Wheeled Mobile Robots
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Lecture - 11
Mobile Robot Dynamics – Part 2

Welcome back to the Wheeled Mobile Robots. So, last class what we have seen lecture 11 this is also lecture 11, but this is part 2; last lecture part 1 what we have seen? So, what is robot dynamics and how we can derive the equation of motion in one of the method what you call Lagrange Euler.

Simply some people call Lagrangian method, but this is what we have seen and we have derived the equation for you can say a non general case in the sense of what you can see like the vehicle body frame and the centroid would be distance apart from there what we have derived the equation we have derived for F_x F_y and M_z and in that what we have seen that the same equation can be obtained by the other method or not.

So, for the other method what we are trying to see in this particular lecture part 2 we are actually like trying to use a Newton equation and the Euler axiom.

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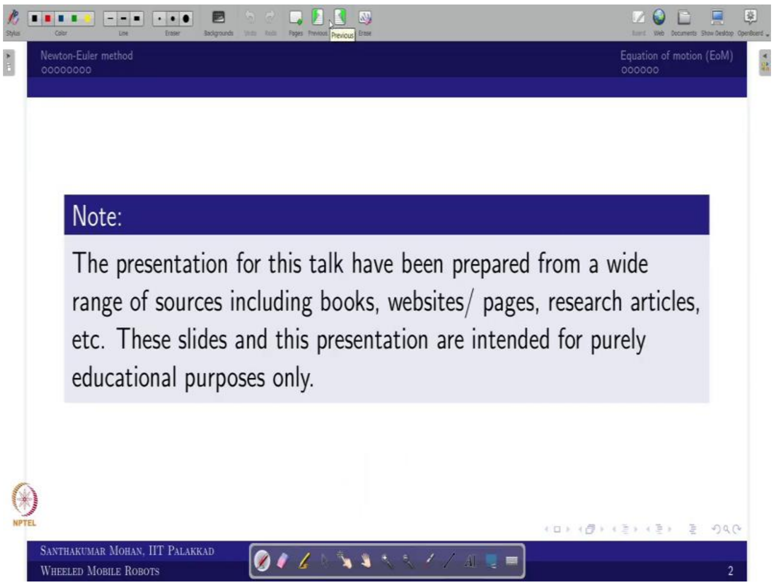

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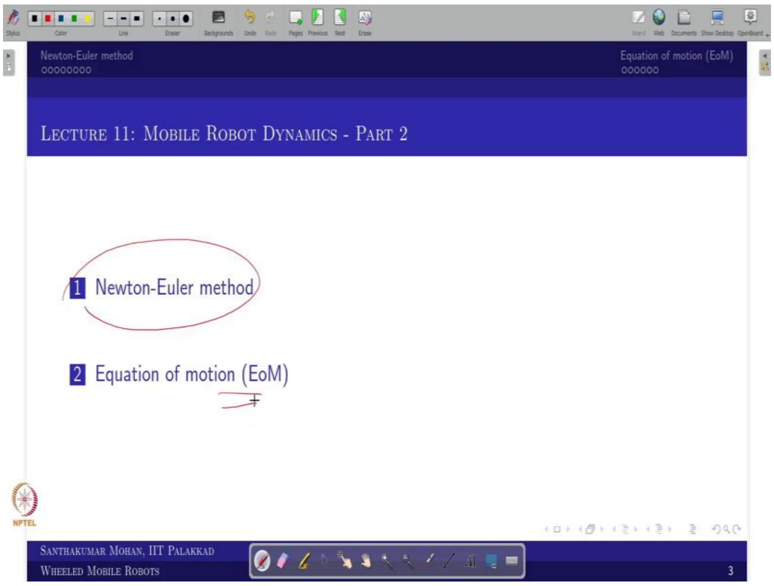

And then we are trying to use how that can be used right. So, further once you know the equations of motion then we will actually like try to write in a matrix and vector format that is what I written as equation of motion.

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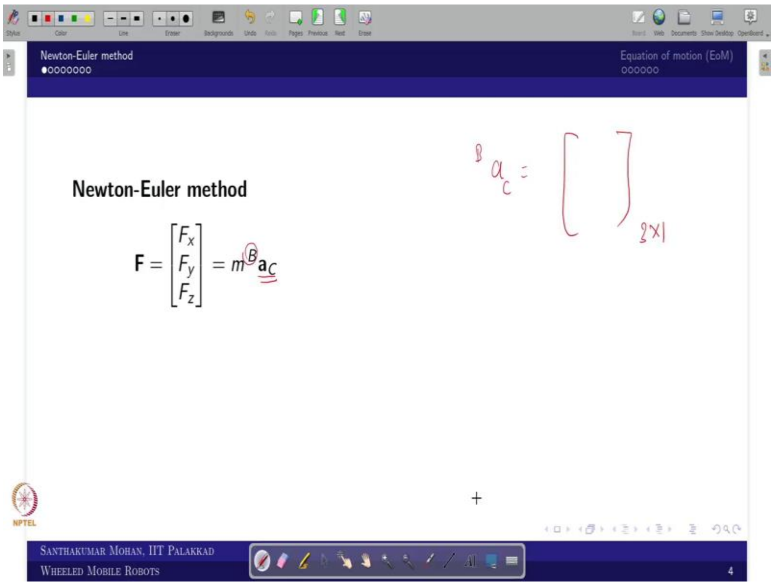

So, with that we will actually like move straight away to the Euler method Newton-Euler method. You know Newton equation. So, the Newton equation we can write in a different way I have written F = m × a, but a I have written as centroidal acceleration. So, it is a linear acceleration, but it is a centroidal acceleration where here I am writing the centroidal acceleration with respect to the body frame.

So, now, in that sense this a_{cb} would be having we can say 3 cross 1 that one should actually like understand ok. So, then you can see the F_{xz} would be 0, but in a planer case, but you can actually like see that this a_c would be having three component.

