Wheeled Mobile Robots

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Lecture – 05
Kinematic Simulation of a Mobile Robot (Land- based)

Welcome back to Wheeled Mobile Robots. So, you know this course actually like took at least small elevation. So, we started talking about what is mobile robot then we were talking about what is degree of freedom then we addressed basic kinematic model for a land based mobile robot. Then we just come and addressed what is degree of maneuverability and then we moved a little forward.

So, now this particular lecture we would be talking more about how to actually like understand the system we have derived the kinematic model. So, how to actually like make it more fruitful way; for example, you want something like the motion need to be simulated or you want to make a close loop, even the basic what you have seen a forward inverse kinematic model.

So, for that what one can actually like see, we will try to do a simulation in the first case. So, the computer based simulation what we are trying to see in this particular lecture. So, that is what the lecture 5. The lecture 5 is all about kinematic simulation of a mobile robot that to specifically what we call land based.

So, we are not giving anything in detail whether it is actually like wheeled mobile base or legged base or track base we are taking a generalized equation which we derived in the lecture 3 and that equation we are like actually using it and going further.

And I hope this is the last lecture of this particular week. So, I hope at the end of this week you can actually like do your own simulation by looking this video and you can actually get more understand you can say; you can get more clarity on this particular topic what we have talk about mobile robot kinematics.

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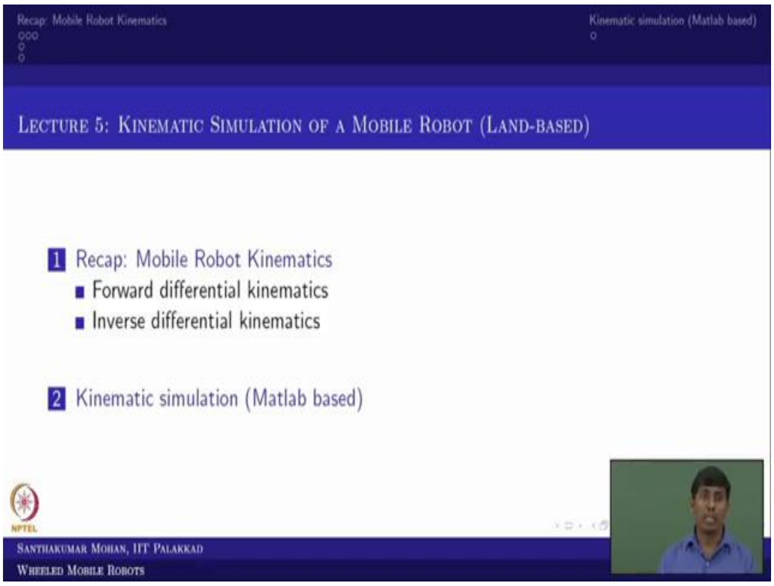

Let us actually like move forward. So, we will actually like take what one can see this particular case, I will just run through the mobile robot kinematics equations I will not be putting any time on that, but what I am actually like interested to show here we would be seeing the kinematic simulation of the land based mobile robot with Matlab you can say equations and the script. So, that is what we are actually like interested.

So, in the sense I may actually like switch over to the screen to a Matlab you can say window and please actually like cooperate sometime the font size may be smaller. So, I have already tried my level. So, we can see that in the upcoming video session ok.

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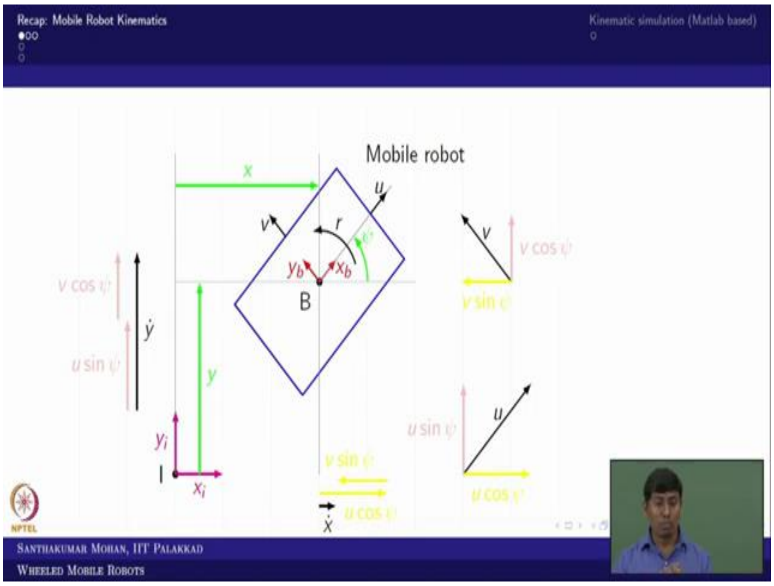

So, now you know this is what we have derived in fact, lecture 3 and lecture 4 I have emphasized this.

