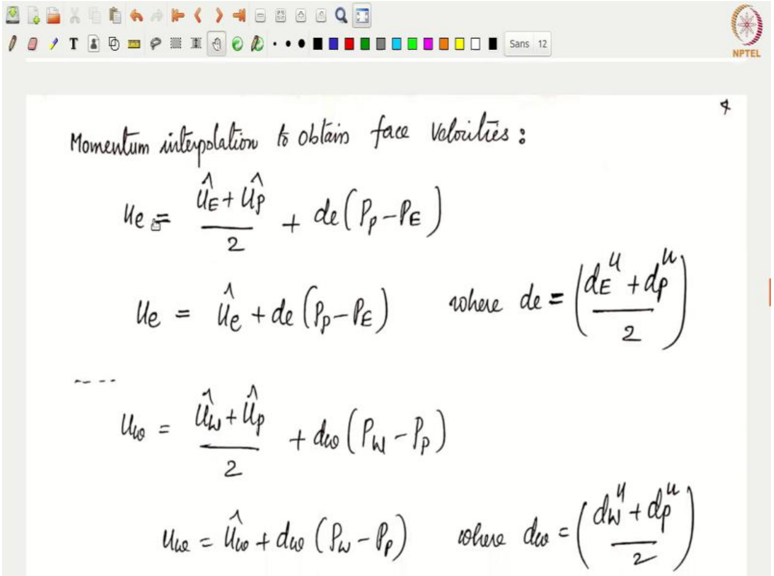

So, we have now the discrete x-momentum and the y-momentum equations they are just written in a slightly different way ok then let us perform momentum interpolation to obtain the face velocities. That means, we already know what is we already know what is u_E and u_P and v_P similarly we can write equations for u_E right that will be u_E would be $\hat{u}_E$ plus d_E hat times the corresponding values here right.

Instead of $P_W - P_E$ you will get something in terms of basically $P_P - P_{EE}$ like what we saw before. So, then we can now write down a momentum interpolated face value for u little e that is $\hat{u}_E$ plus $\hat{u}_P$ by 2 plus with the addition pressure term right this is what we learnt add the hat velocities as the arithmetic average and then you basically add you subtract the non-contiguous pressure and you add the contiguous pressure of these plus d times $P_P - P_E$ right.

So, we can write this as a let us write this as $\hat{u}_e$ that is u_e equals $\hat{u}_e$ plus d_e times $P_P - P_E$, where this d_e is proposed by Rhie and Chow as to be taken as again arithmetic average of the cell coefficients for the ds that means, d_E^u plus d_P^u by 2. What is d_E^u? d E u is basically your $\Delta y/a_P^u$ for u cell right. Similarly, d_P^u would be $\Delta y/a_P^u$ right for the neighboring cells. So, an average of d_s's will give you d on the face which we can use here ok.

Similarly, momentum interpolation can be done for each of the face velocities just like what we have done for u_e; we can do the same thing for u_w right. So, u_w would be basically $\hat{u}_W$ plus $\hat{u}_P$ by 2 plus d_w times $P_W - P_P$ right. And so this can be written as well. Then essentially what we have is you little w equals we can write this as $\hat{u}_w$ plus d_w times $P_W - P_P$, where again d_w would be d_W^u plus d_P^u by 2 alright.

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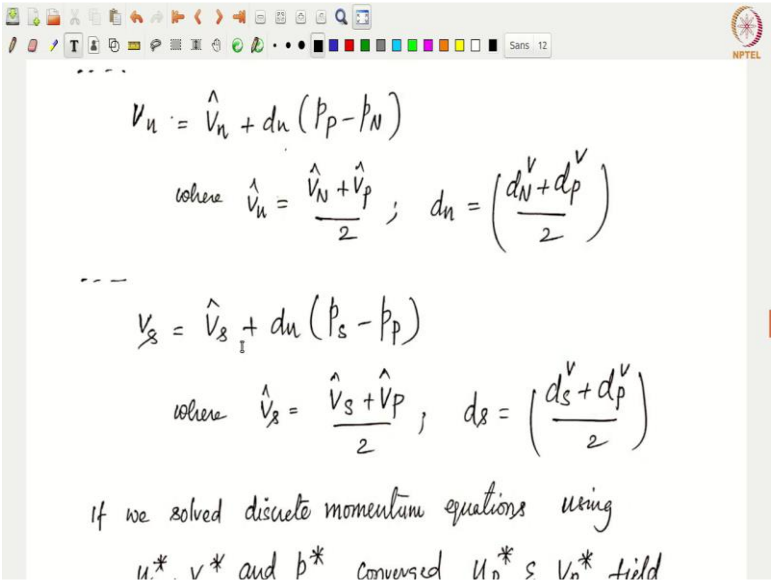

Again now we can write momentum interpolated values for v_n and v_s. So, v_n would be $\hat{v}_n$ plus d_n times $P_P - P_N$, where $\hat{v}_n$ would be $\hat{v}_N$ plus $\hat{v}_P$ by 2, and d_n would be d_N^v plus d_P^v by 2 ok.