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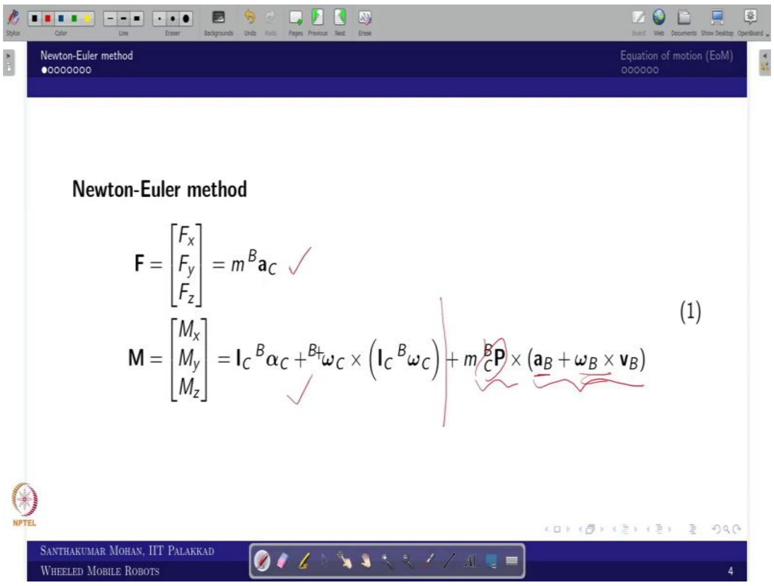

So, further what you know Euler axiom. So, the Euler axiom generally we right this way. So, where I is at into alpha that is what you write as a moment along with what you call ω cross I ω in addition to that you have it you call something like centroidal or inertial force that inertial force since that centroidal point is actually like away from the body frame, then that force also will try to make a you can say moment.

So, that is what we are trying to address here. You can see in that case you are trying to see the slip acceleration and as well as what you can see that coriolis acceleration what it actually like can cause the moment; so, these two equations what we are trying to use as a Newton-Euler method. So, this Newton equation and this Euler axiom we are trying to use.

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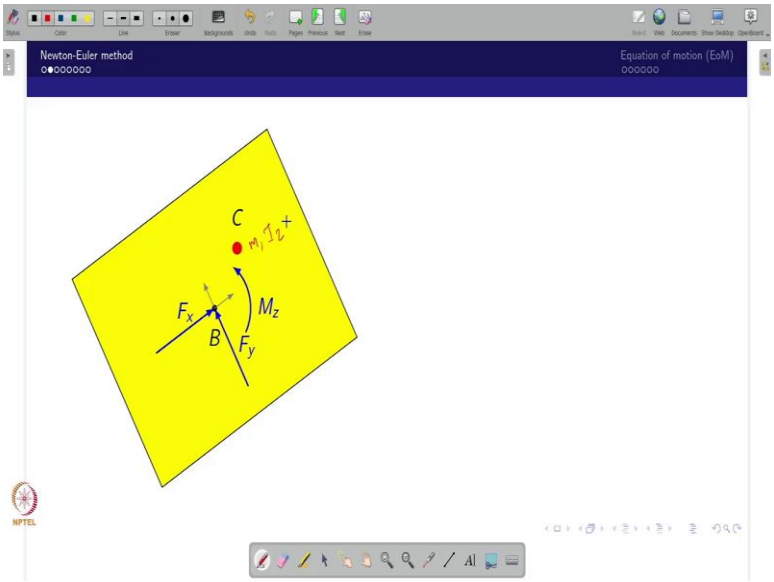

So, if that is the case. So, we will take the same situation. So, we will take a black you can say blank you can say rectangular box as a vehicle. So, were the B is the body frame and now I am putting the C conveniently away in the since I am assuming that the C is actually like not same as B frame.

So, in that sense what you can actually like see there would be a distance, but now if I apply force F. So, what would happen to the point C and similarly if I apply the you can say F_y. What will happen to the point C? And if I actually like apply the moment about z axis what will happen to the point C.

Why I keep on saying point C because the C is actually like what it is representing? It representing the inertia in the centroidal movement of inertia for both you can say mass and rotational case. In the sense this would be consisted of M and I_z right, if that is the case, so, what one can actually like see?

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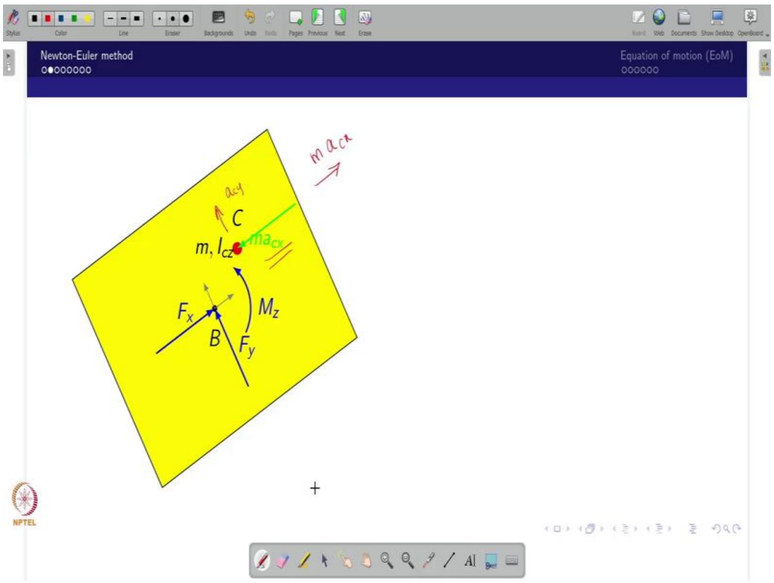

So, one can see that this consists of this mass and second moment of inertia. So, then this will try to resist this forces right. Because this force has tried to move the body, but what you know inertia means it is resistance to the motion right. So, now, this linear inertia and the rotational inertia try to you can say avoid this motion.

So, then what one can see it would give a you can say opposite and you can see equal forces on the point where the C is exact. So, in the sense what you can see? So, if you have a acceleration with generated then the acceleration you can say opposite direction you will generate you can say force that is what you call inertia force.

So, the inertia force I have shown in a green color. So, now, I assume that the acceleration generated on the point C as a cx; so, then ma_{cx} opposite direction that is what your inertia force similarly now I assume that this is the direction of a cy then the forces would be opposite. So, that is what we are trying to show.

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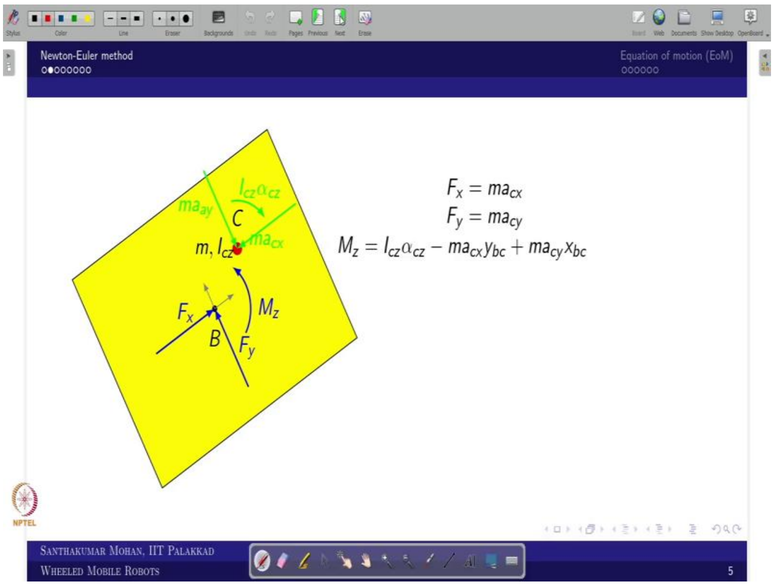

So, in that sense what would be the rotation? This is a rotation which happened. So, now, opposite that you would be having you can see rotational moment which happening in the inertia point or in a C point.