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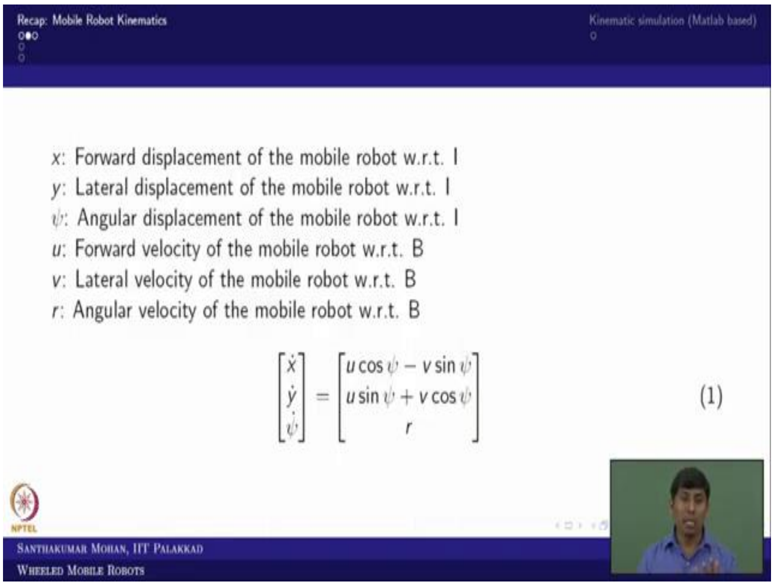

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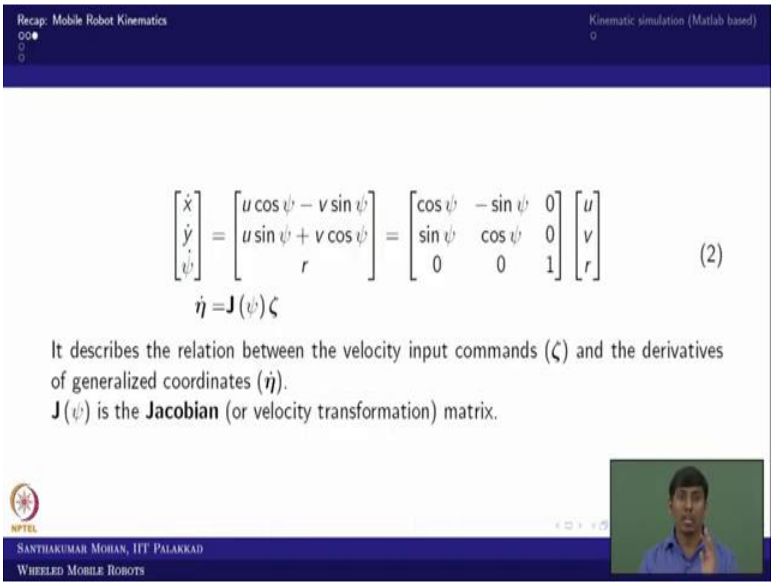

So, what we obtained this equation this equation we are derived in this form. So, now what we are trying to see? If I given this j and if I give a input command can I find $\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}$ and can I integrate that and get the x(t)? So, you know like this is time derivative, I can integrate it and get it. So, in this case it may be look like straightforward, but what would happen if you are having a complexity. So, what one can see? We will do the numerical integration right.

So, in that sense; so, we are going to do the numerical integration in this particular lecture. So, that numerical integration can be done in several way. So, we are going to use one of the simplest method just for understanding not for anything else. So, just for understanding we are going to use simple you can say approximation method we call Euler method ok.

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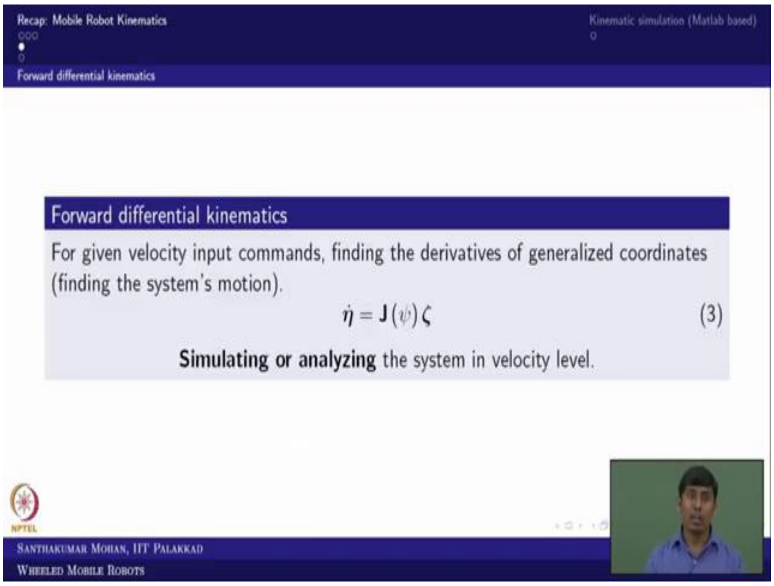

That we will see little later right now what we are trying to understand?

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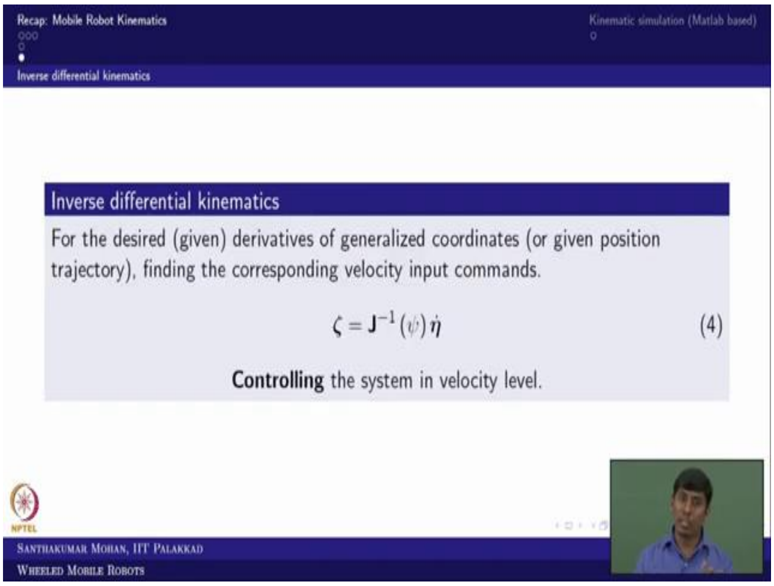

So, we are trying to simulate the system based on the forward differential kinematics and if time permits I will actually like attempt the inverse kinematics for given you call position trajectory can we find the input command velocity that is what we are trying to see. So, we will actually like go little forward.