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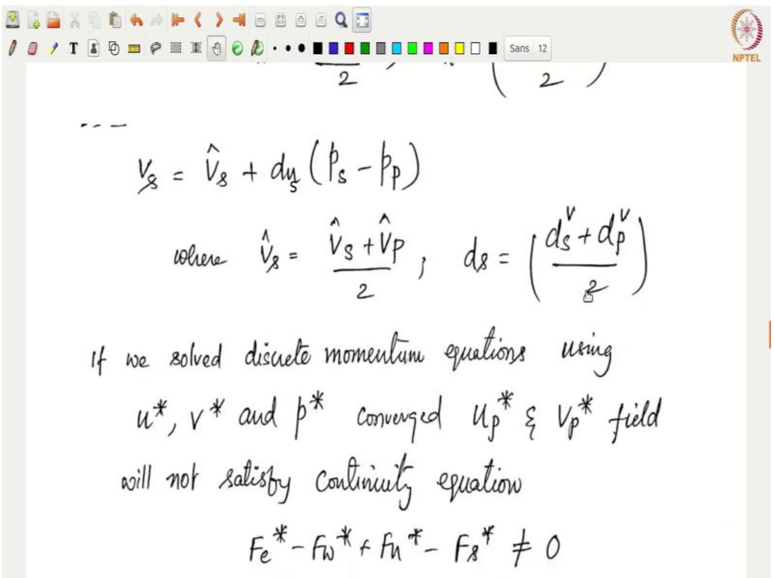

Similarly, v_s would be $\hat{v}_s$ plus oh this should be south is not it this should be, this should be south not north. So, this is south times $P_S - P_P$. So, where v s hat would be $\hat{v}_S$ plus $\hat{v}_P$ by 2. And d_n would be d_S^v plus d_P^v by 2 ok.

So, what we did was basically we wrote from the cell values, we wrote from the cell value and its neighboring values at the cell centers, we wrote this momentum interpolated face velocities ok. Remember these are not linearly, these are not linear interpolation face velocities.

These are the momentum interpolated face velocities right. Because if this were linear interpolated then you would have that alternate pressure term which is not there that is now removed and this is being added ok. So, these are now the momentum interpolated face velocity that is what we have, that means, then we have now the face velocity is on east, west, north, south, all of them.

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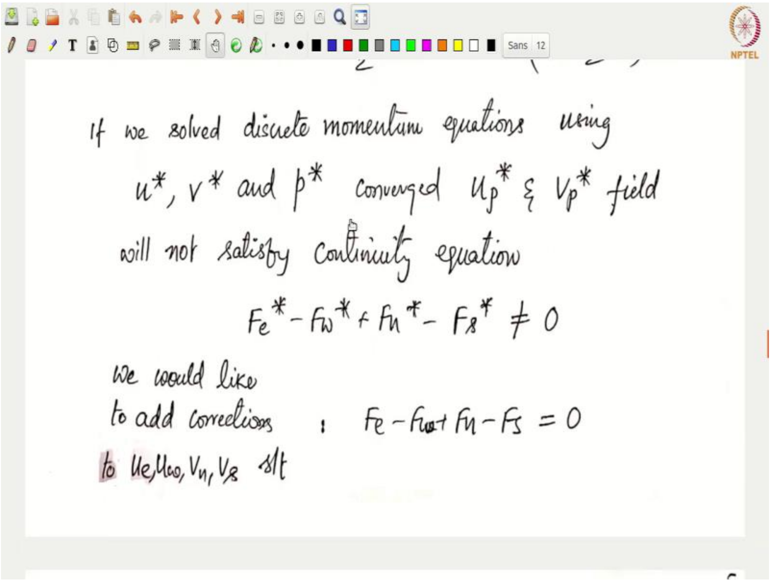

Then what we do is we use these face velocities in the solution of the continuity equation ok. So, what we say is that if we solve the discrete momentum equations, let us say using u^*, u^* and P^*, then the converged u^*, u^* field that is for the cell centroids for u_P^* and v_P^* field.