So, these are the 3 things. So, now, we know this relation, but how this relation would be captured? As $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ or you can say the other quantities which we have written as X_{bc} Y_{bc} and all right; so, for that we will write the generalized equation. So, this is what we can write as a straightforward.

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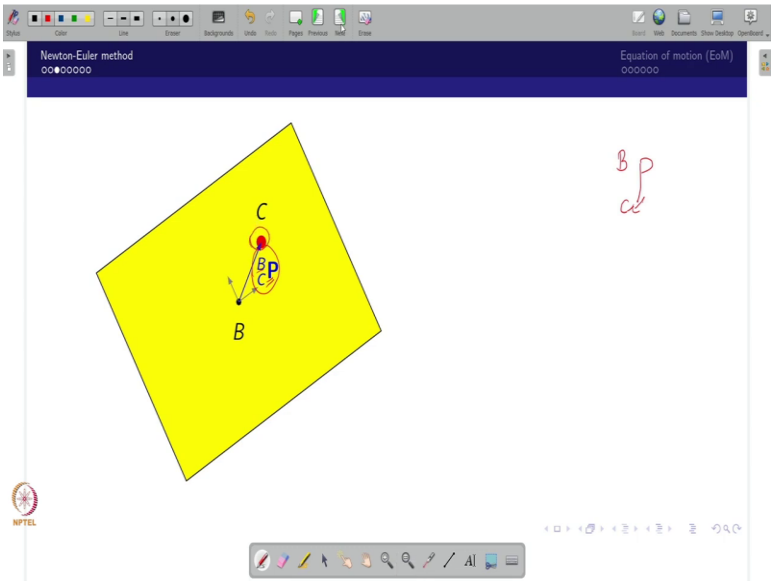

But right now, we will actually like come back to the generalize. What generalize? So, I am writing B, I am writing C, but I am actually like bringing it this as the one of the position vector. So, hear what the meaning is? So, ${}^{B}_{C}P$ that is what this written.

So, whatever I have written in the down that is actually like of and whatever I write on the top that is actually like with respect to. So, ${}^{B}_{C}P$ I am taking it, but this what we are interested we have seen that the C point how that would be moving with respect to inertia ok.

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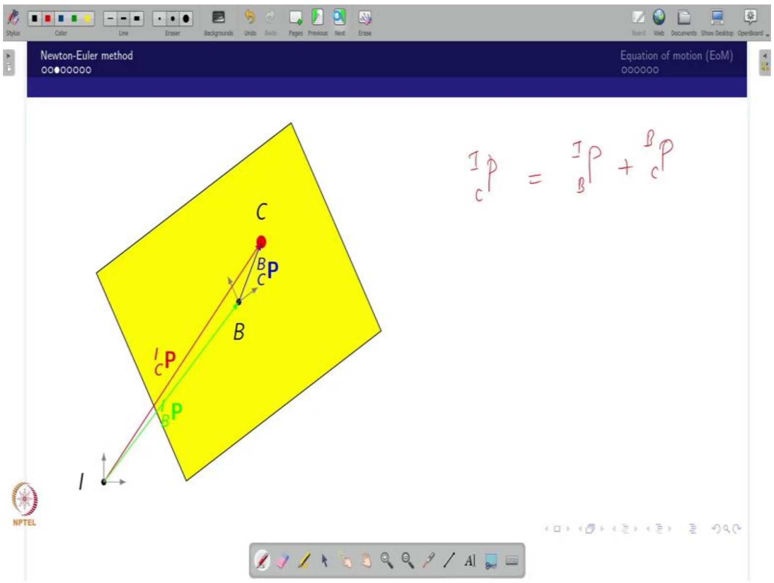

Just I am bringing the initial in the understanding. So, now, this is what my interest in the sense. So, what happened to this position vector, but this position vector I can obtain in another way round right. So, this is what I am actually like interested, but this interest how I can actually like make it I can go along with B right. So, now, what I can write this ${}^{I}_{C}P = {}^{I}_{B}P + {}^{B}_{C}P$ right. So, this is a simple vector edition right. So, how this would help definitely one can ask. So, we will see how that would help.

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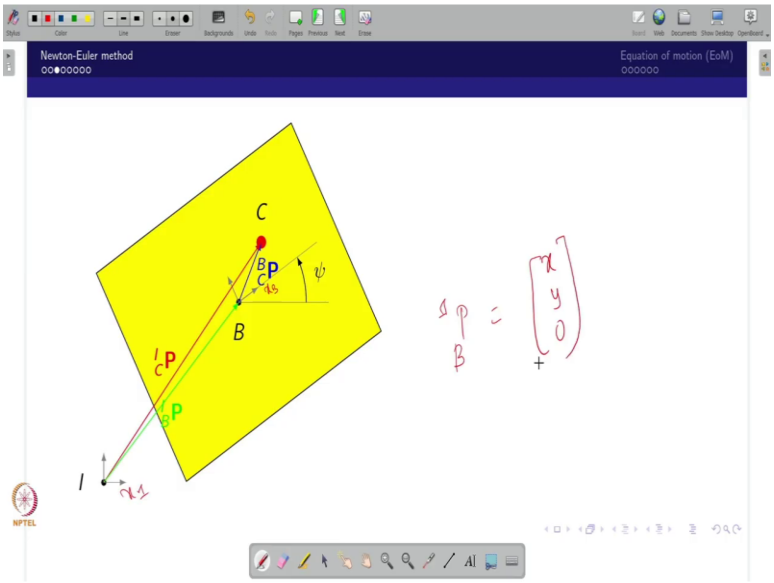

So, further I am assuming that this is actually like rotated with an angle of Ψ with respect to x axis frame of you can say X_I to you can say X_B is actually like Ψ angle this is one of the you can say generalized coordinate which we derived in the very beginning that lecture 3 if you take it. So, we have written x y and Ψ right. So, now, in that sense what would be the B point with respect to I that would be x y and 0 right. So, this is what I am rewriting.