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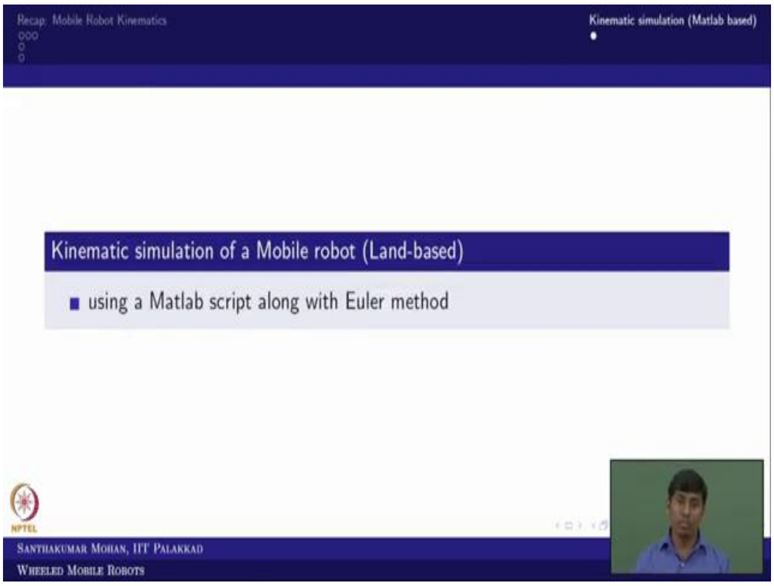

So, in the sense what we are trying to do this particular lecture is mainly focused on the kinematic simulation of a mobile robot. So, what we would be taking? So, we would be taking these equations ok. So, where $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}$ we are writing in a matrix and vector form that is what we are going to integrate and do it.

So, simple sense what we can see simulation here what we are trying to do? We are trying to do a ordinary differential equation solving in the sense we are trying to write a code for ODE solver ok. So, that is what we are trying to do. So, I hope you got what we are trying to do.

So, I will be using the Matlab script later on we can see if actually like over a you can see probably one or two weeks goes, then we can go with probably block diagram based approach what you call Simulink script, but right now this particular lecture would be focusing based on the Matlab code.

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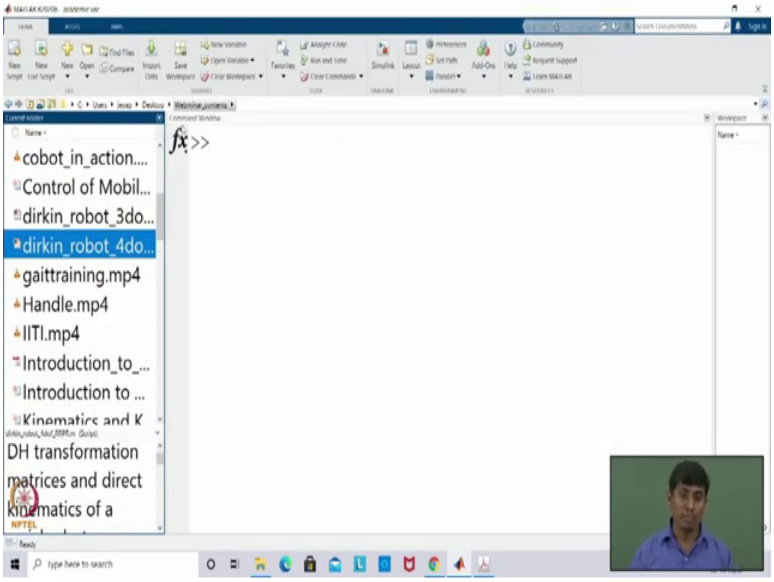

So, for that I am actually like switching the screen ok. So, this is the Matlab window which is actually coming to my screen and I am actually like trying to take everything on the script. So, I am taking the new script ok.

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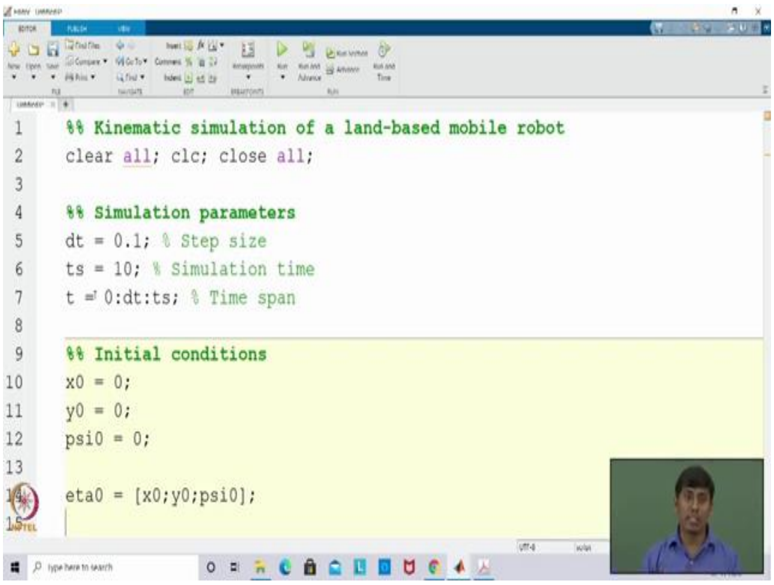

So, I hope you are able to see the new script now. So, now, what I am trying to do? I am trying to do a kinematic model simulation. So, I am actually like writing that as my you can say section title. So, I am writing actually like kinematic simulation of a land based mobile robot. So, this is just for a title and I am just giving so, that you will get an idea what I am writing on the Matlab script later on you should not get a panic thing oh what happened this.