Of course, even if you construct the face velocities will not satisfy the continuity equation, that means, the flow rate that is you would get F_e^*, F_w^*, F_n^* and F_s^*, the mass flow rate continuity equation will not be satisfied for any cell because these are basically coming from the solution of momentum equation for a guess pressure value ok.

So, the idea is basically is similar to essentially same central idea to simple is basically you would wish to have these a flow rates be corrected right or the velocities be corrected such that they satisfy continuity equation. That means, we want to add corrections to u_e^*, u_w^*, v_n^*, and v_s^*

such that F_e minus F_w plus F_n minus F_s equals 0 that means we can split each of these into a star and a prime quantities that is what we do.

And because the star quantities would be known from coming from the solution of momentum equations they would be sent to the right hand side ok. So, minus F_e^* plus F_w^* minus F_n^* plus F_s^* would be there on the right hand side which we denote as b^* ok. And on the left hand side, these prime quantities will remain that is F_e' minus F_w' plus F_n' minus F_s' ok.

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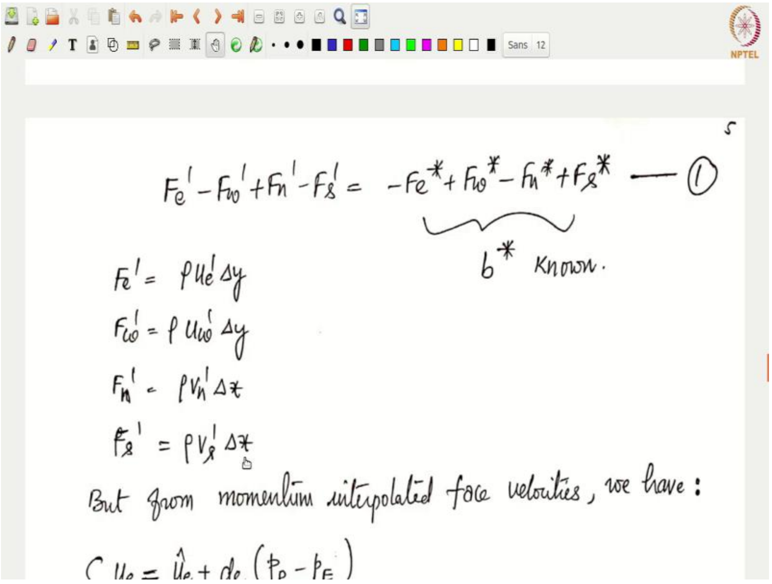

Now, of course, we have these quantities that are the flow rate corrections that is F_w' would be equal to $\rho u_e' \Delta y$, and F_w' would be equal to $\rho u_w' \Delta y$; F_n' would be equal to $\rho v_n' \Delta x$; and F_s' would be equal to $\rho v_s' \Delta x$.

Now, this is where the difference comes between the staggered mesh and the colocated mesh. Essentially, now these u_e', u_w', v_n', v_s', the velocity corrections we will get them from the momentum interpolated face velocities ok. Remember we just had these momentum interpolated velocities that is u_e equals $\hat{u}_e$ plus d_e times $P_P - P_E$ right. And a corresponding equation in terms of stars let us say if we have a guess velocity field and a guess pressure field, there is a star missing here.

So, if you have a guess velocity field and a guess pressure field from which you can write the guess velocity field for the face u_e^* as $u_e^* = \hat{u}_e^* + d_e(P_P - P_E)$ ok. So, we have basically these two equations I mean these are basically the momentum interpolated face velocities for the east face; one is with without stars and one is with star that means we have the guess pressure and the guess velocities and this is what we have.

And we have another one which is basically supposed to satisfy both momentum and continuity the discrete momentum and continuity equations; that means, if you subtract one from the other, that means, u_e minus u_e^* will give u_e' equals $\hat{u}_e$ minus $\hat{u}_e^*$ would give you $\hat{u}_e'$ plus you have d_e times P_P minus P_P^* will result in P_P', P_E minus P_E^* plus then P_E' alright.