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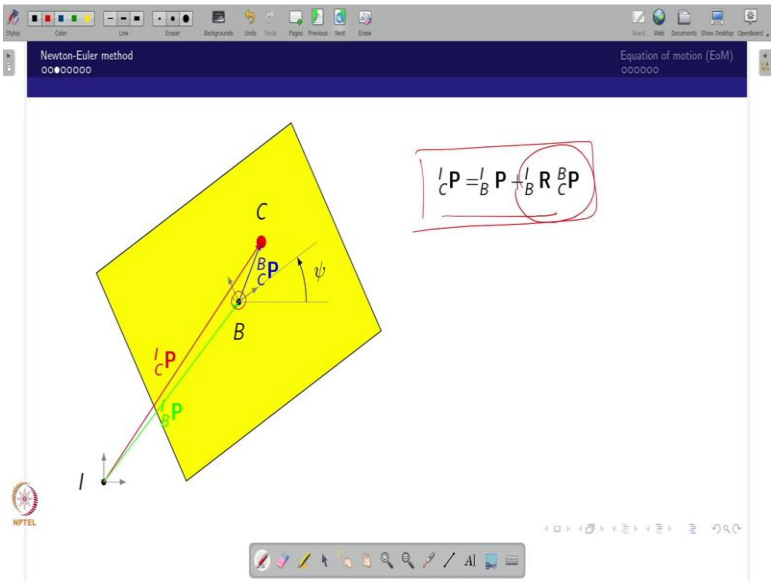

So, in that sense what you can actually like see that this I can write now this B frame is actually like rotating with respect to I then this point what you call the point of C with respite to B also like rotating because the point C is always in the body right. So, when you rotate the body then the point will not change with respect to B, but that is change with respect to I.

So, that is what we have written here. So, this is a general transformation. So now, how this would help? Sorry you have written the generalize equation that is we all know how this would help in the Newton equation this is what probably the question would come in your mind. So, you take the derivative of this ok.

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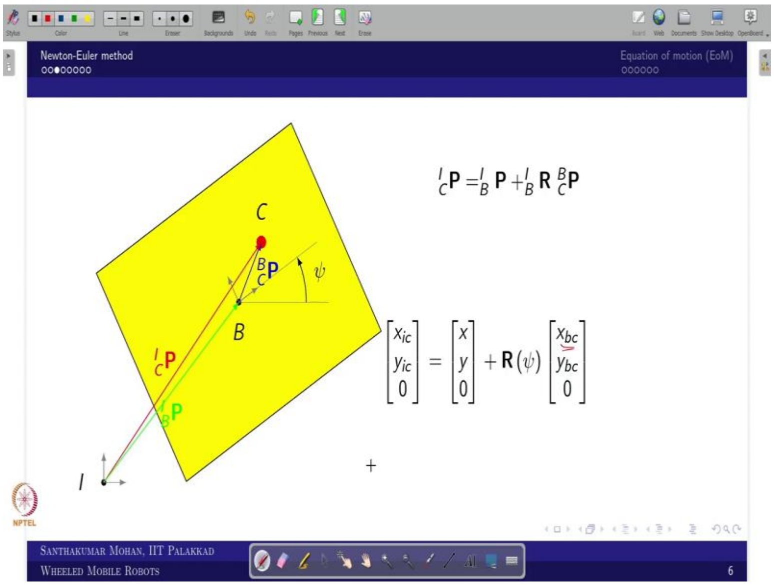

So, this is the generalized way I am writing where X_{ic} is actually like this point with respect to I. So, this is actually like you know with respect to B, but what I am actually like bringing this, but what I am actually like looking at if I take the derivative of this ok.

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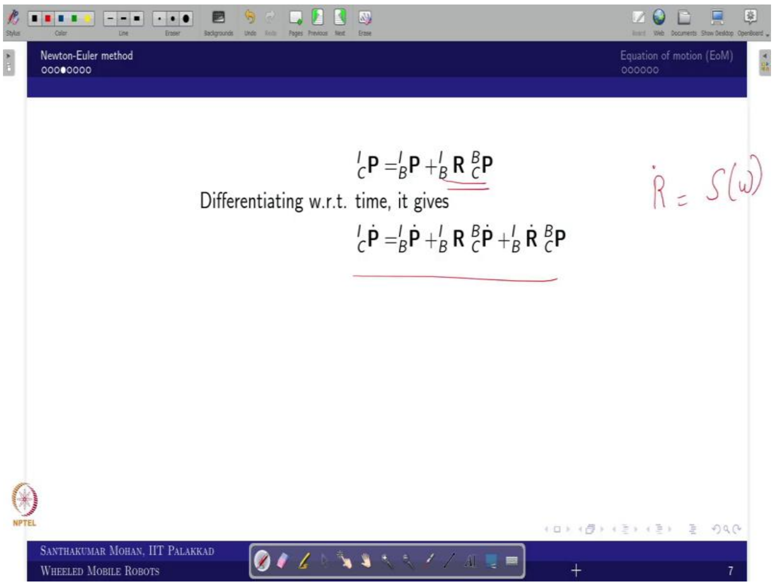

With respect to time I am differentiating. What I will get? This would be actually like my required vector. So, that is what I will get, but what additionally you can see that there is a rotation matrix multiply with a position vector there would be two component. And you know this rotation vector dot product I can write as a skew symmetricity matrix with respect to the angular velocity.

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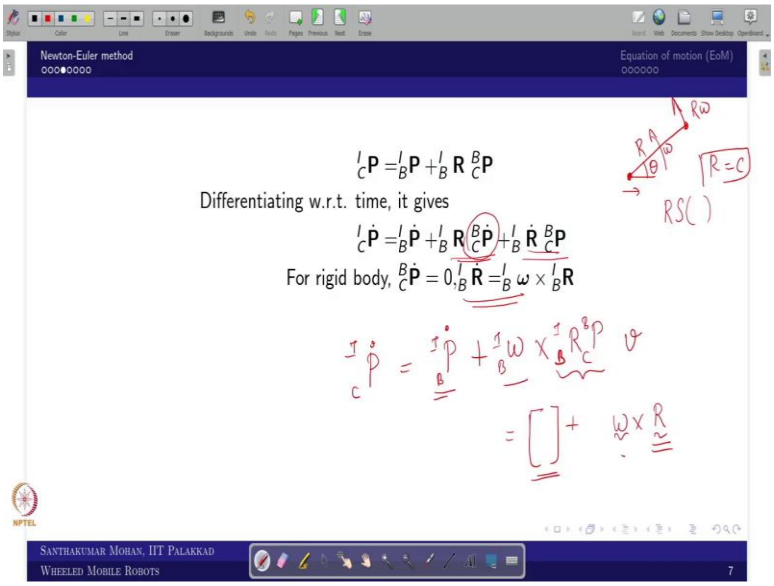

That principle I am bringing it here ok. So, in the sense what I am writing that particular this body dot would be always 0, because with respect to that point with respect to base frame. In the sense it will not get changed right there would not be any velocity which would be realized on that point C with respect to B if you are looking in the you can say inertial frame.