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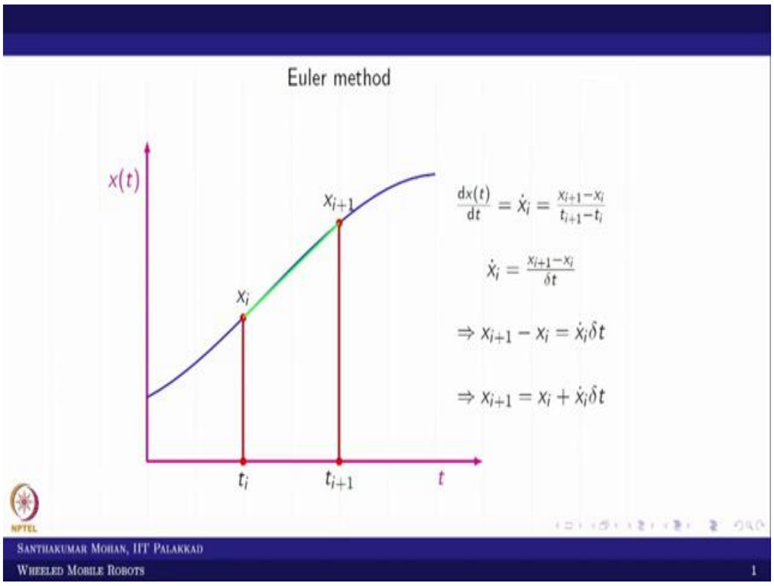

So, in order to get this; so, I hope you know what is Euler method if not I will actually like orally give a small idea. So, that is what I can do in this particular lecture probably when it comes in the dynamic simulation, I may take a slide and then show you right now imagine you take a profile or the function.

So, function is actually like I can write as x. So, that x is actually like depend on the time. So, x is function of t. So, now, that is actually like a draw in a time versus x profile. So, now I am taking a small segment where t_i and t_i + 1.

So, now the segment I am taking further on the curve where it hitting on the curve that is I am taking t_i hitting that is I am calling x_i were t_i + 1 what hitting I am taking as x_i + 1. So, now, you can see this is the segment which is actually like t_i + 1 - t_i this is I am calling as a step size which is δ_t I am taking.

So, now corresponding to that equation or you can say the curve where x_i and x_i + 1 coming you approximate this into a linear in the sense I draw a linear straight line in the sense linear curve between x_i to x_i + 1 what would be the slope? The slope would be x_i + 1 - x_i whole divided by δ_t.

So, now what we would be assuming that, the initial condition is known for solving differential equation because its a ordinary differential equation one of the boundary condition which is we call initial condition where x_i is known and δ_t is known and this is what you call slope of that.

So, in the sense time derivative of that function also known; so, in the sense what you can see; so, $\dot{x}$ of i x_i and δ_t these all are known to us. So, now what you can see? I can rewrite this equation. So, how I can rewrite? So $\dot{x}_i = \frac{x_{i+1} - x_i}{\delta t}$, I can rewrite into what I can write? So, $x_{i+1} - x_i = \dot{x}_i \delta t$. So, which I can rewrite into $x_{i+1} = x_i + \dot{x}_i \delta t$.

So, now what you can see? I can propagate from 0 to 1, 2, 3. So, this is what we are trying to do I think if I write the Matlab code you will get a clarity. So, this is what we call Euler method. So, now based on the step size what you are taking delta t is actually like making the exact solution or you can say awkward solution I can say not approximate ok.

So, in that sense we will come back to the you can see case just for understanding. So, I am just giving the workspace supposed to be clear and the command window also I am clearing and if anything as a image or figure that also I am closing this is just for basic information.

Now, what supposed to be known? This is actually like ordinary differential equation solver. So, I am trying to do that with ODE solver. So, for that I need to have a simulation parameter right. So, this simulation parameter what I call? So, delta t which I am writing as a dt this is what I call step size I am taking 100 millisecond which I call step size ok.

So, then I am taking a simulation time from 0 to what long the simulation supposed to run this I am calling ts which is what I call simulation time I am just taking as a 10 second later on we see how to change. So, this is what you call simulation time in the sense total simulation time also you can take ok.

So, now, what would be the next one? We need to make this as actual like expand I am just taking as a t, t would be spanning from 0 to you can say t s with the interval of say δ_t in the sense here dt. So, this is I am calling time span ok. So, these are a simulation parameter because we are taking a Euler you can say integration method.

So, only delta t and the simulation time is the important case. So, we have come. So, now, what I am trying to do? So, I am trying to give the you can say system parameter here for solving ordinary differential equation what you one can know one should know? So, the initial condition; so, I am just giving the initial condition.

So, initial conditions what I am calling is ξ_0. So, for that I need to know x position y position and Ψ position. So, I am writing x_0 is my initial x position and y_0 is actually like my initial y position I am assuming everything as 0 and Ψ_0 is actually like again the orientation that is also I am assuming as 0.

So, now what one can see? So, these are actually like initial condition that need to be written as in a vector form. So, that is I am writing ξ_0 in the sense it is a initial condition I am just writing as x_0 and its a vector. So, I am putting a semicolon. So, that it would give as actual like vector which is actually like column vector it will give then Ψ_0 right. So, now this is actually like my initial. So, what I need to know I need to actually like propagate. So, x_0 to x_n + 1 right.