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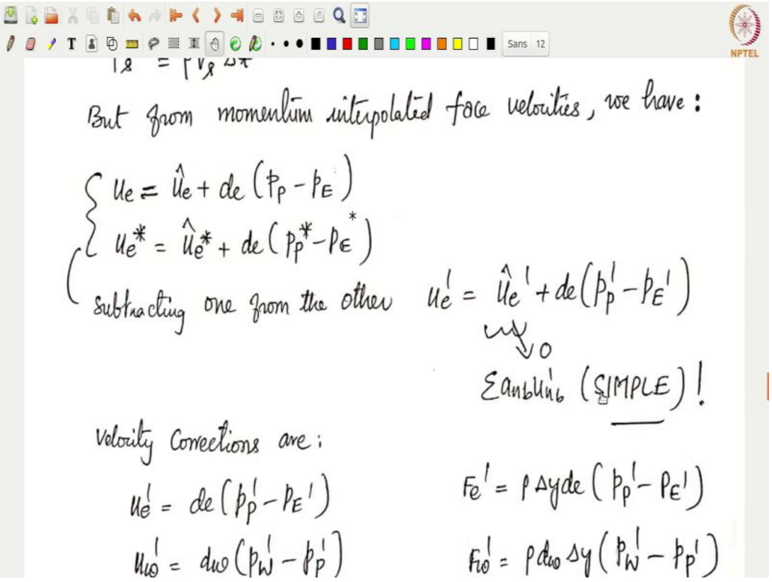

So, we essentially obtained an equation for the velocity corrections that we got here in the continuity equations right $\rho u_e'$ from the momentum interpolated face velocities ok. And what is this $\hat{u}_e'$ this is basically nothing but your $\sum a_{nb} u_{nb}' + b$ right divided by a_P.

Now, of course, this has to be neglected in simple that is the approximation that is made in simple right. The neighbor neighboring velocities corrections are never considered, so that means, the cell velocity corrections are always done by the pressure corrections alone. So, this means we should take $\hat{u}_e'$ equal to 0 ok, that means, $\sum a_{nb} u_{nb}'$ is basically taken to be 0 in simple algorithm as an approximation.

Then what we have is you have the velocity correction on the face u_e' equals d_e times $P_P' - P_E'$ alright, that means u_e' equals d_e times $P_P' - P_E'$. Similarly, we can write similarly we can write u_w' as d_w times $P_W' - P_P'$. And v_n' as d_n times $P_P' - P_N'$. And v_s' as d_s times $P_S' - P_P'$ ok. So, these are the velocity corrections.

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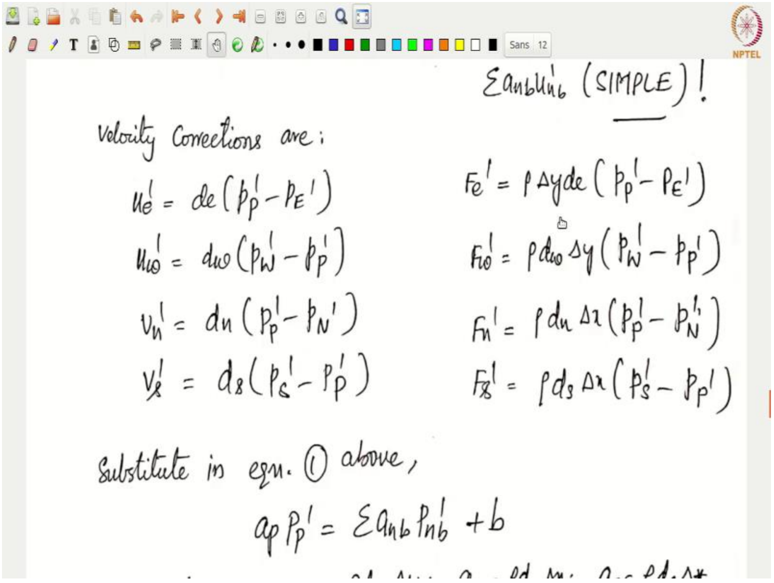

Of course, correspondingly you can write the corrections for the flow rates as F_e' equals $\rho u_e' \Delta y$ right. What we have here $\rho u_e' \Delta y$, but u_e' is now this quantity.

So, if you substitute that, what you get is $\rho \Delta y d_e$ times $P_P' - P_E'$. Similarly, F_W' would be $\rho \Delta y d_w$ times $P_W' - P_P'$. And F_n' is $\rho \Delta x d_n$ time $P_P' - P_N'$. And F_s' is $\rho \Delta x d_s$ times $P_S' - P_P'$ ok.

So, remember this is basically very similar to what we have done in the staggered grid algorithm as well right. We have the prime flow rates. We have now related them to the pressure corrections. Now, what do we do we basically substitute for all these flow rates back into this equation 1, that means, on the left hand side you have everything in terms of pressure corrections, on the right hand side the star velocities are known ok.

That means that means, we have all of these ready. But only difference is between the staggered mesh and the right the and the one we have right now is basically these d_e that we have are different right. And d_n's are different and also these P primes these d_n come as a arithmetic average of d_E and d_P by 2 and so on right.