So, that is what we are actually like looking at. So, this would not get changed, but what you can see? This is actual like would be the product of that to cross product of angular velocity with respect to what you call the rotation matrix. This is what we used to write as actually like R S this is actually a skew symmetricity matrix that we will see in further slides or you can say further lectures, but right now you take this as the case.

So, or even you can actually like see in our general case we write this one. So, I assume that this is actually like a simple system I assume that the rotation only possible. So, this is R and θ where R is constant that is what I said is only rotate. So, now, there is a ω is the rotation. So, what would be the tangential velocity at this point? So, this would be Rω right.

So, in the sense what we can write in a general form? That we can write as $\omega \times$ R and if I assume that this body is having a linear velocity. So, then what would happen that linear velocity would be also added. So, this is what we have written here. So, now, if you rewrite this ${}^{I}_{C}\dot{P}$, I can write as B frame only moving. So, that dot velocity plus this is rotating with respect to I frame. So, then that rotational velocity would come that is actually like with respect to what you call this right.

So, now this is what we are actually like equating right. So, in the sense what you can actually like see this is also coming this is equivalent to R here and this is equivalent to ω vector and this is equivalent to what you have written here. So, this is what you obtain. Still we are not come to the picture right.

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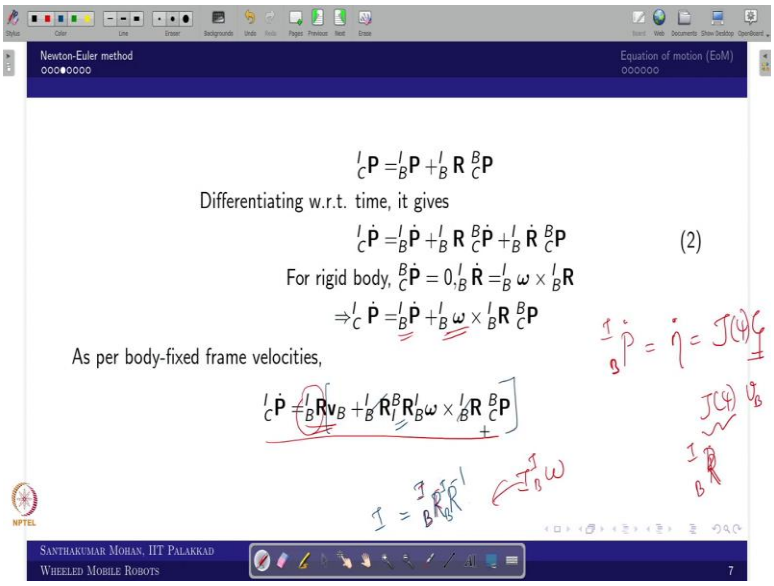

What the picture? The picture is actually like acceleration is required, the centroidal acceleration with respect to inertia what we wanted. So, far that we are differentiating again ok, but before that I m bringing the body fixed velocity relation. So, what this? This relation I know right. So, this is actually like I can write as a kinematic equation which I know already.

So, from there what I can actually like write it. So, this I can write as this is actually like ${}^{I}_{B}\dot{P}$ which is nothing, but $\dot{\eta}$. So, what you have written $\dot{\eta} = J(\Psi) \times \xi$ right. So, this is what we have written now the ξ what I am writing as V_B which is nothing, but the velocity of B with respect to B itself and this J(Ψ) what I am writing as a rotation matrix. This is a rotation matrix of B with respite to I.

So, now you can see like this is we have simplified. Further what we can actually like see? The same simplification you can do it I am just taking it this particular matrix I am multiplying throughout ok. So, why I am actually like doing it I can actually like make it in the sense this is I can write as a identity into ${}^I_B\omega$ this identity matrix I can write as inverse of one to another ok. So, this is actually like what you call inverse.

So, I can write. So, this is what the inverse I will just make it erase otherwise it will give a confusion. So, this is actually like rotation matrix this is actually like inverse. So, this would give a identity, but you know this is actually like ortho normal vectors consist this is the orthogonal matrix then just a transpose the transpose I have written as this form. So, now what one can actually like see? So, this has done. So, I am taking this R as a common ok.

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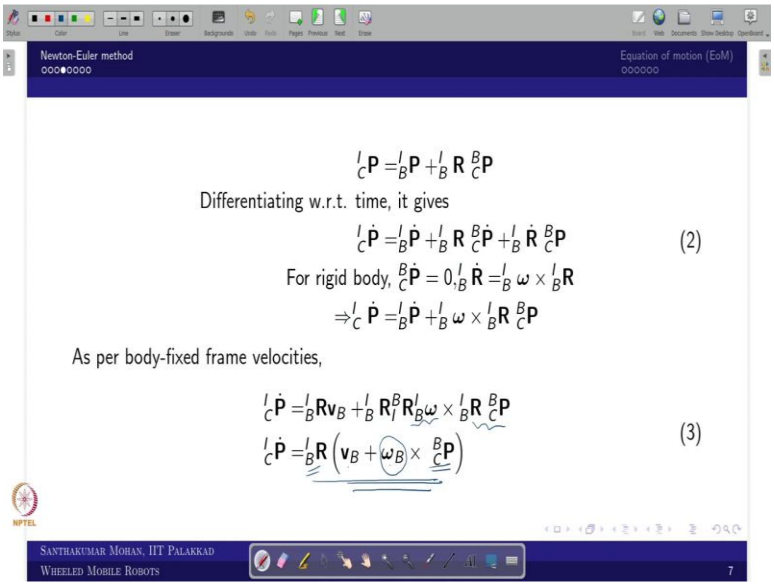

So, now, that is what I am going to write in the next equation I am taking R common. What you get it? So, you got it a very simplified equation right. What the simplified equation? So, you can see like this R I have taken out.

So, this is actually like with respect to I frame, but I take it everything in a B frame and this is actually like I again you can say this I have taken out. So, this is actually like with respect to B frame. So, now what you have here is, everything is body framed only the rotation matrix is that.