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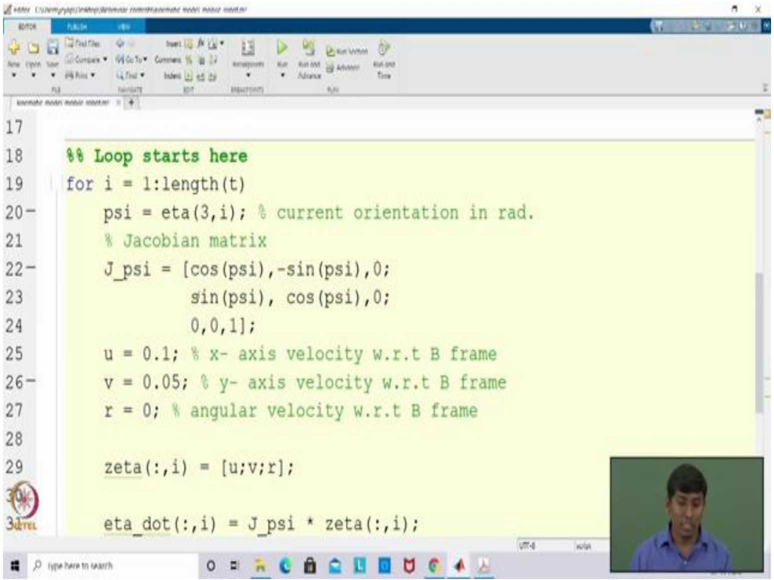

So; obviously, I need to refer as a x. So, here x is nothing, but a vector called ξ. So, I am just writing eta of 1. So, this is a vector. So, that is why I am actually like denoting. So, there would be three cross 1. So, this you call the colon will take forward whatever the number of you can say rows that would take and there is a number of column is 1. So, this is equal to ξ_0 in the sense first value of ξ would we ξ_0 ok.

Now, we were actually like going to the for loop, I am writing loop starts here I am sorry just for understanding. So, now what we did? We gave initial condition and simulation parameter now we are running. So, there are several possibility I am using a for loop for simplicity. So, for loop I am taking a you can say the loop constant or loop you can say variable as i that start with 1 and it goes up to length of t; in the sense I know the t is a span where it starts from 0 to t s with a interval of δ_t.

For example, I take 10 as the ts and the step size is 1 sorry point 1. So, now how many? 101 as the total length. So, 0 to you can say 10 with a interval of point 1. So, that is what for one second it takes 10 sample. So, it starts from 0. So, then you can see that 10 into 10 + 1 as 0. So, 101 times this loop will you can say work that is what the idea.

So, now what we need to know? We need to know what is actually like your Jacobian matrix. So, the Jacobian matrix I am writing as J_{Ψ}. So, this

```
J_psi = [cos(psi),-sin(psi),0;
         sin(psi), cos(psi),0;
         0,0,1];
```

. So, please excuse I would be little slow in typing, but this is the way I can actually like make sure that you are following me.

So, cos psi. So, this is the you can say Jacobian matrix we obtained. So, this is what the Jacobian matrix, now you should know like what is psi right. The psi is actually like a variable that variable is actually like what? The third row of eta right that is what you are psi we represent in the sense I am writing eta and the third row and i th instant for example, if the loop start with 1. So, the (3, 1) would be your Ψ_0 that is what psi right.

Now, it goes the second loop then the psi would be like eta (3, 2) that is what we are writing. So, now you can see that this is what the thing I am writing this is current you can say orient this is the current orientation, but in radians that also should be known ok. So, this is what we got it and this is the Jacobian matrix right. So, now you got a some kind of idea right.

So, the Jacobian matrix we have written. So, what would be the final case? So, you should know like what is eta right eta is actually like integrating with respect to eta dot, but for eta dot what we know the relation? We know the relation is, eta_dot is j_psi * η. So, now I am putting η as the vector ok. So, that would be actually like I call as $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ right. So, this is what we have obtained.

So, for that I need to mention what is u. So, u is actually like I am taking 1. So, which is I will write along; I think I will write as u is actually like x axis velocity that is easy; because longitudinal lateral will not be represented when you go in a proper plane because that would be associated with body x axis velocity with respect to B frame that is what this is the instantaneous velocity right at instant.

So, this is what the velocity; right now I am putting this as 0 this is y axis velocity, B frame right. So, then what you should know r. So, r is actually like your angular velocity. So, since it is a land based I am not writing which axis, but this is actually like with respect to z axis, this is the z axis angular velocity that with respect to B frame right.

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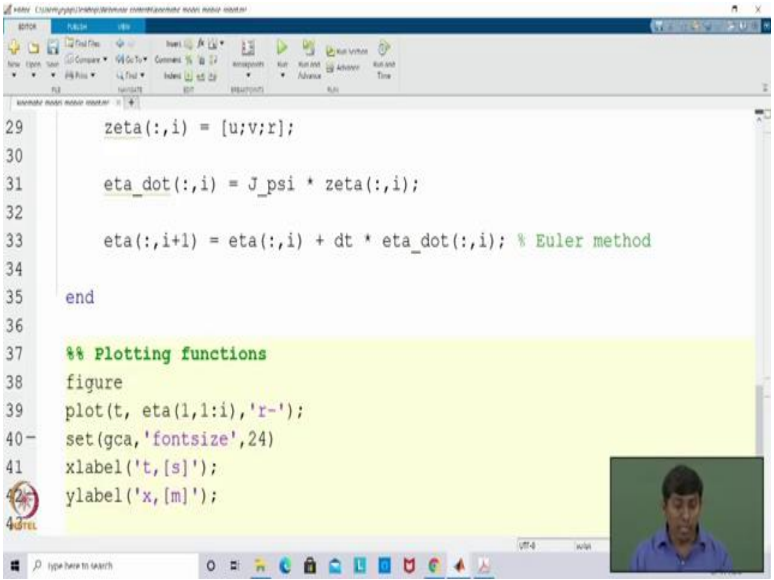