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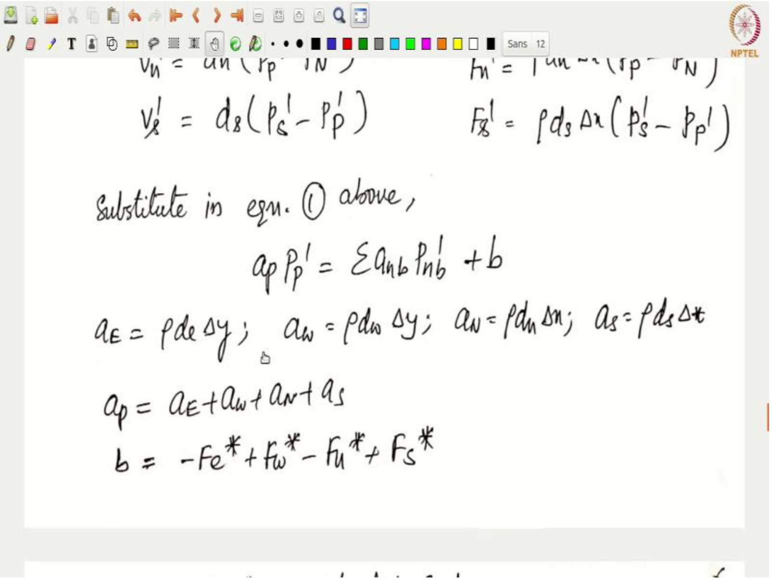

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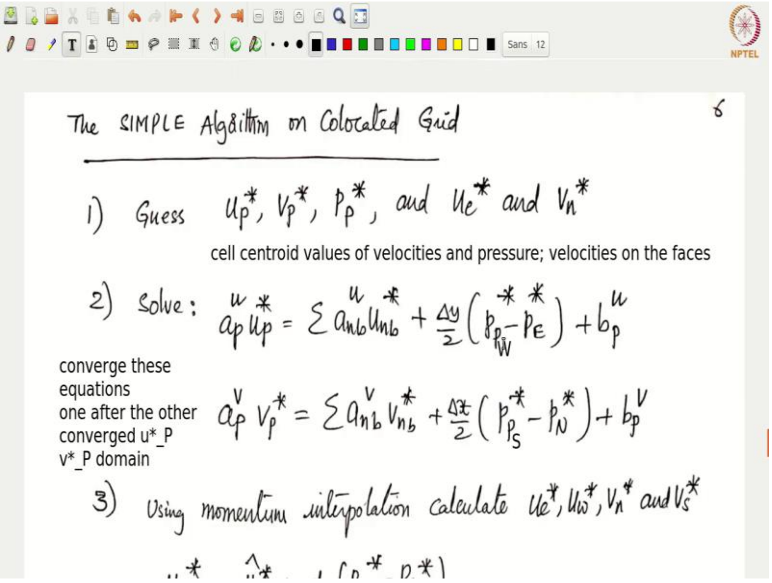

Whereas, this staggered with they are already available because the u little e is already stored at the east face ok. So, essentially we are doing the same thing, but through interpolations not having a grid as such that is created which will be helpful when we go for curvilinear meshes, because then we do not have to solve on any faces, but we kind of solve on the cell centroids and so on alright. So, you essentially substitute for the F_e''s, then what you basically get is $a_P P_P' = \sum \mathrm{a_{nb}} \mathrm{P}_{nb}' + b$.

Where $a_e = \rho d_e \Delta y$; $a_w = \rho d_w \Delta y$; $a_n = \rho d_n \Delta x$; $a_s = \rho d_s \Delta x$. And in addition we have a_P would be equal to basically because there is a P_P' in every term is will be equal to the same sum of all the coefficients $\sum a_{nb}$. And the b term on the right hand side is basically minus F_e^* plus F_w^* minus F_n^* plus F_s^* ok.

So, this is the pressure correction equation. Once you solve this, you would get all the pressure corrections. Then the pressure corrections can be used to correct the velocity corrections to obtain the velocity corrections, and the flow rate corrections. And these corrections can be added to the star values to obtain the correct values for face velocities and face flow rates alright.

So, that is precisely the simple algorithm on colocated grid. So, now let us kind of look at the overall algorithm. Let us put all the things together ok, then we will see how to basically what would be the final simple algorithm ok. So, the simple algorithm on colocated grid; so, we start off with guessing the velocities and pressures at the cell centroid.

So, this is basically guess the cell centroid values of velocities and pressures right. So, guess u_P^*, v_P^*, P_P^*; and in addition we need to also guess u_e^*, v_n^* that means, we are to guess velocities and pressures on the faces as well, oh sorry, only on the velocities on the faces because these are required in calculating the F stars; F stars so; that means, these are required.