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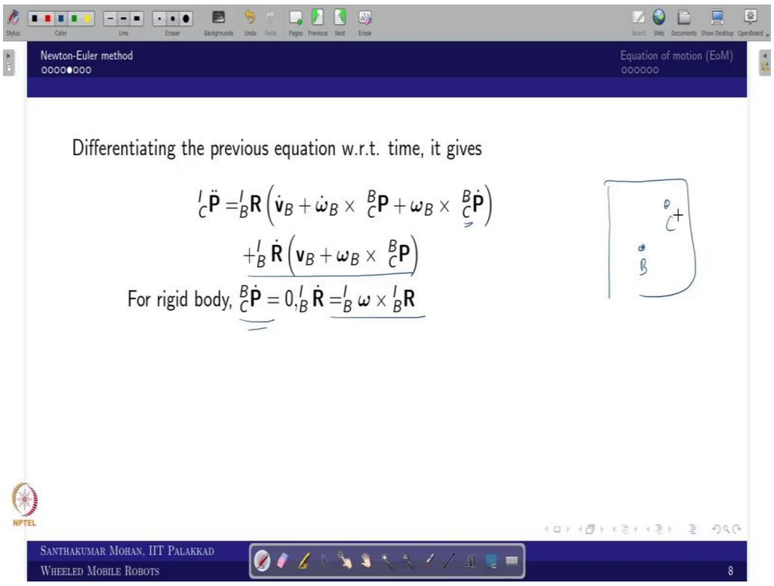

So, now we will actually like move further. So, I am taking the derivative of this again. So, here I have taken a velocity I am taking a acceleration now. So, what I will get? So, you can see like this is there ok. So, I am taking this dot, this dot and this dot in a proper way.

So, this is what going to happen this is one equation and then you will multiply with $\dot{R}$ into the whole equation right. So, that is what happened, but you know like $\dot{R}$ I can write in this form and you know like the ${}^{B}_{C}\dot{P}$ which is actually like in this way because this actually realize C with respect to B both are part of the same body right.

So, this is B and this is C. So, definitely there would not be any relative velocity between these two. So, that is what we have actually like assume and this is the assumption if you substitute what you will get you will get the bigger equation.

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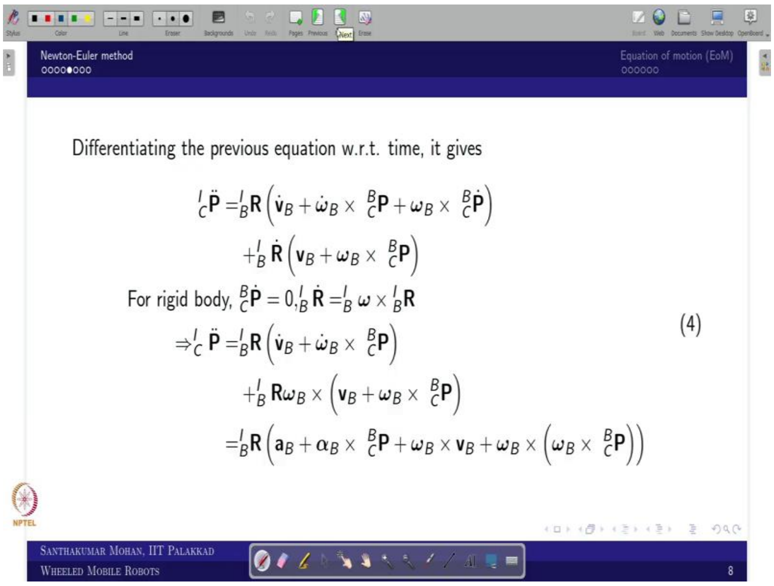

But this bigger equation what you have written in the form of you can say C acceleration with respect to I, but I wanted with respect to B. So, then what I can do I can just multiply with the rotation matrix. So, I first make it everything group.

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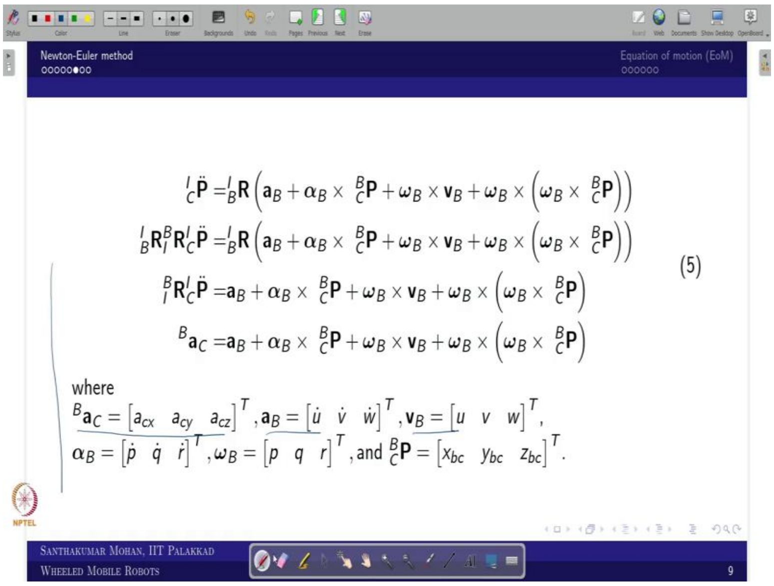

And then I actually like multiply with one of the simplest way. So, then what I can actually like see? So, what I can actually like see? So, I will get the acceleration everything in the body frame. So, that is what I am actually like doing it. So, you can see like this. So, now what you will get the ${}^{I}_{B}\ddot{P}$ will get. So, that is what I call as acceleration of C with response to B.

Now you can see like what you have discussed earlier that has come directly you see this is a slip acceleration and this is actually like you call the tangential acceleration this is nothing but your coriolis and this is what your radial acceleration all the components have come right. So, now you substitute these into a proper vector form and matrix then you will get it; in this case everything is vector.

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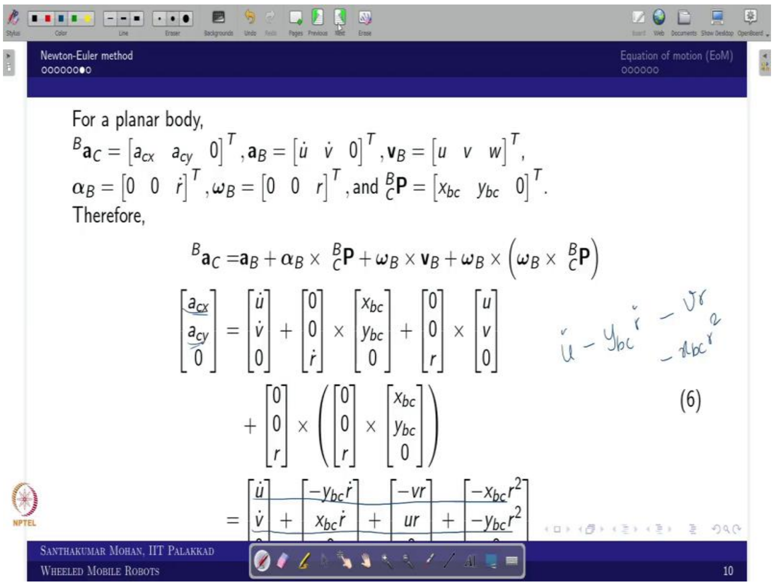

So, you substitute and then you w will get it that is what we are trying to do. You know like this and you know a B and you know V_B V_B is actually like $\begin{bmatrix} u \\ v \\ w \end{bmatrix}$ and w we consider as 0 if here I have put it in a general case it is a 3 dimensional case. So, you substitute this into a 2 dimensional case.