So, now in the sense you can see that these are the things we obtained. So, now I will bring that equation sorry. So, now what we know the eta dot I will write eta underscore dot means its eta dot. So, that is actually like at i^{th} instant. So, I am writing that; that is a vector the i^{th} instant would be equal to. So, zeta this also I will write as i^{th} instant. So, now what would be that? The $J(\Psi) \times \eta$ of i^{th} instant right.

So, this is what we know. So, further what we can actually like see? The ξ of you can say i + 1 we can actually like get it what way? So, $\xi(i) + dt \times \dot{\xi}(i)$ right. So, this is what Euler method ok. So, where the you call dt is the step size and you can see right whatever we have written that has come.

So, now everything has done right. So, that is also you have actually like make it. So, I am just putting end. So, the loop ends here and then if you run it what one can actually like find? So, for given u v r; so, you can find eta time profile what that means?

So, it is actually like the forward differential kinematics you have already solved, but you want to know like how this goes further right. In the sense you want to know like what would be the u profile what would be the $\dot{x}$ profile with respect to time. So, these all need to be known. So, for that what we need to know like I will just put it.

So, plotting; so, plotting functions start here. So, I am just plotting first. So, plot of. So, t. So, t is actually like already defined as a span. So, that would be actually like I am planning to show. So, the eta 1 which is nothing, but the x that would be actually like going further.

So, why I put 1 is to i? Because the eta would be actually like i plus 1 instance would be there, but you know time is actually like up to i instant. So, I am actually like putting ξ as up to i^{th} level so, that you can plot that. So, I am just showing this is red color ok.

So, and I am just putting just you call get I just put the set the figure whatever the fontsize I will just make it so, that you can actually like able to see probably 24 I will put ok. So, xlabel would be here is actually like a time which is in second and ylabel would be you can see x which would be in meter ok.

So, this is just for showing. So, we can see. So, for understanding I am just giving a slow speed because the mobile robot will not go point you can say not more than 0.5 meters per second. So, in the sense I am just taking a very slow speed earlier I gave 1 meter per second now I am taking it is 0.1 because then only the plotting would be actually like reasonable. So, now, if I run I hope there would not be any error.

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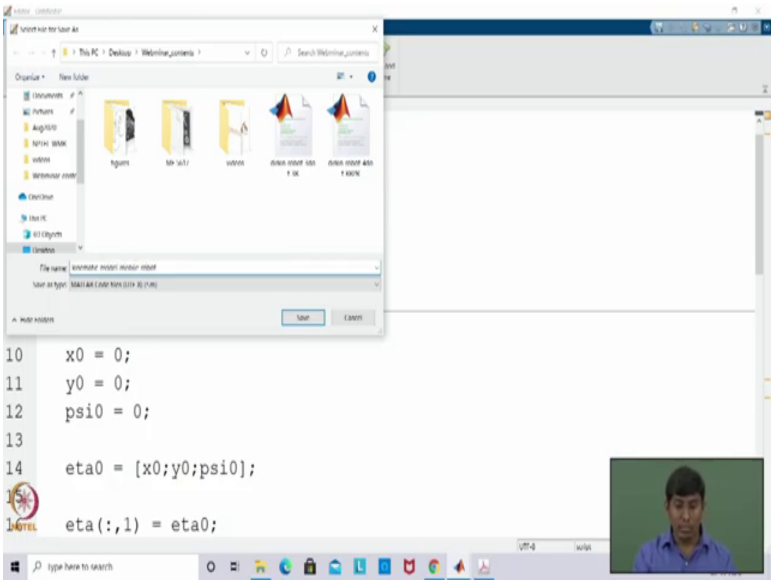

So, I just run so; obviously, Matlab will ask you to save. So, I am just saving it as you can say kinematic model of mobile robot. And one thing I can actually like give. So, while you are saving Matlab code you dont save with starting with the numeral ok. So, anything which is actually like numbers that numbers will not be addressed in Matlab; you start with some you can see alphanumeric ok.

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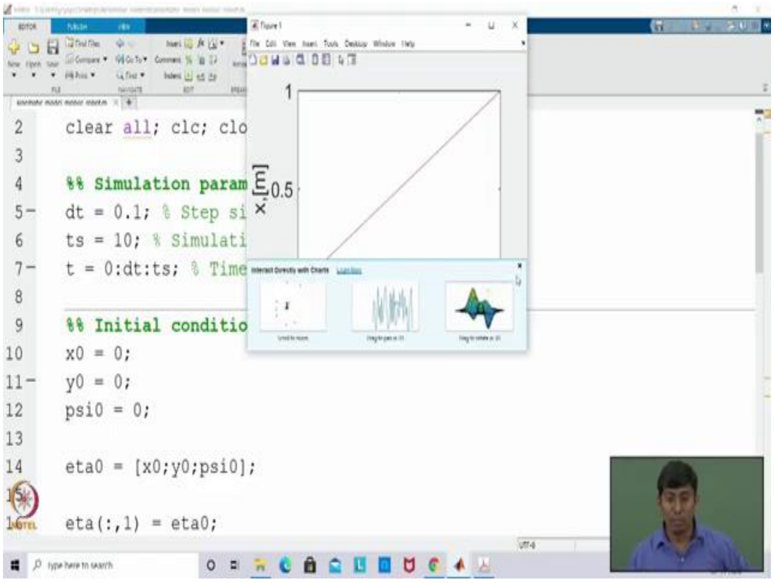

So, now, I saved in a general form you can see that already it was running. So, you can see these are the animations. So, that is not required.