So, these also need to be kind of guessed. Then you solve for momentum equations at the cell centroids right that is basically $a_P u_P^*$, we have a u here to distinguish this guy from this guy right, that means, $a_P^u u_P^*$ equals $\sum a_{nb} u_{nb}^*$ plus $\Delta y/2$, this is of course, not correct this cannot be P_P^* this should be P_W^* right.

And, this should be P_S^* right we are talking about momentum equations where you still have the staggered pressures you still have the non-contiguous pressures which will support pressure checker boarding ok. So, this is $P_W^* - P_E^*$ plus b_P^u. And the y-momentum equation is $a_P^v v_P^*$r equals $\sum a_{nb} v_{nb}^*$ plus $\Delta x/2(P_S^* - P_N^*)$ plus b_P^v right.

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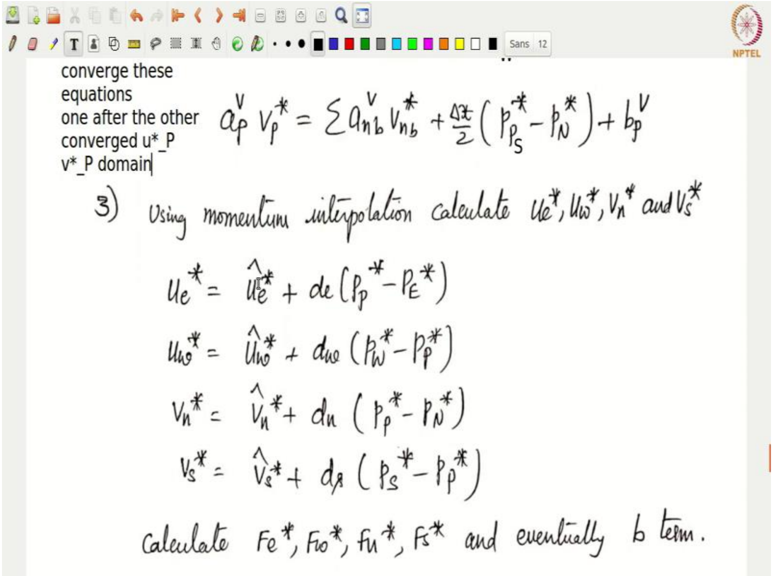

I think these are the correct equations right. We have written somewhere before as well alright. So, essentially you solve two essential you converge these equations one after the other right. And you obtain converge basically converged u_P^* and v_P^* values for the entire domain right ok.

Then essentially what you need to do is you need to now use momentum interpolation and calculate what is the star values ok, ok. So, that means, calculate star values that is basicallyu_e^*, u_w^*, v_n^*, and v_s^*. Essentially you calculate from u_e^* equals $\hat{u}_e^*$ plus $d_e(P_P^* - P_E^*)$. So, these are the guess values and these are the arithmetic average values for the cells which are now converged right. Use those and calculate what is the star values for the faces.

Similarly, u_w^* equals $\hat{u}_e^*$ plus $d_w(P_W^* - P_P^*)$. And v_n^* equals $\hat{v}_n^*$ plus $d_n(P_P^* - P_N^*)$. And v_s^* equals $\hat{v}_s^*$ star plus $d_s(P_S^* - P_P^*)$ ok.

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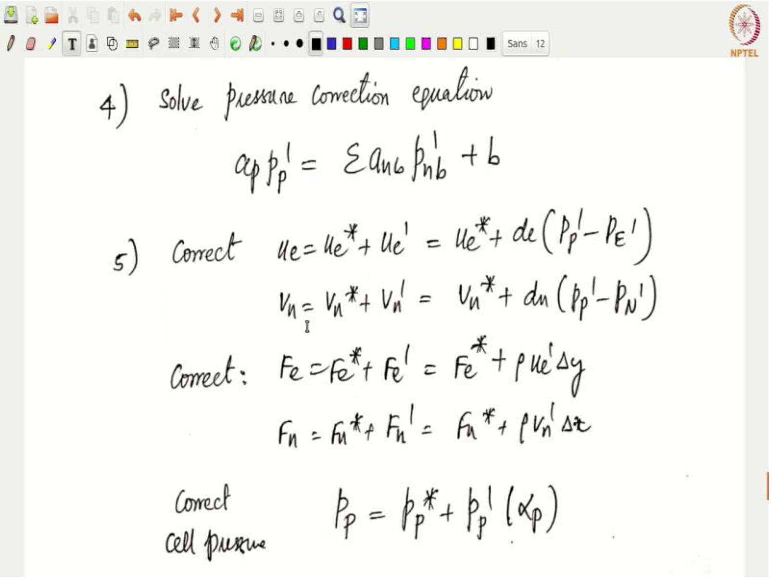

Once you have all the face velocities which are the star face velocities, calculate what is F_e^*, F_w^*, F_n^* and F_s^* that is basically calculate the flow rates, and eventually calculate the b term ok. Then you have when you probably do not have to guess the values ok, so probably do not have to guess the values for faces is not it ok.