So, what you will get? Finally, you substitute these all in the $F = m \times a_x$ $F_x = m \times a_{cx}$ equation and then you can actually like see it for that I am actually like substituting this all and I am trying to find out what is actually like a_{cx} what is a_{cy}, so that I can obtain from here ok.

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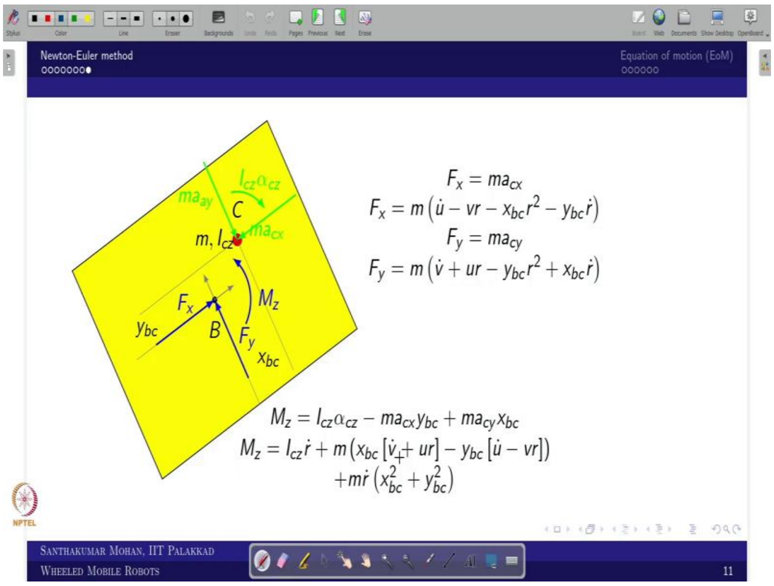

So, I will actually show it in the next one. So, this is what we have already derived. So, now, we can actually like see that this is what a cx. So for understanding that; so, you can actually like see here. So, what it is coming? So, $\dot{u}$ - $Y_{bc}\dot{r}$ - vr - $X_{bc}r^2$ right. So, this is what the first value that is equivalent to a_{cx}.

What is a_{cy}? a_{cy} is actually like equivalent to this right. So, that is what we are actually like substituting here you can see right. So, similarly you can see here. So, now, you have got it what is F_x, what is F_y right. So, you got it everything. Now only left is M_z. So, M_z actually like you can see this is actually like straight forward you can see like ma_{cx} is there ma you can see this is the cy. So, ma_{cy} is here.

So, you can actually like see that this will give a couple and this will also give a couple. So, that is what we have actually like used. So, now M_z also like you can substitute by subduing ma_{cx} and ma_{cy} which is nothing, but F_x and F_y the value is substitute and you will get the final equation what you obtain.

So, now, you see that what you have done in your previous method the same equation you have got it right. So, now, this is what the whole idea. So, now, we will actually like more little forward once you got this equation how to actually like put it in a vector and matrix form.

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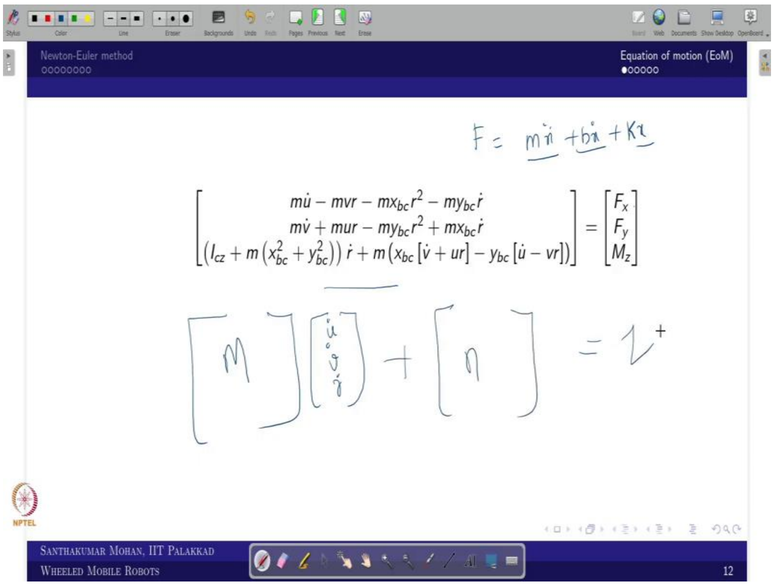

So, now I am just putting in a simple vector form where F_x F_y and M_z and the remaining all I am putting in one side, but right now I am actually like cutting into the other side, what that mean? So, I am trying to make in a stimulating environment. What that mean? If I actually like get the acceleration one side that would be beneficial.

So, in the sense I written equation as like this right. So, I have written the equation in the form where I can write kx separate b$\dot{x}$ separate and m$\ddot{x}$ separate. The same way I am trying to write.

So, since it is actually like $\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{r}\end{bmatrix}$ or the acceleration, but that is in the matrix format. So, you will actually like see a big matrix which is coming that is what you call inertia matrix and the remaining I am all bringing as a simple vector call n and that I am equating toward tau.

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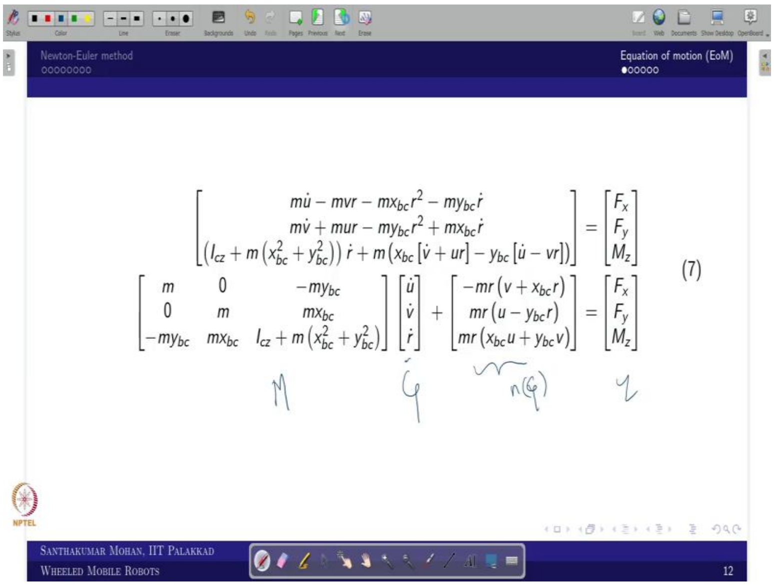

So, that is what I am actually like writing it you can see right I have re written it in a matrix and vector form and the remaining I am writing as a vector. So, now, this is equivalent to n vector ok. So, n(ξ) and this is actually like M × $\dot{\xi}$. So, this I am writing as a Ʈ.