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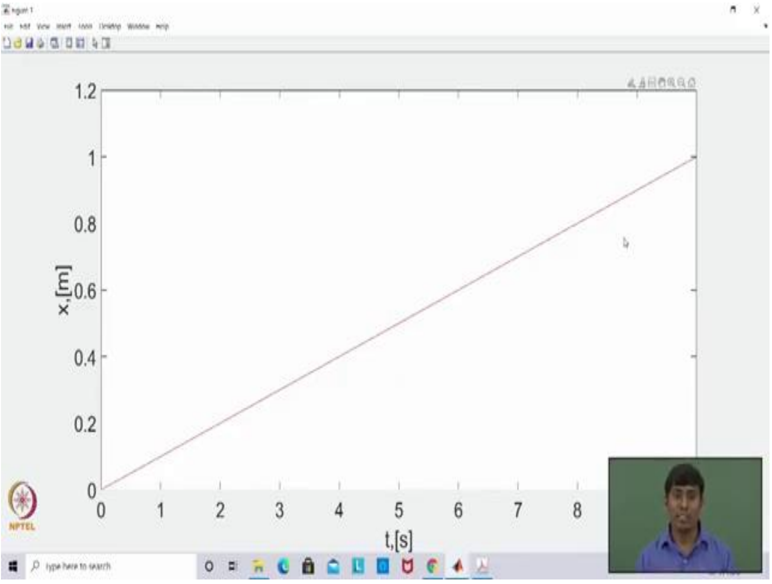

So, you can see that I run the simulation for 10 seconds, I gave you can say longitudinal velocity of 0.1 meter per second, it actually like move forward right you can see 10 into 0.1 how much it would be? 1 meter. So, that is what it is actual travel 1 meter right.

But this is actually like not really the interesting thing why? Because this is not giving any intuition to me right? The intuition will come by you can say animate the system for animation. So, you need some additional tool I hope actually like the animation can be done in the next upcoming lecture, I will show you right now I will show you like the other parameter how you can actually like see.

So, for example, I am giving this is 0.05 and you can see the initial point is actually like 45 degree which is actually like π by 4. So, you can see earlier it starts with 0. So, I am just trying to plot I am just plotting three different plots. So, I am just putting as this is a figure.

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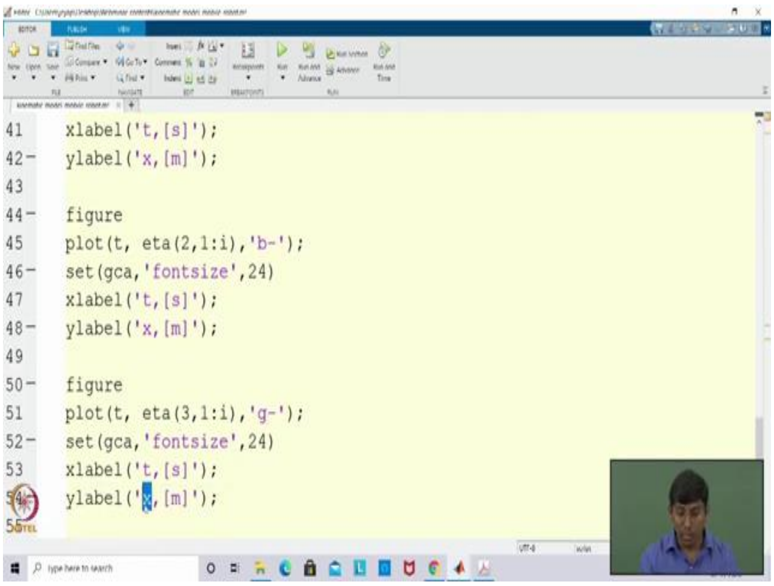

So, I assume that you know already Matlab that is what I am not at all actually like touching anything behind the Matlab picture in the sense Matlab aspect, but if you have actually like issues so you can actually go through several you can say lecture videos are available on Matlab and that to like for engineering cases. So, we will actually like see a little more detail in the Matlab code here itself.

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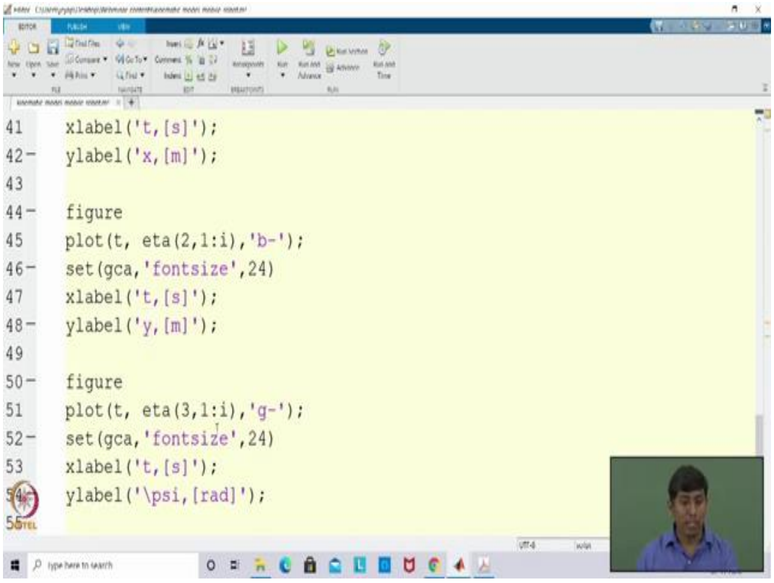

So, right now I am actually like taking it this is a blue color and this is actually like green color and this is actually like not this is actually like Ψ ok and this would be what you call this as in radians and this would be y and this is the case right.