Then you essentially once you have this then you solve for the pressure correction equations, that is $a_P P_P' = \sum \mathrm{a_{nb}} \mathrm{P}_{nb}' + b$ ok. Then once you have this you correct the velocities ok, that means, you correct u_e as u_e^* plus u_e' where u_e' is d_e times $P_P' - P_E'$; v_n equals v_n^* plus v_n' that is v_n^* plus d_n times $P_P' - P_N'$.

So, basically once you converge this you know the pressure correction values. You of course, also have to correct the flow rates the flow rates are F_e, F_n, all though all those things that means, F_e^* plus F_e'. So, F_e^* are were calculated here right these were calculated here now these have to be also corrected, that means, F_e^* plus $\rho u_e' \Delta y$ right that is that is this guy multiplied by Δy.

And similarly F_n would be F_n^* plus F_n' that would be $\rho v_n' \Delta x$. So, correct the both the velocities as well as the flow rates. And of course, you at also correct the pressures ok, that means, correct the cell centroid pressure that is $P_P = P_P^* + P_P'(\alpha_P)$ ok. So, you correct the now you still have to use under relaxation for pressure correction, because we are dropping the $\sum \mathrm{a_{nb}}$ right or we are dropping $\hat{u}_e'$ right in the context of colocated mesh ok.

Then in order to improve convergence, let us also not only correct the face velocities using this formula ok, but also correct these cell centroid velocities ok. So, let us correct the cell centroid velocities as u_P equals u_P^* plus u_P', v_P equals v_P^* plus v_P', this is purely done to improve convergence ok. It will kind of converge faster.

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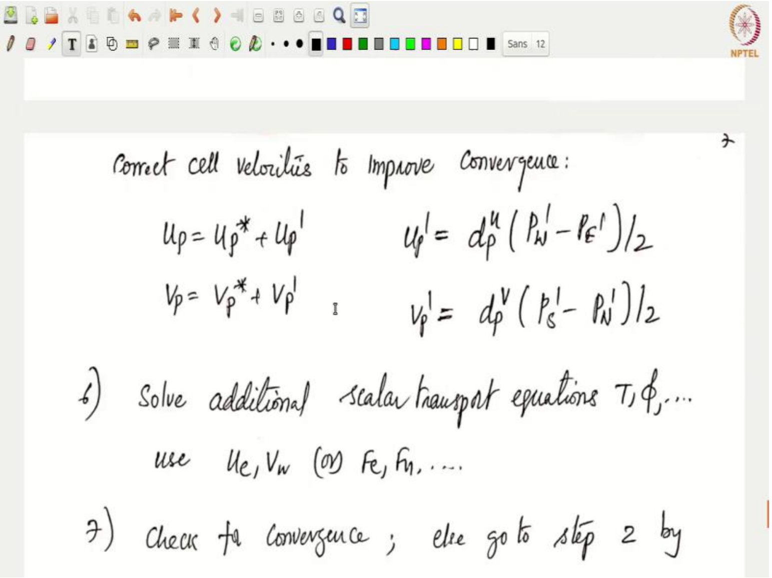

So, by analogy, we will use some u_P' as d_P^u times this is the staggered pressure corrections ok, $P_W' - P_E'$ by 2. And v_P' equals d_P^v these are capital d_P^v times $P_S' - P_N'$ by 2 ok. So, this is purely down to correct the convergence. If you do not do this, it will be slow that is all because we only have to correct the correct the face velocities and the pressures ok.