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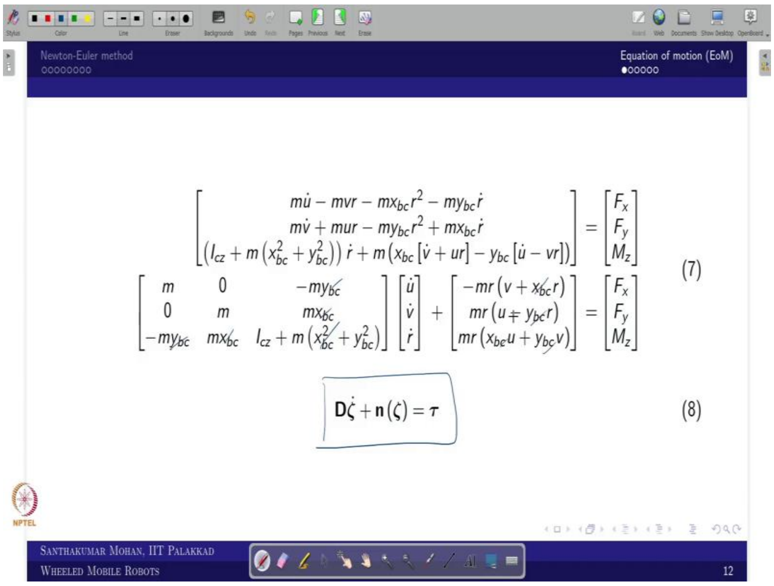

So, you got it already the matrix in vector form right. So, instead of actually like m because I already said M is the moment. So, in order to avoid that I use D as the inertia matrix and the remaining I have keeping it as same.

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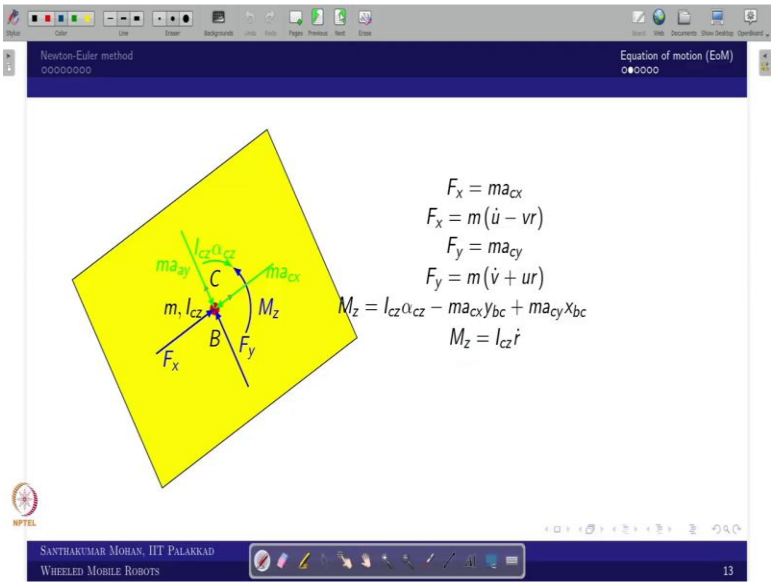

So now, if you see that further; so, if the CG and B frame are coincide. So, what one can expect. So, these all actually like go 0 right. So, that is what you can actually like find it in this. So, you can actually like substitute it and you can actually like see ma_{cx} would be just Coriolis and you call slip acceleration and you can see like the same way and M_z only would be having.

So, these two would go away. So, only would have $I_{cz} \times \dot{r}$ right. So, now, you have seen like even the CG fall under the base frame B you can actually like make the simplified equation most probably like most of the robot design in such a way that the B frame and the C would be coincide even if it is not coincide the B frame you conventionally take on the CG point it will not affect right.

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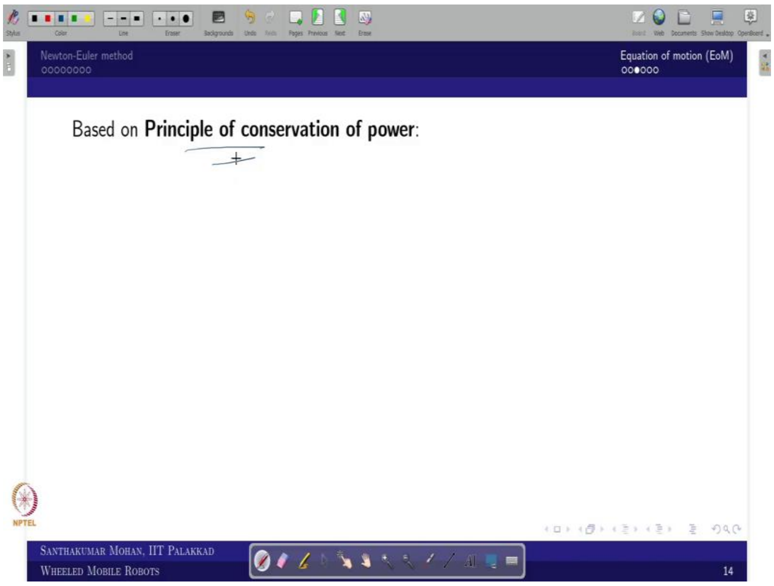

So, that is what the idea. So, in that sense the final few slides on these lectures. So, we are actually like trying to see what is principles of you can say conservation of power. So, this we will actually like try to see in lecture what you call 12.

So, in that sense what you can actually like see that the lecture 11 we have tried to attempt to what is robot dynamics and how that robot dynamics actually like brought in a equation of motion and that equation of motion we can obtain in 2 method that we have seen one is Lagrange Euler the other one is Newton Euler these two we have seen and we have verified that both are giving the same result.

So, in that sense what we can expect in the lecture 12 we can see like how this can be used further and before that I already told in the previous you can say lecture part 1 itself lecture 11 part 1 what I said. So, I told in another way round if we consider everything with respect to inertial frame what would be the equation of motion.

So, we can derive from the base or we can actually like derived equation in body frame can be converted into a inertial frame. So, that is what we are actually like trying to see here; so, that what we are going to see in lecture 12. So, with that what I can say? So, lecture 11 were the robot dynamics we have seen and we can actually like see in lecture 12 the remaining part what supposed to be taken care here. With that I can say thank you and see you in lecture 12 bye.