So, if I actually like run this would be plotting three things. So, one would be time versus x, time versus y and the third thing is time verses Ψ.

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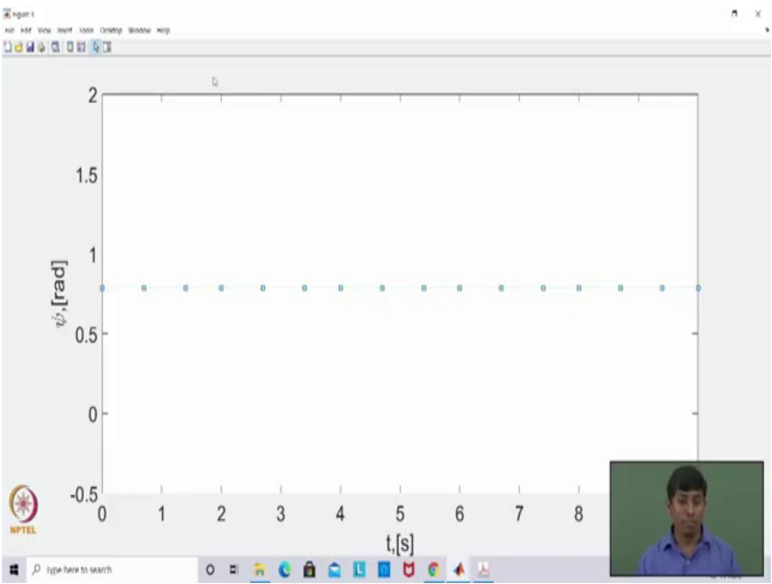

So, we can see whether it is first of all running. So, later on I will show you because we have taken the psi is actually like pi by 4 which is nothing but 0.707 approx not 707 some somewhere it is a 0.89 or something I just check the value. So, I will just I forget what the exact value so, but you can see this is equivalent to pi by 4 and that is what it is actually showing.

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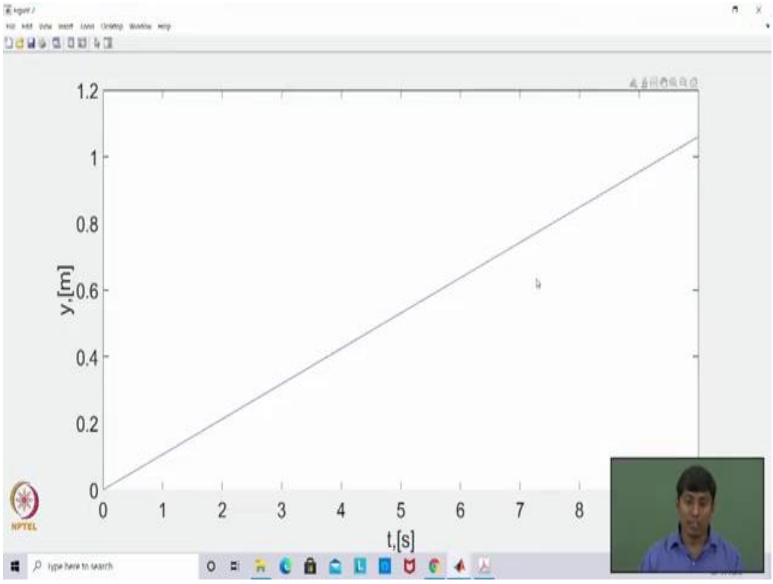

And this is actually like y, y is actually like moving with the velocity of 0.05 or how much I gave? I just check its a 0.05 and x is actually like moving with 0.1 alright. So, these are the cases and you can actually like see how it goes. So, these are the aspect one can actually like see it ok.

So, now you can see that these $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ which are actually like coming directly or giving, but this supposed to be written in the case of what you call based on the angular velocity of the wheels right that is why we address the equation 1 equation we have written right the zeta we can write in terms of w into omega; so, that you need to actually like do it.

So, next class we will actually like address that into very detail and similarly I will be showing the animation. So, how to actually like make one of the block that block is actually like moving on the given plane because it is a planner robot right it is very easy to simulate and animate.

So, the next class I will show you one additional you can say aspect of this kinematic simulation and then actually like we can move forward by introducing the angular velocity relationship of the wheel in the sense you will bring the wheel configuration and then try to simulate.

So, the next class. So, probably we will do the kinematic simulation in the beginning then we will actually like move forward to the you call the wheel model and then we will actually like incorporate the kinematic model along with angular velocity of the wheels. So, with that I am actually like closing this particular what do you call the small simulation session. So, then the next video would come with animated video along with inverse kinematics solution ok.

So, right now we have given simple forward kinematics because I do not know like you are comfortable or not. So, you take a while because I just introduced a Matlab and you can see that we are not done anything, we have started simple Euler integration for the solving of ordinary differential equation and we have taken the initial condition and we have taken the straight forward, forward differential kinematic relation and we substituted and we took the numerical integration and plot the function.

Now, this is not enough for understanding; we will do the animation in the next lecture ok. With that I am saying thank you and see you then bye.