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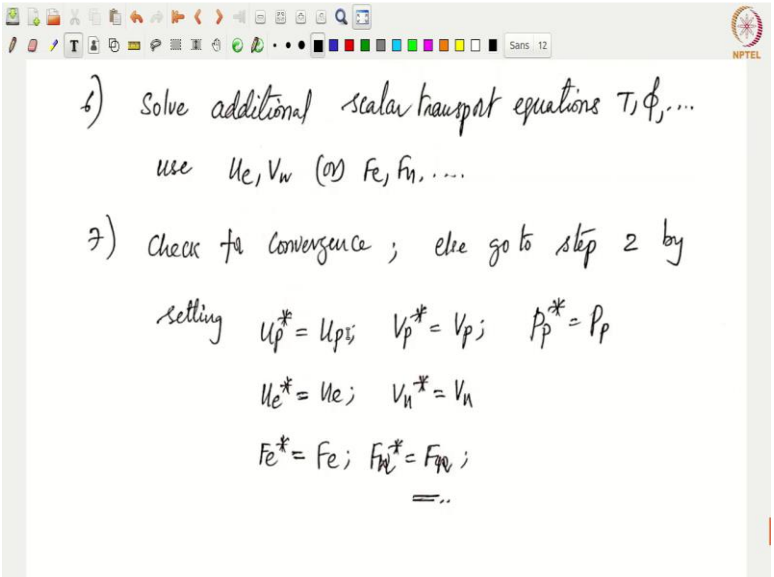

Then at this point the face velocities that we have corrected here satisfy continuity equation right. The face velocities that are corrected here now satisfy the continuity equation, so that means, you can you have a continuity satisfying feel. So, then we can solve for the additional scalar transport equation that you have in the domain, that is equations for T, phi and so on.

So, here use the flow rates the in the corrected flow rate that is F_e, F_w, F_n, F_s which are basically expressed in terms of u_e, v_n,, u_w, v_s ok. Then once you do this thing, then you check for convergence. If it if you have not reached convergence, then you go back to step 2; that means, we go back to this step here ok. Again we solve for the momentum equations.

But then now you have to take the updated u_P^* as whatever is the u_P that is now corrected ok. u_P^* equals u_P, u_P^* equals to v_P, P_P^* star equals P_P. And the velocities on the faces that is u_e^* equals u_e, v_n^* equals v_n. And of course, the flow rates also have to be updated because they are also now corrected and they will be used later on. So, F_e^* equals F_e, F_n^* equals F_n and so on ok.

So, with this we go back and then essentially we come back to step 2, and then continue fine alright. So, that is the simple algorithm on a colocated grid. It is pretty much similar to what we have on a staggered mesh; only thing is that we are now solving only colocated mesh. And we have these hat velocities which should be helpful.

And of course, what you see is that essentially you we are now working with two sets of velocities. One is the velocities at this cell centroid, the other one is the velocities at the faces right. In the staggered grid approach, we only had one set of velocity that is the face velocities always right. Now, we are trying to get away with the staggered grid method. So, as a result we basically have two sets of velocities ok; one is at u_P, v_P and other set is u_e, u_w, v_n and v_s.

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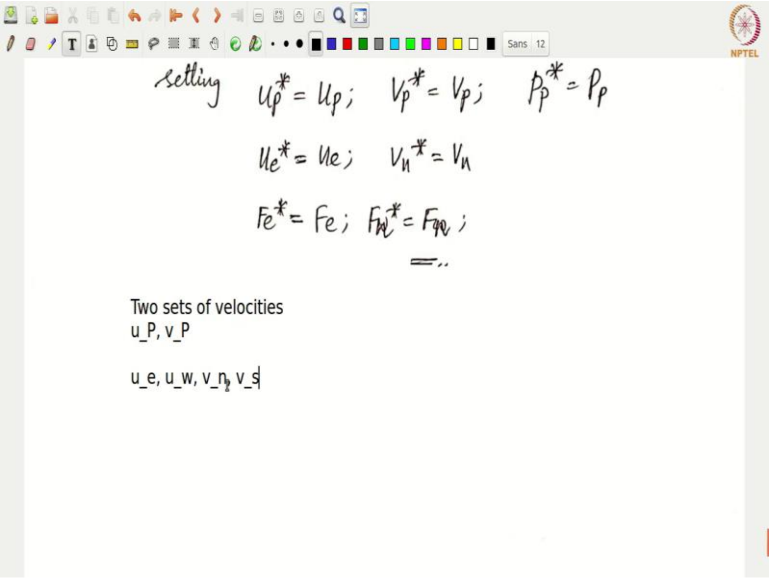

What you see is that basically these face velocities are made to satisfy continuity equation right, because those are the ones which are going into the flow rates whereas, this u_P, v_P are not actually forced to satisfy continuity. As a result we have two sets of velocities one satisfy momentum one satisfies continuity and then we are kind of iterating.

And as you each convergence both of them will come out to be of the similar values when you try to interpolate them and they kind of satisfy both the equations as such ok. So, that is the idea behind colocated grids where we kind of use Rhie-Chow interpolation and interpolate for the face velocities, and avoid pressure velocity checker boarding that comes into play ok.

So, that is the overall algorithm ok. So, I am going to stop here. If you have any questions do let me know through email, and I will reply to your queries. Ok, alright.

Thank you.